Non-monetary Valuation and the Perils of Potential Pareto/Kaldor-Hicks/Cost-Benefit Analysis Decision-Making*

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Abstract. Extending "marginal rate of substitution" to non-marginal changes, we develop a theory of non-monetary valuation of commodities. Value defined this way is fundamentally binary, analogous to compensating and equivalent variation/surplus, or willingness to pay and accept. After treating existence, we investigate Kaldor-Hicks decision-making using non-monetary valuation. Simple indifference curve maps show that it has intransitivities similar to those arising from monetary valuation. The Kaldor criterion can result in impoverishment of one consumer in an Edgeworth Box, and the Hicks criterion, impoverishment of the other. The Hicks criterion can endorse deviations from Pareto optimal allocations.

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JEL Codes: JEL codes

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Other than the marginal rate of substitution, the analysis of economic valuation up to now has only been conducted in an environment with prices. The resulting theory is somewhat complicated, and well understood only by experts such as discussed by Blackorby and Donaldson (1990). Undergraduates are still taught that consumer surplus measures value, graduating without knowing that value has a binary nature. Antitrust economists use consumer surplus without apology, despite efforts such as Hausman's (1981) paper. One Nobel laureate, Angus Deaton, unambiguously pronounces that "there is no valid theoretical or practical reason for ever integrating under a Marshallian demand curve,"[1] while another Nobel laureate, Jean Tirole (1988 pp. 30–1), writes in the introduction to his Ph.D.-level Industrial Organization textbook that "it may be appropriate to assume that demand is downward sloping and that the consumer surplus is a good approximation of welfare," then goes on that basis for the remainder of his book. Environmental economists often still think that a rational consumer's "willingness to accept" should equal or come close to their "willingness to pay,"[2] notwithstanding Hanemann (1991), and defenders of cost-benefit analysis ("CBA")—which U.S. federal regulatory agencies are often legally mandated to use—make decisions assuming "a dollar is a dollar" regardless of who owns it, decisions which welfare economics theorists Hammond and Fleurbaey (2010 p. 15) tell us "lack scientific content." Hammond and Fleurbaey also say, about some decisions made on the basis of cost-benefit analysis, that "most people—especially non-economists—also find them totally unacceptable from an ethical point of view." Nevertheless, standard "law and economics" textbooks teach future legal practitioners that economic justice resides exclusively in that Kaldor-Hicks, cost-benefit approach.

The goal in this paper is to shed light on these issues by studying valuation, and also the many problems with Kaldor-Hicks, "Potential Pareto," "cost-benefit analysis" decision-making, in a radically simpler framework: a framework without prices. Our inspiration is the marginal rate of substitution, the elementary idea of valuing a change in one good only using a consumer's indifference curve, using some other good, rather than money,

[1]Deaton is quoted in Becht (1995 p. 77).

[2]Tietenberg Lewis (2018 p. 81): "Economic theory suggests that differences between WTP and WTA should be small, but experimental findings both in environmental economics and in other microeconomic studies have found large differences. Why? Some economists have attributed the discrepancy to a psychological endowment effect. . . ."

as the numéraire, thus bypassing the entire issue of consumer choice under a price system. We do not venture beyond this simple setting; it is rich enough to illuminate, at least in its own way, most of valuations's most controversial issues. We will not even need to construct the utility possibility curves fundamental to Gorman's (1955) analysis of Scitovsky's (1941) critique of the Kaldor-Hicks criteria.

Addressing valuation the usual way, using a price system, has a long history. The first answer (for example by Jean Baptiste Say) to the question of how to find a cardinal measure of how much a consumer values a commodity was simply the consumer's expenditure on the commodity, but the Paradox of Value (the "Diamond-Water Paradox") shows that answer to be inadequate: consumers value at least some units of water very highly because water is a requirement for life, whereas diamonds are mostly for adornment, yet consumers have higher expenditures on diamonds than on water. An improved answer, consumer surplus, was first proposed by Dupuit in 1844, and was well developed by the time of Alfred Marshall's *Principles*.[3] However, problems with the consumer surplus answer were raised as early as Bordas in 1847.[4] Marshall (1920) understood these problems, but downplayed them, writing, "In regard to different people allowance may have to be made where necessary for differences of sensibility and for differences of wealth: but it is seldom needed in considering large groups of people."[5] The standard modern answers—note the plural—to the valuation question, as it pertains to *price* changes, are Kaldor's "compensating variation" (Kaldor 1939 p. 550; Hicks 1946 Appendix to Ch. 2, p. 40) and Hicks's "equivalent variation" (Hicks, op. cit., Additional Note A, 3); however, the basic question for this paper is instead one of valuing *quantity* changes. Environmental economists have studied valuation of exogenous quantity changes; they call those values "compen-

[3]For Dupuit see Houghton (1958). In some sense, though, the idea goes back to Aristotle: Daly (1991 p. 186) reminds us that "Marx, and Aristotle before him, pointed out that the danger of money fetishism arises when society shifts its focus from use value to exchange value." Aristotle's distinction is between *oikonomia* and *chrematistike* in *The Nichomachean Ethics* Book I Chapter 5 and in *Politics* Book I Parts VIII, IX, and X. Diamonds and water were used by Adam Smith in his discussion of the difference between "value in use" and "value in exchange" in Book I Chapter IV of *The Wealth of Nations.*

[4]Houghton (1958 p. 51).

[5]Marshall (1920), margin notes for pages 130 and 131, Book III ("On Wants and their Satisfaction") Chapter VI ("Value and Utility") Section 3.

sating surplus" and "equivalent surplus," though we will show that that is not the way those terms were originally defined by Hicks (1943). And even in environmental economics treatments such as Hanemann (1991), Hanemann (1999), Kuriyama and Takeuchi (2001), and Hanley, Shogren, and White (2007 pp. 325–6), the quantity change is studied in a context where the consumer has a money income and faces prices for all commodities except for one or more "public goods," whose quantity change is the object of study. Obviously, most consumers do find themselves embedded in a price system for many goods (though not for public goods or externalities), and these other authors' analyses of value assuming prices are certainly useful. This paper supplies a simpler foundation to supplement their more sophisticated analyses (sophisticated in that, for example, they are often based on expenditure functions, which do not exist in a non-monetary model).

Besides its simplicity, non-monetary valuation has another advantage: it strongly discourages the "same yardstick" numéraire fallacy embodied in such incorrect statements as: "it should be emphasized that pure transfers of purchasing power from one household or firm to another *per se* should be typically attributed no value" (Boadway 2020 p. 86); see Ellerman (2014) for an explanation. In this area of microeconomics, money is a veil, which non-monetary valuation dispenses with.

Since we will not use prices, we will not be able to say anything about demand curves, nor, therefore, about consumer surplus, but we will say a great deal about the gap between compensating and equivalent measures of value—illustrating our prior remark that this model can illuminate the important issues "in its own way.".

Section 1 has basic definitions; Section 2 gives valuation results for small changes; Section 3 shows how rational agents' valuation of gains differs from their valuation of losses; Section 4 derives results for the very special case of quasilinear utility; Section 5 studies when the valuation measures fail to exist; Section 6 shows that familiar problems with Kaldor and Hicks project evaluation also occur with non-monetary valuation; Section 7 gives an example where the non-monetary Hicks test endorses a move from a Pareto Optimal point to a non-Pareto Optimal point; and Section 8 gives an example of an Edgeworth Box where making two applications of the Kaldor test endorses confiscating all goods from one agent, while in the same Box, making two applications of the Hicks test endorses confiscating almost all goods from the other agent.

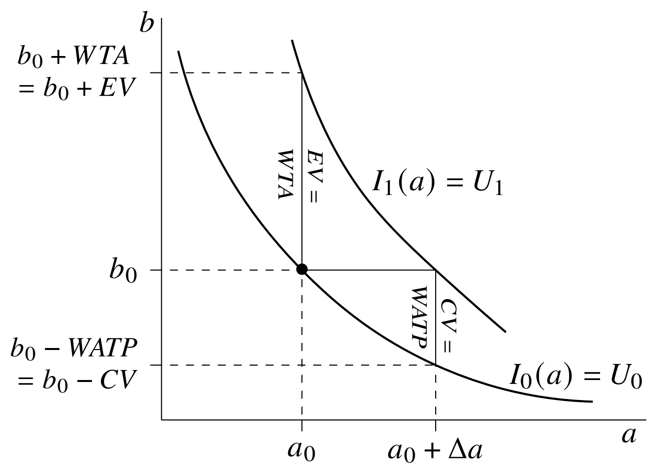

Figure 1. The two values of a policy of $\Delta a > 0$.

1. Definitions

Let $U(a, b)$ be the utility function defined over apples a and bananas b. Throughout the paper, we value a change in apples, Δa, in terms of bananas. If, instead, the consumer's utility depended on $n > 2$ goods, then rather than think of b as being either the numéraire or a composite commodity, one could think of the value of Δa as being an '$n-1$'-dimensional vector, showing Δa's value in terms of each of the other goods. Brekke (1997) points out that in cost-benefit analysis the choice of numéraire is not neutral: "the choice of numéraire is very important to the sign of the sum of net benefits," and "the less valuable the numéraire is to a person, the higher the number required to express his net benefit, and the more will his interest weigh in the total sum. The choice of money as numéraire is systematically favourable to those who value money the least, relative to alternative numéraires" (pp. 117, 118). So in a many-commodity world, it is better not to choose a numéraire, but to think of value as a vector.

