Consumer Surplus, the Coase Theorem, and the Lack of a Unique Objectively Efficient Allocation: An Elementary View

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Feb. 24, 2020

Abstract. Consumer surplus purports to be a value-free welfare measure. The Coase Theorem purports to identify objectively efficient allocations. By fixing a flaw in standard presentations, we require no advanced economics (duality) to construct the foundations of welfare economics, including willingness to pay and equivalent/compensating variations. We illustrate why consumer surplus is unimportant and why the Coase Theorem is false. The set of economically efficient allocations generally shifts whenever the income distribution shifts. Observers with different ethical beliefs about the income distribution will identify different allocations as being efficient. In this sense, there is no objective efficiency.

Keywords: keywords
JEL Codes: JEL codes

Many observers have drawn from the Coase Theorem the lesson that a unique economically-efficient allocation for a society can be objectively identified regardless of its income distribution. Ethics and subjectivism can be banished from the discussion. Furthermore, a simple way to determine which situation is objectively efficient is to determine which one maximizes consumer surplus.

Some academic economists and legal scholars have criticized using the Coase Theorem or consumer surplus in this way. The Achilles Heel of both consumer surplus and the Coase Theorem is that they ignore the "income effect," more properly but less commonly known as the "wealth effect" (or in one article[1] we use, the "welfare effect"), which is the effect that a change in income or in wealth has on the amount of a commodity demanded by a consumer. Most commodities, for most consumers, exhibit non-zero income effects, since for example people of different incomes usually consume very different bundles of goods. The obstacles this poses to the notion of an objectively-arrived at point of economic efficiency divorced from the distribution of wealth cannot be overcome.

These long-standing criticisms are well-understood in some circles but not in others. The arguments by economists—for example, the survey by Slesnick (1998) and monograph of Fleurbaey and Blanchet (2013)—often use advanced methods and are in practice largely inaccessible to most readers without a Ph.D. in economics. To many practitioners, esoteric and possibly nitpicking arguments form little reason to abandon consumer surplus or the Coase Theorem. Alternatively, it is conceivable that in some quarters the devotion to consumer surplus and to this interpretation of the Coase Theorem comes instead because they lead to judicial and public policy decisions which weight the welfare changes experienced by wealthier people more heavily than those experienced by poorer people, since in these approaches "a dollar is a dollar" and wealthier people have more dollars.

In this paper we demonstrate the shortcomings of Consumer Surplus as a welfare measure and the shortcomings of the Coase Theorem taking a new, more elementary approach than that of modern 'duality' theory. Our approach, though not standard, nevertheless leads to all the basic ideas of modern welfare economics. We confine ourselves to the study of only two elementary concrete numerical examples, eschewing generality, and we supply numerous graphs to illustrate the two examples from many viewpoints. It is our hope that by charting a novel path between the largely non-mathematical

[1] Mishan (1971).

discussions in law journals and the modern technical treatments by economists we can make the basic results in this field understandable to a wider audience than has heretofore been the case.

Sections 1, 2, and 3 deal with how to measure changes in an individual's welfare. Section 1 treats consumer surplus, Section 2 treats alternative ways of valuing quantity changes, and Section 3 treats alternative ways of measuring price changes. Section 4 deals with how to measure changes in the welfare of a group of individuals, and Section 5 deals with the Coase Theorem.

As to how "long-standing" the criticisms which are treated in this paper are, the point about "consumer's surplus" that Section 1 illustrates was already alluded to in the second sentence of this excerpt from Alfred Marshall (1920 Bk. III Ch. VI §4 p. 132, emphases added):

> The *substance* of our argument would not be affected if we took account of the fact that, the more a person spends on anything the less power he retains of purchasing more of it or of other things, and the greater is the value of money to him (in technical language every fresh expenditure increases the marginal value of money to him). But though its *substance* would not be altered, *its form would be made more intricate…*

What Section 2 does is to derive exactly what this "more intricate" form actually is. The concerns of Section 4 were first raised by Bordas in 1847[2] and were admitted by Marshall (op. cit. Bk. III Ch. VI §3 pp. 130–1). Section 5's point that the Coase Theorem fails in the face of income effects was noted as early as Mishan (1971 pp. 18, 20–21) and Samuels (1974).

We acknowledge that Marshall added, "But these changes of consumers' rent (being of the second order of smallness) may be neglected, on the assumption, which underlies our whole reasoning, that his expenditure on any one thing, as, for instance, tea, is only a small part of his whole expenditure" (op. cit., Note VI of Mathematical Appendix). Our numerical examples nevertheless make the opposite assumption because Marshall's assumption already underlies the majority of work in this field, and because it is worthwhile to know how welfare changes when the commodity involved is important rather than unimportant.

[2]See p. 52 of Houghton (1958)

1. Consumer Surplus

We would like to know how consumers are affected by changes in their economic environment, but this is hard to do because their utility is subjective, ordinal not cardinal, not observable by outsiders, and not interpersonally comparable. One way forward in spite of this is to try to develop a monetary measure of the change in utility. Consumer surplus is such a measure. It is the area under a demand curve, or, more precisely, the integral under a Marshallian demand function. We will call it "Marshallian consumer surplus" to distinguish it from a more accurate measure we introduce below. Using the change in Marshallian consumer surplus to monetarily measure changes in consumer welfare is, for example, commonly done in the context of antitrust litigation. Becht (1995 p. 77) however points out:

> In his survey article on 'Demand Analysis' for the *Handbook of Econometrics*, Angus Deaton[3] stated that 'there is no valid theoretical or practical reason for ever integrating under a Marshallian demand curve.'

To explain at least one reason why, consider the following example, which we use throughout Sections 1–3.

Suppose a consumer has a utility function of the elementary Cobb-Douglas form, over simply two commodities: $u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$. Let this consumer's fixed income be $m = 2$ and suppose the consumer takes the prices p_1 and p_2 as given. Suppose x_1 is "cheese" and x_2 is "apples." In this paper, p_2 will never change. Maximizing $u(x_1, x_2)$ with respect to the usual budget constraint

$$2 = m = p_1 x_1 + p_2 x_2 \tag{1}$$

leads by standard methods[4] to this consumer's demand curve for cheese,

$$x_1 = \frac{1}{p_1}\,. \tag{2}$$

This is the Marshallian demand curve "MaD" depicted by the solid line in Figure 1.

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[3] Sir Angus Deaton is the co-inventor of the "Almost Ideal Demand System," winner of the 1978 Frisch Medal of the Econometric Society and of the 2015 Nobel Prize in Economics, and was one of the world's most famous economists in developing economic theory for empirical purposes.

[4] The Lagrangian is $\mathscr{L} = x_1^{1/2} x_2^{1/2} + \lambda\,(2 - p_1 x_1 - p_2 x_2)$. The first-order conditions are $0 = \partial\mathscr{L}/\partial x_1 = (1/2)x_1^{-1/2} x_2^{1/2} - \lambda p_1$, $0 = \partial\mathscr{L}/\partial x_2 = (1/2)x_1^{1/2} x_2^{-1/2} - \lambda p_2$, and (1). Solving the first two first-order conditions for λ and setting those expressions equal to each other results in $x_2 = p_1 x_1 / p_2$. Substituting this for x_2 in (1) leads to $2 = 2 p_1 x_1$, and hence (2).

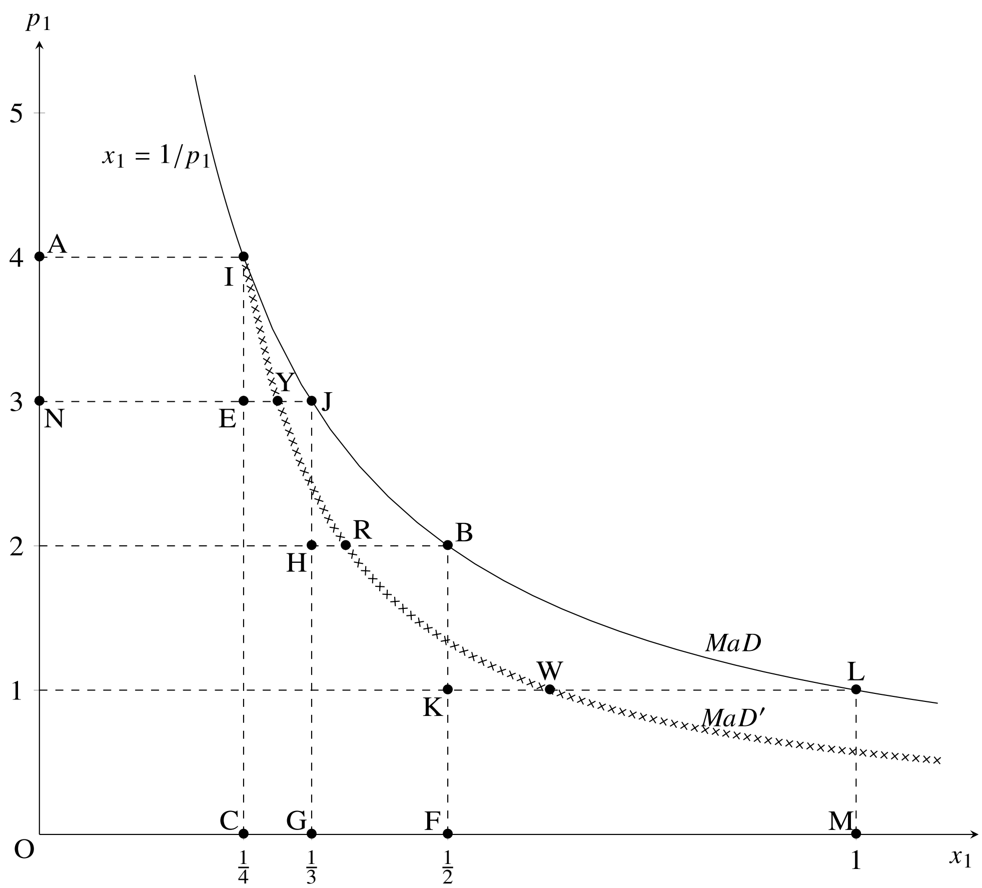

Figure 1. Solid line: standard Marshallian demand curve MaD (includes points I, J, B, and L). Line of crosses: Marshallian demand curve with an initial quantity of cheese x_1 equal to 1/4, "MaD'" (includes points I, Y, R and W).

The standard story told about Marshallian consumer surplus ever since Dupuit (1844, see especially its concluding "Note") has run something like this.

> Consider how much money the consumer whose demand curve appears in Figure 1 would be willing and able to spend to buy a certain total amount of cheese. If the price of cheese were \$4/lb., he would be willing to buy $1/4$ pounds of cheese, and so would be willing and able to spend the amount of money shown by area OAIC in the diagram.
>
> If after making this transaction the price of cheese were to fall to \$3/lb., he would be willing to buy more cheese, raising his total cheese purchases to $1/3$ pounds of cheese, and so would in total have been willing and able to spend the amount of money shown by area OAIEJG in the diagram.
>
> If after making this transaction the price of cheese were to fall further, to \$2/lb, he would be willing to buy more cheese, raising his total cheese purchases to $1/2$ pounds of cheese, and so would in total spend the amount of money shown by area OAIEJHBF in the diagram.
>
> If, finally, after making this transaction the price of cheese were to fall even further, to \$1/lb, he would be willing to buy even more cheese, raising his total cheese purchases to 1 pound of cheese, and so would in total spend the amount of money shown by area OAIEJHBKLM in the diagram. This amount of money is approximately equal to Marshallian consumer surplus, which is the area under the Marshallian demand curve. It follows that, as the increments in price become smaller, the Marshallian consumer surplus measures how much the consumer would be willing and able to spend to buy cheese.

In other words, to derive the monetary valuation, one conducts a thought experiment by starting at an initial point on the standard Marshallian demand curve—point I at a price of four dollars in Figure 1—then removing the consumer from a "uniform price" environment, putting him or her instead into a "facing a price-discriminating seller" environment, and observing how much money such a seller can extract from the (willing) consumer as prices change. This is called the consumer's "willingness to pay" for cheese, though "willingness and ability to pay" would be more accurate.

This basic idea for deriving the monetary valuation of a price change is correct and insightful. In this paper we use it often. However, the particular implementation of the idea described in the preceding story gets one detail wrong and will not generate a numerically correct answer.

To show why, suppose the consumer has already spent the money to purchase, at a price of \$4/lb, the $1/4$ pound of cheese called for at point I of the graph. Suppose the consumer has taken ownership of this amount of cheese but has not eaten it yet. Suppose that before eating this cheese and before buying any x_2, the consumer gets the opportunity to buy more cheese at a price of \$3/lb. Let the extra cheese the consumer buys in response to this offer be denoted by t ("exTra"). Then the consumer's utility is

$$(1/4 + t)^{1/2} x_2^{1/2} . \tag{3}$$

Having already spent \$1 (the area OAIC in the diagram) on the first $1/4$ lb of cheese, of his original income $m = 2$, one dollar remains, so his new budget constraint is

$$\$1 = p_1 t + p_2 x_2 \tag{4}$$

where p_1 will be \$3/lb. Maximizing his utility subject to this budget constraint leads to[5]

$$t = \frac{1}{2p_1} - \frac{1}{8} , \tag{5}$$

so at $p_1 = 3$ he buys $t = 1/24$ additional pounds of cheese, for total cheese purchases of $1/4+1/24 = 7/24 \approx 0.29$, which is less than the $1/3$ represented by points J and G in the diagram, and is located at the diagram's point Y. Consecutively substituting prices of two and then one into (5) generates the points at R and W in the diagram, where the new demand curve (5) is the line of crosses labeled "MaD'." Clearly the demand curve has shifted down to YRW, and the consumer's willingness to pay is not going to be measured by the area under the original demand curve.

One is used to price changes causing a movement along a demand curve, not this kind of shift in the demand curve, but that assumes a consumer facing a uniform price for all units of cheese, and a demand curve derived by assuming a uniform price for all units of cheese. If there is a non-uniform price for some units of cheese then a demand curve derived by assuming a uniform price for all units of cheese does not behave as one is used to expect.

[5] The Lagrangian is $\mathscr{L} = (1/4{+}t)^{1/2} x_2^{1/2} + \lambda\,(1 - p_1 t - p_2 x_2)$. The first-order conditions are $0 = \partial\mathscr{L}/\partial t = (1/2)(1/4{+}t)^{-1/2} x_2^{1/2} - \lambda p_1$, $0 = \partial\mathscr{L}/\partial x_2 = (1/2)(1/4{+}t)^{1/2} x_2^{-1/2} - \lambda p_2$, and (4). Solving the first two first-order conditions for λ and setting those expressions equal to each other results in $x_2 = p_1\,(1/4{+}t)/p_2$. Substituting this for x_2 in (4) leads to $1 = (2t + 1/4)\,p_1$, and hence (5).

The new demand curve is lower than the old one because the consumer had to spend, in order to buy the first $1/4$ lb of cheese, \$4/lb. $\cdot$ $(1/4)$ lb. = \$1, instead of the \$3/lb. $\cdot$ $(1/4)$ lb. = \$$(3/4)$ he would have had to spend had the price been uniform at \$3/lb., meaning his remaining income after buying the first $1/4$ lb. of cheese is \$$(1/4)$ lower than it would have been had he been facing the uniform \$3/lb. price. This lower remaining income has an *income effect* which reduces the demand for cheese because cheese is a normal good in this example. If cheese were an inferior good, point Y would lie to the right of point J instead of to its left, and MaD' would be flatter than MaD, instead of steeper as it is in Figure 1. If the consumer had been able to buy the first $1/4$ lb. of cheese at a price of \$3/lb. instead of \$4/lb., he would have had \$$(5/4)$ left over to spend on apples instead of having only \$1, in which case the budget constraint (4) would instead have been

$$\$5/4 = p_1 t + p_2 x_2 \,, \tag{6}$$

(5) would have been

$$t = \frac{5}{8p_1} - \frac{1}{8} \,, \tag{7}$$

and at $p_1 = 3$, additional cheese t would have been $1/12$ and total cheese purchased would have been $1/4 + 1/12 = 1/3$, in other words, at point J. The demand curve would not have shifted. So it is the income effect coupled with the nonuniform price which caused the demand curve to shift. If a price change has a non-zero income effect but the old and new prices are both uniform, then as usual there will be a movement along the demand curve not a shift in it.

As it is, we get a shift in the demand curve when the price for units beyond $1/4$ lb. falls from \$4/lb. to \$3/lb. If after buying AI at \$4/lb. and EY at \$3/lb. the price were to fall further to \$2/lb., the demand curve would shift down again, below YRW, so the quantity demanded would be less than R; and it would further shift down yet again if after buying some cheese at \$4/lb. and some cheese at \$3/lb. and some cheese at \$2/lb. the price were to fall once more, to \$1/lb, making the actual quantity demanded at \$1/lb much less than W. In the face of all these shifting demand curves it would not be easy to figure out the actual quantities bought at different prices under perfect price discrimination. Section 2 shows how to do that.

Under nonuniform pricing the only situation in which the demand curve would never shift is if this good had no income effect, that is, if the consumer would never change his consumption of this good in response to any change in income. (Any change in income, that is, except one so drastic as to make the prior amount of consumption unaffordable even if all income was directed to purchasing that commodity.) Varian (2010, p. 103) suggests pencils might be such a good:

> Initially I may spend my income only on pencils, but when my income gets large enough, I stop buying additional pencils—all of my extra income is spent on other goods.

Are there any goods, such as Varian's "pencils" (he also suggests salt or toothpaste), on which poor people spend all of their income but on which all other people spend a fixed amount of money irrespective of their income? It would appear to be the rare homeless person who spends all of his or her income on pencils, or on salt, or on toothpaste. So it appears that such commodities—economists refer to them as being characterized by "quasilinear preferences"—are of no practical interest. Even ignoring what happens at low income, few goods of interest have precisely zero income elasticity, which is why authors such as Deaton have little patience for taking quasilinear preferences seriously. Accordingly we completely ignore quasilinear preferences, and the unstated caveat "assuming that preferences are not quasilinear" applies below where appropriate.

2. Valuing Quantity Changes

a. 1/4 lb. of cheese to 1/3 lb. Section 1 illustrated the shortcomings of Marshallian consumer surplus, but it does not imply that there is *no* way to quantify in dollars an individual's welfare change due to a price change. Section 1's basic idea, of using a consumer's behavior when faced with a price-discriminating seller to reveal what the consumer's monetary valuation is, is valid. There is simply a mathematical detail which has to be fixed. We work that out in this section, which enables us to value quantity changes, then apply that knowledge to consumer surplus *per se* in Section 3. Papers that use modern techniques to value quantity changes include Randall and Stoll (1980), Hanemann (1991), and Hanemann (2003).

