Financing Retirement using U.S. Treasury Bonds: Safe Withdrawal Rates, Mean/Standard-Deviation Frontiers, and Endpoint-Dependence of the Safest Maturity[‡]

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December 2018

Keywords: retirement savings, safe withdrawal rates, endpoint dependence
JEL Codes: G11, J26, J32

Abstract. Calculating, for a fixed asset allocation, completely-Safe Withdrawal Rates over various historical periods of one fixed length generates a "SWR" distribution. Combining its minimum, or mean and standard deviation, with those of other allocations implies a maximin portfolio, or a SWR Mean/Standard-Deviation Frontier. Considering portfolios only of Treasurys of one constant maturity, the lowest-withdrawal-variance asset was sometimes not the shortest-maturity, and sometimes was the longest. Constructing most subperiods, these anomalies were very rare for real SWRs and occurred about one-fifth of the time for nominal SWRs, happening when money-market yields changed greatly.

Cash and other money-market instruments are the lowest-risk fixed-income assets for many purposes, such as for a short-term store of value or for rebalancing a portfolio which includes stocks. However what is low-risk for those purposes may not be so for a retiree spending down fixed-income assets over a period of several years. Also, most analyses of retiree spending focus on portfolios including stocks, but some people choose or endeavor to finance expenditures strictly from fixed-income instruments, as was common prior to the mid-twentieth century (Howell 1958 p. 267). For these reasons it is useful to conduct a "safe withdrawal rate" type of analysis for a broad range of fixed-income maturities without including equities. The first section of this paper does this, finding the minimum constant withdrawal rate, but then broadens the analysis to look also at the mean constant withdrawal rate and its standard deviation. The frontiers generated by this analysis show occasional counter-intuitive situations, where longer-term instruments yielded lower-standard-deviation withdrawals than shorter-term instruments. The second section of this paper explores how to explain those anomalies.

Investigating in more detail the fixed-income portion of a retiree's portfolio is suggested in Guyton (2015), following Kitces and Pfau (2015), but this paper goes further because it ignores stocks, as does the (quite different) asset-liability matching framework. This paper also ignores bonds with credit risk or currency risk in order to focus on the role of bond maturity.

1. The Base Case

The instruments we analyze in this paper are U.S. Treasury 13-week ("three month") and one-year bills, three-, five-, and ten-year notes, and 20-year bonds. We may refer to any of these as a "bond" and we abbreviate them, respectively, "3mo," "1yr," "3yr," "5yr," "10yr," and "20yr," dropping the last letter of each when labeling figures. We obtain constant-maturity yields for these instruments for each year from 1955 to 2017 from the Federal Reserve Bank of St. Louis's economic database FRED, `https://fred.stlouisfed.org`. Figure 1 shows these yields converted to continuously-compounded ("logarithmic") rates, and Figure 2 shows those rates adjusted for inflation as measured by CPI-U. Considering data in annual segments as in the graph, nominal yields reached their maximum in (the start of) 1981 (3mo and 1yr bills) or 1982 (the other instruments) and their minimum in 2012 (3mo and 1yr bills) or 2013 (the other instruments). The highest real yields were in 1982 and the lowest were in 1974.

This paper's Appendix shows how to derive the total return of each instrument from its yields. The continuously-compounded total nominal returns which result are shown in Figure 3, and real returns are shown in Figure 4. The

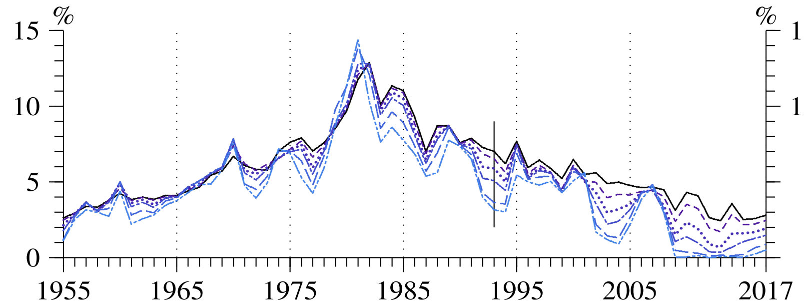

Figure 1. Continuously-compounded nominal yields, 1955–2017. Along the solid vertical line near 1993, from bottom to top: three-month bills, one-year bills, three-, five-, and ten-year notes, and twenty-year bonds.

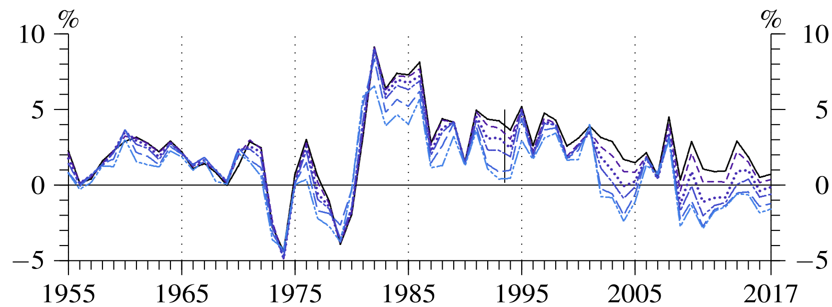

Figure 2. Continuously-compounded real yields, 1955–2017. Along the solid vertical line near 1993, from bottom to top: three-month bills, one-year bills, three-, five-, and ten-year notes, and twenty-year bonds.

mean and standard deviation of each of the six nominal series is illustrated by an open circle in Figure 5 and of each of the real series by an open circle in Figure 6. Measuring risk by standard deviation (acknowledging that can be contested, cf. footnote 6 later), the open circles of Figures 5 and 6 confirm the conventional wisdom about the relative riskiness of different maturities of bonds: the smallest standard deviations were from three-month bills, the largest were from the 20-year bond, and standard deviation was monotonically increasing in bond maturity, although the gap between the three-month bills and the 1-year bills was very small. (The lines joining the open circles in Figures 5 and 6 are not efficient frontiers, which would require a mix of instruments, but simply straight lines drawn to help visualize how withdrawal rate and its standard deviation vary with maturity.)

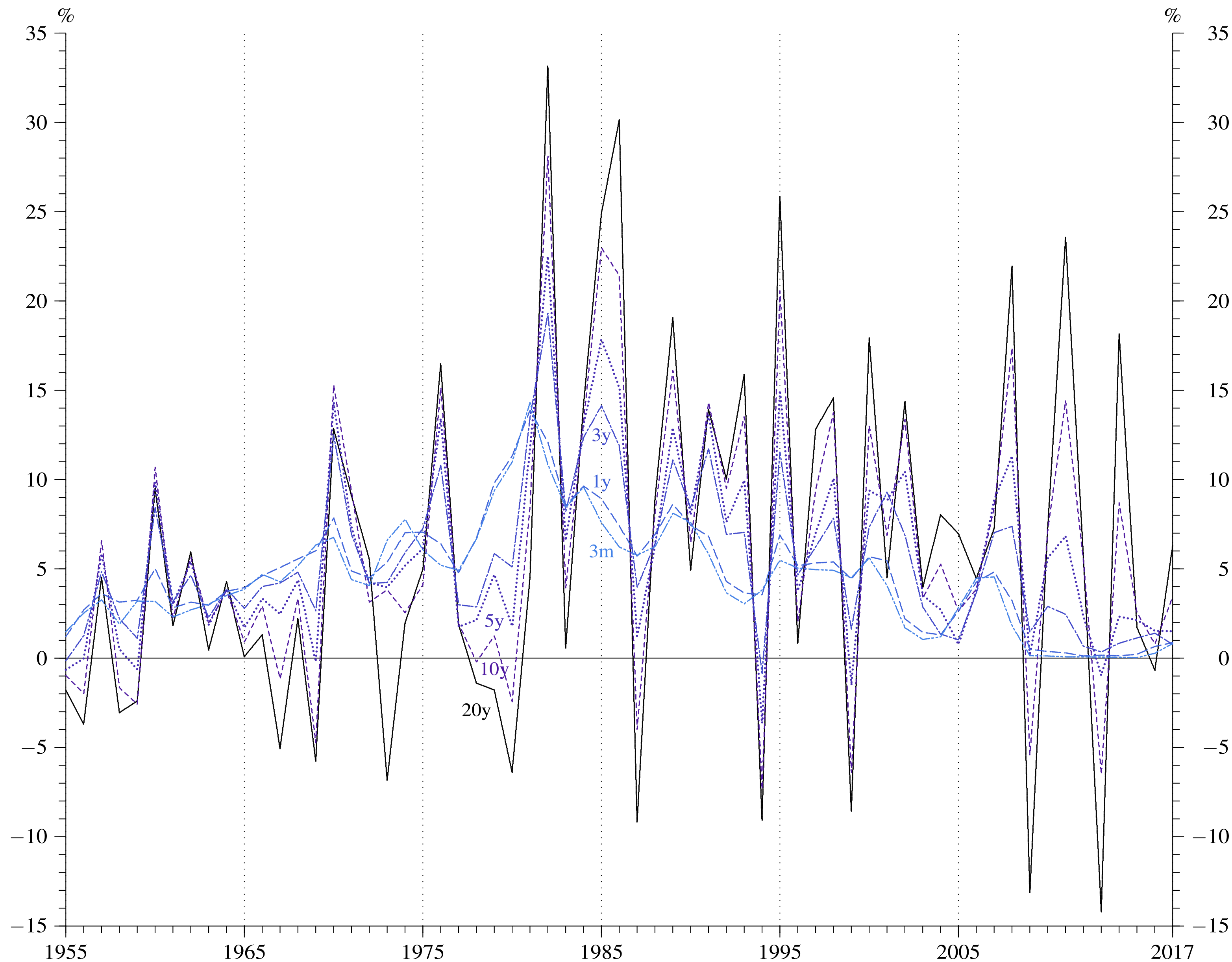

Figure 3. Bond total continuously-compounded nominal returns, 1955–2017.

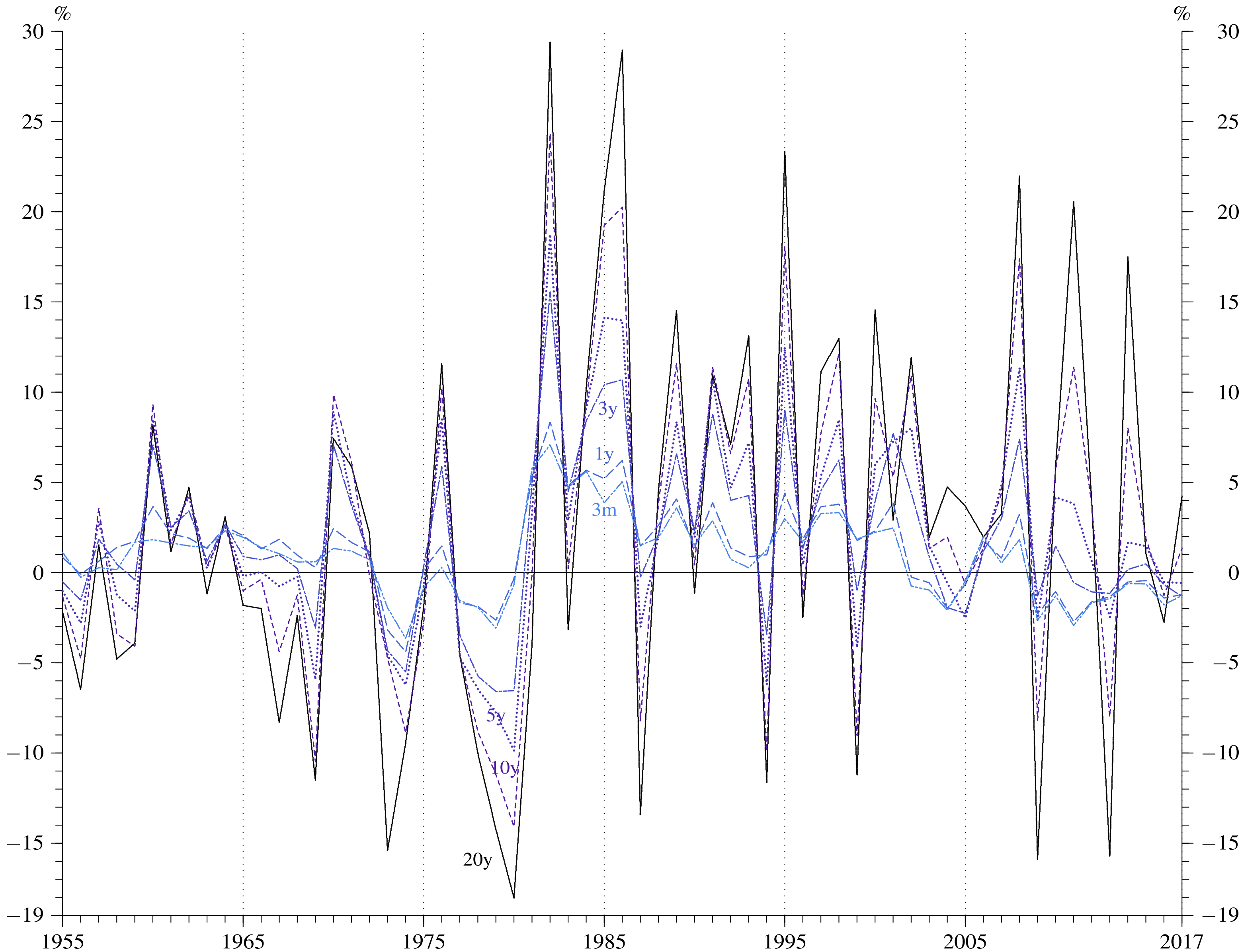

Figure 4. Bond total continuously-compounded real returns, 1955–2017.