Figure 1 illustrates the case of $\Delta a > 0$, with an initial position of (a_0, b_0). The "compensating" measure of the value of Δa presumes the policy *is* carried out, and asks how many bananas the consumer would be willing to give up in exchange. Traditionally, this is called "willingness to pay," but it clearly must be less than or equal to the consumer's initial holdings of bananas, so it is preferable to call it "willingness and ability to pay," to avoid giving the impression that, for example, impoverished African-Americans living in the polluted "Cancer Alley" between Baton Rouge and New Orleans, Louisiana, live there just because they are psychologically predisposed not to care about pollution, when in actuality they live there because of their lack of wealth.[6] Hence the figure's *WATP* label. The next question of terminology is what alternative label containing the word "compensating" to put on Figure 1's *WATP*. "Non-monetary compensating variation" and "non-monetary compensating surplus" suggest themselves: "non-monetary" borrowed from Kuriyama and Takeuchi (2001); "compensating variation" being a widely-used term; and "compensating surplus" referring to a situation where a quantity is not under the consumer's complete control, although the nature of that lack of control differs in Hicks's compensating surplus and in the environmental economics literature's compensating surplus, as alluded to above. "Non-monetary compensating measure of value" would be a neutral possibility. Easiest for the general reader is "compensating variation," so we use it, conceding that environmental economists would find "compensating surplus" more copacetic.[7]

The "equivalent" measure of the value of Δa presumes the policy *is not* carried out, and asks how many bananas the consumer would be willing to accept in exchange for not getting the extra $\Delta a > 0$. Figure 1 labels this *WTA*, for "willing to accept," and *EV* for (non-monetary) "equivalent

[6]See "USA: Environmental racism in "Cancer Alley" must end—experts," United Nations Office of the High Commissioner for Human Rights, 2 March 2021, at `https://www.ohchr.org/EN/NewsEvents/Pages/DisplayNews.aspx?NewsID=26824&LangID=E`.

[7]Hicks never considered exogenous quantity changes. Hicks used "compensating surplus" and "equivalent surplus" in the context of exogenous price changes when goods were lumpy, or, as Brookshire, Randall, and Stoll (1980 p. 480) put it, when the consumer is prohibited from making optimizing adjustments in his consumption. Hicks used "compensating variation" and "equivalent variation" in the context of exogenous price changes when goods were not lumpy or the consumer faced no quantity restrictions. Even Randall and Stoll (1980 p. 452) acknowledge that when the exogenous change is in quantity rather than in price, "the Hicksian compensating and equivalent measures in commodity space are *analogous* to the Hicksian surpluses, not the variations, defined over price changes"—my emphasis, because those measures are not present in Hicks's paper. Tellingly, Randall and Stoll, probably wisely, choose evasion as the best way to handle the terminology quandary: they use "C" and "E" rather than saying anything more about 'variations' and 'surpluses.' Our paper is even further away from Hicks than Randall and Stoll, because we are not using money, so we do not find the fact that our measure is an analog of Randall and Stoll's analog of Hicks's "compensating surplus" to be dispositive. (See also the discussion of Hicks's definitions in Currie, Murphy, and Schmitz (1971 p. 746.)

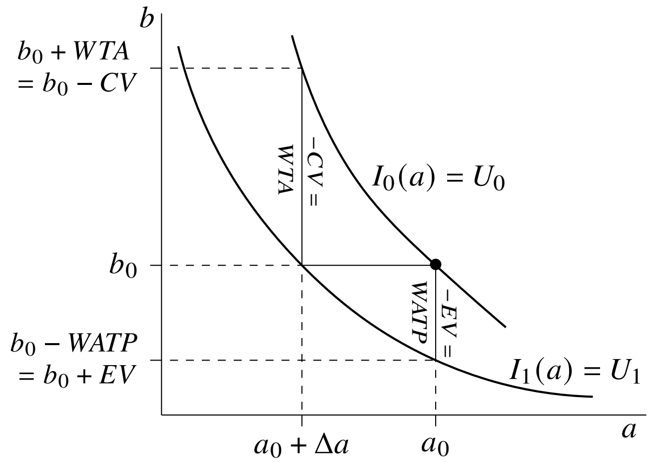

Figure 2. A policy of $\Delta a < 0$.

variation."[8] Formally, when $\Delta a > 0$, *WATP* and *CV* are defined by

$$U(a_0{+}\Delta a, b_0{-}WATP) = U(a_0, b_0) \quad \text{and} \tag{1}$$
$$U(a_0{+}\Delta a, b_0{-}CV) = U(a_0, b_0) \tag{2}$$

and *WTA* and *EV* are defined by

$$U(a_0, b_0{+}WTA) = U(a_0{+}\Delta a, b_0) \quad \text{and} \tag{3}$$
$$U(a_0, b_0{+}EV) = U(a_0{+}\Delta a, b_0) \tag{4}$$.

Figure 2 illustrates the case of $\Delta a > 0$, with initial position of (a_0, b_0) (not the same a_0 as in Figure 1). The "compensating" measure of the value of Δa presumes the policy *is* carried out, and asks how many bananas the consumer would be willing to accept in compensation for the loss of applies; hence the figure's *WTA* and $-CV$, because by convention $CV < 0$ for policies which would cause utility to fall. The "equivalent" measure of the value of Δa presumes the policy *is not* carried out, and asks how many bananas the consumer would be willing and able to pay in exchange for not being subjected to $\Delta a < 0$; hence the figure's *WATP* and $-EV$, because by

[8]We keep the traditional "willingness to accept" term even though *WTA* depends on wealth because wealth does not form a fixed constraint on *WTA* as it does with *WATP*.

	Gains	Losses
CV	$WATP$	$-WTA$
EV	WTA	$-WATP$

	Gains	Losses
$WATP$	CV	$-EV$
WTA	EV	$-CV$

Table 1. Relationships between a policy's *WATP* and *WTA*, and *CV* and *EV*, depending on whether the policy results in a gain or in a loss of utility.

convention $EV < 0$ for policies which would cause utility to fall. Formally, when $\Delta a < 0$, *WTA* and *CV* are defined by

$$U(a_0{+}\Delta a, b_0{+}WTA) = U(a_0, b_0) \quad \text{and} \tag{5}$$
$$U(a_0{+}\Delta a, b_0{-}CV) = U(a_0, b_0) \tag{2}$$

and *WATP* and *EV* are defined by

$$U(a_0, b_0{-}WATP) = U(a_0{+}\Delta a, b_0) \quad \text{and} \tag{6}$$
$$U(a_0, b_0{+}EV) = U(a_0{+}\Delta a, b_0)\,. \tag{4}$$

Clearly Table 1's translation tables hold,[9] and the choice of whether to use *WATP* and *WTA*, or *CV* and *EV*, can be made according to whichever pair is more convenient for the particular situation. Equations (1) and (6) show that the definition of *WATP* is not the same for gains at it is for losses, and equations (3) and (5) show that the definition of *WTA* is not the same for gains at it is for losses, but for both gains and losses, the definition for *CV* is (2) and the definition for *EV* is (4).

In order to prove Table 2, Figure 2's $a_0{+}\Delta a$ was constructed to be equal to Figure 1's a_0, and Figure 2's a_0 was constructed to be equal to Figure 1's $a_0{+}\Delta a$. If $a' > a$, Table 2 directly follows. One can show that Table 2 is also true if $a' < a$.

In an n-commodity world, non-monetary value is best thought of as a $2\times n$-dimensional matrix, where the rows are *CV* and *EV* and the columns are the other goods in which *CV* and *EV* are expressed. However, we continue with $n = 2$.

[9] As they do for the analogous concepts in monetary valuation models of exogenous quantity changes, which are the EG and BA, rather than the EF and BC, of Figure 1 of Randall and Stoll (1980).

a_0 to a'	a' to a_0
$CV =$	$-EV$
$WATP =$	$WATP$
$EV =$	$-CV$
$WTA =$	WTA

Table 2. Valuations of a policy and of the reverse of that policy.

2. Small Changes

Let U'_a be an abbreviation for $\partial U/\partial a$ evaluated at (a_0, b_0), and let U'_b be an abbreviation for $\partial U/\partial b$ evaluated at (a_0, b_0), unless some other evaluation point is explicitly stated. Denote the marginal rate of substitution of a for b at (a_0, b_0), which is $U'_a/U'_b > 0$, by MRS_0. Let the indifference curve passing through (a_0, b_0), considered as a function of a, be $I_0(a)$. Similarly, let the indifference curve passing through $(a_0{+}\Delta a, b_0)$, considered as a function of a, be $I_1(a)$, and let the marginal rate of substitution of a for b at $(a_0{+}\Delta a, b_0)$ be denoted by MRS_1.