As alluded to above, the problem with using the original demand curve MaD in Figure 1 to derive monetary valuation is that it is derived using budget constraint (1), which is not the right budget constraint to use in our thought experiment because (1) takes p_1 to be a constant and our thought experiment takes p_1 to be a variable which is going to be changed by the price-discriminating seller. Demand curve MaD' was derived from budget constraint (4), which has the same problem. Extracting all possible payments from a consumer requires nonuniform pricing, and the correct demand curve to describe consumer behavior under that situation is one which is derived from a budget constraint that explicitly reflects the nonuniform pricing which the consumer knows he or she is facing. We begin this section by showing how to derive that demand curve. This derivation is new and though its economics is pre-modern, in the sense of not using duality theory, some of its mathematics is even now uncommon in economics. Accordingly we omit almost no mathematical details, but the reader not interested in those details may skip ahead to the answer, (16).

Starting in Figure 1 at the initial point "$p_1 = 4$, $x_1 = 1/4$" (point I), again let t denote the additional amount of cheese the consumer buys beyond the first $1/4$ pound. This consumer's budget constraint under "perfect" price discrimination is not (4) but rather the following, where p_1 is a function of t and $p_1(0)$ is at Point I in Figure 1:

$$\$1 = \int_0^t p_1(\hat{t})\, d\hat{t} + p_2 x_2 \,. \tag{8}$$

The integral captures the area under the demand curve ($\hat{t}$ is a dummy variable of integration), and reflects the same basic idea as the integral in Marshall (1920, Mathematical Appendix, Note VI, p. 841), though he never put it into a budget constraint. To solve the problem of maximizing (3) subject to (8), construct the Lagrangian as usual,

$$\mathscr{L} = (1/4 + t)^{1/2} x_2^{1/2} + \lambda \left[1 - \int_0^t p_1(\hat{t})\, d\hat{t} - p_2 x_2\right]. \tag{9}$$

To obtain the first-order condition $0 = \partial\mathscr{L}/\partial t$ one has to differentiate the integral in (9), which can be done using the Leibniz Integral Rule

$$\frac{d}{dt}\left(\int_{a(t)}^{b(t)} f(t,x)\, dx\right) = f\big(t, b(t)\big)\cdot\frac{d}{dt}b(t) - f\big(t, a(t)\big)\cdot\frac{d}{dt}a(t) + \int_{a(t)}^{b(t)} \frac{\partial}{\partial t} f(t,x)\, dx \,.$$

The result is

$$0 = \frac{\partial\mathscr{L}}{\partial t} = \tfrac{1}{2}\,(1/4 + t)^{-1/2} x_2^{1/2} - \lambda\, p_1(t)\,. \tag{10}$$

The other first-order conditions are as usual

$$0 = \frac{\partial\mathscr{L}}{\partial x_2} = \tfrac{1}{2}\,(1/4 + t)^{1/2} x_2^{-1/2} - \lambda\, p_1 \tag{11}$$

and (8). Proceeding in the standard way by solving both (10) and (11) for λ and setting the results equal to each other yields $x_2 = (1/4 + t)\, p_1(t)/p_2$. Substituting this into (8) results in

$$1 = \int_0^t p_1(\hat{t})\, d\hat{t} + (\tfrac{1}{4} + t)\; p_1(t)\,. \tag{12}$$

This is an integral equation in $p_1(t)$ because the unknown, $p_1(t)$, is a function which appears inside an integral. Though the reader may be unfamiliar with integral equations, this one is easy to solve using the same approach applied by Roos (1927 p. 641) and Hotelling (1931 p. 164): differentiate both sides with respect to t (again using the Leibniz Rule) and solve the resulting differential equation:

$$0 = p_1(t) + p_1(t) + (\tfrac{1}{4} + t)\,\frac{dp_1(t)}{dt} \quad \text{so}$$
$$\frac{dp_1(t)}{dt} = \frac{-2p_1(t)}{\frac{1}{4} + t} \quad \text{or} \quad \frac{dp_1}{p_1} = -2\,\frac{dt}{\frac{1}{4} + t}\,. \tag{13}$$

This differential equation can be solved by integrating both sides ($\ln p_1 = -2\ln(t{+}1/4)^{-2}$ plus some constant) and rearranging, yielding

$$p_1(t) = \frac{\text{const.}}{(1 + 4t)^2} \tag{14}$$

where "const." is a (different) constant. To find the constant, substitute (14) back into (12) giving

$$1 = \int_0^t \frac{\text{const.}}{(1 + 4\hat{t})^2}\,d\hat{t} + (\tfrac{1}{4} + t)\,\frac{\text{const.}}{(1 + 4t)^2}$$

so

$$\begin{aligned}
\frac{1}{\text{const.}} &= \tfrac{1}{4}\int_0^t (1 + 4\hat{t})^{-2}(4\,d\hat{t}) + \tfrac{1}{4}(1 + 4t)\,\frac{1}{(1 + 4t)^2} \\
&= \tfrac{1}{4}\,\frac{(1 + 4\hat{t})^{-1}}{-1}\bigg|_0^t + \tfrac{1}{4}\cdot\frac{1}{1 + 4t} \\
\frac{4}{\text{const.}} &= \left(\frac{-1}{1 + 4t} - \frac{-1}{1}\right) + \frac{1}{1 + 4t} = 1
\end{aligned}$$

and therefore const. = 4 and the solution for the inverse demand function under perfect price discrimination is

$$p_1(t) = \frac{4}{(1 + 4t)^2}\,. \tag{15}$$

The corresponding demand function is

$$t = \frac{1}{\sqrt{4p_1}} - \frac{1}{4} \quad \text{and therefore } x_1 = t + \tfrac{1}{4} = \frac{1}{\sqrt{4p_1}}. \tag{16}$$

This demand function is the dash-dotted "perfect price discrimination demand curve through point I" marked as $PPDD_I$ in Figure 2, and areas under it *can* be interpreted as the maximum amount of money this consumer would be willing to pay for increments of cheese above 1/4. $PPDD_I$ lies below MaD and MaD', as expected.[6] Figure 3 is a magnification of the part of Figure 2 of greatest interest from now on. It introduces a new point "T" on $PPDD_I$ (point T would have obscured point Y had it been shown in Figure 2). $PPDD_I$ is steeper than MaD because as price decreases from four to three, the consumer along $PPDD_I$ is poorer than the consumer along MaD (poorer not because of less income but because of facing a price-discriminating seller), and since x_1 is a normal good, being poorer leads the consumer to buy less of it. If x_1 were an inferior good, $PPDD_I$ would be flatter than MaD, as discussed below in connection with Figure 6.[7]

We can now calculate a valuation for increasing cheese consumption from 1/4 to 1/3 lb. One can think of this as being the consumer's "willingness and ability to pay," "WTP," for the additional cheese. We will denote this "*WTPA*" for a reason to be explained shortly. An incorrect valuation based on the Marshallian demand curve would be area IJGC in Figures 3 and 2, whose size is

$$WTPA_{MaD}(1/4, 1/3) = \int_{1/4}^{1/3} MaD(x_1)\, dx_1 = \int_{1/4}^{1/3} \frac{dx_1}{x_1} = \ln\frac{4}{3} \approx 0.288\,. \tag{17}$$

A correct valuation based on $PPDD_I$ would be area IVGC, whose size is the following, noting that the inverse demand function corresponding to (16) is $p_1 = 1/(4x_1^2)$, and abbreviating "$WTPA_{PPDD_I}$" by $WTPA_I$:

$$WTPA_I(1/4, 1/3) = \int_{1/4}^{1/3} PPDD_I(x_1)\, dx_1 = \int_{1/4}^{1/3} \frac{dx_1}{4x_1^2} = \frac{1}{4}\,. \tag{18}$$

The difference between the two *WTPA* figures is $(0.288 - 0.134)/0.134 =$ 15%. (This difference looks larger in Figure 3 than in Figure 2 because both of the axes in the former are interrupted near the origin.)

[6] Attempting to derive a demand curve for this particular consumer facing perfect price discrimination without setting an initial nonzero "reference" point such as $x_1 = 1/4$ fails. The Lagrangian for such a problem would be $x_1^{1/2}x_2^{1/2} + \lambda\,[2 - \int_0^{x_1} p_1(\hat{t})\, d\hat{t} - p_2x_2]$, leading to $x_2 = p_1x_1/x_2$, an integral equation of $2 = \int_0^{x_1} p_1(\hat{t})\, d\hat{t} + x_1p_1(x_1)$, a differential equation $dp_1/dx_1 = -2p_1/x_1$ with solution $p_1(x_1) = \text{const.}/x_1^2$, which with the budget constraint leads to $2 = \int_0^{x_1}(\text{const.}/\hat{t}^2)\, d\hat{t} + \text{const.}/x_1$; however the solution to the indefinite integral is $-1/\hat{t}$, which is undefined at the lower limit of integration, zero.

[7] A proof is in footnote 17. We made a similar point about MaD' a few sentences before (6).

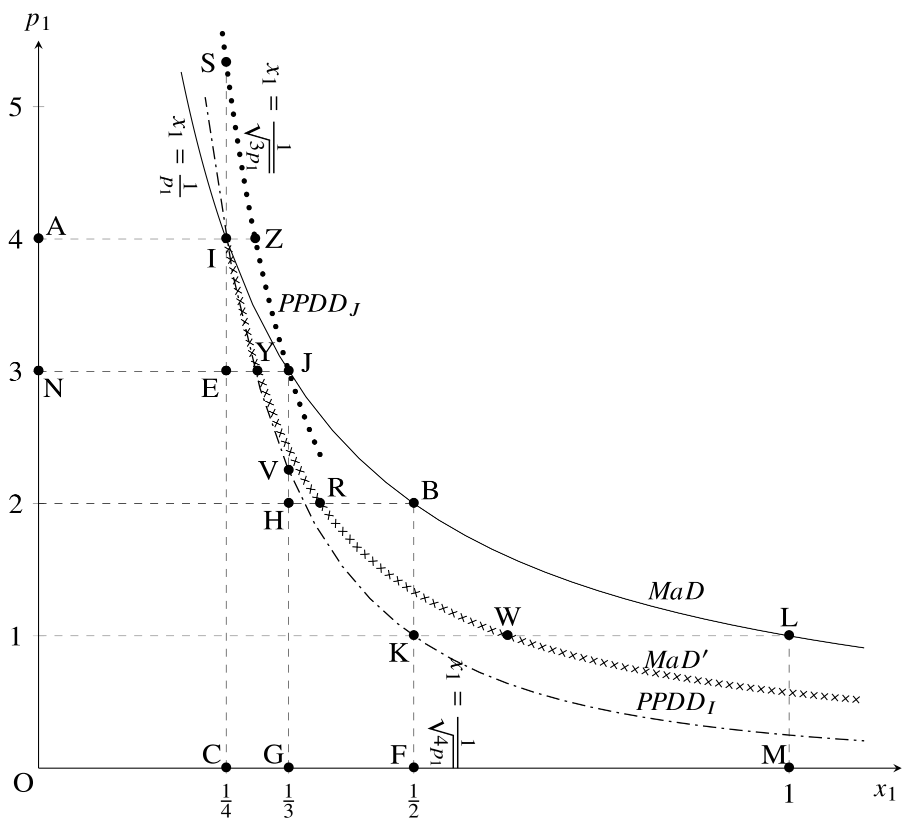

Figure 2. Solid line: standard Marshallian demand curve *MaD* (includes points I, J, B, and L). The label $x_1 = \frac{1}{p_1}$ is rotated because it shows x_1 as a function of p_1, as is proper for a demand curve. Line of crosses: Marshallian demand curve with an initial quantity of cheese x_1 equal to $1/4$, "MaD'" (includes points I, Y, R and W). Dash-dotted line: demand curve under perfect price discrimination keeping utility at point I's level, "$PPDD_I$" (includes points I, V, and, coincidentally, K). Dotted line through J, Z, and S: as discussed in the Appendix, demand curve under perfect price discrimination keeping utility at point J's level, "$PPDD_J$."

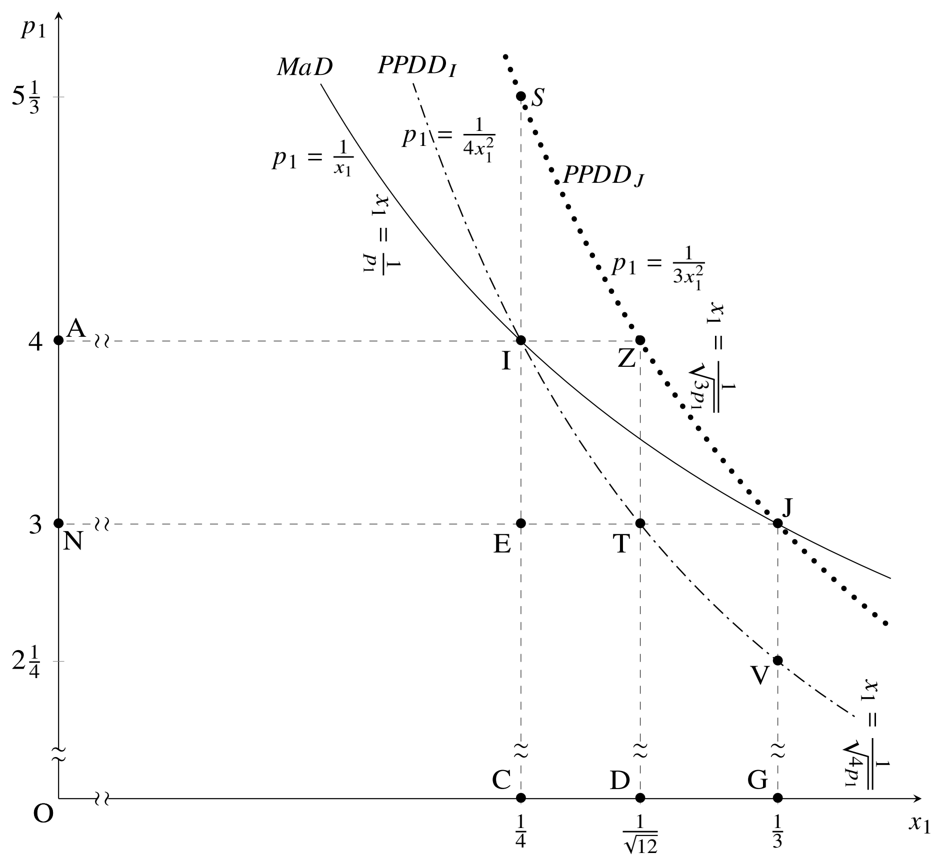

Figure 3. A detail from Figure 2. The area under IV is 0.25 from (18), the area under IJ is 0.288 from (17), and the area under SJ is 0.333 from (18′). Section 3 shows that the area to the left of IT is 0.268 from (29), the area to the left of IJ is 0.288 from (30), and the area to the left of JZ is 0.309 from (29′). The equality of the areas under IJ and left of IJ is an unimportant idiosyncrasy of this particular example.

b. 1/4 lb. of cheese to generic x_1 lb. Instead of moving from 1/4 to 1/3 lb. of cheese, if the final quantity of cheese is an arbitrary value "x_{1f}," then (17) and (18) can be generalized to

$$WTPA_{MaD}(1/4, x_{1f}) = \int_{1/4}^{x_{1f}} MaD(x_1)\,dx_1 = \int_{1/4}^{x_{1f}} \frac{dx_1}{x_1} = \ln(4x_{1f}) \qquad (19)$$

and

$$WTPA_I(1/4, x_{1f}) = \int_{1/4}^{x_{1f}} PPDD_I(x_1)\,dx_1 = \int_{1/4}^{x_{1f}} \frac{dx_1}{4x_1^2} = 1 - \frac{1}{4x_{1f}}. \qquad (20)$$

It is also possible to consider in these functions situations in which $x_1 < 1/4$. The idea would be that the consumer has already purchased, but not yet consumed, 1/4 pound of cheese, and the seller then starts buying back cheese, using its power to price-discriminate by raising the price it pays incrementally, only as the consumer becomes increasingly reluctant to sell cheese. This is called the consumer's "willingness to accept" monetary compensation for the loss of cheese, "WTA." Since (19) and (20) represent WTP if $x_1 > 1/4$ and WTA if $x_1 < 1/4$, it makes sense to denote them by $WTPA$.

Figure 4 graphs $WTPA_{MaD}$ and $WTPA_I$. (Subsection 2e will explain the graph's "CV" notation). As x_{1f} goes to infinity, $WTPA_I$ (willingness to pay) goes to one from below, because the consumer only has one dollar left after buying 1/4 lb. cheese at its price of four.

The relative error of using $WTPA_{MaD}$ instead of $WTPA_I$ is $(WTPA_{MaD} - WTPA_I)/WTPA_I$. This is graphed as the solid line in Figure 5.

Note that if x_1 rises from 1/4 to 1/3 lb., the consumer facing a perfect-price-discriminating firm is prevented from enjoying any increase in utility. Utility stays at the level it had at point I. If x_1 rises from 1/4 lb., the consumer facing a perfect-price-discriminating firm travels along $PPDD_I$. Hence $PPDD_I$ has another important interpretation: along it, x_1 changes but utility stays at the level enjoyed at point I.

c. The second alternative. The next question is, given that the value of increasing the quantity of cheese from 1/4 to 1/3 lb., from (18), is \$0.25 using the *PPDD* curve passing through the *initial* point I in Figures 2 and 3, what would the value be if we measured it using a *PPDD* curve passing through the *final* point, J? Such a *PPDD* curve would keep utility constant at the level enjoyed at point J, not at point I. Valuation along such a curve will not be \$0.25 because, as Figure 3 shows, \$0.25 is the area to below IV and J lies away from that area.

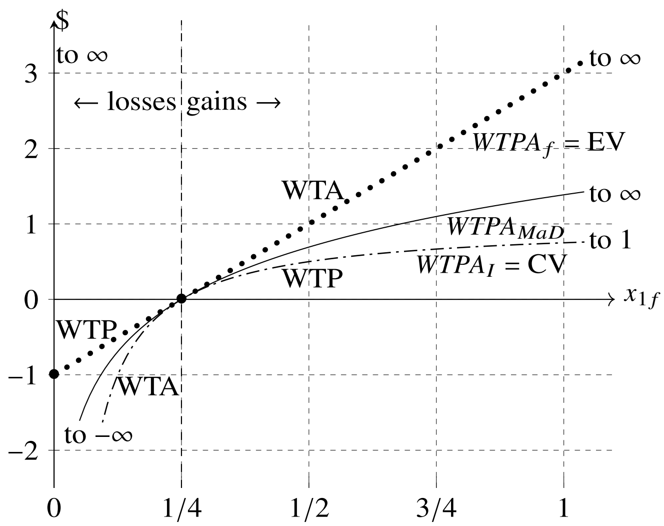

Figure 4. Dot-dashed line: $WTPA_I$ from (20). Along this line, losses are WTA and gains are WTP, both measured as leaving from $x_1 = 1/4$. Solid line: $WTPA_{MaD}$ from (19). Dotted line: $WTPA_f$ from the Appendix's (20′). Along this line, losses are WTP and gains are WTA, both measured as returning to $x_1 = 1/4$. WTP is always limited by this consumer's income, which here is the \$1 left after purchasing the first 1/4 lb. of cheese. WTA is limited by no such constraint, and is larger in absolute value than WTP.