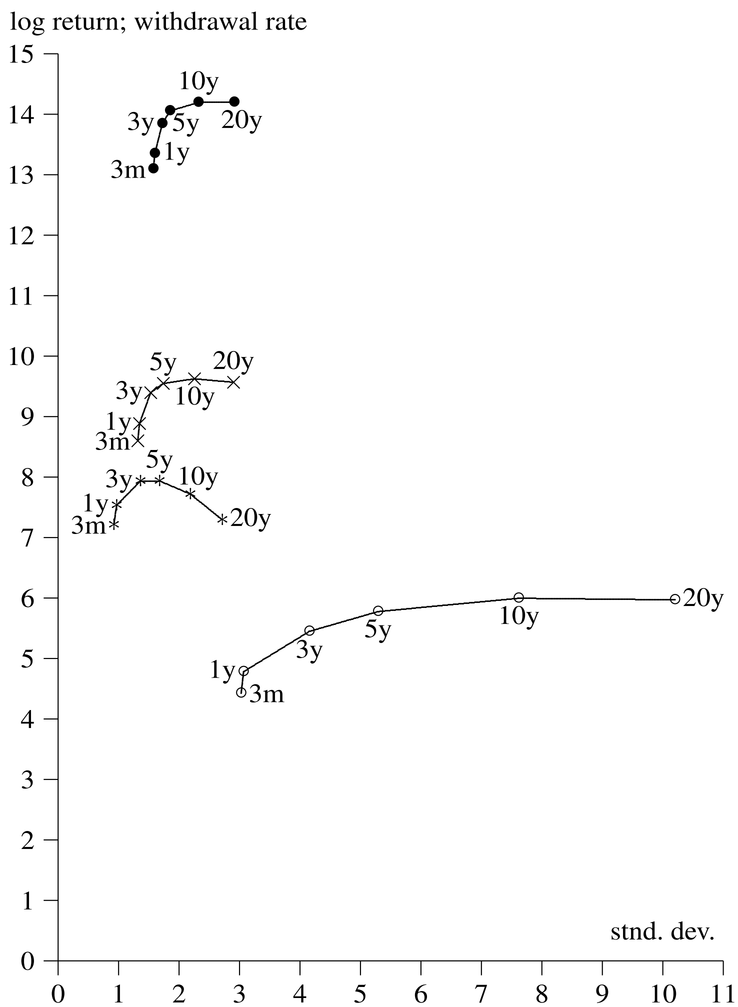

Figure 5. Standard deviation and mean based on the nominal returns of 1955–2017. Open circles: bonds, from Figure 3. Solid circles: $T = 10$ withdrawals, starting dates 1955–2008, from Figure 7. X's: $T = 20$ withdrawals, starting dates 1955–1998, from Figure 9. Asterisks: $T = 35$ withdrawals, starting dates 1955–1983.

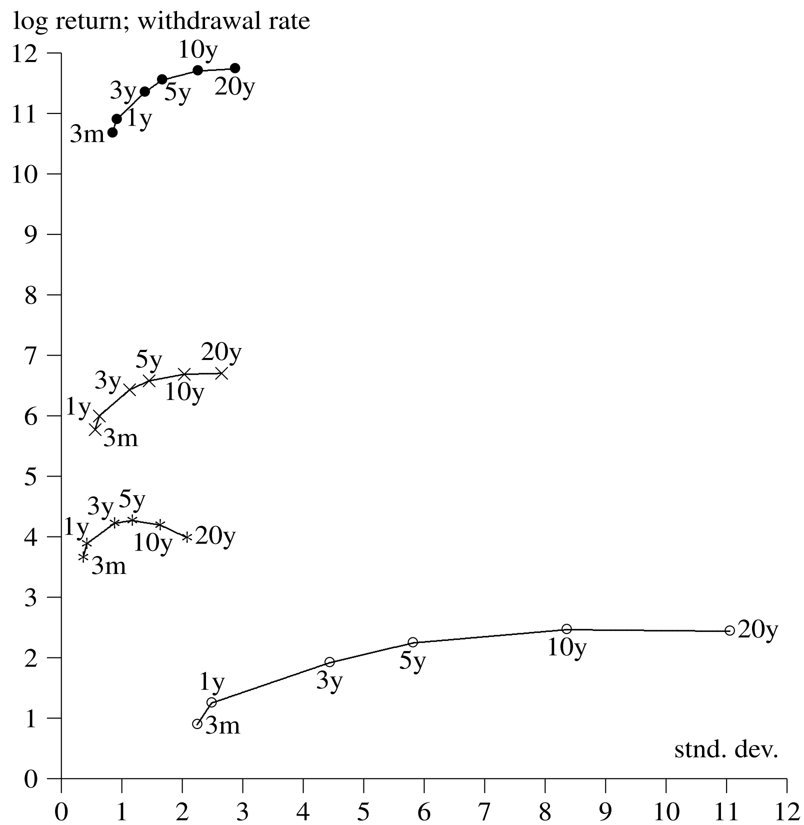

Figure 6. Standard deviation and mean based on the real returns of 1955–2017. Open circles: bonds, from Figure 4. Solid circles: $T = 10$ withdrawals, starting dates 1955–2008, from Figure 8. X's: $T = 20$ withdrawals, starting dates 1955–1998, from Figure 10. Asterisks: $T = 35$ withdrawals, starting dates 1955–1983, from Figure 11.

We wish to study the constant level "w" of end-of-year withdrawals each of these instruments could have supported while depleting an initial \$100 over each 10-, 20-, or 35-year period. A ten-year period would correspond to a rather short-term need for income, such as "tiding over" between retirement and receiving Social Security payments, or when the investor is at a very advanced age, while a twenty-year period resembles how long one might wait for a deferred income annuity to begin, or might be used by an investor of less advanced age. Bengen's original paper on safe withdrawal rates (1994 p. 173) implicitly used a 33-year period, based roughly on life expectancy at age 65. While we include many results for a 35-year period, there are only twenty-nine overlapping 35-year periods in our data, and having so few observations limits our ability to say much about how w varies over time (about "risk"). Bengen's data started in 1926 so he would not have faced that problem, but his data source only had half as many kinds of Treasury bonds.

To formally define the constant withdrawal amount w, let the balance at date t be x_t and let the continuously-compounded ("logarithmic") rate of return in the period leading up to date t be r_{t-1}. Choose w to satisfy:

$$\begin{aligned} x_1 &= 100 \\ x_t &= e^{r_{t-1}}\, x_{t-1} - w \quad \text{for } 2 \le t \le T+1 \text{, and} \\ x_{T+1} &= 0 \quad \text{for } T \text{ equal to 10, 20, or 35.} \end{aligned} \tag{1}$$

Given the returns r_t a root-solving algorithm can solve (1) for w, or one can use the analytical solution, which turns out to be[‡] $w = 100\, e^{\sum_{t=1}^{T} r_t} \big/ \left(1+\sum_{t=1}^{T} e^{\sum_{i=t+1}^{T} r_i}\right)$. Taking $x_1 = 100$ means that, for example, a withdrawal amount of $w = \$10$ implies a withdrawal rate of 10%, so we can use withdrawal "amounts" and "rates" interchangeably. If all the returns were zero then $w = x_1/T$, i.e.., \$10 or 10% for $T = 10$, \$5 or 5% for $T = 20$, and about \$2.86 or 2.86% for $T = 35$. Each time period generates a w, so different time periods generate a distribution of w's. Bengen assumed a withdrawal rate and found a distribution for how long withdrawals could occur. In the notation of (1), this would be like choosing w and for that w finding a distribution of T's. Closely related to Bengen's approach is what is often done now: assuming a withdrawal rate w and finding from the distribution of T's what percentage of them lie beyond a given horizon, that percentage being the strategy's "success rate" for that horizon. By contrast we assume a fixed length of withdrawals and find a distribution on withdrawal rates/returns. None of these methods is inherently superior to the others, but having a distribution on returns rather than on years allows us to use a familiar tool, the mean-vs.-standard-deviation graph with the mean of some return

[‡]Referee: For a not-to-be-published derivation, see the last page of this document.

measure (in our case, w) on one axis and that measure's variance or standard deviation on the other axis.[1]

For ten-year withdrawals ($T = 10$), the first observation represents withdrawals at the end of years 1955, 1956, ..., 1964; the second observation represents withdrawals at the end of years 1956, 1975, ..., 1965; and so forth.[2] These *ex post* feasible constant annual payments over ten years are shown in Figure 7 using nominal returns and thus obtaining withdrawals that are constant in nominal terms, and in Figure 8 for real returns thus obtaining withdrawals that are constant in real terms (withdrawals which would have been increasing in nominal terms at the rate of inflation). The withdrawal rates over twenty-year periods ($T = 20$) are shown in Figure 9 for withdrawals constant in nominal terms and Figure 10 for withdrawals constant in real terms. Figure 11 shows withdrawals constant in real terms for thirty-five year periods ($T = 35$).

It has been common since the pioneering work of Bengen (op. cit.) to focus in this context on the minimum withdrawal rate. The minimum withdrawal rate for each series in Figures 7, 8, 9, 10, and 11, as well as for the unillustrated case of nominal withdrawals for thirty-five-year periods, is given in Table 1. As explained earlier, in the case of the "real" columns this is only the amount of the first withdrawal, and the subsequent ones rose at the rate of inflation. Several values in the table reflect lows that happened quite recently. The highest minimum real withdrawal rates were generated by one-year bills, which also generated the highest minimum nominal withdrawal rates for thirty-five-year periods; three-year notes generated the highest minimum nominal withdrawal rates for ten- and twenty-year periods. These are the investments a maxi-min investor would have done best choosing. The worst (lowest) minimum withdrawal rates for all six categories were generated by twenty-year bonds, in a tie with three-month bills in the $T = 10$ nominal case.

[1] In Bengen's framework one could graph the mean and standard deviation of the "number of years until funds ran out" for different withdrawal rates. Bengen had no need to do that because he was only interested in the minimum of that distribution, not in its mean or standard deviation; we study all those aspects of w's distribution.

[2] Using such overlapping periods has the disadvantage of using observations which are sharing much of the same data, but has the advantage (over using Monte Carlo simulations) of retaining all the time-series properties of the return series. There is no assurance the fact that fixed-income securities pay on a contractually-specified basis is captured either by the autoregressive moving average process of order 1 used by Cooley et al. (2003, see p. 119) nor by the "vector autoregressive specification ["VAR"]... such that *short-term* first-order auto- and crosscovariances are preserved" used by Brouwer and de Ruiter (1997, abstract and pp. 10–11, my emphasis). Sangvinatsos and Wachter (2005 p. 181) write that "estimating bond returns using a VAR gives up the extra information resulting from the no-arbitrage restriction on bonds, namely that bonds have to pay their (nominal) face value when they mature," though following their alternative is beyond the scope of this paper.