We have

Proposition 1. *To a second-order approximation, the willingness and ability to pay for a change $\Delta a > 0$ in apples is*

$$WATP \approx MRS_0 \cdot \Delta a - \tfrac{1}{2} I''_0 \cdot (\Delta a)^2 \,, \tag{7}$$

and the willingness to accept forgoing that change in apples is

$$WTA \approx MRS_1 \cdot \Delta a + \tfrac{1}{2} I''_1 \cdot (\Delta a)^2 \tag{8}$$

where

$$MRS_{0 \text{ or } 1} = \frac{U'_a}{U'_b} \quad \text{and} \quad I''_{0 \text{ or } 1}(a) = \frac{U'_a U''_{ab} - U''_a U'_b}{{U'_b}^2} \tag{9}$$

and the derivatives in (9) are evaluated at (a_0, b_0) and at a_0 for MRS_0 and I_0, respectively, and at $(a_0{+}\Delta a, b_0)$ and $a_0{+}\Delta a$ for MRS_1 and I_1, respectively.

If $\Delta a < 0$, the corresponding results are

$$WATP \approx -MRS_1 \cdot \Delta a - \tfrac{1}{2} I''_1 \cdot (\Delta a)^2 \tag{10}$$

and

$$WTA \approx -MRS_0 \cdot \Delta a + \tfrac{1}{2} I''_0 \cdot (\Delta a)^2 \,. \tag{11}$$

Proof. Denote the second-order Taylor Series approximation of I_0 around a_0 by

$$I_{02}(a) = I_0(a_0) + I_0'(a_0)(a - a_0) + \tfrac{1}{2} I_0''(a_0)(a - a_0)^2 \,.$$

We have $I_0(a_0) = b_0$. Also, $I_0'(a_0) = -MRS_0 = -U_a'/U_b'$, so

$$I_{02}(a) = b_0 - MRS_0 \cdot (a - a_0) + \tfrac{1}{2} I_0''(a_0)(a - a_0)^2 \,.$$

From Figure 1,

$$\begin{aligned} WATP &= b_0 - I_0(a_0{+}\Delta a) \\ &\approx b_0 - I_{02}(a_0{+}\Delta a) \\ &= b_0 - \left[b_0 - MRS_0 \cdot (a_0{+}\Delta a - a_0) + \tfrac{1}{2} I_0''(a_0) \cdot (a_0{+}\Delta a - a_0)^2 \right], \end{aligned}$$

leading to (7). Similarly, from Figure 2, $WTA = I_0(a_0{+}\Delta a) - b_0 \approx I_{02}(a_0{+}\Delta a) - b_0$, leading to (11).

To prove (8), denote the second-order Taylor Series approximation of I_1 around $a_0{+}\Delta a$ by

$$I_{12}(a) = I_1(a_0{+}\Delta a) + I_1'(a_0{+}\Delta a)\big(a - (a_0{+}\Delta a)\big) + \tfrac{1}{2} I_1''(a_0{+}\Delta a)\big(a - (a_0{+}\Delta a)\big)^2 \,.$$

We have $I_0(a_0{+}\Delta a) = b_0$. Also, $I_0'(a_0{+}\Delta a) = -MRS_1 = -U_a'(a_0{+}\Delta a, b_0)/U_b'(a_0{+}\Delta a, b_0)$, so

$$I_{12}(a) = b_0 - MRS_1 \cdot \big(a - (a_0{+}\Delta a)\big) + \tfrac{1}{2} I_1''(a_0{+}\Delta a)\big(a - (a_0{+}\Delta a)\big)^2 \,.$$

From Figure 1,

$$\begin{aligned} WTA &= I_1(a_0) - b_0 \\ &\approx I_{12}(a_0) - b_0 \\ &= \left[b_0 - MRS_1 \cdot \big(a_0 - (a_0{+}\Delta a)\big) + \tfrac{1}{2} I_1''(a_0{+}\Delta a)\big(a_0 - (a_0{+}\Delta a)\big)^2 \right] - b_0 \,, \end{aligned}$$

leading to (8). Similarly, from Figure 2, $WATP = b_0 - I_1(a_0) \approx b_0 - I_{12}(a_0)$, leading to (10).

To confirm (9), use the fact that $I' = -MRS$:

$$I''(a) = \frac{d}{da} I'(a) = \frac{\partial}{\partial a}\left[-\frac{U_a'(a,b)}{U_b'(a,b)} \right] = -\frac{U_{aa}''(a,b)}{U_b'(a,b)} + \frac{U_a'(a,b)\, U_{ab}''(a,b)}{[U_b'(a,b)]^2} \,. \tag{12}$$

∎

Corollary. *To a second-order approximation, the non-monetary measures of the values of* Δa *are*

$$CV \approx MRS_0 \cdot \Delta a - \tfrac{1}{2} I_0'' \cdot (\Delta a)^2 \quad \textit{and} \tag{7}$$

$$EV \approx MRS_1 \cdot \Delta a + \tfrac{1}{2} I_1'' \cdot (\Delta a)^2 \tag{8}$$

regardless of the sign of Δa.

Proof. Use Table 1. ▮

Intuitively, Proposition 1 suggests that the gap between *WTA* and *WATP* as larger when the indifference curve is sharply curved (I'') and when the proposed change in apples (Δa) is large. In Hanemann (1991, (17)) and Kuriyama and Takeuchi (2001, (15) and (16)), the gap between *WTA* and *WATP* is related to the elasticity of substitution and the income elasticity of demand, which has a similar flavor to Proposition 1, although de La Grandville (1997) shows that curvature and elasticity of substitution are not as related as is often thought.

It is difficult to compare *WATP* and *WTA* in Proposition 1 because of the differences between MRS_0 and MRS_1, and between I_0'' and I_1''. To remedy this, the following result, which starts from (2) and (4) instead of from the indifference curves, approximates everything at (a_0, b_0), instead of approximating some things at (a_0, b_0) and other things at $(a_0{+}\Delta a, b_0)$. This result's second-order approximations for *CV* and *EV* can therefore be directly compared to each other. Unfortunately, they have none of the easy geometric appeal of Proposition 1, and numerical calculations are required to determine which approximation is the larger of the two. To first order, on the other hand, the proposition is successful in showing that *CV* and *EV* are equal, and proportional to the marginal rate of substitution. The relationship with the marginal rate of substitution recalls the introduction's remarks about *MRS* being the inspiration for the non-monetary approach to valuation.

Proposition 2. *The non-monetary compensating variation for a change* Δa *in apples satisfies to a second-order approximation*

$$0 \approx \tfrac{1}{2} U_{bb}'' CV^2 - [U_b' + U_{ab}''\Delta a] CV + [U_a'\Delta a + \tfrac{1}{2} U_{aa}''(\Delta a)^2] \tag{13}$$

and to a first-order approximation

$$CV \approx \frac{U_a'}{U_b'} \Delta a = MRS_0 \, \Delta a \, . \tag{14}$$

The non-monetary equivalent variation for that change in apples satisfies to a second-order approximation

$$0 \approx \tfrac{1}{2}U''_{bb}EV^2 + U'_bEV - [U'_a\Delta a + \tfrac{1}{2}U''_{aa}(\Delta a)^2] \tag{15}$$

and to a first-order approximation

$$EV \approx \frac{U'_a}{U'_b}\Delta a = MRS_0\,\Delta a\,. \tag{16}$$

Proof. The second-order Taylor Series expansion for $U(a,b)$ around (a_0,b_0) is

$$U(a,b) \approx U_0 + \nabla U\cdot(a-a_0,b-b_0) + \tfrac{1}{2}(a-a_0,b-b_0)\nabla^2U\cdot(a-a_0,b-b_0)^T$$

where ∇U and $\nabla^2 U$ are evaluated at (a_0,b_0); assume that all other derivatives are also evaluated there unless otherwise explicitly indicated. The left-hand side of (2) is

$$\begin{aligned}
U(a_0{+}\Delta a, b_0{-}CV) &\approx U(a_0,b_0)\\
&\quad + (U'_a,U'_b)\cdot(\Delta a,-CV) + \tfrac{1}{2}(\Delta a,-CV)\begin{pmatrix}U''_{aa} & U''_{ab}\\ U''_{ba} & U''_{bb}\end{pmatrix}\begin{pmatrix}\Delta a\\ -CV\end{pmatrix}\\
&= U_0 + U'_a\Delta a - U'_bCV\\
&\qquad + \tfrac{1}{2}\left[U''_{aa}(\Delta a)^2 - CV\cdot U''_{ab}(\Delta a) - CV\cdot U''_{ba}(\Delta a) + U''_{bb}CV^2\right]\\
&= U_0 + U'_a\Delta a - U'_bCV\\
&\qquad + \tfrac{1}{2}\left[U''_{aa}(\Delta a)^2 - 2CV\cdot U''_{ab}(\Delta a) + U''_{bb}CV^2\right]\\
&= U_0 + U'_a\Delta a - U'_bCV\\
&\qquad + \tfrac{1}{2}U''_{aa}(\Delta a)^2 - CV\cdot U''_{ab}(\Delta a) + \tfrac{1}{2}U''_{bb}CV^2\\
&= \tfrac{1}{2}U''_{bb}CV^2 - [U'_b + U''_{ab}\Delta a]CV + [U_0 + U'_a\Delta a + \tfrac{1}{2}U''_{aa}(\Delta a)^2]\,.
\end{aligned}$$

Equating this to (2)'s right-hand side yields (13). A first-order approximation can be obtained by setting all second derivatives of the utility function equal to zero: $0 = -U'_bCV + U'_a\Delta a$, or (14).