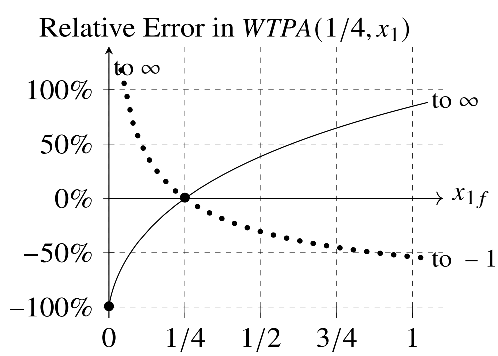

Figure 5. Smooth line: Relative error of using $WTPA_{MaD}$ instead of $WTPA_I$. Dotted line: from the Appendix, the relative error of using $WTPA_{MaD}$ instead of $WTPA_f$.

To construct a *PPDD* curve through J, "$PPDD_J$," start at J and then suppose, as discussed briefly after (20), that the firm starts buying back cheese, using its power to price-discriminate by raising the price it pays incrementally, only as the consumer becomes increasingly reluctant to sell cheese. The details of working out this point J ($p_1 = 3$, $x_1 = 1/3$) reference level of utility are in the Appendix; most of the results simply entail changing 4's to 3's in the calculations we did for point I ($p_1 = 4$, $x_1 = 1/4$). The result is the $PPDD_J$ curve shown as dotted in Figures 2 and 3.

Changing the reference level of utility from I to J increases our value measurements because $PPDD_J$ lies above $PPDD_I$. From (18′) of the Appendix, the consumer has to be paid \$0.333, the area below SJ, to accept the move from 1/3 lb. down to 1/4 lb. (maintaining the utility level achieved at point J). This is larger than \$0.25, the area below IV, which is the most the consumer would be willing to pay for an increase in cheese from 1/4 to 1/3 lb. (maintaining the utility level of point I). The income effect thus means that even for this single consumer, there is no unique answer to the question "what is the value of the 1/12 lb. of cheese separating 1/4 lb. and 1/3 lb.?"

Elaborating on this, a consumer starting at J has a higher utility than one starting at I (because at J the price of cheese is lower). Facing this better-situated consumer starting at J, the price-discriminating seller has to pay 33.3¢ – 25.0¢ = 8.3¢ more to move this consumer from $x_1 = 1/3$ down to $x_1 = 1/4$ than the seller would receive in exchange for moving this consumer up from $x_1 = 1/4$ to $x_1 = 1/3$. So the monetary value the consumer places on the prospect of moving from point I to point J, which has an extra 1/12 pound of cheese, is 4.1¢ smaller than the monetary value the consumer places moving from point J to point I, giving up the already-acquired 1/12 pound of cheese. In behavioral economics this is called an "endowment effect " or a "status quo effect," but here it is a consequence of a perfectly rational consumer's income effect. One is reminded of the maxim "What you see depends on where you stand": the consumer's perception of the value of 1/12 pound change in the amount of cheese consumed depends on whether the consumer is at point I or at point J, or, equivalently, whether the perfect price discrimination holds the consumer to point I's utility or to point J's utility.[8]

[8] The maxim's source is "What you see and what you hear depends a great deal on where you are standing" (C.S. Lewis, *The Magician's Nephew*); cf. "Miles' Law," "Where you stand depends on where you sit."

Having derived $PPDD_I$ and $PPDD_J$ as alternative ways to measure the value of the $1/12$ lb. of cheese between $x_1 = 1/4$ and $1/3$ raises the possibility of using *PPDD* curves anchored at points on the Marshallian demand curve other than I and J, especially with anchoring points between I and J, to form yet more alternative ways of measuring the value of that $1/12$ lb. of cheese. However, such alternative values are not very useful in measuring the value of that $1/12$ lb. because the consumer is going to be either at $x_1 = 1/4$ or at $x_1 = 1/3$, not in between. *PPDD* curves anchored at points other than I or J are, though, extremely useful for a different purpose, namely to value "final" quantities of cheese other than $1/3$ lb. The appendix has the details of how to generalize $WTPA_J$ to utility reference levels other than the one corresponding to point J, so that one could measure the value of an increase from $x_1 = 1/4$ lb. to $x_{1f} = 1/2$ lb. using $p_1 = 2$, $x_1 = 1/2$ (Figure 2's point B) as the reference point, or one could measure the value of an increase from $x_1 = 1/4$ to $x_{1f} = 1$ using $p_1 = 1$, $x_1 = 1$ (Figure 2's point L) as the reference point. The appendix calls this generalized measure "$WTPA_f$" because it represents the value of moving from the initial quantity of one-quarter to the *final* quantity calculated using the area under the *PPDD* curve of the final quantity, instead of using $PPDD_I$ which is the *PPDD* curve of the initial quantity $x_1 = 1/4$. Although in contexts such as "$PPDD_I$" the subscript refers to the point I, one could simultaneously think of the subscript "I" as meaning "initial," in contrast to the subscript "f" which means "final."

$WTPA_f$ is graphed as a dotted curve in Figure 4 (Subsection 2e will explain the "EV" designation). In that figure, $WTPA_{MaD}$ lies between $WTPA_I$ and $WTPA_f$. For values of $x_{1f} > 1/4$, $WTPA_f$ measures the willingness to accept compensation for moving from $x_{1f} > 1/4$ back to $x_1 = 1/4$. For values of $x_{1f} < 1/4$, $WTPA_f$ measures the willingness to pay to moving from $x_{1f} < 1/4$ back to $x_1 = 1/4$. This is why the WTP and WTA designations for $WTPA_f$ are the opposite of those for $WTPA_I$.

The dotted line in Figure 5 shows the relative error of using $WTPA_{MaD}$ instead of $WTPA_f$, and near $x_{1f} = 1/4$ that relative error is near zero, just like the relative error of using $WTPA_{MaD}$ instead of $WTPA_I$.

d. Flaws in the more correct quantity change valuation methods. Although $WTPA_I(1/4, 1/3) = \$0.25$ ((18)) and $WTPA_J(1/4, 1/3) = \$0.333$ ((18′)) are improvements over $WTPA_{MaD}$, the fact that they are different can sometimes pose problems. Suppose the government is contemplating increasing this consumer's cheese consumption from $1/4$ to $1/3$ lb. If this occurs, an extra $1/12$ pound of cheese will be produced. Suppose production of that extra cheese costs \$0.24 and generates pollution that has a social cost of less than \$0.01 (setting aside for now how this social cost is determined). Then its total social cost is less than \$0.25. This is less than either *WTPA*, so society would want this $1/12$ pound of cheese to be produced. On the other hand, if the pollution had a social cost of more than \$0.10, then the total social cost of the extra cheese would be more than \$0.34, which is more than either *WTPA*, so society would not want this $1/12$ pound of cheese to be produced. Finally, suppose the pollution has a social cost between \$0.01 and \$0.0933, for example, \$0.05. Then the total social cost of the extra cheese is \$0.29, which lies between $WTPA_I$ and $WTPA_J$. If the extra cheese already existed, the more relevant of the two measures would be $WTPA_J$, and since that is the higher of the two measures, the net social value of the cheese would be positive, so it was good that the extra cheese was produced. However if the extra cheese does not exist, the more relevant of the two measures would be $WTPA_I$, and since that is the lower of the two measures, the net social value of the cheese would be negative, so it was good that the extra cheese was not produced.[9]

In the preceding paragraph, if $x_1 = 1/4$, society ranked $x_1 = 1/4$ higher than $x_1 = 1/3$, whereas if $x_1 = 1/3$, society ranked $x_1 = 1/3$ higher than $x_1 = 1/4$. If we had used a utility function which, unlike our Cobb-Douglas form $u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$, resulted in x_1 (cheese) being an inferior good, an even stranger situation can occur. In that case it would turn out that, as discussed before (6) and after (16), Figure 3's $PPDD_I$ and $PPDD_J$ curves would be flatter than the Marshallian demand curve, as shown in Figure 6. That would make $WTPA_I(1/4, 1/3)$ *more* than $WTPA_J(1/4, 1/3)$. For example, the $WTPA_I$ might be 35¢ and the $WTPA_J$ might be 25¢. Suppose the social cost of this $1/12$ lb. cheese is 30¢. Then if society was at 'i' and used 'i' as its reference point, by moving to 'j' the society would experience a net gain of 35¢ – 30¢. However upon reaching 'j,' now using 'j' as its reference point, by moving back to 'i' the society would experience a net gain of 30¢ – 25¢. This possibility was first pointed out by Scitovsky (1941) and is known as a Scitovsky reversal.[10] Society would be utterly unable to rank $1/4$ versus $1/3$ lb. of cheese in this situation, always believing whichever situation was the current one to be the worse one. Scitovsky reversals are probably uncommon.[11]

[9] For simplicity we have ignored income effects of the pollution victims, but see Section 5.

[10] Mishan (1971 fn. 33), however, distinguishes between this type of reversal and a reversal in an Edgeworth Box with production, reserving the term Scitovsky reversal for the latter.

[11] Just, Schmitz and Zerbe (2013) show that in economies with production, Scitovsky reversals can occur even if all goods are normal.

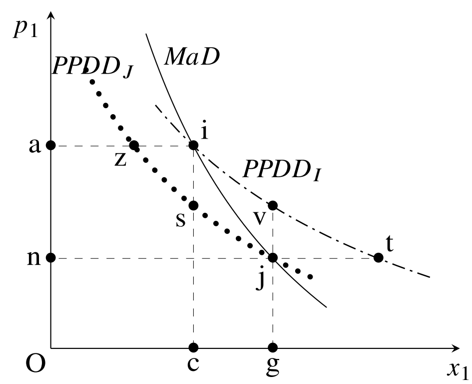

Figure 6. Demands with an alternative utility function that makes x_1 an inferior good. Points are analogous to points in other graphs but are in different locations. The value of extra cheese (the x_1 between 'c' and 'g') if it exists is 'sjgc'; if it does not exist, its value is 'ivgc.' For Section 3: if p_1 = a, the value of reducing price from 'a' to 'n' is 'aitn'; if p_1 = n, the value of increasing price from 'n' to 'a' is 'azjn.'

The difficulties discussed in this section cannot occur if one uses as one's only measure of quantity valuation $WTPA_{MaD}$. This is a drawback of using $WTPA_{MaD}$, not an advantage.

e. Modern Terminology. Although the idea of the $PPDD_I$ and $PPDD_J$ lines and their derivations are new, it turns out that the they are what are called in advanced treatments "compensated" or "Hicksian" demand curves because Hicksian demand curves are defined as keeping utility constant, and that is what the *PPDD* lines do. This point was made by Maxwell (1959), although his analysis did not extend beyond our Figure 1. We relegate the formal proof to a footnote because it involves introducing modern concepts that it has been a key goal of this paper to avoid.[12]

Above we introduced two measures for the value of an increase in x_1 from 1/4 to 1/3 lb., one being the answer $PPDD_I$ gives, $WTPA_I$, and the

[12]Claim: the Hicksian ("compensated") demand function at the original level of utility, $h_1(p_1, p_2, u_0)$ is (16), that is, $PPDD_I$. Proof: Given the standard consumer's problem of maximizing $x_1^{1/2}x_2^{1/2}$ subject to (1), the indirect utility function turns out to be $v = m/(2\sqrt{p_1 p_2})$, the initial level of utility at $m = 2$ and $p_1 = 4$ is $u_0 = 1/(2\sqrt{p_2})$, the expenditure function is $e = 2u\sqrt{p_1 p_2}$, and from Shephard's Lemma the Hicksian demand function for x_1 at u_0 is $h_1 = u_0\sqrt{p_2/p_1} = 1/(2\sqrt{p_1})$, i.e. (16).

The Appendix's footnote 26 shows that $PPDD_J$ is the Hicksian demand function for the "new" level of utility.

other being the answer $PPDD_J$ or similar curves give, $WTPA_f$. The first one, using the original level of utility (point I) as the reference point, is what advanced treatments call the "Compensating Variation," which they define as the decrease in income which, if change of any sort—in our case, cheese increasing from 1/4 to 1/3 lb.—*does occur,* would leave *utility at its original level*. The second one, using the new level of utility (point J) as the reference point, is the same as what advanced treatments call the "Equivalent Variation," which they define as the increase in income which, if change of any sort—again, cheese increasing from 1/4 to 1/3 lb.—*does not occur,* would put *utility at the level it would have had if the change had occurred*. Compensating Variation is abbreviated CV and equivalent variation is abbreviated EV. For a change which is a *ceteris paribus* increase in utility, such as ours when $x_{1f} > 1/4$, both CV and EV are positive, and if $x_{1f} < 1/4$ they would both be negative. (For different sign conventions for CV and EV see Hanemann (1991 fn. 6).)

3. Consumer Surplus: Valuing Price Changes

a. \$4/lb. of cheese to generic p_1/lb. *WTPA* as derived above measures the benefit of a quantity change, such as the area IVGC measuring the willingness to pay to move from 1/4 to 1/3 lb. of cheese in Figure 3. In this section we will be concerned with valuing price changes not quantity changes. $PPDD_I$ and $PPDD_J$ are still used for this, and some of this section's discussions are extremely similar to discussions in Section 2, but there are some differences. For example, the willingness to pay to change from \$4 to \$3 lb. in Figure 3 is not the area under IV but instead the area under IT. The arguments of the *WTPA* function should indicate whether it is valuing a change in quantity or a change in price. For example, describing the area under IT, that is, ITDC:

$$WTPA_I(\$4, \$3) = \int_{PPDD_I(\$4)}^{PPDD_I(\$3)} PPDD_I(x_1)\, dx_1 = \int_{1/4}^{1/\sqrt{12}} \frac{dx_1}{4x_1^2} = 1 - \frac{\sqrt{3}}{2}\,.$$

The geometry is straightforward; the limits of integration are straightforward, with $PPDD_I$ giving quantity demanded as a function of price; but the function in the integrand is actually the inverse of $PPDD_I$ because it gives price as a function of quantity. It follows that for a generic demand curve "$g(p)$" (whether $g(p)$ is $MaD(p)$ or $PPDD_I(p)$ or something else), the "valuation of a price change from p_1 to p_2 according to g" is "the area under g between $g(p_1)$ and $g(p_2)$," properly written as

$$WTPA_g(p_1, p_2) = \int_{g(p_1)}^{g(p_2)} g^{-1}(x_1)\, dx_1\,. \tag{21}$$

The willingness to pay to change from \$4 to \$3/lb. measures the gross benefit of the price change, but not its benefit net of its cost. Another difference between this section and the previous one is that here we would like to move from studying only the *WTPA* of a price change to a study of surplus, which measures the excess of value over "expenditure under uniform pricing." We used nonuniform pricing only to get a correct way to measure value, not because the consumer actually has to face nonuniform pricing; he may not, and the notion of 'surplus' is relevant to the context in which he does not. (If he faced perfect price discrimination for all units of cheese he consumed then his surplus would be zero.)

The gross value of x_1 in Figure 7 is the area under the inverse demand curve $g^{-1}(\hat{x}_1)$, which is the sum of the figure's shaded and hatched areas, and is equal to $\int_0^{x_1} g^{-1}(\hat{x}_1)\, d\hat{x}_1$. The zero lower limit of integration represents quantity demanded at an infinite price, and the upper limit of integration is $g(p_1)$, so the gross value can be rewritten as $\int_{g(\infty)}^{g(p_1)} g^{-1}(\hat{x}_1)\, dx_1$, which according to (21) is $WTPA_g(\infty, p_1)$ (the gross value, as expected). Surplus would then be the gross value, $WTPA_g(\infty, p_1)$, minus the amount paid under uniform pricing, which is $p_1\, g(p_1)$, the hatched area in the figure. So surplus is the shaded area in the figure. This explains (22) below. To obtain (23) use $g(p_1) = x_1$ twice, then rewrite x_1 as $\int_0^{x_1} 1\, d\hat{x}_1$ since the latter is $\hat{x}_1|_0^{x_1} = x_1 - 0$. Equation (24) is the shaded area in the figure, whose caption explains why that area is also equal to (25).

$$\begin{aligned}
\text{surplus} &= WTPA_g(\infty, p_1) - p_1\, g(p_1) = \int_{g(\infty)}^{g(p_1)} g^{-1}(\hat{x}_1)\, d\hat{x}_1 - p_1\, g(p_1) && (22)\\
&= \int_0^{x_1} g^{-1}(\hat{x}_1)\, d\hat{x}_1 - p_1\, x_1 = \int_0^{x_1} g^{-1}(\hat{x}_1)\, d\hat{x}_1 - p_1 \int_0^{x_1} d\hat{x}_1 && (23)\\
&= \int_0^{x_1} [g^{-1}(\hat{x}_1) - p_1]\, d\hat{x}_1 && (24)\\
&= \int_{p_1}^{\infty} g(\hat{p}_1)\, d\hat{p}_1\,. && (25)
\end{aligned}$$

For the rest of this section we will measure surpluses using (25), that is, as areas *to the left* of demand curves. Figure 3 can be used to illustrate this result in the case when g is $PPDD_I$. There $p_{1f} = 3$; $WTPA_I(\infty, p_{1f})$ is the area under $PPDD_I$ to the left of TD; $p_{1f}\, g(p_{1f})$ is $3 \cdot 1/\sqrt{12}$, which is NTDO; and surplus is "the area under $PPDD_I$ to the left of TD" minus NTDO, which indeed is the area to the left of $PPDD_I$ above NT.

When g is the Marshallian demand curve MaD we will call the surplus "Marshallian consumer surplus" or $MaCS$, and when g is $PPDD_I$ we will call the surplus "actual consumer surplus measured from I" or ACS_I.

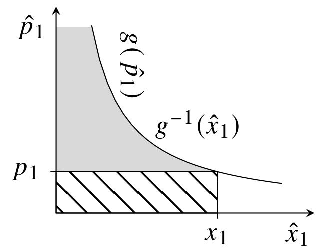

Figure 7. The gross valuation of x_1 is the sum of the shaded and hatched areas. The net, or surplus, valuation is only the shaded area. The shaded area can be expressed either as the area under "$g^{-1}(\hat{x}_1) - p_1$" from zero to x_1 or as the area to the left of $g(\hat{p}_1)$ from p_1 to infinity. Note that $g(p_1) = x_1$.