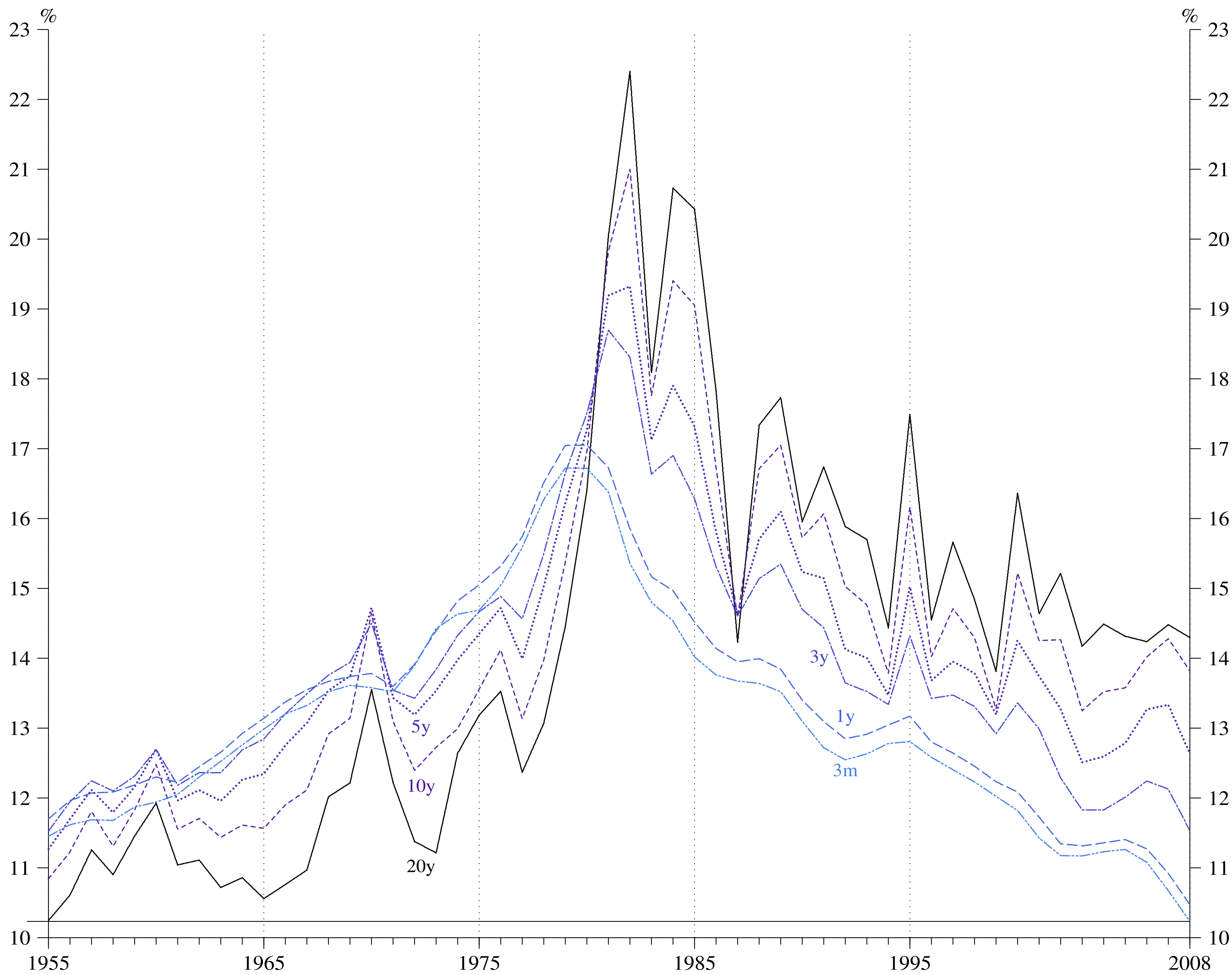

Figure 7. Nominal withdrawals for ten-year periods with starting dates of 1955–2008.

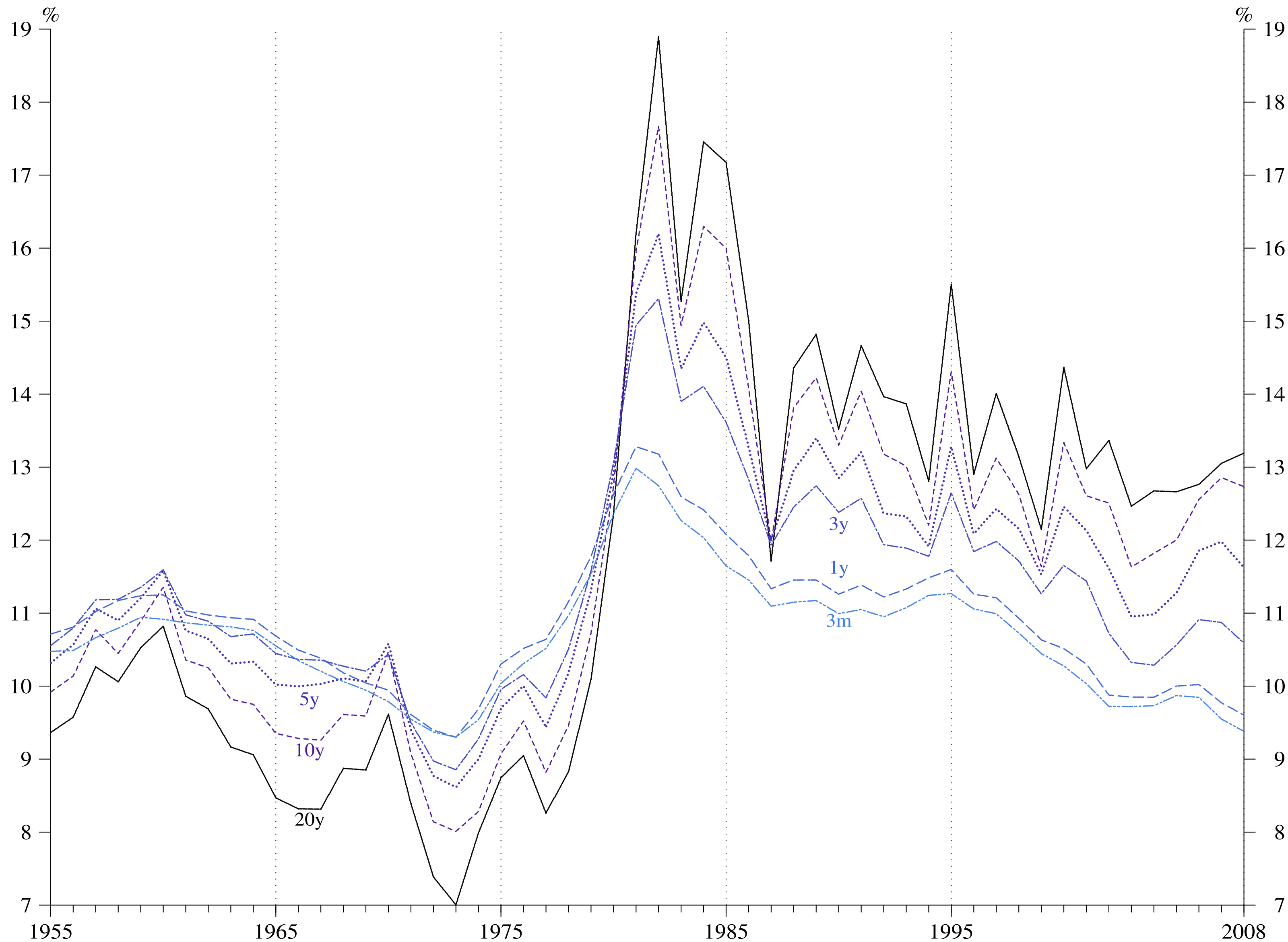

Figure 8. Real withdrawals for ten-year periods with starting dates of 1955–2008.

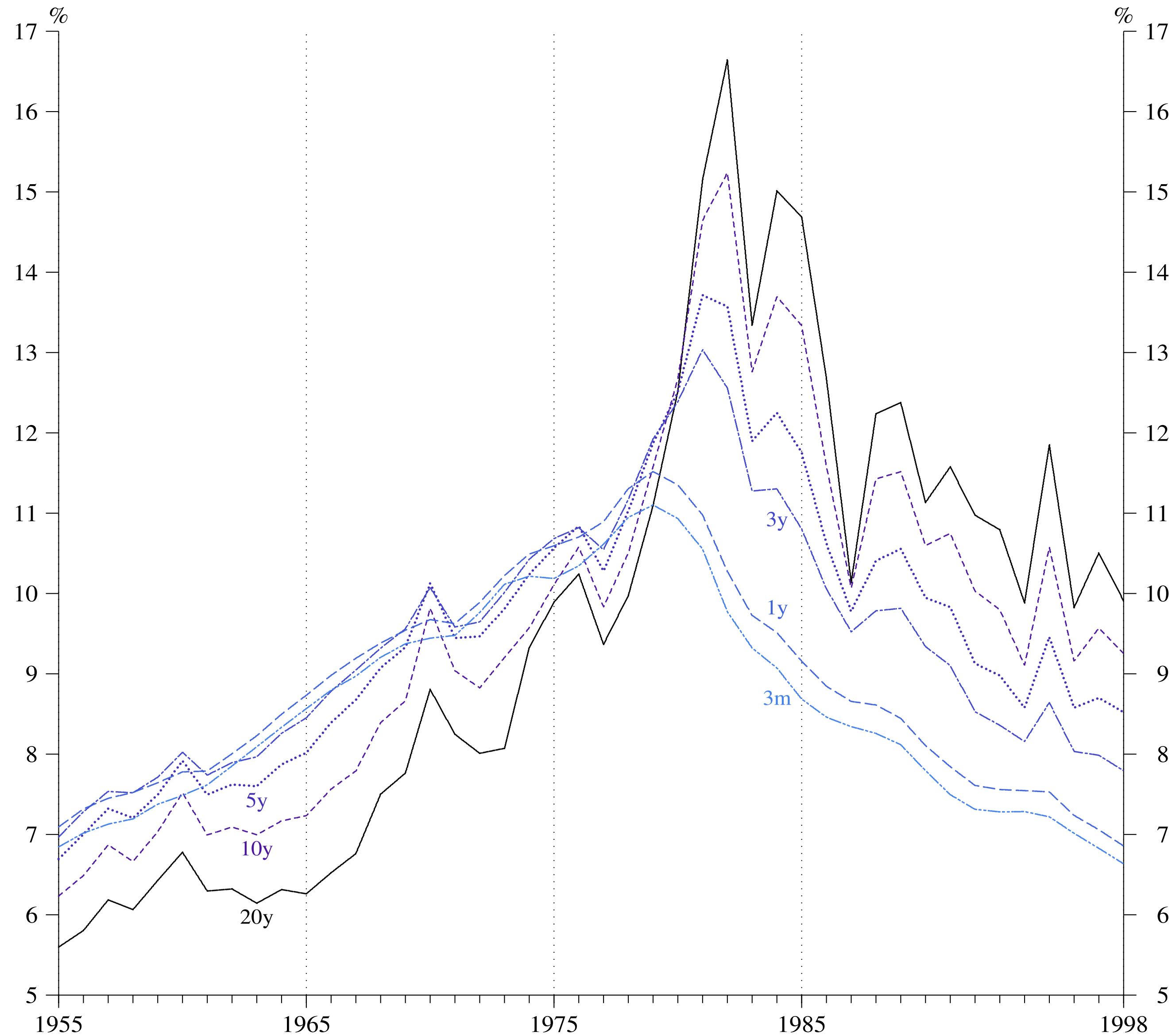

Figure 9. Nominal withdrawals for twenty-year periods with starting dates of 1955–1998.

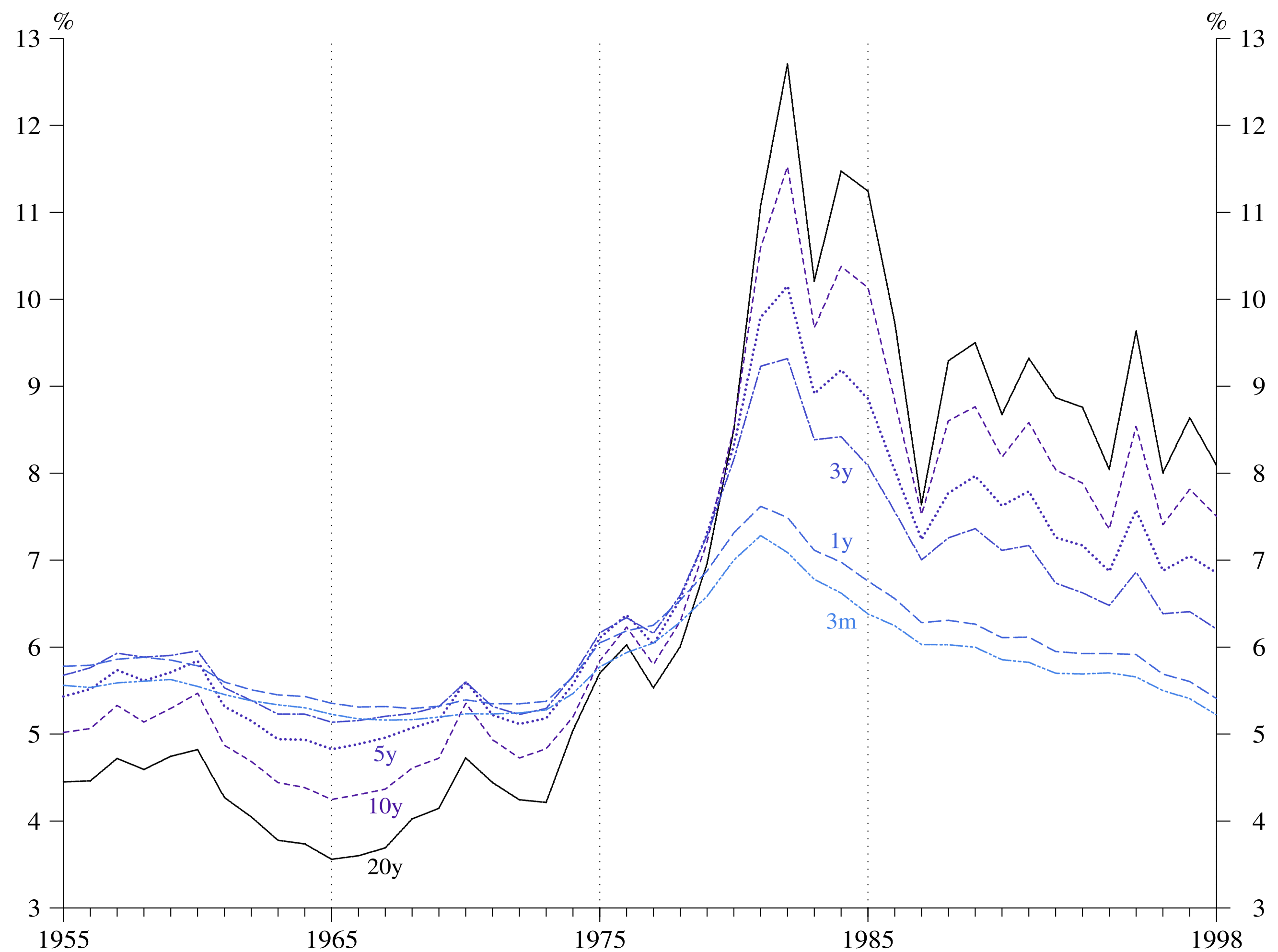

Figure 10. Real withdrawals for twenty-year periods with starting dates of 1955–1998.