The left-hand side of (4) is

$$\begin{aligned}
U(a_0, b_0{+}EV) &\approx U(a_0,b_0)\\
&\quad + (U'_a,U'_b)\cdot(0,EV) + \tfrac{1}{2}(0,EV)\begin{pmatrix}U''_{aa} & U''_{ab}\\ U''_{ba} & U''_{bb}\end{pmatrix}\begin{pmatrix}0\\ EV\end{pmatrix}\\
&= U_0 + U'_bEV + \tfrac{1}{2}U''_{bb}EV^2\,.
\end{aligned}$$

The right-hand side of (4) is

$$\begin{aligned} U(a_0+\Delta a, b_0) &\approx U(a_0, b_0) \\ &+ (U'_a, U'_b) \cdot (\Delta a, 0) + \tfrac{1}{2}(\Delta a, 0) \begin{pmatrix} U''_{aa} & U''_{ab} \\ U''_{ba} & U''_{bb} \end{pmatrix} \begin{pmatrix} \Delta a \\ 0 \end{pmatrix} \\ &= U_0 + U'_a \Delta a + \tfrac{1}{2} U''_{aa} (\Delta a)^2 \, . \end{aligned}$$

Equating (4)'s left-hand and right-hand sides yields (15). A first-order approximation can be obtained by setting all second derivatives of the utility function equal to zero: $0 = U'_b EV - U'_a \Delta a$, or (16). ∎

The results of Proposition 2 can be expressed in terms of *WATP* and *WTA* using Table 1.

3. Valuing Gains versus Losses

List (2004) experimentally studies non-monetary valuation, with coffee mugs and candy bars taking the place of our apples and bananas. He states that neoclassical theory is different from Prospect Theory; that he uses Prospect Theory and the "endowment effect" interchangeably; and that the endowment effect—which is also referred to as "loss aversion" or "status quo bias"—occurs when "a good's value increases once it becomes part of an individual's endowment." List believes that in neoclassical theory, a good's value would not change once it became part of an individual's endowment. However, from Table 2, it is neoclassically completely rational for the *WATP* for an increase from a_0 to $a' > a_0$ to be different from the *WTA* compensation for the reverse movement from a' back to a_0. (Table 2 is silent on whether "different" means "smaller.") Furthermore, the *WATP* for an increase in a, and the *WTA* for a decrease in a, are both *CV* measures, which are straightforward to experimentally test, and which therefore are the value measures mostly likely to be revealed in an experiment. Experimentally determining *EV* measures would be less direct, because one would have to propose an action and then obtain the subjects' valuation for the experimenter *not* to perform that action. Yet to test for rationality, one has to measure at least one *EV*. From Table 2, what would not be consistent with rationality would be for the *WATP* = *CV* for an increase from a_0 to $a' > a_0$ to be different from the *WATP* = −*EV* to avoid the reverse movement from a' back to a_0. From Table 2, it would also not be consistent with rationality for the *WTA* = *EV* compensation for not moving from a_0

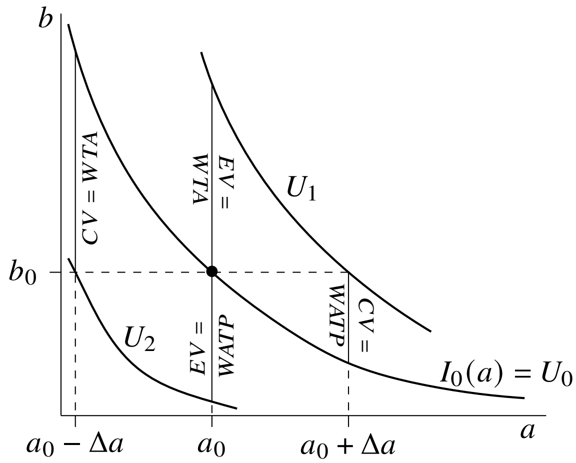

Figure 3. Labels rotated clockwise: the two values of a policy of increasing a. Labels rotated counterclockwise: the two values of a policy of decreasing a.

to a' to differ from the $WTA = -CV$ compensation for moving from a' to a_0 (although if one really did not move from a_0 to a', it would be impossible to move from a' back to a_0).

Table 2, and indeed most of this paper, is concerned with valuing a single policy, moving from a_0 to $a_0 + \Delta a$. However, Table 2 is silent on contrasting the value of two policies. As Hanemann (1999 p. 66 and 1991 note 25) has pointed out, in the context of discussing the endowment effect, a particularly interesting pair of policies to contrast is: one policy increases a and the other policy decreases a by the same amount. We study this policy pair for the rest of this section. Each of these policies, as shown in Figure 3, has two values. Behavioral economists have not been very interested in the *EV* notion of value, so the "gain versus loss" comparison of interest to them which is missing from Table 2 is: the $CV = WATP$ of a gain in a versus the $CV = WTA$ of a loss in a.

We have:

Proposition 3. *Suppose the indifference curve $I_0(a) = U_0$ is a strictly convex function of a, and suppose $\Delta a > 0$. Then the $CV = WATP$ of a policy of moving from a_0 to $a_0 + \Delta a$ is strictly smaller than the $CV = WTA$ of a policy of moving from a_0 to $a_0 - \Delta a$.*

Proof. First consider the policy of moving to $a_0 - \Delta a$. The *WTA* for this policy is $I_0(a_0 - \Delta a) - b_0 = I_0(a_0 - \Delta a) - I_0(a_0)$. By the Mean Value Theorem, there exists a $\delta \in (a_0 - \Delta a, a_0)$ such that $I_0'(\delta) = \big(I_0(a_0 - \Delta a) - I_0(a_0)\big)/(a_0 - \Delta a - a_0) = -WTA/\Delta a$. Therefore, the *WTA* for this policy is $-I_0'(\delta)\,\Delta a$.

Next consider the policy of moving to $a_0 + \Delta a$. The *WATP* for this policy is $b_0 - I_0(a_0 + \Delta a) = I_0(a_0) - I_0(a_0 + \Delta a)$. Again by the Mean Value Theorem, there exists a $\gamma \in (a_0, a_0 + \Delta a)$ such that $I_0'(\gamma) = \big(I_0(a_0) - I_0(a_0 + \Delta a)\big)/(a_0 - (a_0 + \Delta a)) = -WATP/\Delta a$. Therefore, the *WATP* for this policy is $-I_0'(\gamma)\,\Delta a$.

The difference between these two *CV*'s is the *WTA* of the harmful policy minus the *WATP* of the beneficial policy, which is $-I_0'(\delta)\,\Delta a - (-I_0'(\gamma)\,\Delta a) = [I_0'(\gamma) - I_0'(\delta)]\Delta a$. This is strictly greater than zero because $\gamma > \delta$ and $I_0'(a)$ is an increasing function because I_0 is strictly convex. ▮

A related result is that no quasiconcave utility function will have the property that "the *CV* value of a gain of $\Delta a > 0$ is strictly greater than the *CV* value of a loss of Δa."

In conclusion, a neoclassical consumer with a strictly convex indifference curve would *CV*-value a loss from a_0 more highly than they would *CV*-value a gain from a_0 of the same absolute amount of a.

4. Quasilinear Utility

Figure 4 illustrates the case when the utility function has the quasilinear form $U(a, b) = a + \hat{u}(b)$ for a function $\hat{u}$. In this case, indifference curve U' is obtained by shifting U_0 to the right, as shown in Figure 4. With this quasilinear utility function, the price-taking consumer's first-order condition for optimal b is $\hat{u}(b^*) = p_b/p_a$ for prices p_a (apples) and p_b (bananas), so the income elasticity of demand for b is exactly zero. That is clearly a knife-edge, not particularly interesting case, but it is mentioned often in the monetary valuation literature because with this utility function, the monetary *WATP* for a fall in the price of b is equal to the monetary *WTA* for a fall in the price of b. To illustrate, in Figure 4 suppose the initial position is A and the final position after the fall in the price of b is C. The monetary $CV = WATP$ is the value of BC. Note that BC is also equal to the income effect of the fall in the price of b. The monetary $EV = WTA$ is the value of AD. (Note that AD would be equal to the income effect of a rise in the price of b if the initial position were C and the final position were A). Because U' is just a shifted version of U, the length of AD (Δa_a) is equal

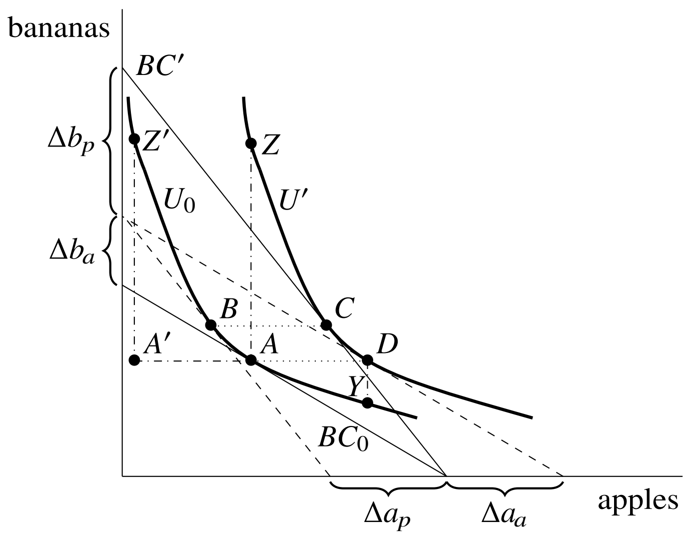

Figure 4. Monetary and non-monetary valuation in the very special case of quasilinear utility. The a subscript denotes *WTA* and the p subscript denotes *WATP*.

to the length of BC (Δa_p). Since the price of apples is unchanging, there is no ambiguity about stating that the monetary value of AD is equal to the monetary value of BC, proving that monetary CV is equal to monetary EV.[10]

These monetary valuations of a change in the price of b cannot be literally compared to non-monetary valuations, which never concern price changes. What we can say is Proposition 4. As usual in this paper, the valuation measures are non-monetary unless explicitly stated otherwise.