From (25), the *change* in surplus due to a price change from p_{1i} to p_{1f} would be

$$\Delta(\text{surplus}) = \int_{p_{1f}}^{\infty} g(p_1)\,dp_1 - \int_{p_{1i}}^{\infty} g(p_1)\,dp_1 = \int_{p_{1f}}^{p_{1i}} g(p_1)\,dp_1\,, \tag{26}$$

which is the way we will measure surplus changes for the rest of this section: the area to the left of the *MaD* or *PPDD* curve between the initial and final price. For example, in Figure 3, $\Delta ACS_I(\$4,\$3)$ is the area to the left of IT, namely AITN.[13]

There is a relationship between change in surplus and *WTPA*. Using $PPDD_I$, $\Delta ACS_I(\$4,\$3)$, and $WTPA_I(\$4,\$3)$ and Figure 3 to illustrate:

$$\begin{aligned}\Delta ACS_I(\$4,\$3) - WTPA_I(\$4,\$3) &= \text{AITN} - \text{ITDC}\\ &= (\text{AIEN} + \text{ITE}) - (\text{ITE} + \text{ETDC})\\ &= \text{AIEN} - \text{ETDC} = \text{AIEN} + \text{NECO} - \text{NECO} - \text{ETDC}\\ &= \text{AICO} - \text{NTDO} = 4 \cdot \tfrac{1}{4} - 3 \cdot \tfrac{1}{\sqrt{12}}\\ &= p_{1i} \cdot PPDD_I(p_{1i}) - p_{1f} \cdot PPDD_I(p_{1f})\,. \end{aligned} \tag{27}$$

This illustrates the general relationship between change in surplus and *WTPA*, which is (28):

$$\begin{aligned}\Delta(\text{surplus}) &= WTPA_g(\infty, p_{1f}) - p_{1f}g(p_{1f}) - WTPA_g(\infty, p_{1i}) + p_{1i}g(p_{1i})\\ &= \int_0^{g(p_{1f})} g^{-1}(x_1)\,dx_1 - p_{1f}g(p_{1f}) - \int_0^{g(p_{1i})} g^{-1}(x_1)\,dx_1 + p_{1i}g(p_{1i})\end{aligned}$$

[13]Because we will only be interested in surplus changes, not in the absolute amount of surplus, we need not be disturbed by cases when surplus, from (25), is infinite.

$$
\begin{aligned}
&= \int_{g(p_{1i})}^{g(p_{1f})} g^{-1}(x_1)\,dx_1 - p_{1f}g(p_{1f}) + p_{1i}g(p_{1i}) \\
&= WTPA_g(p_{1i}, p_{1f}) + p_{1i}g(p_{1i}) - p_{1f}g(p_{1f})\,. \qquad (28)
\end{aligned}
$$

If the price falls from four to three the change in actual consumer surplus, the area to the left of the dashed-dotted line IT in Figure 3, is (26) with $g(p_1) = 1/\sqrt{4p_1}$:

$$
\Delta ACS_I(\$4, \$3) = \int_3^4 \frac{dp_1}{\sqrt{4p_1}} = 2 - \sqrt{3} \approx 0.268\,. \qquad (29)
$$

By contrast conventionally-measured consumer surplus changes by the area to the left of the solid line IJ, which is $g(x_1) = 1/p_1$, so by (26),

$$
\Delta MaCS(\$4, \$3) = \int_3^4 \frac{dp_1}{p_1} = \ln\frac{4}{3} \approx 0.288\,, \qquad (30)
$$

a difference of $(0.288 - 0.268)/0.268 \approx 7\%$.

If price changed from \$4/lb. to a generic level of p_{1f}, the monetary value correctly measured as in (29), "ΔACS_I," is

$$
\Delta ACS_I = \int_{p_{1f}}^4 \frac{dp_1}{\sqrt{4p_1}} = 2 - \sqrt{p_{1f}}\,. \qquad (31)
$$

This holds even for levels of p_{1f} above \$4/lb. ΔACS_I for different values of p_{1f} is graphed in Figure 8. Note that at $p_{1f} = 0$, which is in the WTP not WTA range, $\Delta ACS_I = 2$, which is the maximum WTP because it is the consumer's entire income.

The corresponding change in Marshallian consumer surplus, "$\Delta MaCS$," is

$$
\Delta MaCS = \int_{p_{1f}}^4 \frac{dp_1}{p_1} = \ln(4/p_{1f})\,. \qquad (32)
$$

$\Delta MaCS$ is also graphed in Figure 8. The relative error resulting from using $\Delta MaCS$ instead of ΔACS_I is

$$
\frac{\Delta MaCS - \Delta ACS_I}{\Delta ACS_I} = \frac{\ln(4/p_{1f})}{2 - \sqrt{p_{1f}}} - 1\,. \qquad (33)
$$

This is illustrated by the solid curve of Figure 9. For values of p_{1f} near four, the relative error is small. This is related to the famous "Consumer's Surplus Without Apology" paper of Willig (1976). On the other hand, from (33),

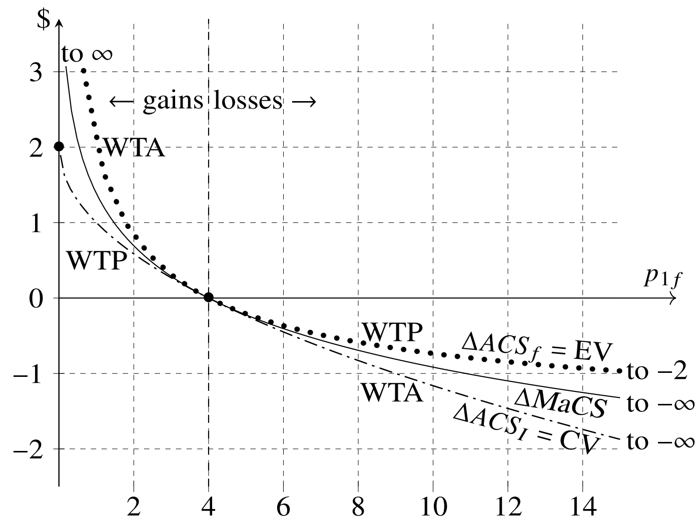

Figure 8. Dot-dashed line: ΔACS_I from (31); in Subsection 3d its equivalence to CV is explained. Along this line, losses are WTA and gains are WTP, both measured as leaving from $p_1 = 4$. Solid line: $\Delta MaCS$ from (32). Dotted line: ΔACS_f from the Appendix's (31′); in Subsection 3d its equivalence to EV is explained. Along this line, losses are WTP and gains are WTA, both measured as returning to $p_1 = 4$. WTP is always limited by this consumer's income, which here is \$2. WTA is limited by no such constraint, and is larger in absolute value than WTP.

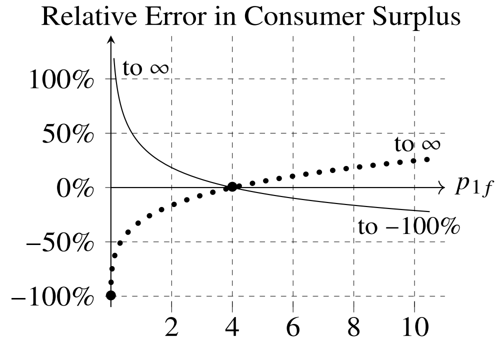

Figure 9. Smooth line: Relative error, from (33), of using Marshallian consumer surplus instead of the correct area measure, ΔACS_I. Dotted line: from the Appendix, the relative error, from (33′), of using Marshallian consumer surplus instead of the correct area measure, ΔACS_f, with reference prices other than four.

as p_{1f} becomes small the relative error from using Marshallian consumer surplus is literally unbounded,

$$\lim_{p_{1f}\to 0} \frac{\Delta MaCS - \Delta ACS_I}{\Delta ACS_I} = \frac{\infty}{2} - 1 = \infty\,,$$

since in this limit $\Delta MaCS$ goes to infinity but the correct measure ΔACS_I goes only to two (this consumer's maximum WTP). As p_{1f} goes to infinity the limit of relative error (33) is −100%.[14]

In examples more complicated than ours, using the Marshallian measure $\Delta MaCS$ can give even worse results than in Figure 8. In our example, $\Delta MaCS$ is well-defined, but in examples having multiple price changes, $\Delta MaCS$ is path-dependent, as discussed by Hotelling (1938 p. 247). In our example, $\Delta MaCS$ always has the same sign as ΔACS_I, but in examples having Giffen goods, $\Delta MaCS$ can have the opposite (i.e., wrong) sign (Facchini, Hammond and Nakata 2001).

b. Deadweight loss of a price change. There is one further flaw of using areas defined by the Marshallian demand curve that can be illustrated even in our simple model. It is the one first discussed by Hausman (1981 p. 672–3), namely that is does a poor job in measuring deadweight loss. To recall, given a generic demand curve $D(p)$ or its perfect-price-discrimination counterpart, a *PPDD* curve, as shown in Figure 10, the deadweight loss of a rise in price from an initial price P_0 to "P_m" is the area to the left of the demand curve

[14]This can be proven by applying L'Hôpital's Rule to the first term of (33).

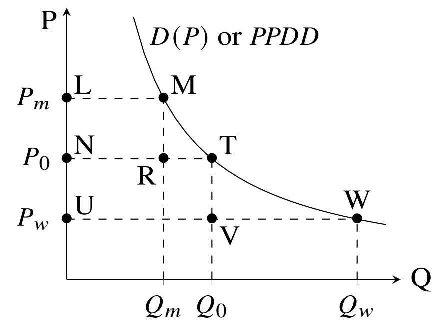

Figure 10. Deadweight loss decreases when a price drops: for example, TVW, that is, to the right of the original quantity Q_0 and to the left of the demand curve. Deadweight loss increases when a price rises: for example, MRT, that is, to the right of the new quantity Q_m and to the left of the demand curve.

MT and to the right of the *new* quantity Q_m. When price rises from P_0 to P_m, consumer surplus shrinks by the area to the left of MT but the rectangle LMRN is a mere transfer, in this case to the firm, which now receives it as part of the payment for Q_m. Only the approximately-triangular area MTR is a net-to-society loss in value. Similarly, the amount by which deadweight loss shrinks accompanying a fall in price from P_0 to "P_w" is the area to the left of the demand curve TW and to the right of the *original* quantity Q_0. When price falls from P_0 to P_w, consumer surplus expands by the area to the left of TW but the rectangle NTVU is a mere transfer, in this case from the firm, which used to receive it as part of the payment for Q_0, to the consumers. Only the approximately-triangular area TWV is a net-to-society gain in value.

Mathematically expressed, then, the deadweight loss when price changes from P_0 to P' is

$$\begin{aligned} DWL &= \int_{P_0}^{P'} \Big[D(P) - D\big(\max(P_0, P')\big) \Big] \, dP \\ &= \int_{P_0}^{P'} D(P) \, dP - (P' - P_0) \; D\big(\max(P_0, P')\big) \, . \end{aligned} \tag{34}$$

It follows that in Figures 2 and 3, a correct deadweight loss measure can be obtained by using (34) with demand curve $PPDD_I$ (equation (16)), that is,

$$DWL_I = \int_4^{p_{1f}} \frac{dP}{\sqrt{4P}} - \frac{p_{1f} - 4}{\sqrt{4 \max(4, p_{1f})}}$$

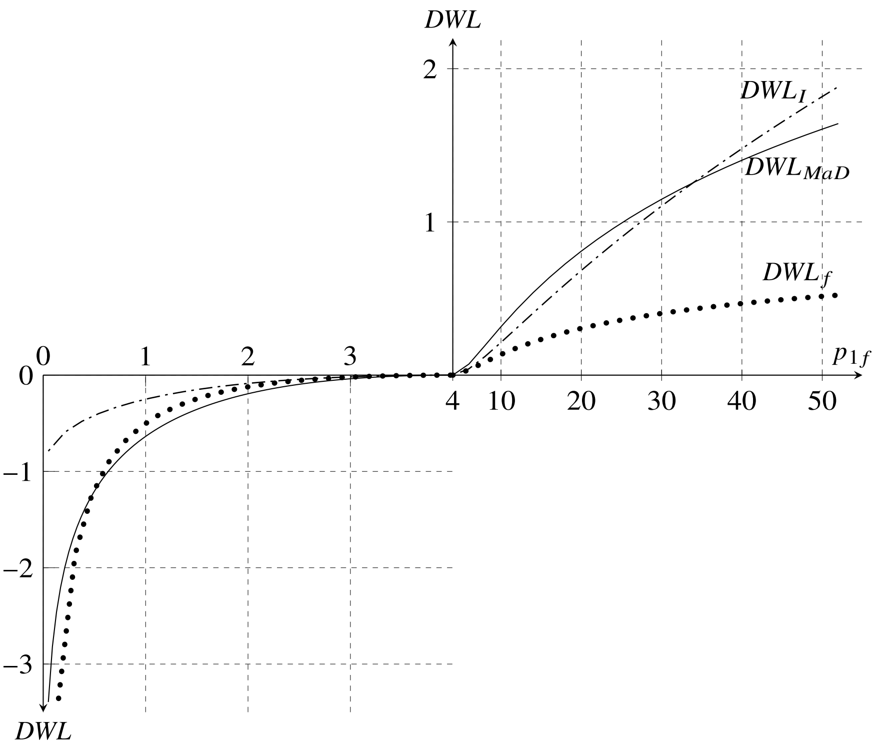

Figure 11. Dot-dashed line: DWL_I from (35), Solid line: DWL_{MaD} from (36). Dotted line: DWL_f from the Appendix's (49). (Different scales in use for $p_{1f} < 4$ and $p_{1f} > 4$.)

$$= \sqrt{p_{1f}} - \sqrt{4} - \frac{p_{1f} - 4}{\sqrt{4 \max(4, p_{1f})}} . \tag{35}$$

An incorrect measure is (34) with the Marshallian demand curve (2):

$$DWL_{MaD} = \int_4^{p_{1f}} \frac{dP}{P} - \frac{p_{1f} - 4}{\max(4, p_{1f})} = \ln\left(\frac{p_{1f}}{4}\right) - \frac{p_{1f} - 4}{\max(4, p_{1f})} . \tag{36}$$

These are graphed in Figure 11. When $p_{1f} = 4$, both DWL_I and DWL_{MaD} are zero, so at that point the absolute error from using the incorrect rather than a correct measure of DWL is zero. However the relative error, $(DWL_{MaD} - DWL_I)/DWL_I$, in the limit as p_{1f} goes to four is 100%, as is shown as the solid line in Figure 12. Furthermore, the relative error is not close to zero except around the uninteresting value of $p_{1f} \approx 34$. For modest deviations

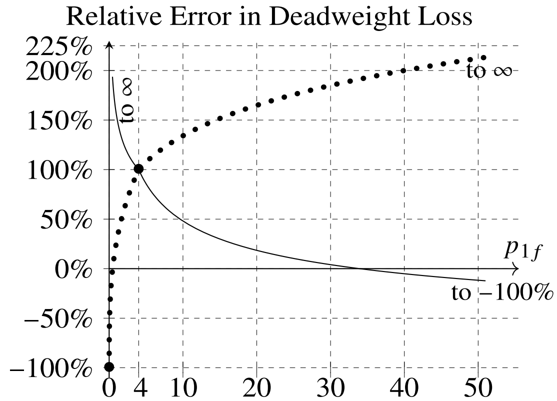

Figure 12. Smooth line: Relative error of using deadweight loss calculated from the Marshallian demand curve, instead of DWL_I, as price moves away from \$4/lb. Dotted line: as described in the Appendix, the relative error of using deadweight loss calculated from the Marshallian demand curve, instead of DWL_f, as price moves back to \$4/lb.

of price from \$4/lb. one could not use this Marshallian demand curve to measure 'deadweight loss' without apology. We will revisit this case in the next subsection.

This concludes our analysis starting from the initial point of \$4/lb., point I in Figures 2 and 3.

c. The second alternative. The next question is, given that the value of decreasing the price of cheese from \$4 to \$3 is \$0.268 using the *PPDD* curve passing through the initial point I in Figures 2 and 3, what would the value be if we measured it using the *PPDD* curve passing through the final point, J, which we derived in Subsection 2c? Changing the reference level of utility from I to J increases our value measurements because $PPDD_J$ lies above $PPDD_I$. The consumer has to be paid \$0.309, the area to the left of JZ ((29′)) to accept the move from \$3/lb. up to \$4/lb. (maintaining the utility level achieved at point J). This is larger than \$0.268, the area to the left of IT, which is the most the consumer would be willing to pay for a reduction in price from \$4 to \$3 (maintaining the utility level of point I). The income effect thus means that even for this single consumer, there is no unique answer to the question "what is the value of a price difference between \$4 and \$3?"

Elaborating on this, a consumer starting at J has a higher utility than one starting at I (because at J the price of cheese is lower). Facing this better-situated consumer starting at J, the price-discriminating seller has to pay 30.9¢ – 26.8¢ = 4.1¢ more to move this consumer from $p_1 = 3$ to $p_1 = 4$ than the seller would receive in exchange for moving this consumer from $p_1 = 4$ to $p_1 = 3$. Said another way, the monetary value the consumer places on the prospect of a price reduction from \$4 to \$3 is 4.1¢ smaller than the monetary cost the consumer places on a price increase from \$3 to \$4. As in Subsection 2c, this consequence of the income effect is related to behavioral economics' endowment effect.

In Subsection a above, price changes not just from $p_1 = 4$ to 3 but also from four to many other prices, as illustrated by ΔACS_I, DWL_I, and their error measures. Analogously, the appendix shows how to generalize ΔACS_J to utility reference levels other than the one corresponding to point J, so that one could measure the value of a fall from $p_1 = 4$ to $p_{1f} = 2$ using $p_1 = 2$, $x_1 = 1/2$ (Figure 2's point B) as the reference point, or one could measure the value of a fall from $p_1 = 4$ to $p_{1f} = 1$ using $p_1 = 1$, $x_1 = 1$ (Figure 2's point L) as the reference point. The appendix calls this generalized measure "ΔACS_f" because it represents the value of moving from the initial price of four to the *final* price calculated using the area to the left of *PPDD* curve of the final price, instead of using $PPDD_I$ which is the *PPDD* curve of the initial price $p_1 = 4$. Although in contexts such as "$PPDD_I$" the subscript refers to the point I, one could simultaneously think of the subscript "I" as meaning "initial," in contrast to the subscript "f" which means "final."