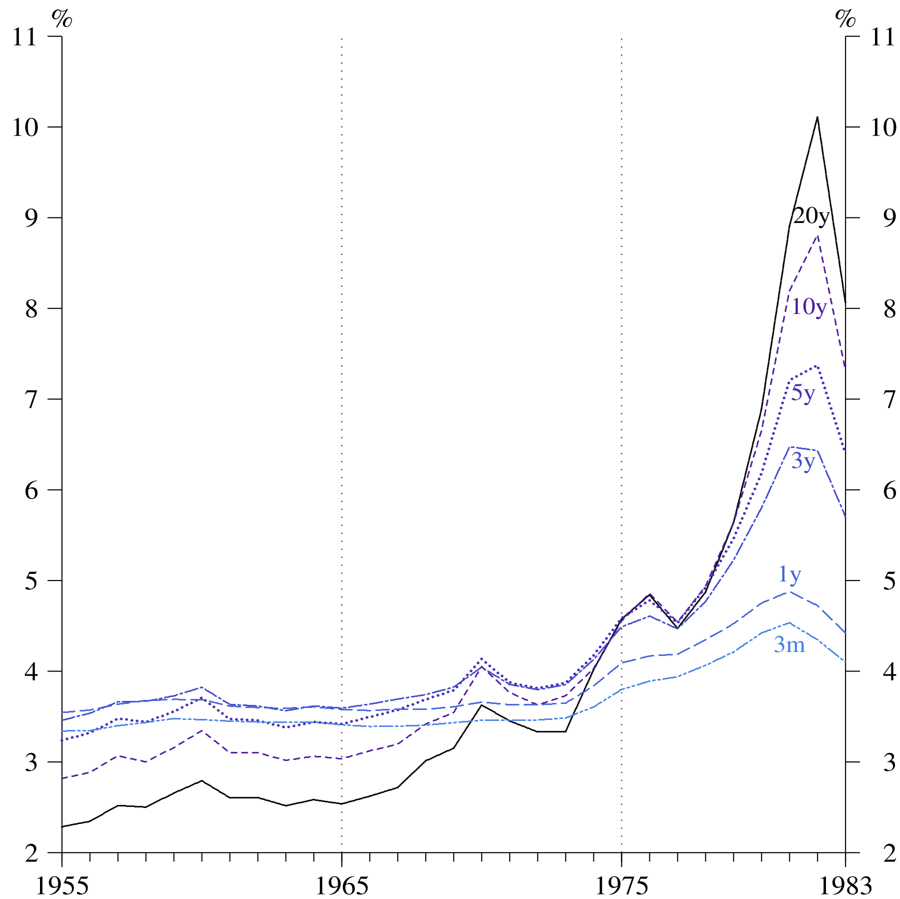

Figure 11. Real withdrawals for thirty-five-year periods with starting dates of 1955–1983.

	T = 10 nominal		T = 20 nominal		T = 35 nominal	
20yr	10.2	’55–’64	5.6	’55–’74	4.1	’55–’89
10yr	10.8	’55–’64	6.2	’55–’74	4.8	’55–’89
5yr	11.3	’55–’64	6.7	’55–’74	5.3	’55–’89
3yr	11.5*	’55–’64	7.0*	’55–’74	5.6	’55–’89
1yr	10.5	’08–’17	6.9	’98–’17	5.7*	’55–’89
3mo	10.2	’08–’17	6.6	’98–’17	5.4	’55–’89
	T = 10 real		T = 20 real		T = 35 real	
20yr	7.0	’73–’82	3.6	’65–’84	2.3	’55–’89
10yr	8.0	’73–’82	4.2	’65–’84	2.8	’55–’89
5yr	8.6	’73–’82	4.8	’65–’84	3.2	’55–’89
3yr	8.9	’73–’82	5.1	’65–’84	3.5*	’55–’89
1yr	9.3*	’73–’82	5.3*	’68–’87	3.5*	’55–’89
3mo	9.3*	’73–’82	5.2	’67–’86	3.3	’55–’89

Table 1. Minimum withdrawal rates, 1955–2017. The accompanying years show the period when the minimum occurred. Asterisks denote the highest minimum withdrawal rate in each column.

Some (quite risk-averse) investors may only care about the minimum withdrawal rates of Table 1. For other investors, in particular those whose attitudes towards risk can be captured by standard deviation, note that since each of the series in Figures 7, 8, 9, 10, and 11 has a mean and standard deviation, one can form for each one a “withdrawal” mean-standard-deviation frontier. These are graphed in Figures 5 (nominal returns) and 6 (real returns), using solid circles for $T = 10$, X’s for $T = 20$, and asterisks for $T = 30$. In these two figures the gap between the standard deviation of *withdrawals* (w) from a portfolio of 20-year bonds and a portfolio of 3-month bills was much less than the gap between the standard deviation of *returns* from a portfolio of 20-year bonds and a portfolio of 3-month bills.[3] Otherwise, for $T = 10$ and $T = 20$ there is little surprising in these results: the mean return and standard deviation of withdrawals was monotonically increasing in bond maturity, and the gain to extending maturity beyond five years was quite modest but still positive.

Because the $T = 35$ case has relatively few observations, as illustrated in Figure 11, it is unclear how much importance to put on the fact that return in the $T = 35$ graphs of Figures 5 and 6 was increasing in maturity and in standard deviation only until the five-year maturity, beyond which return was decreasing in maturity and in standard deviation. Perhaps the more interesting observation

[3]This is not because bond returns are mean-reverting; Campbell and Viceira (2002 Fig. 4.2(a) and p. 108) found that they were mean-averting.

from those results is the extent to which the initial withdrawal rate falls from Figure 5 to Figure 6. Inflation from 1955 to 2017 averaged a continuously-compounded 3.53%, which is the difference between the height of the open circles in Figure 5 and in Figure 6. The difference between Figure 5 and 6's mean $T = 35$ withdrawal rate was, for three-month, one-year, three-year, five-year, ten-year, and twenty-year instruments, respectively, approximately that much: 3.6, 3.7, 3.7, 3.7, 3.5, and 3.3 percent. In other words, keeping real rather than nominal withdrawals constant decreased initial withdrawals by roughly one half.[4] Protection from inflation for shorter time periods was less expensive. For $T = 20$, the difference between Figure 5 and 6's mean withdrawal rate was, in the order of instruments used above, 2.8, 2.9, 3.0, 3.0, 2.9, and 2.9 percent, and for $T = 10$ it was 2.4, 2.5, 2.5, 2.5, 2.5, and 2.5 percent.[5] In the market for single-premium immediate annuities, it is difficult to find providers of inflation-indexed versions, presumably because of lack of customer demand, which is turn is presumably driven by how much less inflation-indexed SPIAs pay initially compared to non-inflation-indexed ones. On the other hand the U.S. government's Social Security retirement program, which is inflation-indexed, is one of the government's most popular programs, despite the fact that if it were not inflation-indexed its initial payments could be quite a bit higher (keeping its actuarial fairness constant). It could be that consumers generally do not know how very much less money one could have afforded to spend in the last several decades if one had wanted to protect one's future self from inflation.

2. Anomalies and Endpoint-Dependence

Because even broad conclusions in finance are often endpoint-dependent, we reran some of the analyses using less than the full data set. Figures 12 (nominal) and 13 (real), on the one hand, and Figures 14 (nominal) and 15 (real), on the other hand, show the results of dividing the data into two parts, pre-1982 and post-1981: an initial era of mostly rising interest rates and inflation and a final era of mostly declining interest rates and inflation.

These graphs have unsurprising results for mean withdrawal amounts: the early period's rising interest rates were bad for withdrawals from long bonds and the later period's falling interest rates were good for withdrawals from long bonds; vice versa for short bonds.

In fact the results are so strong that we can say something even about investors who do not satisfy the rather restrictive assumptions needed to assume

[4] The ratios, in the order given above, were: 0.51, 0.52, 0.53, 0.54, 0.54, 0.55.

[5] The ratios were 0.67, 0.67, 0.68, 0.69, 0.69, and 0.70 for $T = 20$ and 0.82, 0.82, 0.82, 0.82, 0.82, and 0.83 for $T = 10$.

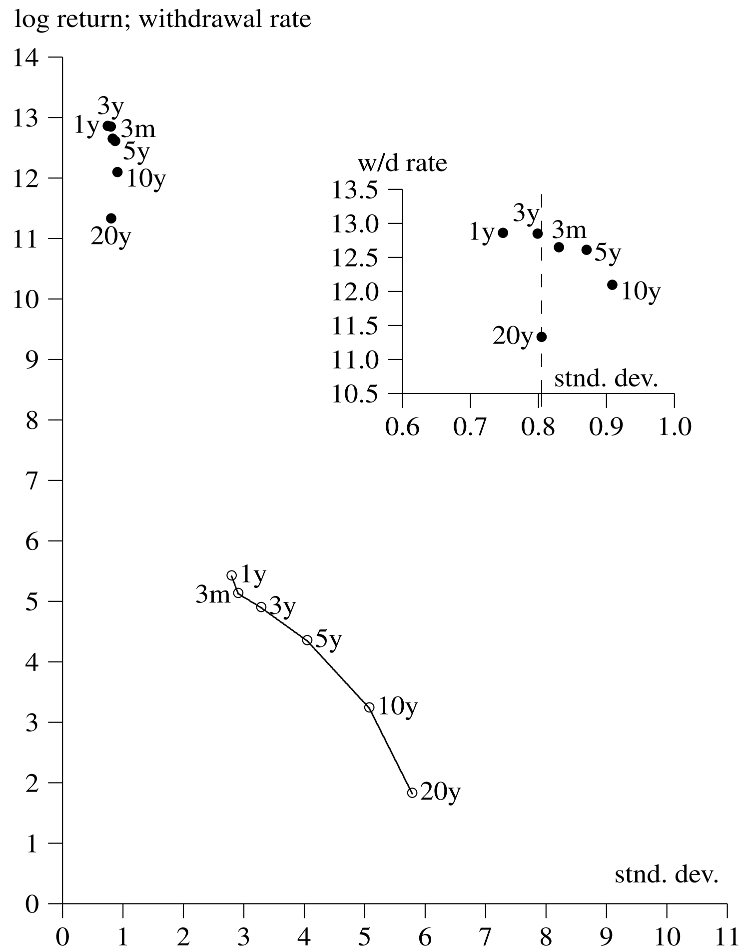

Figure 12. Standard deviation and mean based on the nominal returns of 1955–1981. Open circles: pre-1982 bonds, from the early part of Figure 3. Solid circles: pre-1982, $T = 10$ withdrawals, starting dates 1955–1972, from the early part of Figure 7.

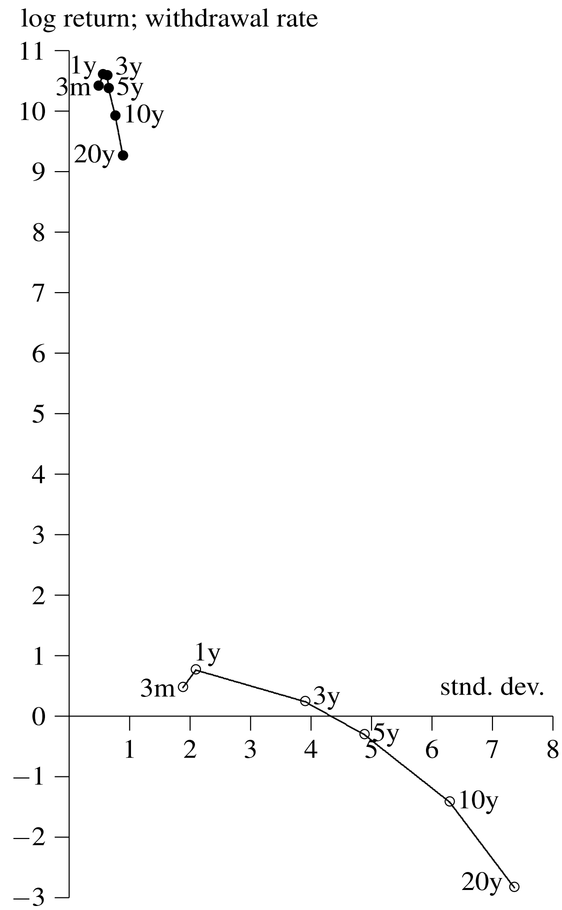

Figure 13. Standard deviation and mean based on the real returns of 1955–1981. Open circles: pre-1982 bonds, from the early part of Figure 4. Solid circles: pre-1982, $T = 10$ withdrawals, starting dates 1955–1972, from the early part of Figure 8.