Proposition 4. *Suppose utility takes the quasilinear form $U(a, b) = a+\hat{u}(b)$ for a function $\hat{u}$. Also suppose that the policy is an increase in b. Then the policy's CV = WATP is equal to its EV = WTA.*

If instead the policy is an increase in a, suppose that the indifference curve $I_0(a) = U_0$ is a strictly convex function of a. Then that policy's CV = WATP is strictly smaller than that policy's EV = WTA.

[10]Quasilinearity of the utility function, with the form $U = a+\hat{u}\mathbf{b}$, is a sufficient condition for the CV of a change in the price of one of the components of $\mathbf{b}$ to equal the EV of that change, but related work by Chipman and Tian (2012) on the Coase Theorem suggests that the necessary condition is weaker "yet still restrictive" (p. 310)—restrictive because after all, it has to generate a vertical or horizontal income expansion path.

Proof. First assertion: suppose the consequence of the policy is to shift from an initial position of A to a final position of Z. Then $CV = WATP$ is $Z'Z$, and $EV = WTA$ is AD. These both equal the constant horizontal shift between U_0 and U'.

Second assertion: suppose the consequence of the policy is to shift from an initial position of A to a final position of D. Then $CV = WATP$ is DY, and $EV = WTA$ is AZ. Since U' is a horizontal shift of U, the triangle DAZ has the same size and orientation as the triangle $AA'Z'$, so $A'A = AD$ and $AZ = A'Z'$. The latter means it suffices to prove that $A'Z' > DY$; the former means that that can be proven in the same way as in the proof of the analogous result of Proposition 3. ∎

5. Existence Problems

In the limit as the policy change's magnitude goes to zero, monetary CV and EV of a price change exist as long as the Slutsky decomposition, in its differential form, holds, because as mentioned in Section 4, the monetary CV and EV of a price change are just the income effects of that price change and its reverse. However, for non-infinitesimal price changes, it is possible for the income effect to be undefined. Figure 5 shows an example where monetary $WATP$ does not exist: the price of apples falls, but one looks unsuccessfully for a point on the original indifference curve U_0 having the same slope as BC'. Figure 6 shows an example where monetary WTA does not exist: the policy is a fall in the price of apples, and if it does not happen, one looks unsuccessfully for a point on the new indifference curve U' having the same slope as BC_0.

It is easy to construct an example of non-existence of finite non-monetary $WTA = EV$ in Figure 1: move the upper part of U_1 to the right so that it never intersects the vertical dashed line at $a = a_0$. Similarly shifting U_0 in Figure 2 generates non-existence of a finite non-monetary $WTA = -CV$. Such graphs would look identical, except for the interpretation of the goods on their axes, to Amiran and Hagen's (2003) infinite WTA example, their Fig. 1B.

For more on existence problems of non-monetary valuation measures, refer to Figure 7, which is an Edgeworth Box for two agents, Smith and Jones.

Apples are on the horizontal axis and bananas on the vertical axis. The policy is a move from o to n, which hurts Smith by taking bananas away.

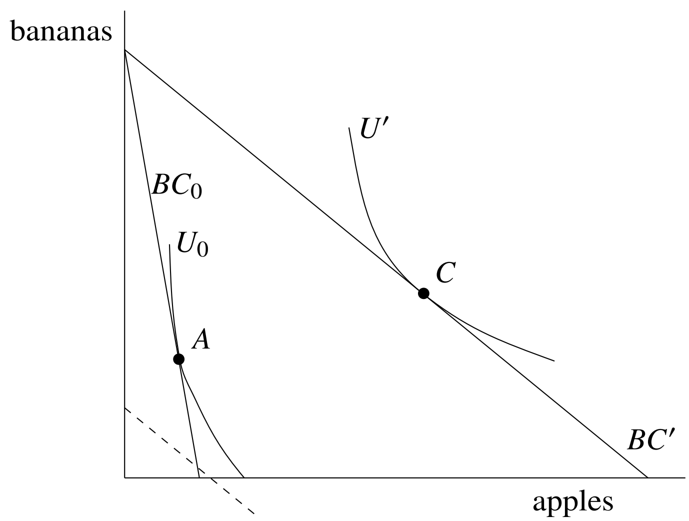

Figure 5. Undefined *WATP*.

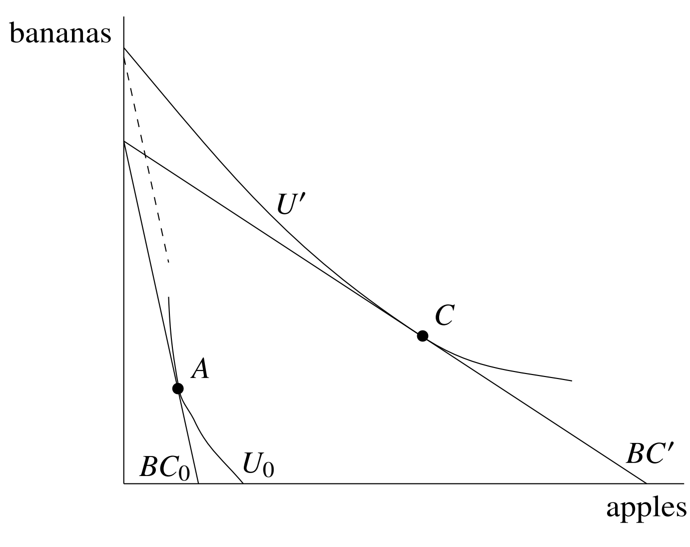

Figure 6. Undefined *WTA*.

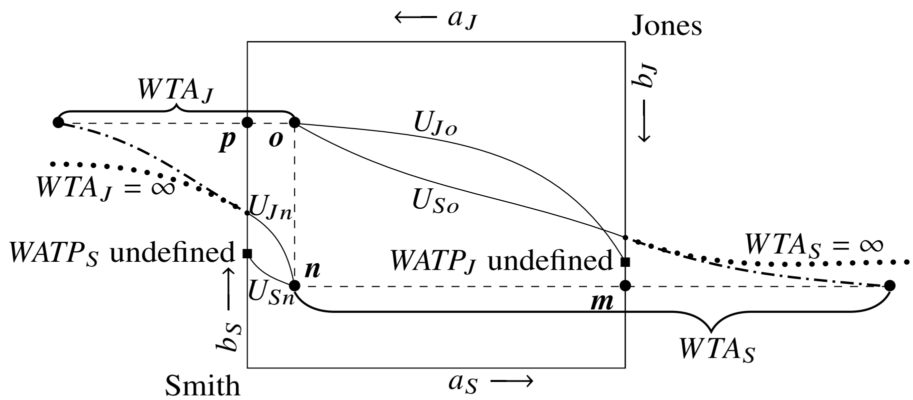

Figure 7. Valuation existence problems in an Edgeworth Box.

Smith's *WATP* = *EV* would be the number of apples he would be willing to give up, starting at *o*, to avoid being subjected to the move to *n*, and that would be illustrated by moving horizontally left from *o* until intersecting U_{Sn}, Smith's indifference curve through *n*. However, as drawn, U_{Sn} never intersects a line drawn left from *o*. Therefore Smith's *WATP* does not exist.

Smith's *WTA* = *CV* would be the number of extra apples needed to get him back to his original indifference curve, U_{So}, if the move to *o* happens. This is the horizontal distance between point *n* and the intersection of U_{So} and a line drawn left from *n*. If U_{So} continues past the right edge of the Edgeworth Box along the dash-dot line, this *WTA* exists, but it is infeasible because it extends beyond the edge of the Edgeworth Box. In other words, there are not enough apples in this economy to compensate Smith for the move from *o* to *n*. If, on the other hand, U_{So} continues past the right edge of the Edgeworth Box along the dotted line, this *WTA* is infinite.

Similarly, Jones's *WATP* = *CV* would be the number of apples she would be willing to give up in return for moving from *o* to *n*. This is the loss of apples resulting in Jones returning to her original indifference curve, U_{Jo}. This is the horizontal distance between point *n* and the intersection of U_{Jo} and a line drawn to the right from *n*. However, as drawn, U_{Jn} never intersects a line drawn to the right from *o*. Therefore Jones's *WATP* does not exist.

Jones's *WTA* = *EV* measures, starting from *o*, how many more apples Jones would need if she is prevented from moving from *o* to *n*. This is the horizontal distance between point *o* and the intersection of U_{Jn} and a line drawn to the left from o. If U_{Jn} continues past the left edge of the Edgeworth Box along the dash-dot line, this *WTA* exists, but it is infeasible because it extends beyond the edge of the Edgeworth Box. If, on the other hand, U_{Jn} continues past the left edge of the Edgeworth Box along the dotted line, Jones's *WTA* is infinite.

