Geographic Population Structure

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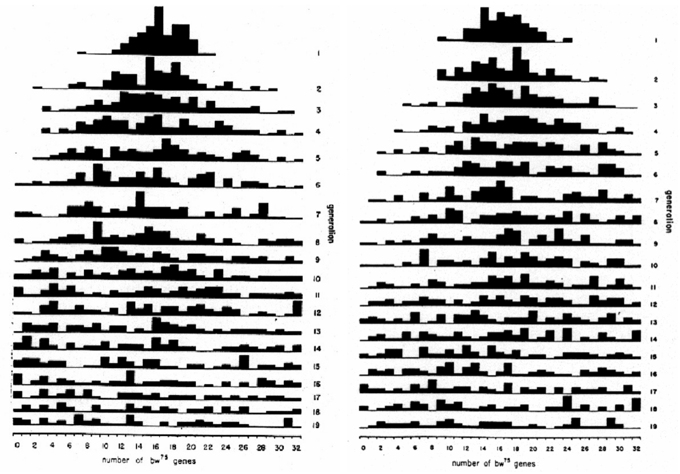
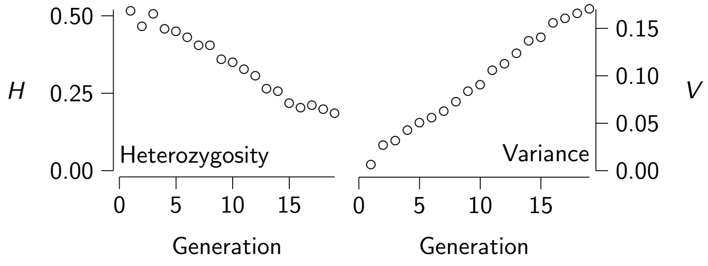
Buri's (1956) data

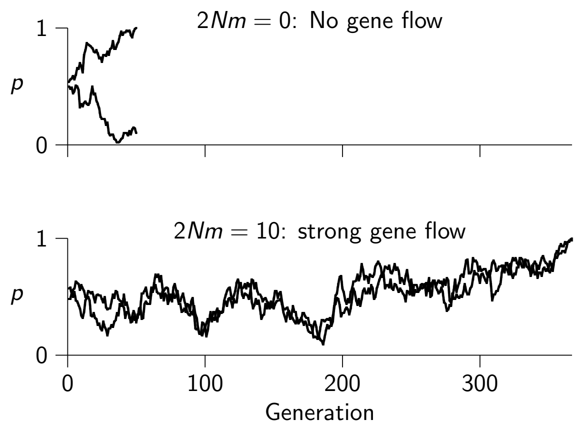


Drift reduces heterozygosity w/i groups while increasing variance among groups

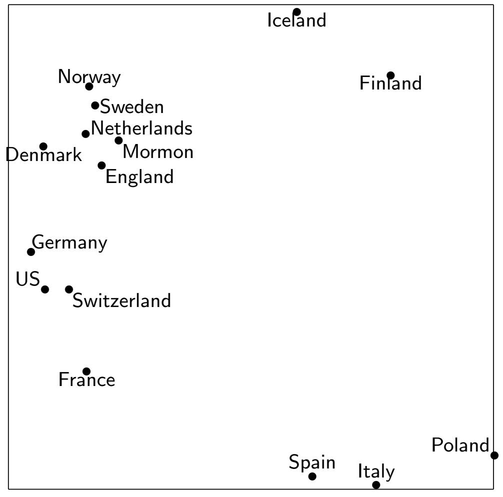


Data: Buri (1956)

Gene flow (migration) reduces population differences

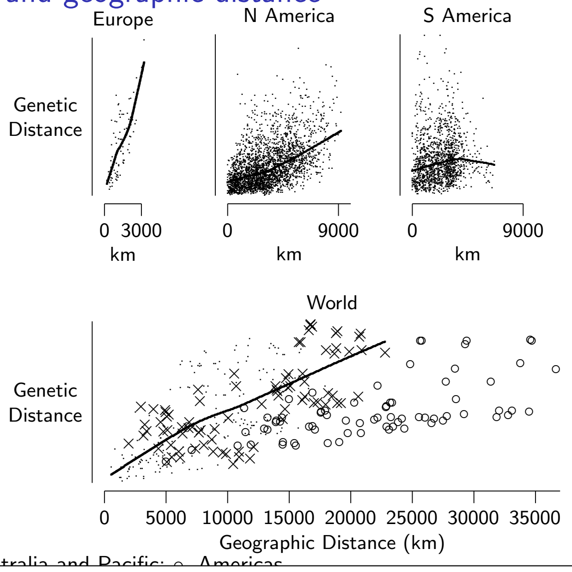


PC map of European genetic distances



(McLellan et al 1984)