ΔACS_f is graphed as a dotted curve in Figure 8. In that figure, the change in Marshallian consumer surplus lies between ΔACS_I and ΔACS_f. This underlies Willig's observation that Marshallian consumer surplus is an approximation of the correct measurements. The dotted line in Figure 9 shows the relative error of using $\Delta MaCS$ instead of ΔACS_f, and near $p_{1f} = 4$ that relative error is near zero, just like the relative error of using $\Delta MaCS$ instead of ΔACS_I.

In Figure 11, the value of the dotted line at $p_{1f} = 3$ represents deadweight loss measured along $PPDD_J$. The other points on the dotted line represent a generalization, deadweight loss "DWL_f" (equation (49)) measured along curves like $PPDD_J$ but ending at prices other than $p_{1f} = 3$. In contrast to Figure 8, in Figure 11, for values of p_{1f} between approximately 0.47 and 34, deadweight loss measured by the Marshallian demand curve is larger in absolute value than *either* of the two correct deadweight loss measures. Auerbach (1985 pp. 71–2) explains why. In Figure 3, the deadweight loss measured along *MaD*, which is flatter than either *PPDD* curve, is IJE. This is considerably greater than the deadweight loss with $PPDD_I$, which is ITE, or with $PPDD_J$, which is ZJT, since these *PPDD* curves are steeper than MaD when x_1 is a normal good. This in turn explains why, in Figure 12 (where the dotted line shows the relative error of using DWL_{MaD} instead of DWL_f), the relative error in using DWL_{MaD} near $p_{1f} = 4$ is far from zero, both when comparing with DWL_I, as observed earlier, and when comparing with DWL_f. In our example, while it is true that the Marshallian demand curve generates small *absolute* errors in deadweight loss near $p_{1f} = 4$ as shown in Figure 11, it generates very large *relative* errors in deadweight loss there and everywhere else except near the uninteresting points of $p_{1f} \approx 0.47$ and 34.

If x_1 were an inferior good, then as discussed after (16) in connection with Figure 6 and footnote 17 below, the *PPDD* curves would be flatter than MaD, making deadweight loss measured by the Marshallian demand curve smaller in absolute value than either of the two correct deadweight loss measures, at least for small price changes.

d. Modern Terminology. Above we introduced two measures for the change in actual consumer surplus of a fall in p_1 from \$4 to \$3/lb., one being the answer $PPDD_I$ gives, ΔACS_I, and the other being the answer $PPDD_J$ gives, ΔACS_f. It can be shown that the first one, using the original level of utility (point I) as the reference point, is the same as what advanced treatments call the "Compensating Variation (of a price change)," which they define as the amount of money one has to subtract from income m when prices are at their new level (namely prices $p_1 = 3$ and the unchanging p_2 in our example) to make utility equal to what it was when prices were at their old level (namely $p_1 = 4$ and our unchanging p_2) and income was m. It is abbreviated CV, and is a specific application to price changes of the general idea of CV introduced in Section 2. It can be shown that the second one, using the new level of utility (point J) as the reference point, is the same as what advanced treatments call the "Equivalent Variation (of a price change)," which they define as the amount of money one has to add to income m when prices are at their old level (namely $p_1 = 4$ and our unchanging p_2) to make utility equal to what it is when prices are at their new level (namely prices $p_1 = 3$ and the unchanging p_2 in our example) and income is m. It is abbreviated EV, and as with CV, EV is a specific application to price changes of the general idea of EV introduced in Section 2. The equivalence of CV and EV with the areas to the left of the constant-utility demand curves $PPDD_I$ and $PPDD_J$ may be obvious but a formal proof of the first claim is given in a footnote here[15] and a formal proof of the second claim is given

[15]The standard result (Varian (1992, p. 167, (10.2))) is that the area to the left of the Hicksian demand function at the original level of utility is compensating variation, so (29)'s

in the Appendix's footnote 27.

One consequence is that while in Section 2, CV = $WTPA_I$, in this Section, CV = $\Delta ACS_I \neq WTPA_I$, as shown in (27). The reason is that in this Section we have developed CV in the context of a surplus measure, while surpluses played no role in Section 2. Similar comments apply with respect to EV.

Compensating variation, then, is the increase in surplus when the price of cheese falls from \$4 to \$3, \$0.268 in our example, whereas equivalent variation is decrease in surplus when the price of cheese increases from \$3 to \$4, \$0.309 in our example.

e. Flaws in the more correct price change valuation methods. Compensating variation and equivalent variation replace Marshallian consumer surplus as key components underlying modern welfare economics because they give convincing, though different, ways of measuring a single individual's utility changes in terms of money. The fact that they are different can sometimes pose problems. Suppose the government is contemplating reducing the price of cheese from \$4 to \$3/lb. If this occurs, an extra $1/12$ pound of cheese will be produced. Suppose production of that extra cheese costs \$0.010 and generates pollution that has a social cost of less than \$0.258 (setting aside for now how this social cost is determined). Then its total social cost is less than \$0.268. This is less than either EV or CV, so society would want this $1/12$ pound of cheese to be produced (would want to adopt the policy of decreasing the price of cheese). On the other hand, if the pollution had a social cost of more than \$0.299, then the total social cost of the extra cheese would be more than \$0.309, which is more than either CV or EV, so society would not want this $1/12$ pound of cheese to be produced (would not want to adopt the policy of decreasing the price of cheese). Finally, suppose the pollution has a social cost between \$0.258 and \$0.299, for example, \$0.280. Then the total social cost of the extra cheese is \$0.290, which lies between CV (\$0.268) and EV (\$0.309). The net social value of the cheese is positive if it already exists, because then it would be valued at the higher number, but the net social value of the cheese is negative if it does not yet exist and so is valued at the lower number. The social value

area to the left of the dashed-dotted line should be equal to CV. To prove this, note that with a final price vector of $\mathbf{p}_1$, the definition of compensating variation is $v(\mathbf{p}_1, m_1 - CV) = u_0$. Using the results in footnote 12, the left-hand side is $(2-CV)/(2\sqrt{3p_2})$, and equating it to the u_0 of footnote 12 yields $CV = 2 - \sqrt{3}$, which indeed is (29). (Hence compensating variation is measured using the *PPDD* curve whose reference point is the utility at the initial price (and initial quantity) even though the mathematical definition of compensating variation involves only the final price—what is of overriding importance is using the original level of utility.)

is history-dependent (exhibits “hysteresis”). From another viewpoint, suppose the \$0.280 represents the cost the pollution victim personally places on the pollution (their willingness and ability to pay to not have pollution). Suppose society assigns the pollution victim the right to no pollution. Starting at point I, the consumer would value a fall in the price of cheese to \$3 (leading to him buying an extra 1/12 pound of cheese) at \$0.268, but giving \$0.010 of this to the cheese seller to cover the cost of producing the extra cheese and the rest of it to the pollution victim would be insufficient to compensate the pollution victim for the production of the extra cheese, so the price should not fall, the 1/12 extra pound of cheese would not be produced, and the consumer would remain at point I. However, suppose instead that society gave the property right to the lower price of cheese to the consumer (minus a payment to the seller) and no property right to the pollution victim. The initial situation would be at J, where the consumer would value the \$1 fall in the price of cheese (leading to him buying 1/12 pound more cheese) at \$0.309, \$0.299 after payment to the cheese seller to cover the cost of producing the extra cheese. The pollution victim would not be able to entice the consumer to give up the lower price of cheese even if they offered to pay the maximum they are willing, \$0.280, so the consumer would remain at J. The assignment of property rights thus would determine whether the price of cheese would be \$3 or \$4, and thus whether or not the extra cheese would be produced, a theme we return to in Section 5.[16]

In the preceding paragraph’s final example, if $p_1 = 4$, society ranked $p_1 = 4$ higher than $p_1 = 3$, whereas if $p_1 = 3$, society ranked $p_1 = 3$ higher than $p_1 = 4$. If we had used a utility function which, unlike our Cobb-Douglas form $u(x_1, x_2) = x_1^{1/2} x_2^{1/2}$, resulted in x_1 (cheese) being an inferior good, an even stranger situation can occur. In that case it would turn out that, as discussed after (16), Figure 3’s $PPDD_I$ and $PPDD_J$ curves would be flatter than the Marshallian demand curve ‘ij,’ as shown in Figure 6.[17]

[16]See Zerbe and Dively Section 6.8, “Benefit-Cost Analysis and the Assignment of Property Rights.” For simplicity we have ignored income effects of the pollution victims, but see Section 5. Advanced texts would describe this example by saying that the move from I to J does not pass the Hicks compensation test (an *ex ante* test) but it does pass the Kaldor compensation test (an *ex post* test). For example, see Jones (2005 pp. 10–13), who calls this an example of a “Kaldor contradiction.” Jones calls the example with a different utility function that is set out in the next paragraph of the text, where the move from the analogue of I to the analogue of J passes the Hicks test but fails the Kaldor test, a “Hicks contradiction.”

[17]Proof: using the Slutsky equation, with h denoting the Hicksian demand curve and x the Marshallian: $\partial h/\partial p = \partial x/\partial p + x\, \partial x/\partial m < \partial x/\partial p < 0$ when $\partial x/\partial m < 0$ (x is an inferior good); then $|\partial h/\partial p| > |\partial x/\partial p|$ and recall that derivatives with respect to p which are larger in absolute value will imply flatter slopes when p is graphed on the vertical axis.

That would make the CV for a fall in price from \$4 to \$3 at ‘i’ *more* than the EV for a rise in price from \$3 to \$4 at ‘j.’ For example, at ‘i’ the CV for the price fall might be 35¢ and at ‘j’ the EV for the price rise might be 25¢. Suppose the social cost of this $1/12$ lb. cheese is 30¢. Then at ‘i,’ by moving to ‘j’ the society would experience a net gain of 35¢ – 30¢. However upon reaching ‘j’, by moving back to ‘i’ the society would experience a net gain of 30¢ – 25¢. This possibility was first pointed out by Scitovsky (1941) and is known as a Scitovsky reversal.[18] Society would be utterly unable to rank a \$4 versus a \$3 price of cheese in this situation, always believing whichever situation was the current one to be the worse one. The closing comments of Subsection 2d apply here, *mutatis mutandis*.

What these monetary valuation methods show is that there is no definitive answer to a question such as how much a move between \$4/lb. and \$3/lb. of cheese is valued, even in the case of a single individual. However, these methods are certainly more satisfactory than Marshallian consumer surplus, which among other shortcomings completely obscures the fact just pointed out. Hammond and Fleurbaey (2010) reflect the modern situation:

> . . . one should be able to calculate or estimate each individual’s net benefit from any policy decision. In principle, it is usually possible even to construct a money metric measure of net benefit. This is done by finding what increase or decrease in wealth would have exactly the same effect on the individual’s welfare as the policy decision being contemplated, provided that private good prices and public good quantities remained fixed at their status quo values. It is not done, except possibly very inaccurately, by calculating consumer surplus based on the area under an uncompensated Marshallian demand curve. For details, see Hammond (1994) or Becht (1995), amongst others. The measure that results is closely related to Hicks’ equivalent variation. It tells us how much each particular individual gains or loses from a policy change, which is immensely valuable information.

The notion of monetary damages in tort law is based to some extent on just this idea, namely that there does exist *some* monetary equivalent to any particular change in welfare.

[18]Mishan (1971 fn. 33), however, distinguishes between this type of reversal and a reversal in an Edgeworth Box with production, reserving the term Scitovsky reversal for the latter.

4. Valuation by and of Multiple Individuals

While judging welfare changes experienced by a single individual is not simple, we have seen there are tools to measure such changes which work satisfactorily in many situations. This section addresses the problem of how to aggregate the welfare changes of different individuals to arrive at a measure of total, society-wide welfare change. A simple idea would be to solve social decision-making problems by just adding up the compensating variation or equivalent variation of different individuals to get a social welfare measure. That however treats \$1 of damages suffered by any person in society as being equal to \$1 of damages suffered by any other person in society, which is a peculiar aggregation method having no particularly compelling ethical basis. Similarly, even if Marshallian consumer surplus were a good measure of an individual's consumer welfare, there would be no particular justification for adding up the unweighted Marshallian consumer surplus changes of different individuals to arrive at a measure of the welfare change of the group of consumers, which is what standard consumer surplus arguments do.

Zerbe and Dively (1994 pp. 94–5) explain, "the sad fact is that the aggregations of CVs or EVs over several individuals, unlike the CVs or EVs of one person, have no meaning as indicators of ordinal utility, unless we are willing to make additional and restrictive assumptions." Paraphrasing their discussion, suppose Project A shows a CV of +5 for Robert and −2 for Alice, for a total of +3. Suppose Project B shows a CV of −2 for Robert and +4 for Alice, for a total of +2. They write, "it is not possible to say whether Project A or B is superior without a scheme to compare Robert's and Alice's CV. That is, the net total of +3 for Project A and the net total of +2 for Project B are net totals of what? They are income measures that when broken apart into the CV for Alice and the CV for Bob serve to rank Alice's and Bob's choices but when the CVs of Bob and Alice are combined they do not rank their choices. Roberts and Alice's CVs are not measured in units of utility. Rather, they only indicate ordinal ranking for the individuals. Alice's CV of +4 in Project B may yield her much more satisfaction than Robert's CV of +5 in Project A. Any solution or approach to comparing the welfare of different individuals must clearly and directly involve ethical choices." Even if utility were cardinal and we only wanted to maximize the sum of Alice's and Robert's utilities, we cannot because we can never know what those utilities are. (If utility were in fact ordinal, any sum of utilities would have no meaning.) Whether we want to or not, we have no choice but to introduce our own weights on Alice and Robert. It is no wonder that

Zerbe and Dively have an entire chapter on "The Ethical Foundations of Benefit-Cost Analysis," discussing various kinds of social welfare functions one might want to use.

Auerbach (1985 pp. 82–3) echoes these sentiments:

> A... response might be that we are only interested in efficiency, not distribution, and so will assign equal distributional weights to individuals, thereby allowing the interpretation of the aggregate measures derived above as "efficiency-only" social welfare measures. Such is the approach suggested by Harberger (1971). Unfortunately, this will not work either. We can certainly imagine a social welfare function of the form
>
> $$w(U^1, \ldots, U^H) = \sum_{i=1}^{H} U^i , \tag{3.23}$$
>
> and can even choose a normalization for the individual utility functions so that, in the initial state, the marginal utility of income and hence the social marginal utility of income for each individual is one ("money metric" utility). However, once prices change, as they will when taxes are introduced, the changes in real income, and hence the marginal utility of income, will generally be different.... Thus, it will generally not be possible to make welfare comparisons on the basis of aggregate measures of excess burden, no matter what our attitude is about the relative importance of equity and efficiency.

For yet another reference along the same lines, see Jones (2005, 1.1.2). And for another, Blackorby and Donaldson (1990 p. 490):

> Harberger does not claim, however, that he is indifferent to income distribution. His claim is rather that economists are not 'professionally qualified to pronounce'... on such issues.... But such views require a notion of efficiency that is independent of the distribution of income—an idea that makes no sense in real-world economies. Costless lump-sum transfers (and taxes) are not feasible, and, in addition, governments cannot be counted on to pursue distributive justice rigorously and effectively. As a result, the social ethics implicit in rules such as the compensating-variation test must be subjected to serious scrutiny and judged on their own merit.

Specialized texts are in agreement on these points.

It is possible that misuse of individual welfare measures to form measures of group welfare arose due to mere misunderstandings. Whether that is true or not, such misuse has the effect of magnifying the importance of the preferences of the wealthy and reducing the importance of the preferences of the poor. Hammond and Fleurbaey (2010) write:

> "A dollar is a dollar", they might say, regardless of how deserving is the recipient. Implicitly, they attach equal value to the extra dollar a rich man will spend on a slightly better bottle of wine and to the dollar a poor woman needs to spend on life-saving medicine for her child. Of course, any such judgement is a value judgement, even an interpersonal comparison, which lacks scientific foundation. . . . Thus, the "surplus economists" who just add monetary measures, often of consumer surplus rather than individual welfare, make their own value judgements and their own interpersonal comparisons. Moreover, their comparisons not only lack scientific content, but most people—especially non-economists—also find them totally unacceptable from an ethical point of view.

To be more explicit, making social or legal judgments based on adding up individual monetary measures of welfare means assuming that the current wealth and income distribution, which gives rise to those measures, is just. One might associate this viewpoint with the right-libertarian Robert Nozick's "entitlement theory of justice" (see e.g. De Gregori 1979), but the adding-up approach is actually more protective of the status quo because Nozick endorses "rectification" for historical injustices—slavery, say, or colonialism (De Gregori p. 20–22; Collste 2010 p. 86)—which the adding-up approach ignores.[19] If Nozick represents mainstream right-libertarianism then it seems most accurate to call the ethical foundation of the adding-up approach "extreme right-libertarianism." Hammond and Fleurbaey are probably be correct that "most people" do not share this ethical point of view, but welfare analysis using extreme right-libertarianism could defended as at least being self-consistent if most wealthier people do.

Adherents to ethical points of view other than extreme right-libertarianism would need to make explicit ethical adjustments to the analysis. This could

[19] Collste (p. 86) traces the idea of rectificatory justice back to Aristotle's *Nichomachean Ethics*. For the divergence between Nozick's theory of rectification and the application of it by him and his followers to existing societies, see Mills (2018 pp. 81–2).

involve making Gini-coefficient adjustments as in Fleurbaey and Blanchet (2013 equation 3.4), or applying any of the many other approaches suggested in that book. If one has an econometric model of willingness to pay as a function of (among other things) income, it could involve computing "willingness to pay holding income constant," what might be called "Democratic Willingness To Pay." It could be the "marginal-utility-weighted C[ost]-B[enefit] A[nalysis]" discussed, although not rigorously defined, by Matthew Adler and Eric Posner (2006 p. 144). The article by Blackorby and Donaldson (1990), whose very title proclaims their distaste for simply adding up compensating or equivalent variations, adds:

> We have investigated other methods for performing distributionally sensitive cost-benefit analysis. One is the employment of welfare ratios (ratios of household incomes to the household's poverty lines) as indexes of well-being (Blackorby and Donaldson 1987). Another is the employment of household equivalence scales in estimated utility functions (Blackorby and Donaldson 1988c). The latter method has been used by Jorgenson and Slesnick (1984a, 1984b). In general, the procedure is straightforward. It requires an econometric procedure for estimating household preferences, a way to move from household well-being to individual well-being (such as equivalence scales), and a family of social-welfare functions with a parameter that allows for different degrees of inequality aversion. The impact of the project on incomes and prices must be forecast (with, perhaps, some aggregation into income classes) and the project evaluated with different values of the inequality-aversion parameter. The results of these procedures are approximate, of course, but there is no underlying difficulty with the social ordering, and the ethics are explicit. (pp. 492–3)

Other progress in developing tools for distributionally-sensitive cost-benefit analysis is reported in the survey article of Slesnick (1998).