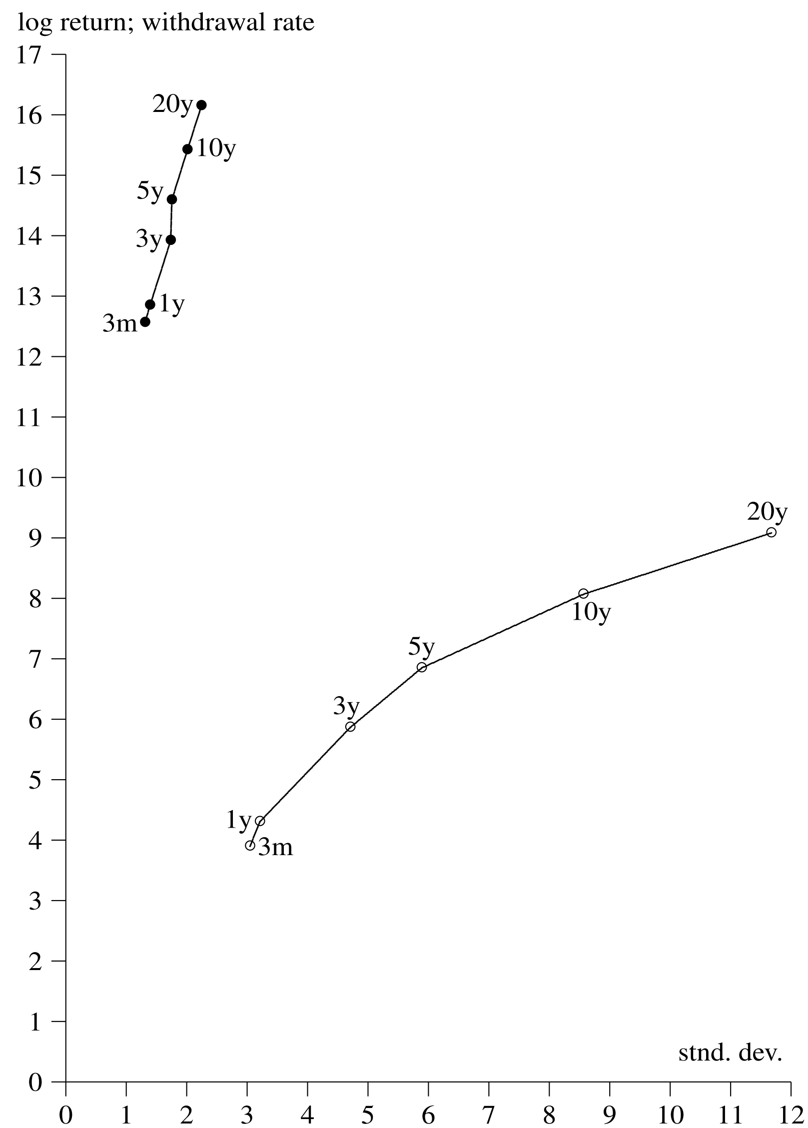

Figure 14. Standard deviation and mean based on the nominal returns of 1982–2017. Open circles: post-1981 bonds, from the late part of Figure 3. Solid circles: post-1981, $T = 10$ withdrawals, starting dates 1982–2008, from the late part of Figure 7.

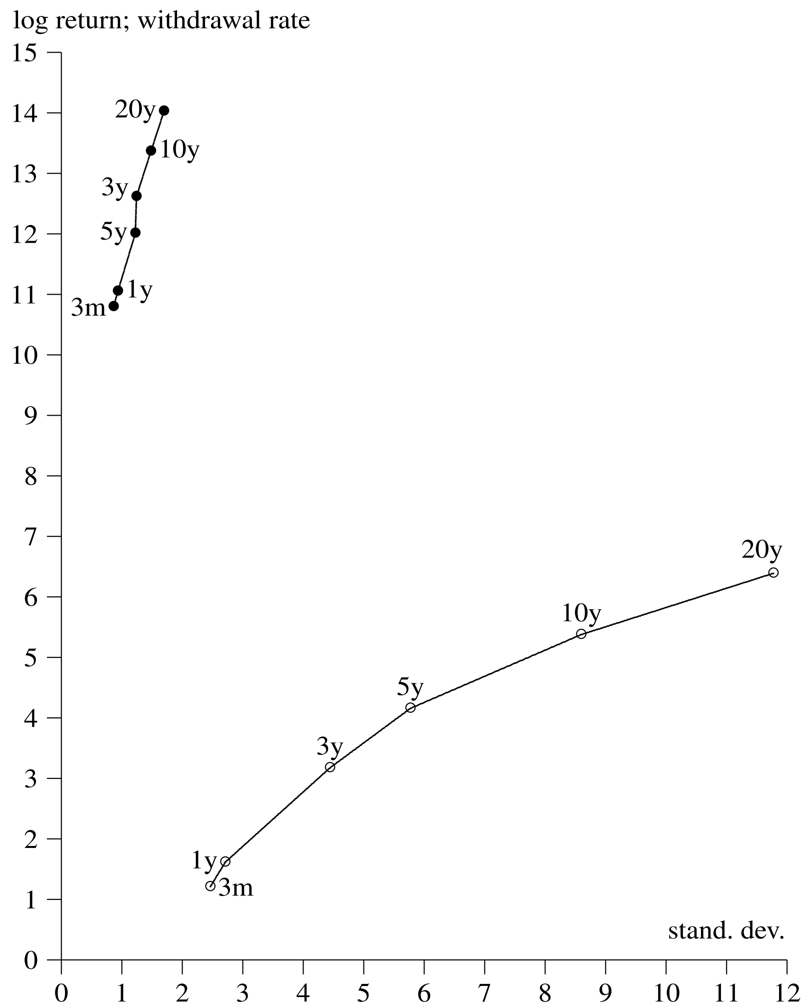

Figure 15. Standard deviation and mean based on the real returns of 1982–2017. Open circles: post-1981 bonds, from the late part of Figure 4. Solid circles: post-1981, $T = 10$ withdrawals, starting dates 1982–2008, from the late part of Figure 8.

that they care only about the mean and variance of outcomes.[6] All expected-utility-maximizing investors, regardless of other characteristics of their utility functions, will prefer a distribution F over another distribution G if F exhibits "first degree stochastic dominance" ("FDSD") over G, where "F exhibits FDSD over G" when for *any* fixed return $\bar{r}$, the probability that the realized return r is greater than $\bar{r}$ is larger under F than under G. In this case the cumulative distribution function for F lies everywhere under (or, to the right—to larger r's) of G's.[7] In the pre-1982 era, in both nominal and real analyses for $T = 10$, twenty-year bonds were FDSDominated by all other instruments and ten-year bonds would have been FDSDominated by all instruments (other than twenty-year bonds) except for one data point. The other bonds could not be ranked by FDSD. In the post-1981 era, in both nominal and real analyses for $T = 10$, all of the instruments could be ranked by FDSD, with every maturity dominating all shorter maturities. Since financial instruments usually cannot be ranked by FDSD, that is a remarkably strong result. It is illustrated for the case of real returns in Figure 16. The problem will all such results, even as strong as these, is that they are so period-dependent that they may be useless in the future—as indeed these post-1981 results would have been in the pre-1982 period.

As far as standard deviation of withdrawal amounts is concerned, Figures 13, 14, and 15 again have unsurprising results: standard deviation of withdrawal rates was increasing in maturity. However, Figure 12 has a very surprising ordering from smallest-to-largest standard deviation of withdrawals, as can be read off of the inset within the graph: 1yr, 3yr, 20yr, 3mo, 5yr, and 10yr. Three-month bills were "riskier" (or at least had a higher standard deviation) than twenty-year bonds, when viewed as source of generating income (generating withdrawals).

This raises a question of how common it was for three-month bills not to be the "safest" (lowest standard deviation) source of withdrawals. Because of the aforementioned sensitivity of results to endpoints, the most comprehensive answer to that question involves investigating all possible choices of endpoints.

[6]Sufficient conditions for "only caring about the mean and variance of outcomes" are having quadratic utility, or facing outcomes which follow a multivariate normal distribution. The graphs in this paper use standard deviation because it has the same units as return, and it is monotonically related to variance, but any analysis requiring a more exact frontier should be graphed as a function of variance instead of standard deviation.

[7]See e.g. `https://en.wikipedia.org/wiki/Stochastic_dominance`, which points out that FDSD of F over G does not necessarily imply FDSD of "F mixed with another asset H" over "G mixed with the same amount of H," and that without assuming expected-utility-maximization, "all investors preferring more return to less return," regardless of other characteristics of their utility functions, will prefer F over G if F exhibits "statewise dominance" over G; "statewise dominance" implies but is not implied by FDSD.

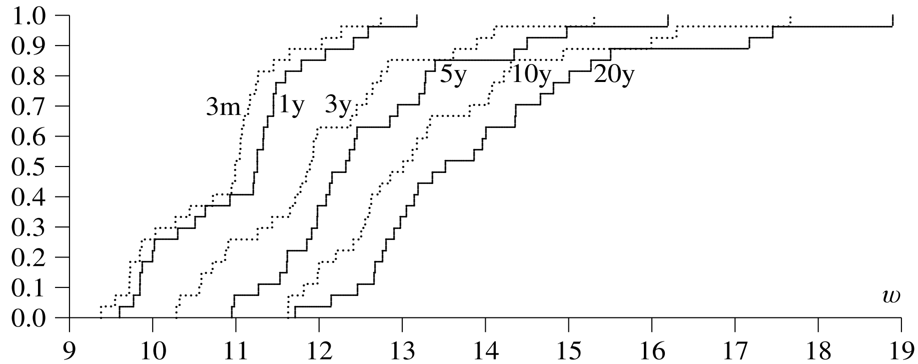

Figure 16. Cumulative distribution of $T = 10$ withdrawals (1982–2008) for, from left to right, three-month bills (dotted), one-year bills, three-year notes (dotted), five-year notes, ten year-notes (dotted), and twenty-year bonds. Data 1982–2017.

The next five figures answer the question (for $T = 10$ and $T = 20$, nominal and real, and $T = 35$ nominal) "which instrument generated the lowest-standard-deviation withdrawals?" for every possible choice of endpoints in our data set giving at least ten observations.[8] In the case of ten-year withdrawals ($T = 10$), there are 1035 such periods, each shown in Figure 17 and 18. For example, in Figure 17, in the very bottom left-hand corner, the first ten-year period begins in 1955 and ends in 1964 (where each of those dates represents the beginning of a T-length-long series of years). In the case of twenty-year withdrawals ($T = 20$) there are 630 such periods, and in the case of thirty-five year withdrawals ($T = 35$) there are 210 such periods. The bond which generated the lowest-standard-deviation withdrawals for each choice of endpoints is shown in Figure 17 for the case of fixed nominal withdrawals and $T = 10$; in Figure 18 for the case of fixed real withdrawals and $T = 10$; in Figure 19 for the case of fixed nominal withdrawals and $T = 20$; in Figure 20 for the case of fixed real withdrawals and $T = 20$; and in Figure 21 for the case of fixed nominal withdrawals and $T = 35$. The unillustrated case of $T = 35$, real, gives a figure like Figure 21 but filled everywhere with "C's" except for two places, 1961–1973 and 1964–1973, which both had a "1." These five figures together with the unillustrated case are summarized in Table 2.

For withdrawals fixed in real terms, in more than 97% of cases for $T = 10$, for $T = 20$, and for $T = 35$, three-month bills gave the smallest standard

[8]With fewer than ten observations it is difficult to conclude much about standard deviation, which is also the reason the barely-twenty-one-years-old Treasury Inflation-Protected Securities are not analyzed in this paper.