Clearly, existence and infeasibility can be problems for the non-monetary valuation measures.

6. Potential Pareto, Kaldor-Hicks, CBA Decisions and Problems

Kaldor (1939 p. 550) wrote:

> In all cases, therefore, where a certain policy leads to an increase in physical productivity, and thus of aggregate real income, the economist's case for the policy is quite unaffected by the question of the comparability of individual satisfactions; since in all such cases it is possible [...] to make some people better off without making anybody worse off. There is no need for the economist to prove—as indeed he never could prove—that as a result of the adoption of a certain measure nobody in the community is going to suffer. In order to establish his case, it is quite sufficient for him to show that even if all those who suffer as a result are fully compensated for their loss, the rest of the community will still be better off than before. Whether the landlords, in the free-trade case, should in fact be given compensation or not, is a political question on which the economist, *qua* economist, could hardly pronounce an opinion.... This argument lends justification to the procedure, adopted by Professor Pigou in *The Economics of Welfare*, of dividing "welfare economics" into two parts: the first relating to production, and the second to distribution.

This captures the basic idea of the Potential Pareto criterion of social decision-making, later operationalized in the Kaldor and Hicks Criteria, which underlie Cost-Benefit Analysis. The notion is plagued with measurement problems. In a world with more than one good, any measurement of "productivity" or "aggregate real income" requires a method of aggregating the output of different goods, and for economic purposes, the only aggregation method that makes sense is weighting the vector of physical outputs **y** by a corresponding vector of "values" **v**. Neoclassical value, it is obvious by now, is binary, so even if there is only one consumer, there are two measures of value, and hence there are two measures of $\mathbf{y} \cdot \mathbf{v}$; which should be used? Next arises the question of aggregating the **v** or **v**'s when there is more than one consumer. Sometimes the market price vector **p** is used to measure value. That might be useful for some accounting purposes, but it falls short of being a good measure of value for making social decisions for reasons Dupuit explained in 1844. Even worse, the market price **p** obviously depends on the distribution of income (as do each consumer's monetary *EV* and *CV*—their non-monetary *EV* and *CV* depend on their initial endowment), so Pigou's division of welfare economics is impossible. In symbols, if D denotes the income distribution, Pigou's "division" is "production" $\mathbf{p}(D) \cdot \mathbf{y}$ and "distribution" D, which is not a division, just two manifestations of D. Ignoring the dependence of **p** on D results in misinterpreting **p** as an "objective" measure of value, whereas in truth, **p** is not only a reflection of D, it is an ethically-compromised reflection of D in which the preferences of a currently-wealthy person typically have a stronger influence than the preferences of a currently-poor person.

Speaking of ethics, the Potential Pareto criterion also has a fatal ethical flaw, nicely phrased by Coleman (1980 note 62): there is absolutely no reason to think that people would consent to a policy that harms them "in virtue of its potential to be something other than it is." It was ethically acceptable for nineteenth century England to repeal the Corn Laws, hurting landlords, because England could have done something else which would not have hurt the landlords? Few things are more seemingly contradictory than the eager adoption of the Potential Pareto criterion, which justifies State confiscation of property rights, by conservative scholars of the "Law and Economics" movement, who are famous for embracing respect for property rights.[11]

[11]The contradiction disappears if there is one social class, for example the poor, whose property the State regularly confiscates using the Potential Pareto criterion as justification, and another social class, for example the wealthy, to whom that property is regularly transferred and who rest easy knowing that their property rights will always be protected by the State. In the U.S. "Cancer Alley" mentioned earlier, any property right the poor residents had to a healthful environment was removed a long time ago, while the petrochemical plants were given the property right to operate within lax parameters.

This is the counterargument to "Package [Potential-]Paretianism" (Wonnell 2001 p. 660), which asserts that "if each policy had different winners and losers so that in

Set aside such considerations. The "Kaldor criterion" (or "Kaldor test") version of the Potential Pareto criterion holds that a policy should be adopted if, after adoption, it would be possible for the beneficiaries to fully compensate the losers and still be better off than they (the winners) were originally. No compensation needs to actually be paid. The "Hicks criterion" (or "test") version of the Potential Pareto criterion holds that a policy should be adopted if, supposing the policy is not adopted, it would not be possible for those whom the policy would hurt to fully compensate those whom the policy would help for the decision not to adopt the policy. In our non-monetary economy, a policy passes the Kaldor criterion if and only if the sum of the *CV*s of all members of the society is positive, and a policy passes the Hicks criterion if and only if the sum of the *EV*s of all members of the society is positive.

In a monetary economy, things are much more complicated: a policy passes the Kaldor criterion only if the sum of the monetary *CV*s of all members of the society is positive, but sufficiency fails, and a positive sum of *EV*s is neither necessary nor sufficient for the Hicks criterion to be satisfied (Blackorby and Donaldson 1990 Section V). And in a monetary economy, Potential-Pareto social decision-making based on monetary *CV* and *EV* are plagued by ranking reversals, as first pointed out by Scitovsky (1941). We now show that Potential Pareto social decision-making based on non-monetary *CV* and *EV* suffers from the same kinds of reversals.

Figure 8 illustrates two policies, a movement from o to n_1, called Policy 1 (for Smith's final amount of b), and a movement from o to n_3, called "Policy 3" (also for Smith's final amount of b). "1" subscripts refer to Policy 1 and "3" subscripts refer to Policy 3. "L" subscripts refer to the policies' loser, Smith, and "W" subscripts refer to the winner, Jones. To save space, two incompatible possibilities for the location of indifference curve U_{W1} are shown by dotted lines, and two incompatible possibilities for the location of indifference curve U_{W3} are shown by dash-dotted lines.

the long run everyone were a winner as often as he or she were a loser, the unfairness of individual policies taken separately might wash out" (Hausman and McPherson 2007 p. 247). Those authors continue: "But the bias built into cost-benefit analysis against the preferences of the poor suggests that the unfairness will not wash out" (ibid.). Bebchuk (1980 p. 672) makes the same point. Hackinen (2012) shows in a mathematical model that "that repeated application of the Kaldor-Hicks criterion concentrates wealth. In the long run, this can lead to a winner-takes-all process where one lucky individual accumulates almost all of the wealth" (ibid., abstract).

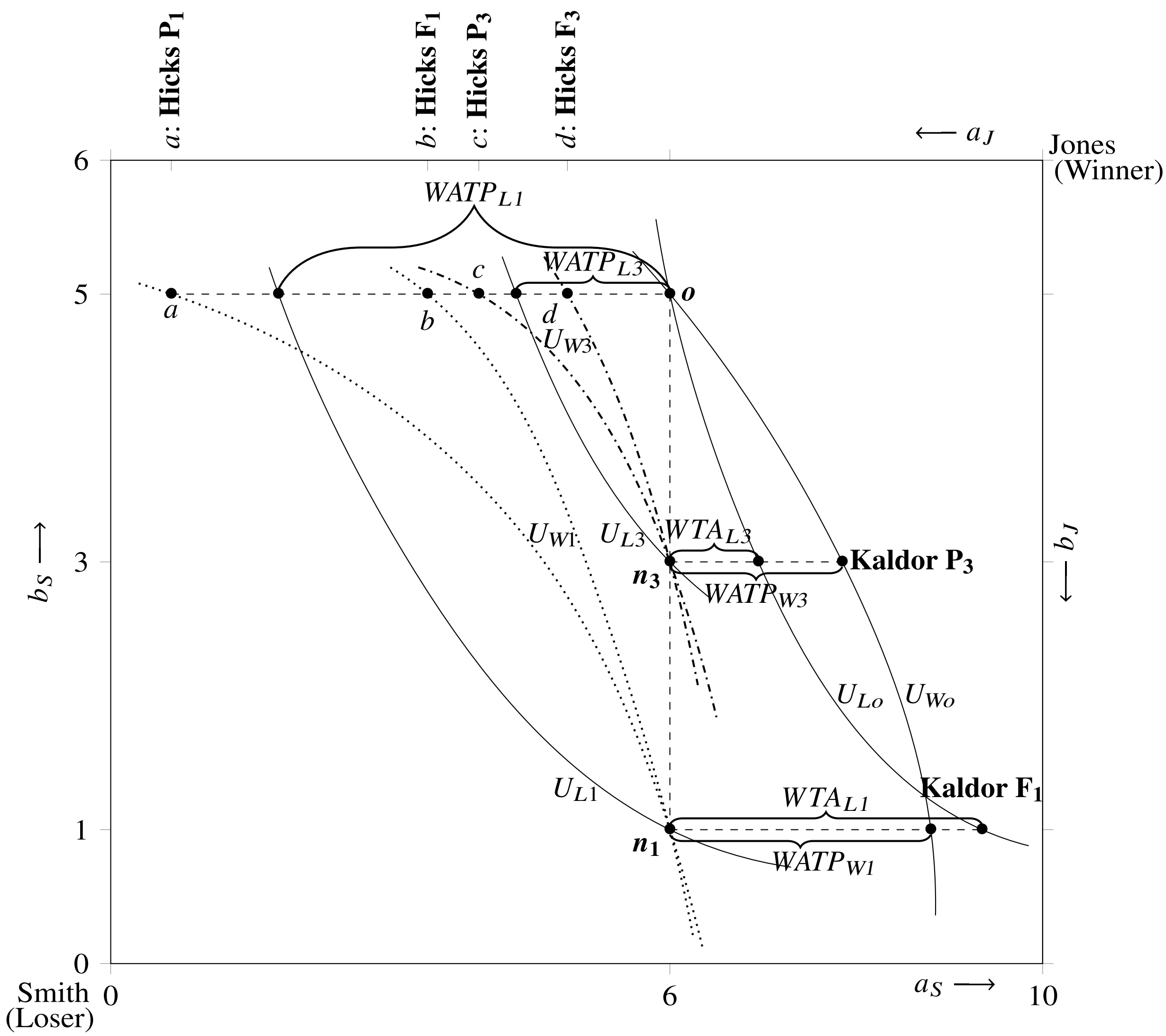

Figure 8. Various possibilities for the Kaldor and Hicks tests.