There are economic arrangements which are objectively inefficient ("not Pareto Optimal"), but unless it is for some reason impossible to use better allocations, these have no advocates and excluding them is not contested. When making choices between other arrangements, our conclusion is that evaluation requires taking an ethical position. There is no such thing as objective, "ethics-free" determination of a single economically efficient allocation.

5. The Coase Theorem

The sentence ending Section 4 probably strikes readers coming from the "law and economics" tradition as having something wrong with it. After all, as Richard Posner (1993 p. 195) put it:

> . . . the Coase Theorem [states that]: If transaction costs are zero, the initial assignment of a property right—for example, whether to the polluter or to the victim of pollution–will not affect the efficiency with which resources are allocated.

This is certainly an assertion that objective, "ethics-free" economic decision-making—for example, ethics-free identification of a unique efficient allocation—is possible. In Section 1, a change in income/wealth—the sort of thing which would happen if "the initial assignment of a property right" were changed—affected the monetary valuation of a commodity, and thus would affect the calculation of which allocation would be deemed socially optimal. This raises the question of which result is wrong: the result of Section 1 or the Coase Theorem. The purpose of this section is to show that it is the Coase Theorem which is wrong because, like Marshallian consumer surplus, the Coase Theorem depends on commodities having no income effects.[20] We work with a framework of a single polluter and a single pollution victim because otherwise Coase Theorem analyses are subject to all the ethical critiques of aggregation already well-covered in Section 4, the most important being that if they do not make explicit ethical adjustments then they implicitly reflect extreme right-libertarian ethics.

The customary presentation of the Coase Theorem goes along these lines.

> In Figure 13, *MEC* stands for the "marginal external cost" which production of a polluting commodity "*Q*" inflicts on bystanders. The curve "*MΠ*" is the marginal profit of the polluting firm (or, more generally, the "marginal net private benefit" of a marginal increase in production of *Q*, where marginal net private benefit includes both marginal profit and any marginal net value to consumers of *Q*). If the property rights to the environment are assigned to the firm then it wishes to go to its profit-maximizing point Z. However, at Z, the pollution victim, whose willingness and ability to pay for pollution reductions is represented by *MEC*,

[20] For criticisms of the Coase Theorem along other lines see Usher (1998), Posin (1999), McCloskey (1998), and, of particular note, Coase (1991), who prepends the epithet "infamous" to it. As McCloskey and Coase explain, the theorem itself is due to Stigler.

could offer the firm a payment as high as *MEC* in return for a marginal reduction in Q, and the firm would accept because its marginal benefit from Q is only $M\Pi$. For all Q's for which *MEC* lies above $M\Pi$, such mutually-beneficial payments in return for marginal reductions in Q are possible, so with a sufficiently unsophisticated model of bargaining[21], the agents will end up at Q^*, the socially-optimal level of output. If on the other hand the property rights to the environment are assigned to the pollution victim, the initial situation is at O, zero output. However, the firm, whose willingness and ability to pay for output increases is represented by $M\Pi$, could offer the pollution victim a payment as high as $M\Pi$ in return for a marginal increase in Q, and the victim would accept because his marginal cost from Q is only *MEC*. For all Q's for which *MEC* lies below $M\Pi$, such mutually-beneficial payments in return for marginal increases in Q are possible, so with a sufficiently unsophisticated model of bargaining, the agents will again end up at Q^*, the socially-optimal level of output. Hence the socially-optimal level of output will be achieved regardless of the assignment of property rights. The assignment of property rights does affect the distribution of income—because the assignment of property rights determines who pays whom, and thus who gets richer and who gets poorer—but in this telling, that is a mere matter of distribution and is unrelated to efficiency, which is the only thing that is important, since distribution could be adjusted using lump-sum taxes if one cared to do so.[22]

[21]A sophisticated model of bargaining would admit asymmetric information, which opens up many more possibilities for the outcome; see Medema and Zerbe (2000, Section 3.4) and Samuelson (1985).

[22]Douglas W. Allen (1998, see "Must Wealth be Held Constant?") interprets the Coase Theorem as not actually making the "invariance" claim "that the final allocation of resources will be invariant under alternative assignments of rights" (Medema and Zerbe 2000 p. 71) because Allen believes allowing (uncompensated) alternative assignment of rights is "theft" (Allen op. cit. p. 111) and so violates the Coase Theorem's assumption of costlessly-enforceable property rights. (See also Medema and Zerbe 2000 Section 5.2.) This interpretation is not shared by most others, who, as do Medema and Zerbe (p. 71) and Poser in the passage quoted above, consider the "invariance" claim one of the precisely two assertions the Coase Theorem makes, the other being the "efficiency" claim that efficiency results regardless of the assignment of rights. A reader agreeing with Allen will not disbelieve my disproving the "invariance" claim, but will simply think that I have proven no violation of the Coase Theorem because they believe the Coase Theorem never made an "invariance"

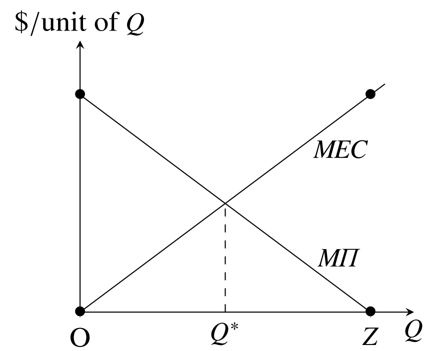

Figure 13. The social optimum does not shift with a reassignment of property rights.

This story sounds correct but deriving the curves from first principles reveals that it is wrong, for exactly the same reason Marshallian consumer surplus was a wrong measure of monetary valuation: the income effect causes curves to move which the standard story assumes are fixed.

Begin by supposing that a (potential) pollution victim's welfare depends on the number of apples a which he consumes and on the amount of clean air in his environment. There is a potentially polluting firm in the pollution victim's environment and the amount of air pollution it would emit is proportional to the level of its output Q. For some fixed level of output $\overline{Q} > 0$,

$$u(a, Q) = a \cdot (\overline{Q} - Q)$$

is a simple and reasonable specification for this pollution victim's utility function, reasonable because it is decreasing in Q. It is a member of the Cobb-Douglas family just like the utility function introduced near the beginning of Section 1 which underlay Sections 1 to 3. For a more conventional appearance at the cost of slightly more algebra we could equivalently have used its square root, $a^{1/2} \cdot (\overline{Q} - Q)^{1/2}$.

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claim in the first place.

Perhaps the best critique of Allen's definition that "transactions costs are the costs of establishing and maintaining property rights" (op. cit. p. 108) is that under such a definition, assigning a property right in *any* way means that people to whom the rights were not assigned face a high cost of asserting their property right, hence there would exist no transactions-cost-free societies even in principle and the Coase Theorem would apply nowhere. The technical term is that the Coase Theorem would be a "vacuous truth" because it would be an assertion that all members of an empty set have a certain property (cf. `https://en.wikipedia.org/wiki/Vacuous_truth`).

Suppose this pollution victim sets out one day with m dollars to visit the marketplace to buy some apples. Before he gets to the marketplace, he encounters the owner of the polluting firm. He may strike up a conversation with this owner in the hopes of affecting how much the firm pollutes. Perhaps he and the firm owner will exchange money for a change in Q. Let m_a denote the amount of money the pollution victim has when he takes leave of the firm owner and proceeds to the marketplace, at which time the amount of Q, and therefore air pollution, becomes irrevocably fixed.

Having m_a funds remaining at his disposal and only apples to spend them on, if we denote the price of apples by p_a, then the pollution victim will purchase m_a/p_a apples, and his utility will be

$$\frac{m_a}{p_a}\,(\overline{Q} - Q)\,.$$

Now suppose that in this country, firms have the property right to pollute. Furthermore suppose that in the absence of any interaction or bargaining between the firm and the pollution victim, amount of output which the firm sees fit to produce is $\overline{Q}/2$. Then in an initial, no-bargaining situation, the (indirect) utility of the customer would be

$$v_0 = \frac{m}{p_a}\,(\overline{Q} - \tfrac{1}{2}\,\overline{Q}) = \frac{m\overline{Q}}{2p_a}\,. \tag{37}$$

Upon meeting the firm owner, the pollution victim contemplates offering the firm owner money in return for a reduction of Q. If the pollution victim offered the firm owner T dollars ("T" for "transfer") and in return the firm owner reduced output to Q, the pollution victim would, after making the bargain and then buying apples, have a utility level of

$$v' = \frac{m - T}{p_a}\,(\overline{Q} - Q) \tag{38}$$

since $m - T = m_a$.

The maximum T which the pollution victim would be willing to pay for a given Q would satisfy the property that $v' = v_0$, because if v' were any lower than v_0, the pollution victim would prefer to stay at v_0.[23] Accordingly setting (37) equal to (38), some algebra leads to

$$T = m\,\frac{2Q - \overline{Q}}{2Q - 2\overline{Q}}\,. \tag{39}$$

[23]The analogous idea in game theory is the "participation constraint," also called the "individual rationality constraint."

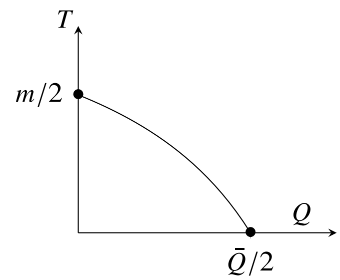

Figure 14. A sketch of T versus Q, equation (39). (The graph is drawn with $m = 4$ and $\overline{Q} = 2$.)

This is valid for reductions in output below its initial level of $\overline{Q}$, that is, for $Q \leq \frac{1}{2}\overline{Q}$. Note that this makes both T's numerator and its denominator have identical signs, so $T > 0$. Figure 14 illustrates (39) for the case of $m = 4$, $\overline{Q} = 2$.

It is worthwhile for the pollution victim to spend T to reduce output to Q because output imposes "external costs" (denote them by "$EC(Q)$") on the pollution victim. Zero output imposes no external costs on the pollution victim, so $EC(0) = 0$. If on the other hand output were $\overline{Q}/2$, the pollution victim would be willing to pay whatever $T(0)$ is in order to reduce output to zero; hence $EC(\overline{Q}/2) = T(0)$. It follows that the connection between T and external cost is

$$EC(Q) = T(0) - T(Q)\,. \tag{40}$$

This implies that the marginal external cost

$$MEC = \frac{dEC}{dQ} = \frac{m\overline{Q}}{2\,(Q - \overline{Q})^2} > 0\,. \tag{41}$$

This is the curve "*MEC* if the firm has the property right" in Figure 15. In that figure, to represent the behavior of the firm we have drawn a marginal profit curve *MΠ* consistent with its desire to produce $\overline{Q}/2$ in the absence of bargaining with the pollution victim.

Now contrast this to the situation under a different constitution in which (potential) pollution victims have the right to clean air and firms cannot pollute the air without obtaining permission from the (potential) pollution victim. (Here (potential) pollution victims have the "property right" to clean air.) In the absence of any interaction or bargaining between the firm and

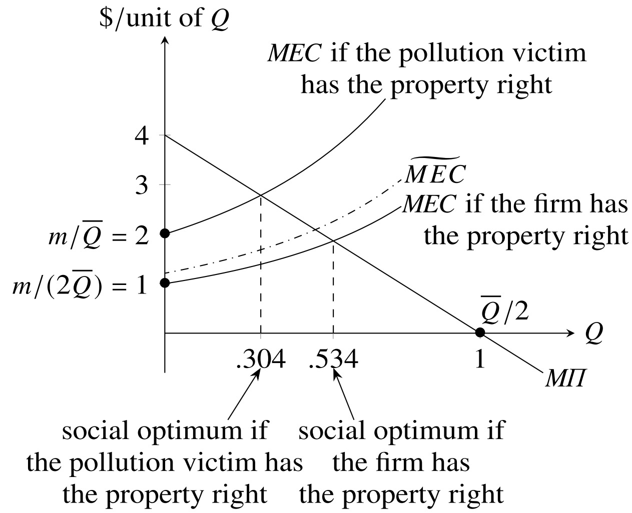

Figure 15. The social optimum shifts with a reassignment of property rights. The lowest *MEC* curve is (41), the highest one is (46), and $\widetilde{MEC}$ is (47). The graph is drawn with $m = 4$, $\overline{Q} = 2$, and $M\Pi = 4 - 4Q$.

the (potential) pollution victim, initial value of output would be $Q_0 = 0$, and the (indirect) utility of the customer would be

$$\hat{v}_0 = \frac{m}{p_a}\,(\overline{Q} - 0) = \frac{m\overline{Q}}{p_a}\,. \tag{42}$$

Upon meeting the firm owner, the pollution victim contemplates offering to allow the firm to increase output to Q in return for the firm paying the pollution victim $\widehat{T}$ dollars (a "hat" accent will denote quantities in this "pollution victims have the right to clean air" world). The pollution victim would, after making the bargain and then buying apples, have a utility level of

$$\hat{v}' = \frac{m + \widehat{T}}{p_a}\,(\overline{Q} - Q)\,. \tag{43}$$

The minimum $\widehat{T}$ which the pollution victim would be willing to accept for a given Q would satisfy the property that $\hat{v}' = \hat{v}_0$, because if $\hat{v}'$ were any lower than $\hat{v}_0$, the pollution victim would prefer to stay at $\hat{v}_0$. So we set (42) equal to (43) and obtain, after some algebra, that

$$\widehat{T} = \frac{mQ}{\overline{Q} - Q} > 0\,. \tag{44}$$

In this situation, external cost

$$\widehat{EC}(Q) = \widehat{T}(Q) \tag{45}$$

because the reason the pollution victim is willing to accept $\widehat{T}$ and not less is because Q causes $\widehat{EC}(Q)$ in damage. Using this,

$$\widehat{MEC} = \frac{d\widehat{EC}}{dQ} = \frac{d\widehat{T}}{dQ} = \frac{m\overline{Q}}{(\overline{Q} - Q)^2}\,, \tag{46}$$

which is "*MEC* if the pollution victim has the property right" curve of Figure 15. The socially-optimal level of output—which occurs where the marginal benefit to society of an extra unit of output (*MΠ*) is equal to the marginal cost to society of an extra unit of output (*MEC*)—is smaller, by a factor of approximately 1.75, if the pollution victim has the property right than if the firm has it. This is sufficient to disprove the Coase Theorem. In addition, which level of output is deemed socially-optimal shifts in favor of the party which is given the property right and hence has become richer and against the party which has become poorer, just as Section 4 discussed.

The conclusion of Figure 15, that there are not one but two social optima, suffices to disprove the Coase Theorem but drastically understates the case, because of Figure 15's simplifying assumption that the pollution victim always pays the maximum he or she is willing to pay to reduce Q. The firm only requires payment at or above *MΠ*, so depending on the deal or deals struck, the pollution victim may pay less that what is assumed in order to draw the *MEC* curves, so the actual *MEC* curves would be different.

To make this concrete, suppose the firm has the property right and suppose that an interim bargain is struck between the pollution victim and the firm, reducing Q to $3/4$. Suppose that the pollution victim, rather than paying the maximum amount (their willingness to pay, which is the area under the *MEC* curve, $\int_{3/4}^{1}((m\overline{Q}/2)/(Q-\overline{Q})^2)\,dQ = 4/5 = T(3/4)$), only had to pay the firm's minimum acceptable amount, the area under the *MΠ* curve, to obtain this reduction in Q. This transfer amount is $\tilde{T} = \int_{3/4}^{1}(4-4Q)\,dQ = 1/8$, which is much less than the $4/5$ area under the *MEC* curve. The pollution victim's remaining income is $4 - 1/8 = 31/8$, which we call $\tilde{m}$. Denoting the pollution victim's utility after this transfer as $\tilde{v}_0$, we have

$$\tilde{v}_0 = \frac{\tilde{m}}{p_a}(\overline{Q} - \tfrac{3}{4})\,.$$

Suppose a second round of bargaining begins from this $Q = 3/4$ point. In this second round of bargaining, the pollution victim transfers $\tilde{T}$, ending up with a utility level of

$$\tilde{v}' = \frac{\tilde{m} - \tilde{T}}{p_a}(\overline{Q} - Q) \quad \text{for } Q < 3/4.$$

If in this second round of bargaining the pollution victim pays the maximum transfer (his willingness to pay) rather than the minimum transfer as he paid in the first round, then $\tilde{v}'$ will equal $\tilde{v}_0$ and therefore, after some algebra,

$$\tilde{T} = \tilde{m}\frac{Q - 3/4}{Q - \overline{Q}} = \tilde{m}\frac{2Q - 3/2}{2Q - 2\overline{Q}} \quad \text{for } Q < 3/4$$

(cf. (39)). Then proceeding as in (40) and (41),

$$\widetilde{MEC} = \frac{dEC}{dQ} = -\frac{d\tilde{T}(Q)}{dQ} = \tilde{m}\frac{\overline{Q} - 3/4}{(Q - \overline{Q})^2} \quad \text{for } Q < 3/4\,. \tag{47}$$

In Figure 15 this dash-dotted curve is higher than "*MEC* if the firm has the property right" because $\widetilde{MEC}$ corresponds to a pollution victim with higher income because he paid less to reduce Q from 1 to 3/4 (and because for this pollution victim, environmental quality is a normal good, so when his income is larger, his demand for environmental quality is higher).

Next, suppose the pollution victim paid something in between the minimum, namely 1/8 dollars, and the maximum, namely 4/5 dollars, to reduce Q from 1 to 3/4. (Theorizing about where in between 4/5 and 1/8 the pollution victim might be would divert us beyond the scope of this paper and into game theory, for which see the references in footnote 21.) Each of the infinitely many different possible transfers between 4/5 and 1/8 will generate a different *MEC* curve, each such curve lying somewhere between "*MEC* if the firm has the property right" and $\widetilde{MEC}$. So there are an infinite number of possible *MEC* curves.

Next, suppose the first round of bargaining resulted, not in $Q = 3/4$—remember we picked 3/4 in a completely arbitrary way—but in some other Q between 1 and 0.534. There are an infinite number of such Q. For each such Q, the above analysis can be repeated (*mutatis mutandis*) and that will generate its own new set of infinitely many possible *MEC* curves.

Next, suppose the number of rounds of bargaining is not two—we picked two in a completely arbitrary way—but some number larger than two. This number of rounds could even be infinity if bargains are always made over marginal decreases in Q. Each of the rounds will generate, for each of the infinitely many possible intermediate Q's temporarily arrived at, one *MEC* curve for each of the infinitely many possible pollution victim transfers between the pollution victim's willingness to pay (the largest incentive-compatible transfer from the pollution victim to the firm) and the firm's *MΠ* (the smallest incentive-compatible transfer from the pollution victim to the firm).