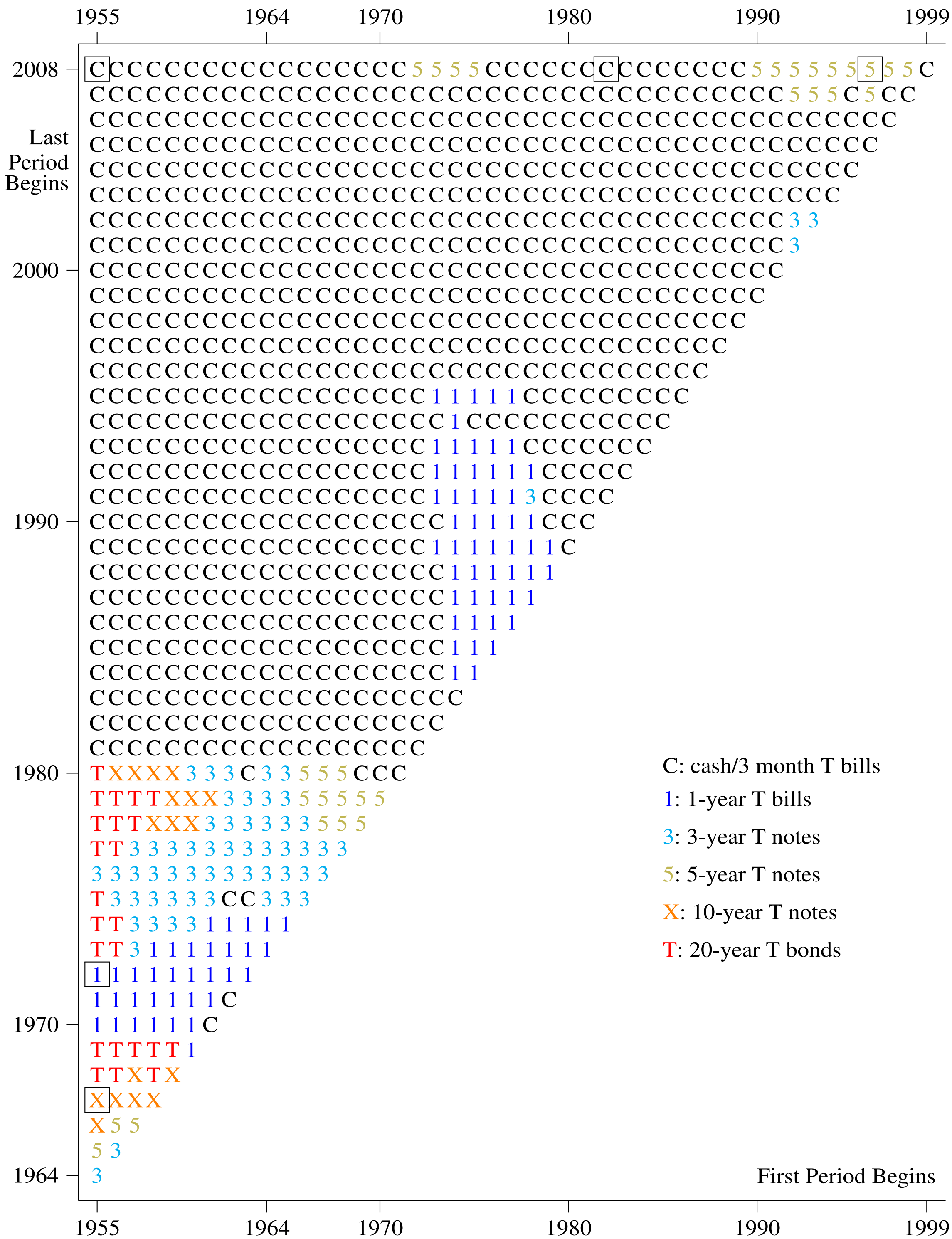

Figure 17. Instrument giving lowest standard deviation (among the choices listed) for ten-year withdrawals fixed in nominal terms. Reading clockwise from the upper left-hand corner, the date pairs with a box around them were or will be illustrated in: Figure 5 (solid circles), starting dates 1955–2008; Figure 14, 1982–2008; Figure 22, 1996–2008; Figure 25, 1955–1967; Figure 12, 1955–1972.

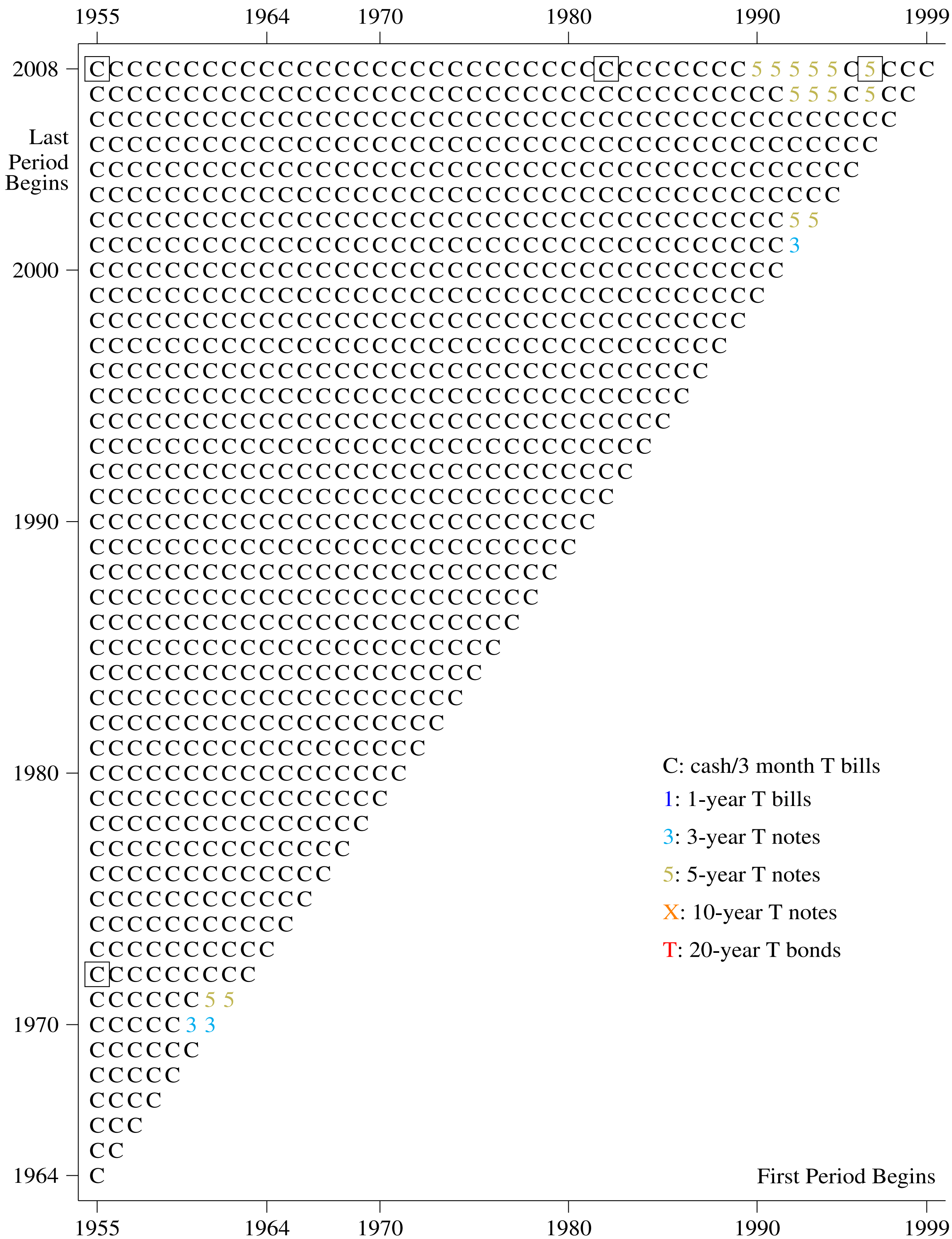

Figure 18. Instrument giving lowest standard deviation (among the choices listed) for ten-year withdrawals fixed in real terms. Reading clockwise from the upper left-hand corner, the date pairs with a box around them were or will be illustrated in: Figure 6 (solid circles), starting dates 1955–2008; Figure 15, 1982–2008; Figure 23, 1996–2008; Figure 13, 1955–1972.

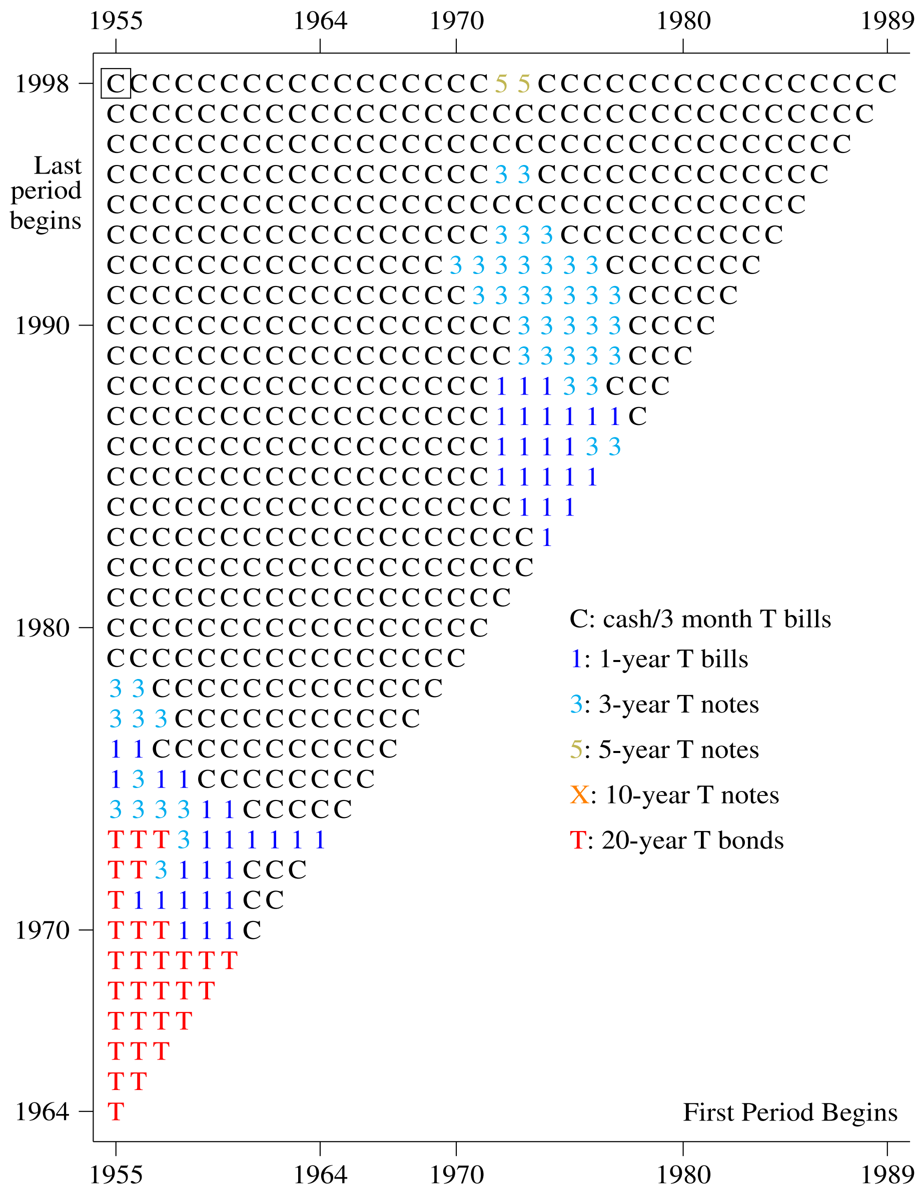

Figure 19. Instrument giving lowest standard deviation (among the choices listed) for twenty-year withdrawals fixed in nominal terms. The date pair with the box around it was illustrated in Figure 5 (X's), starting dates 1955–1998.

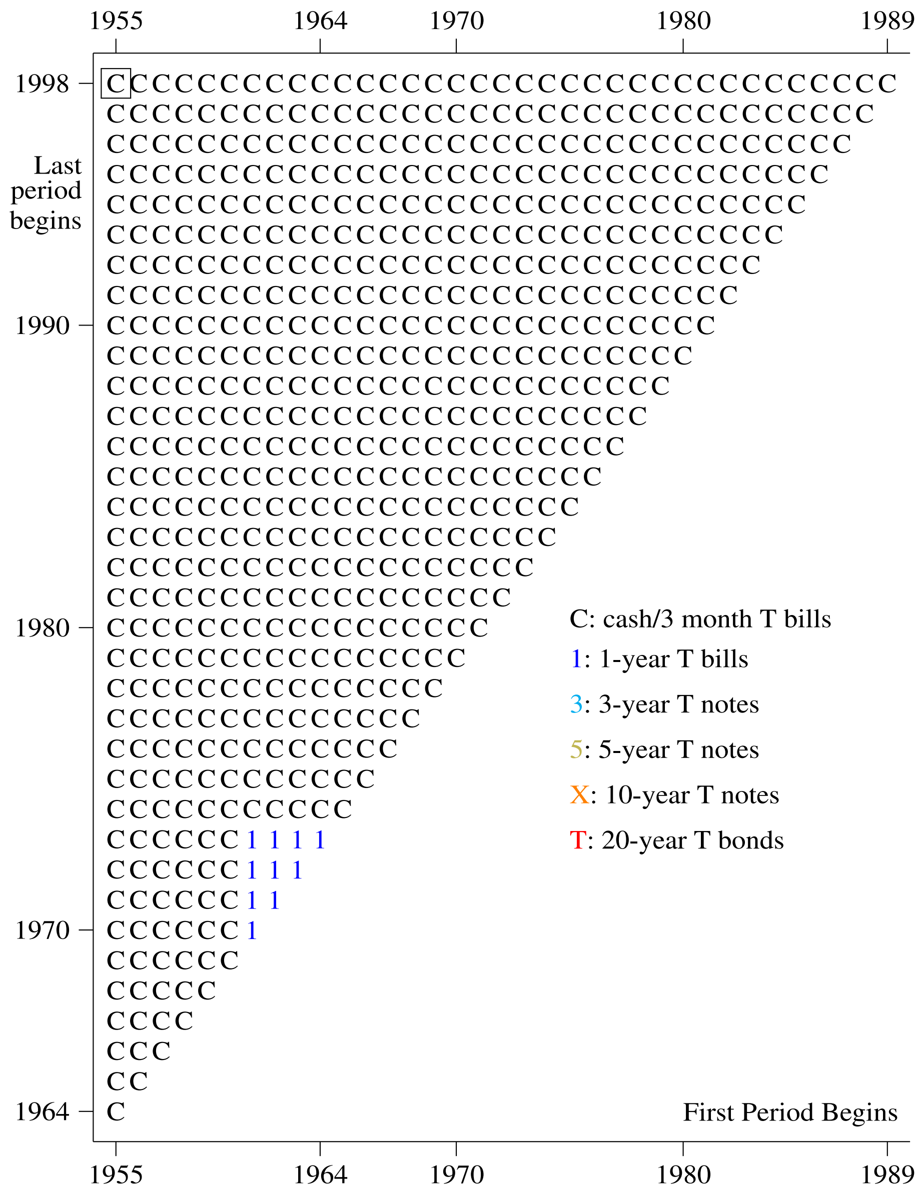

Figure 20. Instrument giving lowest standard deviation (among the choices listed) for twenty-year withdrawals fixed in real terms. The date pair with a box around it was illustrated in Figure 6 (X's), starting dates 1955–1998.