Genetic and geographic distance



Wahlund principle: subdivision reduces heterozygosity

	A_1	A_1A_1	A_1A_2	A_2A_2
Pop 1	4/16	1/16	6/16	9/16
Pop 2	12/16	9/16	6/16	1/16
Species	8/16	5/16	**6/16**	5/16
Hardy-Weinberg	8/16	4/16	**8/16**	4/16

Amount of reduction: 2/16

	$p_i - \bar{p}$	$(p_i - \bar{p})^2$
Pop 1	–4/16	1/16
Pop 2	4/16	1/16
Variance:		1/16

$$\mathcal{H}_S = \mathcal{H}_T - 2V \qquad \text{(Wahlund 1928)}$$

- Wahlund showed that heterozygosity is reduced by group differences in allele frequencies.
- Buri's experiment illustrates that
  - Drift reduces heterozygosity
  - Drift increases group differences
- We need a theory to connect these facts.
- Let us build one on top of what we have already.

What we already know about heterozygosity

$$\begin{aligned} E[\mathcal{H}_t|p_t] &= 2p_tq_t && \text{(Hardy-Weinberg)} \\ E[\mathcal{H}_t|p_0] &= 2p_0q_0\left(1-\tfrac{1}{2N}\right)^t && \text{(Ch. 2)} \end{aligned}$$

How can these both be true?
It must be true that

$$E[p_tq_t|p_0] = p_0q_0\left(1-\frac{1}{2N}\right)^t$$

On the other hand, it is also true that

$$E[p_tq_t|p_0] = p_0q_0 - V_t \quad \text{(Wahlund)}$$

where V_t is the variance of p_t about p_0.

Rearranging gives the variance among groups

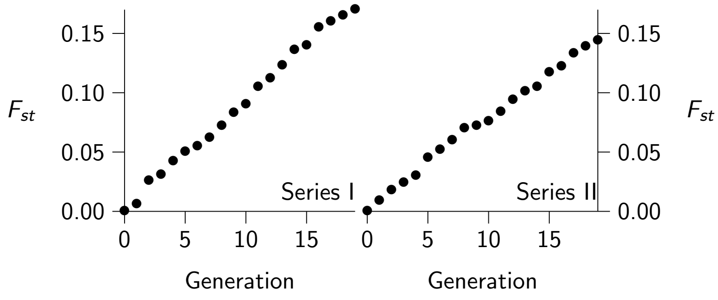
$$V = p_0q_0\left[1-\left(1-\frac{1}{2N}\right)^t\right]$$

We usually normalize this expression by dividing both sides by p_0q_0. The result is called F_{ST}:

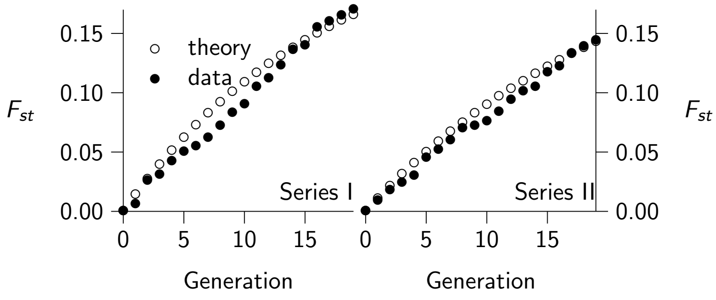
$$F_{ST} = \frac{V}{p_0q_0} = 1 - \left(1-\frac{1}{2N}\right)^t$$

In data analysis, we take $p_0 \approx \bar{p}$, the current population mean.

Data from Buri (1956)



Data from Buri (1956)



Application to differences among human races

Theory assumes *no* migration. Seems unlikely, but let's see where it leads.

For major human populations, $F_{ST} \approx 0.1$. Modern humans have been in Europe and E Asia for 50,000 years (2000 generations). Plug this into

$$F_{ST} = 1 - \left(1 - \frac{1}{2N_e}\right)^t$$

and solve for N_e.

Answer: $N_e \approx 10,000$: agrees with genetic diversity within populations.

The role of migration

- Drift increases group differences.
- Migration (m) reduces them
- Eventually, an equilibrium is reached.

$$F_{ST} = \frac{1}{4N_e m + 1}$$

Depends only on $N_e m$. Small if $N_e m > 1$.

Human value ($\approx 1/9$) implies $N_e m \approx 2$.

Summary

- Drift increases variance among groups and reduces that within them.
- Wahlund's principle: $\mathcal{H}_S = \mathcal{H}_T - 2V$
- F_{ST} measures between-group variance relative to total variance.
- If major human populations have been isolated for 50,000 y, then $N_e \approx 10,000$.
- If they are at migration-drift equilibrium, then pairs of populations exchange $\sim$2 migrants per generation.