The conclusion is that, in a model with nonzero income effects, it is in principle impossible to tell which of an infinite number of possible *MEC* curves will be the right one, and therefore impossible to define a socially-optimal level of Q.

A completely analogous situation would obtain if we began from the pollution victim having the property right.

Clearly, then, the Coase Theorem completely breaks down with nonzero income effects: its basic object of interest, " 'the' socially-optimal level of output," becomes a term with no meaning. As Samuels wrote (1974 p. 4), "There is no unique relevant optimum."[24]

We are almost done, but the subtitle of Posin (1999), "Of Judges Hand and Posner and *Carroll Towing*," raises a point which we have not yet addressed. *Both* the plaintiff and the defendant in the *Carroll Towing* case were firms. In the standard neoclassical theory of the firm, firms have no budget constraints, and therefore experience no income effect/wealth effect. The Coase Theorem would then seem to be rescued (at least from the criticisms of this paper—not from those raised but not evaluated in this paper in footnote 20, nor from criticisms of Section 4's type) in cases when every party concerned is a firm.[25]

To this there are two responses. The first is, "standard neoclassical theory" notwithstanding—and thus notwithstanding the way I have modeled the polluting firm above—many firms are constrained in the amount of inputs they can purchase by a limit to the amount of credit available to them. Without here going into the reasons such credit constraints exist, simply note that this credit constraint plays for the firm an analogous role to that played by the budget constraint for the pollution victim. If at least one of the

[24]This is not a criticism of Coase, it is a criticism of Stigler (see footnote 20. One form of a converse of the Coase Theorem would be "*only* if transactions costs are zero and income effects are zero will the initial assignment of a property right not affect the socially-optimal level of output." This remains true, and this, not the "Coase Theorem" itself, seems to be what Coase himself was primarily interested in, because he thought it was easy to see that transactions costs in the "real world" are not zero.

[25]See also Schwab (1989 pp. 1182–1183).

firms involved in a dispute is credit-constrained, the income-effect critique of the Coase Theorem holds.

A more general response is that from an economic point of view, firms are legal fictions. The benefits and costs of governmental decisions are, ultimately, solely borne by individuals. It may take effort to determine who they are and how they will be affected. Besides owners and customers, managers and other employees may be affected, and so may contractors, vendors, individuals connected to this firm via positive or negative market imperfections (such as externalities like pollution but also such as oligopolistic behavior which would affect competing firms), people who are not currently members of any of these groups but would become members under one of the alternatives being considered by the governmental decision-makers, and the web of individuals touched by all these groups (such as the contractors of the vendors). A correct economic analysis of governmental decisions affecting firms pierces the veil of "the firm" and imputes costs and benefits to the natural persons who ultimately bear all the costs and benefits.

As a hypothetical example, suppose Firm A ekes out a profit of $1 million making a medicine of vital concern only to poor people, while Firm B makes private jets. Suppose the production of the medicine emits a substance into the atmosphere whose only effect is to decrease the profits of producing private jets by $100 million, and that the only way to ameliorate this effect is to cease production of the medicine. A neoclassical social planner unaware of results like those discussed earlier in this paper would choose to shut down Firm A, the "least cost," "most efficient" solution. In a laissez-faire world in which Firm B was assigned the property right, it would offer to pay Firm A's owners a sum between $1 million and $100 million to shut down production, and Firm A's owners would accept that offer (assuming a one-shot game). In a laissez-faire world in which Firm A was assigned the property right, Firm B would offer to pay Firm A's owners a sum between $1 million and $100 million to shut down production, and Firm A's owners would accept that offer (assuming a one-shot game). This is not the efficient solution which would result if the government imposed large taxes on people who were (previously) wealthy enough to buy private jets and redistributed this money to the poor, who are more numerous and thus even in the new situation did not demand private jets, because this would change the demand for private jets and for the medicine.

6. Scope and General Equilibrium

Since as explained at the end of Section 1 quasilinear preferences are unrealistic, the sections above completely ignored them. However, to eliminate quasilinear preferences as a last refuge of proponents of using surplus changes to evaluate public policy, in this section we reverse course and adopt them. The income effects of the buyer and seller of the good in question then vanish, and it would appear that the conventional interpretation of "deadweight losses" (reductions in surplus) would then be valid.

Deadweight losses, however, arrive from cost reductions, and while cost reductions are welcomed by the buyer and seller of the commodity immediately at hand—corn, let us say—they will not be welcomed by the input suppliers—suppliers of fertilizer or of labor, let us say—whose income comprises the costs of producing corn. From the viewpoint of social welfare, *all* people affected by a policy should be included in its policy analysis. This means taking into account not only consumers of the product and owners of the firm which produces the product, but also persons who supply inputs to the production of the product—firm managers, lower-level employees, suppliers of non-labor inputs, and, for that matter, *competing firms'* customers, managers, owners, employees, and suppliers, and so forth, and then everyone affected by changes in the economic position of all of these people. For example, an antitrust decision which lowers a product's price modestly, slightly benefitting its many consumers and shareholders but throwing some of the newly-merged company's employees permanently out of work, could easily decrease social welfare using a distributionally-sensitive, non-extreme-right-libertarian approach. Also, while curves on a demand-and-supply diagram may snap into new positions effortlessly, adjustments actually take time, generate costs including sometimes external costs, and affect a wider net of people than may be apparent at first glance. Considering "deadweight losses" as always being bad and their elimination as always being praiseworthy is a normative, value-laden position which has masqueraded as a 'scientific' principle far too long.

The standard response to these objections would be to conduct a general equilibrium analysis of the surplus implications of the policy under consideration, but this immediately runs up against two severe theoretical difficulties with fallacies of composition.

The first is that while *some* commodities have no income effect if the consumer has quasilinear preferences, it is impossible for *every* commodity to have no income effect even if the consumer has quasilinear preferences.

Recall that the quasilinear form of utility is

$$U(x_1, x_2, \dots, x_n) = x_1 + u(x_2, x_3, \dots, x_n)\,.$$

Maximizing this subject to the standard budget constraint $p_1 x_1 + p_2 x_2 + \cdots + p_n x_n = m$ for income m, one can solve the budget constraint for $x_1 = (m - p_2 x_2 - p_3 x_3 - \cdots - p_n x_n)/p_1$, and substitute to turn the problem into its unconstrained version

$$\max_{x_2, x_3, \dots, x_n} \frac{(m - p_2 x_2 - p_3 x_3 - \cdots - p_n x_n)}{p_1} + u(x_2, x_3, \dots, x_n)\,.$$

For $i \in \{2, 3, \dots, n\}$, the first-order condition for x_i is

$$0 = \frac{-p_i}{p_1} + \frac{\partial u(x_2, x_3, \dots, x_n)}{\partial x_i} \tag{48}$$

which does not depend on income, hence no income effect. However this is not true for the first commodity. For $i \in \{2, 3, \dots, n\}$, (48) gives the optimal amount of x_i, "x_i^*." The optimal amount of x_1 is then given by a residual,

$$x_1^* = m - p_2 x_2^* - p_3 x_3^* - \cdots - p_n x_n^*$$

(subject to $x_1^* \geq 0$). Clearly x_1^* does depend on income. The only situation in which *no* commodity's consumption depends on income is the situation when budget constraints are irrelevant to consumers' decisions, a situation so unusual—e.g., living under a vow of poverty, or, perhaps, being a multi-billionaire—that economists almost never study it.

To illustrate the second fallacy-of-composition problem, consider the Edgeworth Box in Figure 16. The agents are Smith "S" and Jones "J" and the goods are apples "a" and bananas "b." There is no problem calculating Smith's WTP to move from g to f': it is the amount of bananas $f'r'$ he would be willing to pay in return for the increase in apples which "g to f'" represents. Similarly, there is no problem calculating Smith's WTA a move from g to f: it is the amount of bananas $f'h$ he would be willing to accept in return for the decrease in apples which "g to f" represents. These movements are like those treated in Section 2, "Valuing Quantity Changes," above, rather than Section 3's price changes. However it is unclear how to value a *diagonal* change, say from f' to f. Jones's WTP to move from f' to f could be expressed in terms of apples or bananas. Expressing it in terms of bananas, the answer is fq, but that answer tells us nothing more than "Jones's WTP to move horizontally from f to g is gq" because bananas gf' are merely subtracted in the move from f' to f and then added back again in the move from f to q. Diagonal motions *per se* cannot be valued, only horizontal or vertical motions can be. Similarly, with $N > 2$ commodities, WTP or WTA for motions changing all N commodities cannot be valued using some commodity as the metric, only those changing fewer than N commodities can be.

Calculating the EV or CV for price changes, as in Section 3, can formally be done, and in Appendix 2 I explain in detail how to it for the utility functions which generated Figure 16. Solving equations (56)–(59) for the movement from h' to h in Figure 16 yields (after calculating the equilibrium prices at h' and at h):

$$CV_S = -2.07887 \qquad CV_J = 2.22818$$
$$EV_S = -2.17026 \qquad EV_J = 2.32614\,.$$

Both according to the CV (Kaldor) test and according to the EV (Hicks) test, Jones, who is the winner of the move from h' to h, can compensate the loser, Smith, and still be better off. However these numbers lack all meaning. In partial equilibrium, there certainly is a possibility that if the consumer is actually given EV or CV units of money, the consumer will overall be no worse off after a change, because the consumer can use the money to purchase more of other commodities than he was purchasing before. However in the Edgeworth Box (that is, in general equilibrium), given the lack of any goods besides apples and bananas, it is in principle impossible to compensate Smith for the movement from h' to h. What would Smith do with a side payment from Jones of $2.20? By assumption, Smith is now fixed at h, so he has nothing to spend the $2.20 on. How could Jones afford to pay Smith $2.20? By assumption, Jones's wealth is fixed at h; if he gave Smith $2.20, he would not be at h any more. This example, illustrating "Boadway's Paradox," shows that sometimes it is possible to calculate CV and EV and apply the Kaldor or Hicks "potential Pareto Improvement" tests when in fact no Pareto Improvement could ever be made and CV and EV are meaningless numbers.

Conclusion

There is no single, unambiguous measurement of economic value for an individual, and that multiplicity is greatly magnified when measuring economic value for a group of individuals. In practice, the measurement of welfare changes and the determination of a socially-optimal level of an economic activity depends inescapably on the observer's ethical position (taken wittingly or not) concerning the distribution of income and wealth in a society.

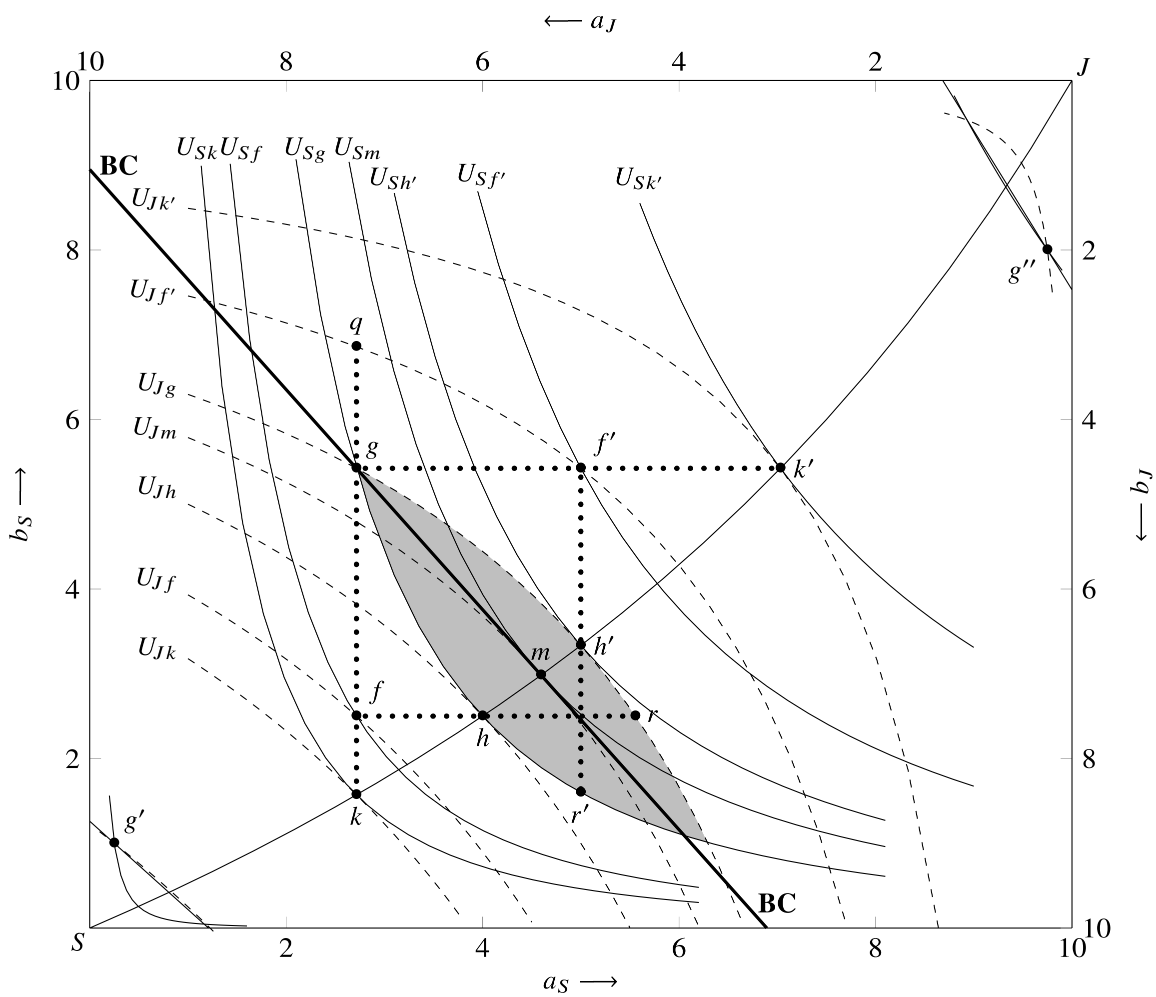

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Figure 16.

As Baker (1975 p. 16) wrote, "No neutral efficiency analysis exists which can avoid or segregate the distributive question." Ignoring the dependence of welfare measures on the income and wealth distribution gives wealthier citizens a greater weight in these measures, a "special advantage" (Adler and E. Posner 2006 p. 125). As Hausman and McPherson (2007 p. 247) put it, "If each policy had different winners and losers so that in the long run everyone were a winner as often as he or she were a loser, the unfairness of individual policies taken separately might wash out. But the bias built into cost-benefit analysis against the preferences of the poor suggests that the unfairness will not wash out."

The influence of Richard Posner on the legal system of the USA has meant that many court decisions in the last few decades implicitly reflect this extreme right-libertarian ethical position. Baker (1975 p. 5) predicted its wide-ranging effects in many areas of the law:

> . . . in the case of an alleged nuisance, the law could grant the right to the party to whom the right was most valuable. Negligence rules could be designed to allocate rights and duties so that parties will use the cheapest methods of avoiding economic costs. Property rules could define rights such that transfers to the highest valued use are made easier (cheaper). Contract rules could help to 'minimize the breakdowns in the process of exchange' and reduce the costs of exchange. [citations to R. Posner's *Economic Analysis of Law* omitted.]

As Adler and E. Posner (2006) note, "Cost-benefit analysis (CBA) has been an important policy tool of government since the 1980s, when the Reagan administration ordered that all major new regulations be subjected to a rigorous test of whether their projected benefits would outweigh their costs." Hence both the executive and the judicial branch of the USA's federal government have had this systematic preference for the wealthy over the poor—and it is true both that some judicial decisions can indirectly influence the composition of the legislative branch, and that the legislative branch exercises important control over the composition of the judicial branch. Baker (p. 9) foretold the result: as decisions privileging the wealthy accumulate, the wealthy become wealthier, making their preferences weigh even more heavily than before, leading to more government decisions in their favor, making them even wealthier, in a feedback loop.

In his review of James R. Hackney, Jr.'s 2007 book *Under Cover of Science: American Legal-Economic Theory and the Quest for Objectivity*, Ejan Mackaay (2009 p. 242) writes:

> The debate this book relates about distributional concerns appears dated and out of touch with what seem to be the current concerns within the law-and-economics movement: what place history, institutions and behavioral economics are to occupy in our economic understanding of law and how to design sophisticated empirical studies. We may yet come to realize that ignoring distributional concerns has introduced a congenital flaw in our research paradigm. But no indices currently point that way.

Setting aside the question of whether this was true in 2009, in 2020 does the evolution of the Gini coefficient of income distribution not point that way? Does the falling life expectancy in the USA caused by the increasing "deaths of despair" among the not-rich not point that way? Considering that the very poorest people are those who are not yet alive, does the change of global mean temperature not point that way?

We have gotten to this point after decades of evaluating efficiency along the flawed path by laid down by Kaldor and Hicks and it is time to take a different path. The proper criterion for evaluating efficiency is Pareto's, full stop. The proper role for economists in public policy lies in designing policies which are actual, not potential, Pareto Improvements. Perhaps we can excuse Kaldor and Hicks because in 1940 it was difficult to design such policies but that excuse should no longer apply. Moreover, it is not easy to argue that designing Pareto-improving policies is more difficult than correctly measuring the welfare change of other kinds of policies.

In the legal system, this reorientation would change the role economists play from being advocates for one adversarial party or the other—an ironic role given that the advocates of conventional applied welfare economics claim it is impartial and value-neutral—to facilitating mediation, seeking actual Pareto-improving resolutions.

Although constructing actual Pareto Improvements will finally allow economists to recommend policies that actually improve efficiency, sometimes in addition an economist may be very legitimately interested not only in efficiency but also in distribution—after all even Robert Nozick endorses rectification for historical injustices, as noted above. Coleman (1980 p. 547) reminds us:

> Generally, a policy that makes A better off and no one worse off would be Pareto superior, even if A had no right to be made better off, or if he deserved to be made worse off, or even if B not A should have been better off. Until we know something about the rights and deserts of individuals affected by alternative

> courses of conduct, we should remain agnostic about the moral value of those policies that would otherwise be recommended to us as Pareto superior.

In such cases, if the economist designed an actual-Pareto-Improving policy component first, then advocated for it in conjunction with a redistributive policy component to rectify injustices (including injustices generated by economists' own past embrace of the Kaldor/Hicks approach), one would actually achieve the clear separation between "efficiency" and "distribution" which advocates of the current approach have long claimed they have but which they actually lack.

APPENDIX

In this appendix we study the consumer's value when the reference point is not point I of Figure 3 but is some other point on *MaD*.