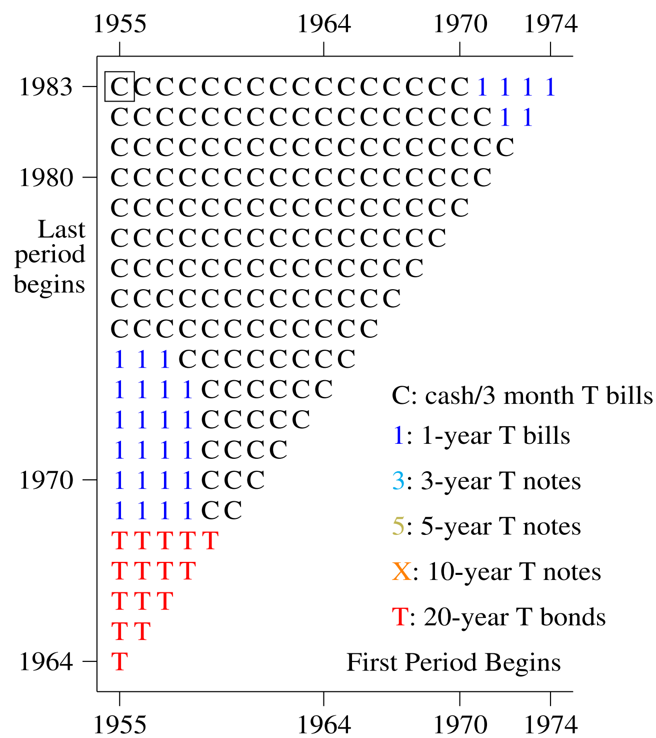

Figure 21. Instrument giving lowest standard deviation (among the choices listed) for thirty-five withdrawals fixed in nominal terms. The date pair with a box around it was illustrated in Figure 5 (asterisks), starting dates 1955–1983.

	$T = 10$ nom.		$T = 20$ nom.		$T = 35$ nom.	
	obs.	%	obs.	%	obs.	%
3mo	815	79	507	80	166	79
1yr	89	9	46	7	29	14
3yr	60	6	45	7	0	0
5yr	31	3	2	0	0	0
10yr	17	2	0	0	0	0
20yr	23	2	30	5	15	7
obs.	1035	101	630	99	210	100
	$T = 10$ real		$T = 20$ real		$T = 35$ real	
	obs.	%	obs.	%	obs.	%
3mo	1018	98	620	98	208	99
1yr	0	0	10	2	2	1
3yr	3	0	0	0	0	0
5yr	14	1	0	0	0	0
10yr	0	0	0	0	0	0
20yr	0	0	0	0	0	0
obs.	1035	99	630	100	210	100

Table 2. Number of subperiods in which each instrument gave the lowest standard deviation (summary of Figures 17, 18, 19, 20, 21, and the unillustrated case of $T = 35$, real). Percentages sometimes fail to total one hundred due to rounding.

deviation, as they did in Figures 6, 13, and 15. For withdrawals fixed in nominal terms, in only roughly 80% of cases (for $T = 10$, for $T = 20$, and for $T = 35$) did three-month bills gave the smallest standard deviation, as they did in Figures 5 and 14 but as they did not do in Figure 12. In Figures 17–21, anomalies are most likely to occur the further one is from the upper left-hand corner, which means anomalies are most likely to occur with a smaller number of observations (the number of observations increases going up and to the left).

In each one of Figures 7, 8, 9, 10, and 11, withdrawals from long-term bonds appear more jagged than withdrawals from shorter-term bonds, so it is of interest to investigate why riskiness as measured by standard deviation sometimes gives the opposite conclusion.

Start by looking at cases using recent data, 1996 to 2017, corresponding to the boxed entry near the upper-righthand corner of Figures 17 and 18. For these years and $T = 10$, Figures 22 (nominal) and 23 (real) both show standard deviations of withdrawals to be almost the same for all of the bond maturities. Detailed analysis in Figure 22 shows three-month bills having a higher standard deviation of withdrawals than five-year notes, ten-year notes, and (barely) even twenty-year bonds. Figure 23 is not as extreme, with only five-year and ten-year notes having lower withdrawal standard deviation than three-month notes, but nevertheless this is not conventionally-anticipated behavior. The left-hand panel of Figure 24 shows two of the time paths underlying Figure 22 and the middle panel of Figure 24 shows two of the time paths underlying Figure 23. In those panels one can see that although twenty-year withdrawals are intuitively more jagged than the T Bills' withdrawals, it is possible for them to have a smaller standard deviation.

As these panels of Figure 24 show, this anomalous behavior was during a time of strongly falling short-term yields (which is why the withdrawals based on three-month bills fell sharply). To show that anomalous behavior can also occur during a time of strongly rising short-term yields, consider $T = 10$ and starting dates from 1955 to 1967 (the lowest "boxed" observation in Figure 17). That time is illustrated in the right-hand panel of Figure 24 and in Figure 25. The latter shows the anomalous behavior it generates: the withdrawals based on three-month bills have a higher standard deviation than withdrawals based on any of the other instruments. In this case, the high standard deviation of withdrawals from three-month bills was due to a welcome rise in their yields, a rise which kept down withdrawals from longer-term instruments (due to their capital losses)—decreasing the longer-term's standard deviation in an unwelcome way. In the period with 1996–2008 starting dates shown in Figures 22 and 23, by contrast, the high standard deviation of withdrawals from three-month bills was due to the precipitous fall in their yields, a fall which kept up withdrawals from

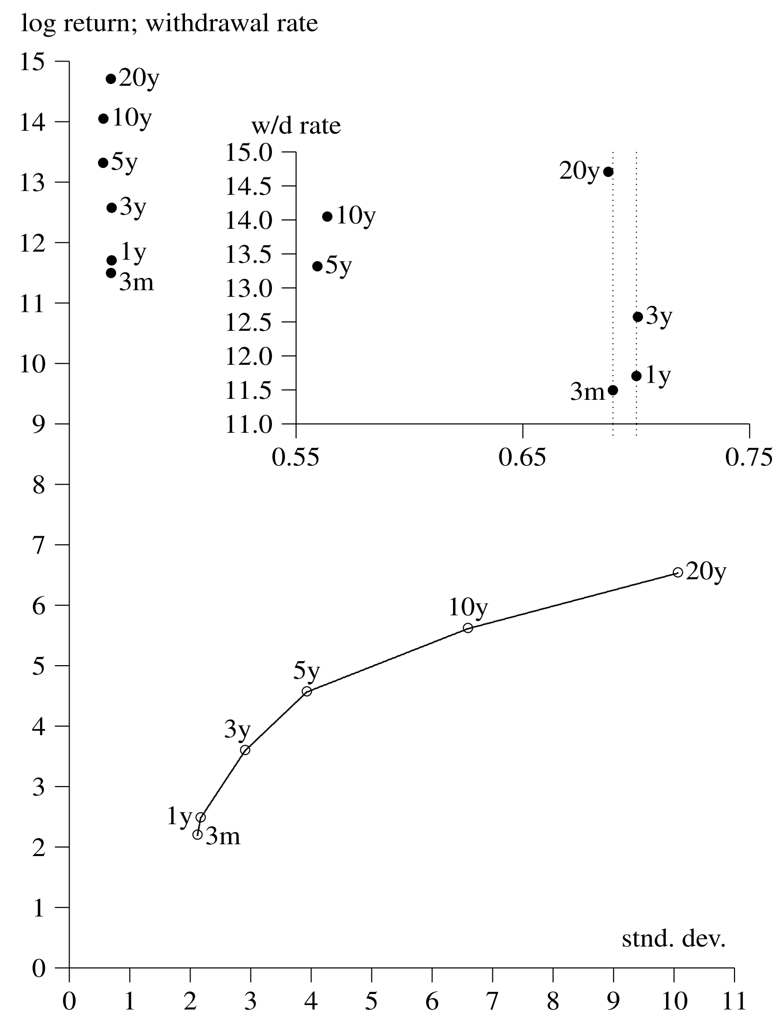

Figure 22. Standard deviation and mean based on the nominal returns of 1996–2017. Open circles: bonds, from near the end of Figure 3. Solid circles: $T = 10$ withdrawals, starting dates 1996–2008, from near the end of Figure 7. Standard deviations for 1y and 3y withdrawals are 0.7001 and 0.7007, respectively.

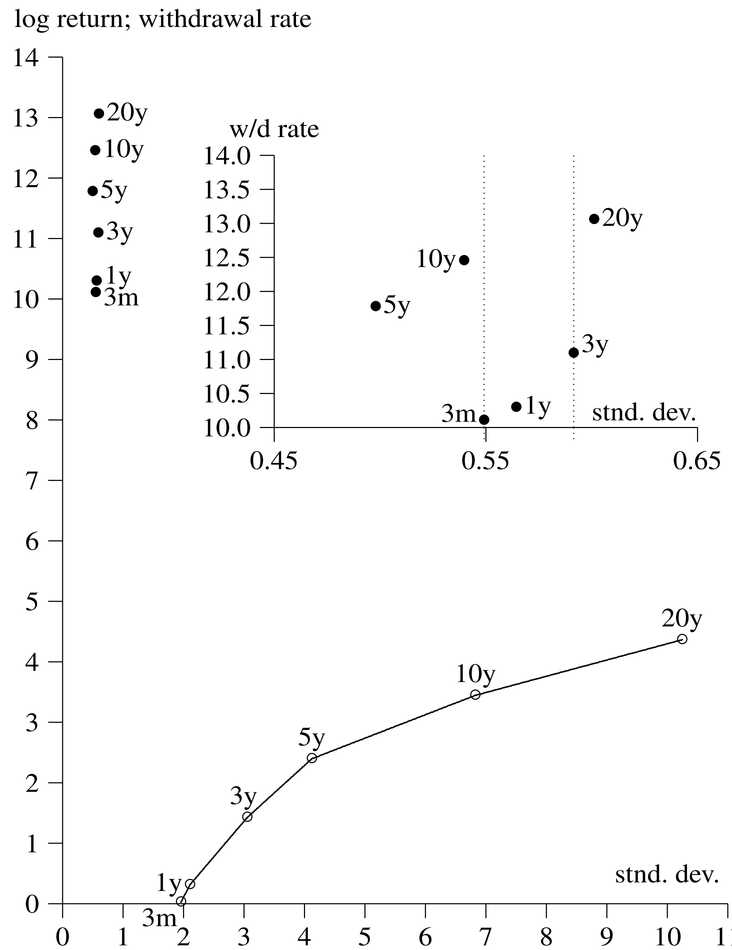

Figure 23. Standard deviation and mean based on the real returns of 1996–2017. Open circles: bonds, from near the end of Figure 4. Solid circles: $T = 10$ withdrawals, starting dates 1996–2008, from near the end of Figure 8.

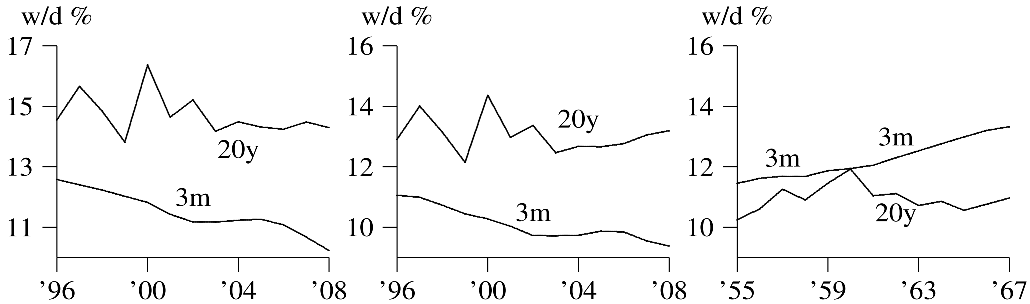

Figure 24. Examples of situations when withdrawals from twenty-year bonds had a lower standard deviation than withdrawals from three-month bills. Left panel: Excerpt of the 3m and 20y lines near the end of Figure 7, underlying Figure 22. Middle panel: Excerpt of the 3m and 20y lines near the end of Figure 8, underlying Figure 23. Right panel: Excerpt of the 3m and 20y lines near the beginning of Figure 7, underlying Figure 25.

longer-term instruments (due to their capital gains)—decreasing the longer-term's standard deviation in a welcome way. The 1996–2008 period shows not only in a mathematical sense but also in an intuitive sense that short-term bonds can be a risky source of withdrawals. With the collapse of short-term yields during the Great Recession, three-month and one-year bills during that time gave a lower nominal withdrawal amount in Figure 7 ($T = 10$) than any other instruments at any other time except roughly tying with twenty-year bonds at the beginning of the data set. By contrast, short-term instruments at that time did not do quite as badly, relatively speaking, when considering real returns (Figure 8), or when considering $T = 20$ (Figures 9 and 10), but they still did not do well. Living on the returns of short-term instruments during the Great Recession was not easy and these results reflect that.