	All positions	position of U_{W3} c	d	position of U_{W1} a	b
Policy 3	Kaldor P_3	Hicks P_3	Hicks F_3		
Reverse of Policy 3	Hicks F_{r3}	Kaldor F_{r3}	Kaldor P_{r3}		
Policy 1	Kaldor F_1			Hicks P_1	Hicks F_1
Reverse of Policy 1	Hicks P_{r1}			Kaldor F_{r1}	Kaldor P_{r1}
Summary					
Hicks:		P_3, F_{r3}	F_3, F_{r3}	P_1, P_{r1}	F_1, P_{r1}
Kaldor:		P_3, F_{r3}	P_3, P_{r3}	F_1, F_{r1}	F_1, P_{r1}

Table 3. Contradictions between the Kaldor and Hicks Tests occur in columns d and a but not in columns c or b. "P" denotes pass and "F" denotes fail. Subscripts 1 and 3 denote Policies 1 and 3, and subscripts $r1$ and $r3$ denote the reverse of Policies 1 and 3.

Reading counterclockwise from the bottom right, the boldface labels note that Policy 1 fails the Kaldor test; Policy 3 passes the Kaldor test; Policy 3 fails the Hicks test if U_{W3} passes through point d; Policy 3 passes the Hicks test if U_{W3} passes through point c; Policy 1 fails the Hicks test if U_{W1} passes through point b; and Policy 1 passes the Hicks test if U_{W1} passes through point a. These results are listed in the "Policy 3" and "Policy 1" rows of Table 3. Table 2 shows that the Kaldor test (CV) of a policy yields the opposite judgment of the Hicks test (EV) of the reverse policy. This allows us to fill in Table 3's "Reverse of Policy 3" and "Reverse of Policy 1" rows.

For Table 3's column c, Policy 3 passes the Hicks test and the reverse of Policy 3 fails the Hicks test, and Policy 3 passes the Kaldor test and the reverse of Policy 3 fails the Kaldor test: so Policy 3 is endorsed by all the Potential Pareto Criteria. For column b, Policy 1 fails the Hicks test and the reverse of Policy 1 passes the Hicks test, and Policy 1 fails the Kaldor test and the reverse of Policy 3 passes the Kaldor test: so Policy 1 is rejected by all the Potential Pareto Criteria.

However, for column d, Policy 3 fails the Hicks test and passes the Kaldor test, a contradiction. Note also that in this column, Policy 3 fails the Hicks test but the reverse of Policy 3 also fails the Hicks test. This is an inconsistency within the Hicks test: the Hicks test says not to adopt

Policy 3, but if Policy 3 were adopted anyway, the Hicks test says that decision should not be reversed. ("The grass is always greener on this side of the fence," regardless of which side one is on.) Again for column *d*, Policy 3 passes the Kaldor test but the reverse of Policy 3 also passes the Kaldor test. This is a reversal within the Kaldor test: the Kaldor test says to adopt Policy 3, but if Policy 3 is adopted, the Kaldor test says that decision should be reversed. ("The grass is always greener on the other side of the fence.")

For column *a*, Policy 1 passes the Hicks test and fails the Kaldor test, a contradiction. Note also that in this column, Policy 1 passes the Hicks test but the reverse of Policy 3 also passes the Hicks test, which is a reversal within the Kaldor test. In column *a*, Policy 1 fails the Kaldor test but the reverse of Policy 3 also fails the Kaldor test, an inconsistency within the Kaldor test.

Clearly, in the context of non-monetary valuation one can construct examples where the Kaldor and Hicks tests, even when all the *CV*s and *EV*s are finite and lie in the interior of the Edgeworth Box, are unhelpful in making social decisions.

7. Potential Pareto's Hicks Test does not Deserve its Name

The Hicks and Kaldor tests suffer not only from positive problems but also, we we noted earlier, from normative ones. Our hypothetical Policies 1 and 3 both involve State confiscation of bananas from Smith and the transfer of the confiscated bananas to Jones. The main moral ground justifying using the Kaldor or Hicks Tests to decide whether to carry out such confiscation is that they identify moves which, combined with another move, could make some people better off without harming others. However, Figure 9 shows that a prospective policy move which satisfies the Hicks Test is no guarantee that the move could actually be paired with another move in a way that raised utility for everyone (or lowered it for no one). Without paying any attention to the indifference curves passing through point *o*, one can determine in Figure 9 that the move from *o* to *n* passes the Hicks Test. However, if the indifference curves passing through point *o* were tangent to each other at *a*, as shown in Figure 9, then no actual Pareto improvement would be possible if society moves away from *o*, because *o* is already Pareto Optimal. In this example, the Hicks criterion lacks normative justification of any kind.

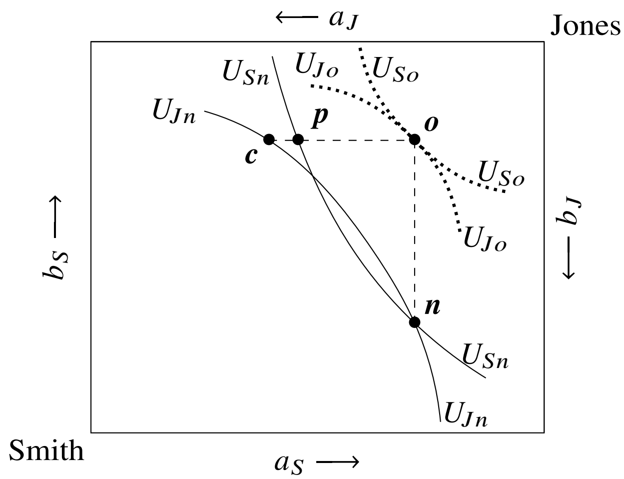

Figure 9. Moving from *o* to *n* passes the Hicks test regardless of the orientation of the dotted indifference curves, but given the orientation shown, *o* is Pareto Optimal and *n* is not.

8. Arbitrariness and Great Inequality of some Potential Pareto Decisions

In Figure 10, moving from *o* to n_1 passes the Kaldor test ($CV_{J1} = WATP_{J1} > WTA_{S1} = CV_{S1}$), and further moving from n_1 to Smith's origin, n_2, also passes the Kaldor test ($CV_{J2} = WATP_{J2} > WTA_{S2} = CV_{S2}$). Therefore, if Jones can set the society's agenda, and if society approves all projects which pass the Kaldor test, the government will confiscate all of Smith's possessions and give them to Jones. (It is unclear from the graph whether the moves from *o* to n_1, or from n_1 to n_2 could be prevented by requiring them to pass the Hicks test because neither person has a feasible *EV* for those moves. In any event, with unfeasible *EV*s, the "Potential" part of "Potential Pareto" could never actually be realized in practice, so it is not really a potential.)

On the other hand, moving from *o* to n_3 passes the Hicks test ($EV_{S3} = WTA_{S3} > WATP_{J3} = EV_{J3}$), and further moving from n_3 to n_4 near Jones's also passes the Kaldor test ($EV_{S4} = WTA_{S4} > WATP_{J4} = EV_{J4}$. Therefore, if Smith can set the society's agenda, and if society approves all projects which pass the Hicks test, the government will confiscate all of Smith's apples and most of Smith's bananas and give them to Jones. (Extending to the left the indifference curves S_0 and J_0 assuming typical convexities, the