Consider first a reference point of Figure 3's point J. At point J the consumer has bought 1/3 pound of cheese at \$3/lb., and so has spent \$1 and has \$1 left over. Let t denote the amount of cheese the consumer buys in excess of the first 1/3 pound. This consumer's budget constraint under "perfect" price discrimination is (8) with this reinterpretation of t as being zero at 1/3 instead of at 1/4. Also, 1/3 should replace 1/4 in the utility function (3) because when $t = 0$ one-third pound of cheese is consumed (point J), whereas before, when $t = 0$ one-fourth pound of cheese was consumed (point I). To solve the maximization problem construct the Lagrangian as usual,

$$\mathscr{L} = (1/3 + t)^{1/2} x_2^{1/2} + \lambda \left[1 - \int_0^t p_1(\hat{t})\, d\hat{t} - p_2 x_2\right]. \tag{9'}$$

The first-order conditions are obtained analogously to (10) and (11), and yield $x_2 = (1/3 + t)\ p_1(t)/p_2$. Substituting this into the budget constraint (8) leads to an integral equation which is the same as (12) except that 1/4 is replaced by 1/3. Differentiating it leads to a differential equation which is the same as (13) except that 1/4 is replaced by 1/3. Its solution is the same as (14) except that four is replaced by three. Substituting that back into the version of (12) with 1/4 replaced by 1/3 and solving for the constant results in

$$p_1(t) = \frac{3}{(1+3t)^2}, \tag{15'}$$

which differs from (15) only in 3's replacing 4's. The corresponding demand function is

$$t = \frac{1}{\sqrt{3p_1}} - \frac{1}{3} \quad \text{and therefore } x_1 = t + \tfrac{1}{3} = \frac{1}{\sqrt{3p_1}} \tag{16'}$$

where $\sqrt{3}$ replaces (16)'s $\sqrt{4}$ and $1/3$ replaces $1/4$. This demand function is the dotted curve called $PPDD_J$ in Figures 2 and 3. It can also be expressed as $p_1 = 1/(3x_1^2)$. Along this curve, utility will stay constant at the level it has at point J.

If the quantity increased from $1/4$ to $1/3$ lb., a correct valuation using J as the reference point would be SJGC, that is,

$$WTPA_J(1/4, 1/3) = \int_{1/4}^{1/3} PPDD_J(x_1)\, dx_1 = \int_{1/4}^{1/3} \frac{dx_1}{3x_1^2} = \frac{1}{3}\,. \tag{18$'$}$$

From (17), this is a difference of $(0.288 - 1/3)/(1/3) \approx -1.5\%$ compared to $WTPA_{MaD}$.

If, as in Section 3, the price fell from four to three, using this reference point, ΔACS_J would be the area to the left of the dotted line JZ, that is,

$$\int_3^4 \frac{dp_1}{\sqrt{3p_1}} = \frac{4}{\sqrt{3}} - 2 \approx 0.309\,. \tag{29$'$}$$

By contrast conventionally-measured consumer surplus changes by the area to the left of the solid line IJ, which is approximately 0.288 from (30), a difference of $(0.288 - 0.309)/0.309 \approx -7\%$.

It can be formally proven[26] that the dotted line $PPDD_J$ is what advanced treatments call a "compensated" or "Hicksian" demand curve (as was $PPDD_I$). It can also be formally proven[27] that the answer $PPDD_J$ gives for ΔACS_J for a price change from \$4/lb. to \$3/lb., (29′), is exactly the same as what more advanced treatments call the "Equivalent Variation."

At reference point J, $p_1 = 3$, $x_1 = 1/3$, and $PPDD_J = 1/(3x_1^2) = (1/3)/x_1^2$ as mentioned after (16′). By analogy, using a different reference point whose quantity of cheese is $\hat{x}_1$ and whose price of cheese is $\hat{p}_1$, the $PPDD$ demand function using that reference point (that is, maintaining its level of utility) would be

$$p_1(x_1) = \frac{\hat{x}_1}{x_1^2}\,. \tag{16$''$}$$

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[26]Claim: (16′) is the Hicksian demand function at the "final" level of utility, $h_1(p_1, p_2, u_1)$. Proof: Given the standard consumer's problem of maximizing $x_1^{1/2} x_2^{1/2}$ subject to (1), as in footnote 15 the indirect utility function turns out to be $v = m/(2\sqrt{p_1 p_2})$; the "final" level of utility at $m = 2$ and $p_1 = 3$ is $u_1 = 1/\sqrt{3p_2}$; the expenditure function is $e = 2u\sqrt{p_1 p_2}$; and from Shephard's Lemma the Hicksian demand function for x_1 at u_1 is $h_1 = u_1\sqrt{p_2/p_1} = 1/\sqrt{3p_1}$, i.e., (16′).

[27]The standard result (Varian (1992, p. 167, (10.2))) is that the area to the left of the Hicksian demand function at the final level of utility is equivalent variation, so (29′)'s area to the left of the dotted line should be equal to EV. To prove this, note that with an original price vector of $\mathbf{p}_0$, the definition of equivalent variation is $v(\mathbf{p}_0, m_0{+}EV) = u_1$. Here the left-hand side is $(2 + EV)/(2\sqrt{4p_2})$, and equating it to the u_1 of the preceding footnote yields $EV = 4/\sqrt{3} - 2$, which indeed is (29′). (Hence equivalent variation is measured using the $PPDD$ curve whose reference point is the utility at the final price (and final quantity) even though the mathematical definition of equivalent variation involves only the initial price—what is of overriding importance is using the final level of utility.)

We are only interested in the case where the reference point (which has $x_1 = \hat{x}_1$) and the "final point" (whose x_1 we call x_{1f}) are the same point, so setting $x_{1f} = \hat{x}_1$, the area of interest is

$$WTPA_f(1/4, x_{1f}) = \int_{1/4}^{x_{1f}} \frac{x_{1f}\, dx_1}{x_1^2} = 4x_{1f} - 1\,. \tag{20$'$}$$

This is the monetary value of a change from an initial quantity of 1/4 lb. to a final quantity of x_{1f} with a reference point of x_{1f}. Figure 4 graphs $WTPA_f$ as a dotted line. The relative error of using $WTPA_{MaD}$ instead of $WTPA_f$ is $(WTPA_{MaD} - WTPA_f)/WTPA_f$, which is graphed as the dotted line in Figure 5.

The area to the left of $PPDD_J$ starting from a price of four and going to a final price of p_{1f} is $\int_{p_{1f}}^{4} dp_1/\sqrt{\hat{p}_1\, p_1}$. We are only interested in the case where the reference point (which has $p_1 = \hat{p}_1$) and the final point (which has $p_1 = p_{1f}$) are the same point, so setting $p_{1f} = \hat{p}_1$, the area of interest is

$$\Delta ACS_f = \int_{p_{1f}}^{4} \frac{dp_1}{\sqrt{p_{1f}\, p_1}} = \frac{2}{\sqrt{p_{1f}}}\left(\sqrt{4} - \sqrt{p_{1f}}\right) = \frac{4}{\sqrt{p_{1f}}} - 2\,. \tag{31$'$}$$

This is the monetary value of a change from an initial price of \$4/lb. to a final price of p_{1f} with a reference point of p_{1f}. This is depicted in Figure 8 by the dotted curve. As p_{1f} goes to infinity this goes to -2, corresponding to the fact that the most this consumer is could ever pay to avoid any price increase is his entire income of \$2.

The change in Marshallian consumer surplus, "Δ*MaCS*" is (32). The relative error resulting from using Δ*MaCS* instead of ΔACS_f is

$$\frac{\Delta MaCS - \Delta ACS_f}{\Delta ACS_f} = \frac{\ln(4/p_{1f})}{\frac{4}{\sqrt{p_{1f}}} - 2} - 1\,. \tag{33$'$}$$

The behavior of this relative error is illustrated in the dotted curve of Figure 9. We have

$$\lim_{p_{1f}\to\infty} \frac{\Delta MaCS - \Delta ACS_f}{\Delta ACS_f} = \frac{-\infty}{-2} - 1 = \infty\,,$$

since in this limit Δ*MaCS* goes to $-\infty$ but the correct measure ΔACS_f goes only to -2. The limit of (33$'$) as p_{1f} goes to zero is -100% (as can be shown via L'Hôpital's Rule).

Things are even worse than in Figure 9 if one uses *MaD* to measure the deadweight loss of a price change. Expressing (16$''$) as $x_1(p_1) = 1/\sqrt{\hat{p}_1\, p_1}$, making the $p_{1f} = \hat{p}_1$ notation change of (31$'$) so that $x_1(p_1) = 1/\sqrt{p_{1f}\, p_1}$, and substituting into (34) yields a correct deadweight loss measure

$$\begin{aligned} DWL_f &= \int_{4}^{p_{1f}} \frac{dp_1}{\sqrt{p_{1f}\, p_1}} - \frac{p_{1f} - 4}{\sqrt{p_{1f}}\,\max(4, p_{1f})} \\ &= \frac{2}{\sqrt{p_{1f}}}\left(\sqrt{p_{1f}} - \sqrt{4}\right) - \frac{p_{1f} - 4}{\sqrt{p_{1f}}\,\max(4, p_{1f})}\,. \end{aligned} \tag{49}$$

This is graphed as the dotted line in Figure 11. The relative error resulting from using the incorrect measure of deadweight loss, DWL_{MaD} from (36), rather than this correct measure is $(DWL_{MaD} - DWL_f)/DWL_f$, which is the dotted curve in Figure 12. For small deviations around $p_1 = \$4/\text{lb.}$, the relative error of deadweight loss is approximately 100%, and the relative error is not within ±5% except in the small and uninteresting interval of approximately $0.4 \leq p_{1f} \leq 0.55$.

Appendix 2

This appendix gives an example illustrating how to compute CV and EV in general equilibrium.

Consider a two-person two-commodity economy with persons named Smith ("S") and Jones ("J") and commodities named apples ("a") and bananas ("b"). Suppose the utility functions are $u_S = 2\ln a_S + \ln b_S$ and $u_J = \ln a_J + \ln b_J$, respectively. Let the price of apples be p_a, and take the price of bananas to be the numéraire. Let Smith's endowment vector be $\omega_S = (\omega_{Sa}, \omega_{Sb})$, and let Jones's endowment vector be $\omega_J = (\omega_{Ja}, \omega_{Jb})$. Let Smith's and Jones's income be

$$m_S = p_a\omega_{Sa} + (1)\,\omega_{Sb} \quad \text{and} \tag{50}$$

$$m_J = p_a\omega_{Ja} + (1)\,\omega_{Jb}\,, \tag{51}$$

respectively.

The solution to Smith's problem of maximizing $2\ln a_S + \ln b_S$ subject to $p_a a_S + (1)\,b_S = m_S$ is

$$b_S = m_S/3 \quad \text{and} \quad a_S = 2m_S/(3p_a)\,, \tag{52}$$

leading to indirect utility of $v_S = 2\ln[2m_S/(3p_a)] + \ln[m_S/3] = \ln[4m_S^3/(27p_a^2)]$. The solution to Jones's problem of maximizing $\ln a_J + \ln b_J$ subject to $p_a a_J + (1)\,b_J = m_J$ is

$$b_J = m_J/2 \quad \text{and} \quad a_J = m_J/(2p_a)\,, \tag{53}$$

leading to indirect utility of $v_J = 2\ln[m_J/(2p_a)] + \ln[m_J/2] = \ln[m_J^3/(8p_a^2)]$.

Given an initial situation denoted with "0" subscripts and a new situation denoted by "1" subscripts, footnote 15 defines CV as $v(\mathbf{p}_1, m_1 - CV) = u_0$, and footnote 27 defines EV as $v(\mathbf{p}_0, m_0 + EV) = u_1$. Hence for Smith, $\ln[4\,(m_{S1} - CV_S)^3/(27p_{a1}^2)] = u_{S0}$ and $\ln[4\,(m_{S0} + EV_S)^3/(27p_{a0}^2)] = u_{S1}$, whereas for Jones, $\ln\,[(m_{J1} - CV_J)^3/(8p_{a1})^2] = u_{J0}$ and $\ln\,[(m_{J0} + EV_J)^3/(8p_{a0})^2] = u_{J1}$. Expressed differently,

$$\begin{aligned}
\ln\,[4\,(m_{S1} - CV_S)^3/(27p_{a1}^2)] &= \ln\,[4m_{S0}^3/(27p_{a0}^2)] \\
\ln\,[4\,(m_{S0} + EV_S)^3/(27p_{a0}^2)] &= \ln\,[4m_{S1}^3/(27p_{a1}^2)] \\
\ln\,[(m_{J1} - CV_J)^3/(8p_{a1}^2)] &= \ln\,[m_{J0}^3/(8p_{a0}^2)] \\
\ln\,[(m_{J0} + EV_J)^3/(8p_{a0}^2)] &= \ln\,[m_{J1}^3/(8p_{a1}^2)]
\end{aligned} \tag{54}$$

One can rewrite the demands for apples as

$$a_S = \frac{2\,(p_a\omega_{Sa} + (1)\,\omega_{Sb})}{3p_a} \quad \text{and} \quad a_J = \frac{p_a\omega_{Ja} + (1)\,\omega_{Jb}}{2p_a}\,.$$

If we denote the total number of apples and bananas in the economy by A and B respectively then equilibrium in the market for apples implies

$$\begin{aligned}
\frac{2p_a\omega_{Sa}+2\omega_{Sb}}{3p_a}+\frac{p_a\omega_{Ja}+(1)\,\omega_{Jb}}{2p_a} &= A\\
\frac{4p_a\omega_{Sa}+4\omega_{Sb}+3p_a\omega_{Ja}+3\omega_{Jb}}{6p_a} &= A\\
4p_a\omega_{Sa}+4\omega_{Sb}+3p_a(A-\omega_{Sa})+3\,(B-\omega_{Sb}) &= 6p_aA\\
p_a\omega_{Sa}+\omega_{Sb}+3p_aA+3B &= 6p_aA\\
\omega_{Sb}+3B &= 6p_aA-p_a\omega_{Sa}-3p_aA\\
\frac{3B+\omega_{Sb}}{3A-\omega_{Sa}} &= p_a\,.
\end{aligned}$$

Then the original and new equilibrium prices in terms only of exogenous variables are

$$p_{a0}=\frac{3B_0+\omega_{Sb0}}{3A_0-\omega_{Sa0}}\quad\text{and}\quad p_{a1}=\frac{3B_1+\omega_{Sb1}}{3A_1-\omega_{Sa1}}\,. \tag{55}$$

Substituting these into (50) and (51) will give m_{S0}, m_{S1}, m_{J0}, and m_{J1}. Substituting these into (52) and (53) gives the initial and final amounts of apples and bananas for Smith and Jones, and substituting m_{S0}, m_{S1}, m_{J0}, and m_{J1} and (55) into (54) implicitly defines the CV's and EV's in terms only of the exogenous variables, completing their computation.

To illustrate the example numbers given in the text, take the utility functions for Smith and Jones as above, suppose $A_0=A_1=B_0=B_1=10$, and describe coordinates in the Edgeworth-Bowley Box as $(\text{apples}_S, \text{bananas}_S)$. To find the location of the contract curve, note that on it, the marginal rate of substitution of bananas for apples for Smith and for Jones are equal. This marginal rate of substitution is $-(\partial u/\partial a)/(\partial u/\partial b)$. For Smith this is $-2b_S/a_a$ and for Jones it is $-b_J/a_J$. Setting these equal and writing $b_J=B-b_S$ and $a_J=A-a_S$ yields the contract curve

$$b_S=\frac{B}{\frac{2A}{a_S}-1}\quad\text{for } 0\le a_S\le A.$$

One can show that both $\omega_{S0}=(5,3\frac{1}{3})$ and $\omega_{S1}=(4,2.5)$ are on the contract curve (they are points h' and h, respectively, in Figure 16). Using the procedure outlined in the previous paragraph one could calculate the CV and EV of moving from ω_{S0} to ω_{S1}.

$$\begin{aligned}
&\ln\left[4\,(\frac{3B_1+\omega_{Sb1}}{3A_1-\omega_{Sa1}}\omega_{Sa1}+\omega_{Sb1}-CV_S)^3\;/\;\left(27(\frac{3B_1+\omega_{Sb1}}{3A_1-\omega_{Sa1}})^2\right)\right]\\
&\qquad=\ln\left[4\,(\frac{3B_0+\omega_{Sb0}}{3A_0-\omega_{Sa0}}\omega_{Sa0}+\omega_{Sb0})^3\;/\;\left(27(\frac{3B_0+\omega_{Sb0}}{3A_0-\omega_{Sa0}})^2\right)\right]\\
&\ln\left[4\,(\frac{3B_0+\omega_{Sb0}}{3A_0-\omega_{Sa0}}\omega_{Sa0}+\omega_{Sb0}+EV_S)^3\;/\;\left(27(\frac{3B_0+\omega_{Sb0}}{3A_0-\omega_{Sa0}})^2\right)\right]
\end{aligned} \tag{56}$$

$$= \ln \left[4 \left(\frac{3B_1 + \omega_{Sb1}}{3A_1 - \omega_{Sa1}} \omega_{Sa1} + \omega_{Sb1} \right)^3 \bigg/ \left(27 \left(\frac{3B_1 + \omega_{Sb1}}{3A_1 - \omega_{Sa1}} \right)^2 \right) \right] \tag{57}$$

$$\ln \left[\left(\frac{3B_1 + \omega_{Sb1}}{3A_1 - \omega_{Sa1}} \omega_{Ja1} + \omega_{Jb1} - CV_J \right)^3 \bigg/ \left(8 \left(\frac{3B_1 + \omega_{Sb1}}{3A_1 - \omega_{Sa1}} \right)^2 \right) \right]$$
$$= \ln \left[\left(\frac{3B_0 + \omega_{Sb0}}{3A_0 - \omega_{Sa0}} \omega_{Ja0} + \omega_{Jb0} \right)^3 \bigg/ \left(8 \left(\frac{3B_0 + \omega_{Sb0}}{3A_0 - \omega_{Sa0}} \right)^2 \right) \right] \tag{58}$$

$$\ln \left[\left(\frac{3B_0 + \omega_{Sb0}}{3A_0 - \omega_{Sa0}} \omega_{Ja0} + \omega_{Jb0} + EV_J \right)^3 \bigg/ \left(8 \left(\frac{3B_0 + \omega_{Sb0}}{3A_0 - \omega_{Sa0}} \right)^2 \right) \right]$$
$$= \ln \left[\left(\frac{3B_1 + \omega_{Sb1}}{3A_1 - \omega_{Sa1}} \omega_{Ja1} + \omega_{Jb1} \right)^3 \bigg/ \left(8 \left(\frac{3B_1 + \omega_{Sb1}}{3A_1 - \omega_{Sa1}} \right)^2 \right) \right] . \tag{59}$$

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