Conclusion

We have shown that a maxi-min investor spending from a portfolio of constant-maturity Treasuries would have done best in real terms using one-year bills based on 1955–2017 data. An investor who instead cared about the mean and variance of potential income levels typically faced a tradeoff of a higher expected payoff with a greater standard deviation, although about 20% of the time, this risk/return tradeoff was not present when analyzing nominal returns. When analyzing real returns, the tradeoff was almost always present. When the tradeoff was present, extending maturity from 10 to 20 years was usually accompanied by a large increase in standard deviation and only a very small

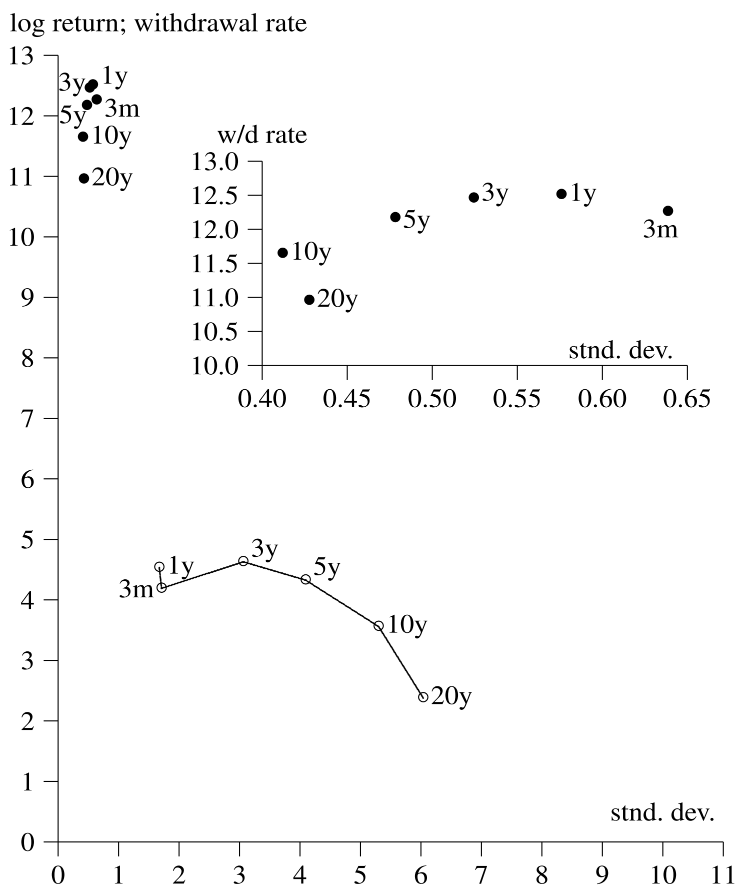

Figure 25. Standard deviation and mean based on the nominal returns of 1955–1976. Open circles: bonds, from near the beginning of Figure 3. Solid circles: $T = 10$ withdrawals, starting dates 1955–1967, from near the beginning of Figure 7.

increase in average return. When the tradeoff was absent, it was usually a time of great change in short-term interest rates, either a large increase, as in the 1960's and 1970's, or a large decrease, as during the Great Recession.

An important restriction of the strategies we have investigated is that they keep fixed-income maturity constant. By contrast, if one thinks of retirement spending as a sequence of liabilities, one could (if one has sufficient assets, given current interest rates) ensure that each one is met by separately immunizing each with a zero-coupon Treasury created via the STRIPS program, or, in a more complicated way, by using coupon bonds. (Immunization of *real* liabilities using TIPS likely has to be done using coupon bonds because very few TIPS are available stripped.) Such a portfolio is utilizing "cash-flow matching." Moving forward in time, the duration (or maturity) of the liabilities will shrink, and so will the duration (or maturity) of the matched assets. The same shrinking of duration will occur if the approach taken to immunization is to match investor's assets and liabilities in a less precise, easier-to-implement way, "duration matching," as in Brown and Jones (2011). These falling-maturity bond portfolios form an important alternative to the fixed-maturity bond portfolios of this paper when it comes to funding retirement spending, and they may put longer-term bonds in a more positive light, as proposed by Campbell and Viceira (2001).

Appendix: Constructing Total Returns from Yields

To construct total returns we use the following constant-maturity series of annually-compounded yields from the Federal Reserve Bank of St. Louis's economic database FRED mentioned at the start of Section 1: for 3-month bills, TB3MS (quarterly "end of period," 10/1/1954 to 10/1/2017); and for 1-year, 3-year, 5-year, 10-year, and 20-year instruments, we use GS1, GS3, GS5, GS10, and GS20, respectively, all "annual 'end of period' 1/1/1954 to 1/1/2017." (See the Board of Governors of the Federal Reserve System (2018).) For all of these series, the "end of period" is not the last day but the average of all the days in the last month. This is not ideal but series with daily information do not start until 1962; for consistency we will not switch to them for post-1962 results. This means our returns will differ from returns obtained using "December 31 to December 31" or "January 1 to January 1" yields. Also note that the constant-maturity yields are not directly observed in the market but are interpolated by the Treasury from market data, which inevitably adds uncertainty to the figures.

The 20-year series is missing 1987–1992, so for those years we interpolate the 20-year yield using the other maturities and FRED's 30-year yield GS30. To avoid oscillations in the interpolation we use the Steffen Interpolation method, which is a cubic interpolation method guaranteed to be monotonic where the data are monotonic, a property many other polynomial interpolation methods lack.[9]

[9] See `https://mathematica.stackexchange.com/questions/14023/joining-and-interpolating-data-points/14040#14040`; some alternatives are discussed in `https://math`

From these yields, which are on an annually-compounded basis, we need to calculate total returns. Treasuries other than Treasury bills pay semiannual coupons. For those coupon Treasuries let y be the annually-compounded rate of return, c be the semiannual coupon paid, M be the maturity of the bond in years, and 100 be the bond's face value. Present Value "PV" is identical to price. Although the theoretical explanation for the initial price of a bond is

$$PV_0 = \frac{c}{1+y_1/2} + \frac{c}{(1+y_2/2)^2} + \frac{c}{(1+y_3/2)^3} + \cdots + \frac{c}{(1+y_{2M-1}/2)^{2M-1}} + \frac{c+100}{(1+y_{2M}/2)^{2M}}$$

for zero-coupon yields $y_1, y_2, y_3, \ldots, y_{2M}$, it is also true that if y_t is instead defined as the "yield to maturity" (an internal rate of return), which is what the Treasury reports, then by that definition, PV_0 equals

$$PV_0 = \frac{c}{1+\frac{y_0}{2}} + \frac{c}{(1+\frac{y_0}{2})^2} + \frac{c}{(1+\frac{y_0}{2})^3} + \cdots + \frac{c}{(1+\frac{y_0}{2})^{2M-1}} + \frac{c+100}{(1+\frac{y_0}{2})^{2M}}\,.$$

We assume the bond is originally at par, so $c = 100 * (y/2)$. One year later,

$$\begin{aligned}
PV' &= c\,(1+\tfrac{y'}{2}) + c + \frac{c}{(1+\frac{y'}{2})} + \frac{c}{(1+\frac{y'}{2})^2} + \cdots + \frac{c}{(1+\frac{y'}{2})^{2M-3}} \\
&\quad + \frac{c+100}{(1+\frac{y'}{2})^{2M-2}} \\
&= (1+\tfrac{y'}{2})^2 \Bigg\{ \frac{c}{1+\frac{y'}{2}} + \frac{c}{(1+\frac{y'}{2})^2} + \frac{c}{(1+\frac{y'}{2})^3} + \frac{c}{(1+\frac{y'}{2})^4} \\
&\quad + \cdots + \frac{c}{(1+\frac{y'}{2})^{2M-1}} + \frac{c+100}{(1+\frac{y'}{2})^{2M}} \Bigg\} \\
&= (1+\tfrac{y'}{2})^2 \left\{ c \sum_{t=1}^{2M} (1+\tfrac{y'}{2})^{-t} + 100\,(1+\tfrac{y'}{2})^{-2M} \right\} \\
&= (1+\tfrac{y'}{2})^2 \left\{ c\frac{2}{y'} \left[1 - \left(1+\tfrac{y'}{2}\right)^{-2M}\right] + 100\,(1+\tfrac{y'}{2})^{-2M} \right\}
\end{aligned}$$

and using $c = 100y/2$,

$$\frac{PV'}{PV_0} = \frac{PV'}{100} = (1+\tfrac{y'}{2})^2 \left\{ \frac{y_0}{y'} \left[1 - \left(1+\tfrac{y'}{2}\right)^{-2M}\right] + (1+\tfrac{y'}{2})^{-2M} \right\}\,. \tag{2}$$

Since y' is the yield of a bond of maturity $M-1$, we do not have data on it, but we do have data on the yields of bonds of maturities M and less than M at date t', so the yield of a bond of maturity $M-1$ at t' can be interpolated. Again we use Steffen

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`ematica.stackexchange.com/questions/14662/monotone-periodic-1d-interpolation-with-continuous-1st-order-derivative` and in `https://math.stackexchange.com/questions/45218/implementation-of-monotone-cubic-interpolation/51412#51412`. There are other more advanced approaches used in finance to fit the term structure.

interpolation. (A (positive) capital gain results from this "rolling down the yield curve" from maturity M to $M-1$ whenever the yield curve is upward-sloping unless it has shifted upwards considerably in the intervening year; an advantage of our method, as opposed to approximating the yield of a bond of maturity $M-1$ with the yield of a bond of maturity M, is that it captures that roll-down return.) Total return can be expressed either using an annually compounded rate "r_a" or continuously compounded rate ("logarithmic returns") "r_c," where equation (2)'s $PV'/PV_0 = 1 + r_a = e^{r_c}$; we use the continuously-compounded return from now on unless otherwise specified.

One-year and three-month Treasury bills are zero-coupon securities. The annual return of one-year bills is simply the initial yield. The annual return of three-month bills is the sum of the four initial continuously-compounded quarterly yields.

To calculate real continuously-compounded returns "r_r" from nominal continuously-compounded returns "r_n" given the levels of a price index I, we have

$$\begin{aligned} e^{r_r} &= \frac{PV'/I'}{PV_0/I_0} = \frac{PV'}{PV_0} \cdot \frac{I_0}{I'} = e^{r_n} \cdot \frac{I_0}{I'} \quad \text{so} \\ r_r &= r_n + \ln(I_0/I')\,. \end{aligned}$$

The price index we use from FRED is CPIAUCSL, the "Consumer Price Index for All Urban Consumers" (see Bureau of Labor Statistics (2018)).

Calculating real yields in this way means using the second of the two approaches described by Girola (2005, page 8): "There are many different ways to convert a nominal interest rate to real. Perhaps the best-known approach is to subtract a distributed lag on inflation from the nominal rate. The lag represents adaptive inflation expectations and the resulting real rate represents the expected real rate." He then describes the second approach, which is "for each year the real interest rate is derived from... the price indexes for that year and the following year." He defends the second approach because it "shows the actual real earnings that were realized from holding the bond, while the use of lags is an estimate of the expected real earnings," and notes that the second approach is the one used in the 2005 Social Security Administration's Trustees Report.

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Not-to-be-published Appendix for Referee

The following *Mathematica* code gives the analytical solution for w when $T = 10$. The analytical solution given in the paper generalizes this.

```
In[1]:= x[1] := 100;
        x[t_] := Exp[r[t - 1]] * x[t - 1] - w;

In[3]:= YearsOfPayments = 10;

In[4]:= Solve[x[YearsOfPayments + 1] == 0, w]
```

Out[4]= $\left\{\left\{w \to \left(100\, e^{r[1]+r[2]+r[3]+r[4]+r[5]+r[6]+r[7]+r[8]+r[9]+r[10]}\right) \big/ \left(1 + e^{r[10]} + e^{r[9]+r[10]} + e^{r[8]+r[9]+r[10]} + e^{r[7]+r[8]+r[9]+r[10]} + e^{r[6]+r[7]+r[8]+r[9]+r[10]} + e^{r[5]+r[6]+r[7]+r[8]+r[9]+r[10]} + e^{r[4]+r[5]+r[6]+r[7]+r[8]+r[9]+r[10]} + e^{r[3]+r[4]+r[5]+r[6]+r[7]+r[8]+r[9]+r[10]} + e^{r[2]+r[3]+r[4]+r[5]+r[6]+r[7]+r[8]+r[9]+r[10]}\right)\right\}\right\}$