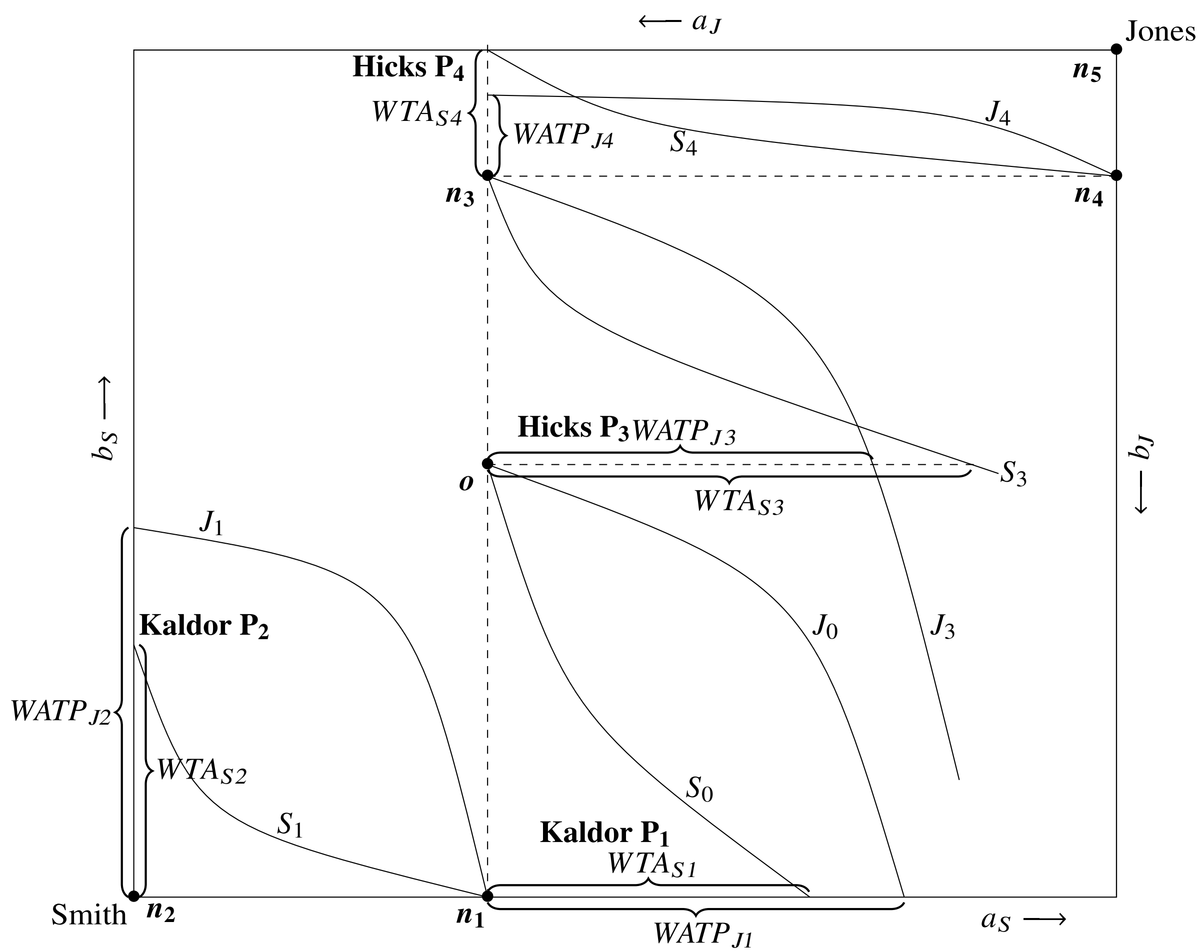

Figure 10. Extreme allocations endorsed by different Potential Pareto criteria. The J and S subscripts refer to Jones and Smith; the 1, 2, 3, and 4 subscripts refer to moving to n_1, n_2, n_3, and n_4.

move from o to n_3 would not pass the Kaldor test; Jones's required compensation for the move might be outside the Edgeworth Box. Extending to the right the indifference curves S_3 and J_3 assuming typical convexities, the move from n_3 to n_4 presumably would not pass the Kaldor test, since Jones's required compensation for the move would be outside of the Edgeworth Box.)

Cost-benefit decision-making has usually been applied to only one decision. Figure 10 is just one example, but raises the possibility that repeated application of cost-benefit analysis could give rise to extreme results, and even more concerning, results that could vary widely depending on who has the power to set the agenda of the State. Gorman (1955 p. 26) wrote, "It would appear that Scitovsky did not visualize the routine use of his criterion." It would appear that routine use of Kaldor or Hicks criteria can lead to problems beyond intransitivities.

Conclusion

Expressing the value of a commodity in terms of another commodity is logically prior to expressing that value in terms of money once the consumer has been placed in a market economy. Working out this more primitive, non-monetary definition of value makes highlights the binary natue of value. Non-monetary valuation is able to explain much "gains versus losses" behavior within a simple rational-agent framework. However, another simple framework, quasilinear utility, is an uninteresting, knife-edge case which has prevented the binary nature of value from being part of the understanding of every student of the field.

Although the Potential Pareto, Kaldor-Hicks, Cost-Benefit Analysis approach to making social decisions, based on monetary valuation, has flaws that are well-documented in articles such as Blackorby and Donaldson (1990), this has not prevented widespread, continuing use of those techniques. By showing similar problems in the simpler, non-monetary setting, we hope to draw more attention to the deeply problematic nature of Potential Pareto decision-making.

Bibliography

Bebchuk, Lucian A. (1980), "The Pursuit of a Bigger Pie: Can Everyone Expect a Bigger Slice?," *Hofstra Law Review* **8**: 671–709.

Blackoby, Charles, and David Donaldson (1990), "The Case against the Use of the Sum of Compensating Variations in Cost-Benefit Analysis," *The Canadian Journal of Economics* **23**: 471–494.

Boadway, Robin (2020), "Economic Evaluation of Projects." Chapter 3, pages 59–106 of Anwar Shah, editor, *Policy, Program and Project Evaluation: A Toolkit for Economic Analysis in a Changing World.* Cham, Switzerland: Palgrave Macmillan.

Brekke, Kjell Arne (1997), "The Numéraire Matters in Cost-Benefit Analysis," *Journal of Public Economics* **64**: 117–123.

Chipman, John S., and Guoqiang Tian (2012), "Detrimental Externalities, Pollution Rights, and the 'Coase theorem'," *Economic Theory* **49/2**: 309–327.

Coleman, Jules L. (1980), "Efficiency, Utility, and Wealth Maximization," *Hofstra Law Review* **8**: 509–551.

Currie, John Martin, John A. Murphy, and Andrew Schmitz (1971), "The Concept of Economic Surplus and Its Use in Economic Analysis," *The Economic Journal* **81/324**: 741–799.

Daly, Herman E. (1991), *Steady-State Economics: Second Edition with New Essays.* Washington D.C.: Island Press.

Ellerman, David (2014), "On a Fallacy in the Kaldor–Hicks Efficiency–Equity Analysis," *Constitutional Political Economy* **25/2**: 125–136.

de La Grandville, Olivier (1997), "Curvature and the Elasticity of Substitution: Straightening It Out," *Journal of Economics/Zeitschrift für Nationalökonomie* **66/1**: 23–34.

Gorman, W. M. (1955), "The Intransitivity of Certain Criteria used in Welfare Economics," *Oxford Economic Papers (new series)* **7/1**: 25–35.

Hackinen, Brad (2012), "Does Repeated Application of the Kaldor-Hicks Criterion Generate Pareto Improvements?." Unpublished Honours thesis, Department of Economics, University of Victoria. Downloaded from `https://www.uvic.ca/socialsciences/economics/assets/docs/honours/Hackinen.pdf`.

Hanemann, W. Michael (1991), "Willingness to Pay and Willingness to Accept: How Much Can They Differ?," *The American Economic Review* **81/3**:635–647.

Hanemann, W. Michael (1999), "The Economic Theory of WTP and WTA." Pages: 42–96 of Ian J. Bateman and Kenneth G. Willis, editors, *Valuing Environmental Preferences: Theory and Practice of the Contingent Valuation Method in the US, EU, and Developing Countries.* Oxford: Oxford University Press.

Hanley, Nick, Jason F. Shogren, and Ben White (2007), *Environmental Economics: In Theory and Practice, Second Edition.* New York: Palgrave Macmillan.

Hausman, Daniel M., and Michael S. McPherson (2007), "The Philosophical Foundations of Mainstream Normative Economics." Pages 226–250 of Daniel M.

Hausman, editor, *The Philosophy of Economics: An Anthology (Third Edition)*. Cambridge: Cambridge University Press.

Hausman, Jerry A. (1981), "Exact Consumer's Surplus and Deadweight Loss," *American Economic Review* **71**: 662–676..

Hicks, J.R. (1946), *Value and Capital: An Inquiry into some Fundamental Principles of Economic Theory (Second Edition)*. Oxford: Oxford University Press.

Houghton, R. W. (1958), "A Note on the Early History of Consumer's Surplus," *Economica (New Series)* **25**: 49–57.

Kaldor, Nicholas (1939), "Welfare Propositions of Economics and Interpersonal Comparisons of Utility," *The Economic Journal* **49/195**: 549–552.

Kuriyama, Koichi, and Kenji Takeuchi (2001), "The Disparity between WTP and WTA with or without Money." Downloaded from `https://www.fun.ac.jp/~kawagoe/paper/kuriyama.pdf`.

List, John A. (2004), "Neoclassical Theory versus Prospect Theory: Evidence from the Marketplace," *Econometrica* **72/2**: 615–625.

Marshall, Alfred (1920), *Principles of Economics (eighth edition)*. London: Macmillan.

Randall, Alan, and John R. Stoll (1980), "Consumer's Surplus in Commodity Space," *The American Economic Review* **70/3**: 449–455.

Scitovsky, Tibor (1941), "A Note on Welfare Propositions in Economics," *The Review of Economic Studies* **9/1**: 77–88. (Last name also styled De Scitovszky.)

Tietenberg, Thomas H., and Lynne Lewis (2018), *Environmental and Natural Resource Economics (Eleventh edition)*. New York: Routledge.

Tirole, Jean (1988), *The Theory of Industrial Organization*. 1988 Cambridge, MA: MIT Press.

Wonnell, Christopher T. (2001), "Efficiency and Conservatism," *Nebraska Law Review* **80/4**: 643–714.