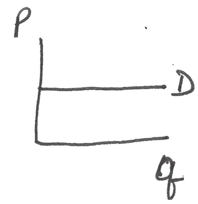
(13.1) Le market price be $\bar{p}$ (exogenously fixed). Then the firm faces

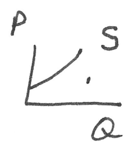
$$D(p) = \begin{cases} 0 & \text{if } p > \bar{p} \\ \text{any amount} & \text{if } p = \bar{p} \\ \infty & \text{if } p < \bar{p} \end{cases}$$

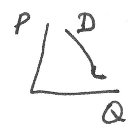


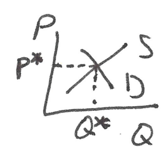
(meaning of competitive price-taking behavior)

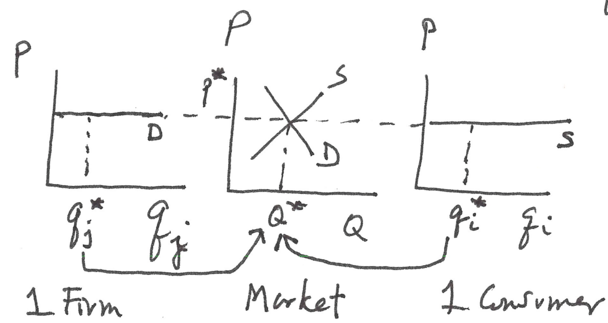
(p. 216 ¶2 l 1 wrong.)

This D curve is counterfactual.

These counterfactual beliefs generate  .

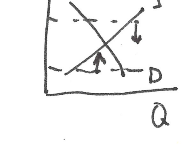
Similar " " by consumers generate  .

So the counterfactuals together generate 



All plans are realized; no one discovers the beliefs aren't true.

The equ. is generated by the beliefs. (Repeated Prisoner's Dilemma w/ drastic punishment ⇒ "cooperate, cooperate" equ. in which you never find out if your belief that the other guy will kill you is true.)

- Joan Robinson: can't do  ∵ p's are taken as given, & such an assumption generated these S & D curves

13.2

We've studied the supply function before.

$$y(\underset{\sim}{w}, p) = f(\underset{\sim}{x}(\underset{\sim}{w}, p))$$

$\Rightarrow$ really $\underset{\sim}{y}(\underset{\sim}{p})$

Supressing $\underset{\sim}{w}$ dependence,

$$y(p);$$

the inverse supply function is

$$p(y).$$

"π" profit equation, not profit function $\pi(\underset{\sim}{p})$

Fig. 13.1: $\max_y \; py - c(y) \Rightarrow p = c'(y)$

MR = MC

We studied π w/ vectors; w/ cost now understood, π can be modelled w/ just the scalar y.

Notwithstanding $C(y, \underset{\sim}{w})$'s concavity in $\underset{\sim}{w}$.

S.O.C. $-c''(y) < 0 \Rightarrow c''(y) > 0$. $\frac{d}{dy} c'(y) > 0 \Rightarrow$ MC rises.

If AVC is U-shaped then

$\Rightarrow c''$ not > 0 $\forall y$.

$(VC'' = TC'')$

Read last ¶ on p. 217.

(13.3)

industry supply function

$$Y(p) = \sum_i y_i(p)$$

Ex. $c_1(y) = y^2$ $c'' > 0$ $p = c' = 2y \Rightarrow y = \frac{1}{2}p$

$c_2(y) = 2y^2$ $c'' > 0$ $p = c' = 4y \Rightarrow y = \frac{1}{4}p$

$Y = \frac{3}{4}p.$

Not:

$2p = 6y$

$p/3 = y$

easier is

$c_1(y_1) = y_1^2$

$c_2(y_2) = 2y_2^2$

Varian uses the worse notation.

(13.4) Market Equilibrium

$$\sum_i x_i(p) = \sum_j y_j(p)$$

Work example at the bottom of p. 219.

[...]

$$X(p) = m\,y(p) \qquad \begin{Bmatrix} dm \\ dp \end{Bmatrix}$$

← # of firms, not income! (from p. 210 Q.3)

$$X'\,dp = y\,dm + m y'\,dp$$

$$(X' - m y')\,dp = y\,dm$$

$$\frac{dp}{dm} = \frac{y}{X' - m y'} < 0$$

X': (−) demand curve

my': (+) supply curve

(17.1) pure exchange economy $\begin{cases} \text{no production (vs. Marx)} \\ \text{no coercion} \end{cases}$

general equilibrium

k # commodities

n # consumers

initial endowment $\underset{\sim}{\omega}_i$ for consumer i

Consumption bundle $\underset{\sim}{x}_i$

allocation $\underset{\sim}{x} = (\underset{\sim}{x}_1, \underset{\sim}{x}_2, \ldots, \underset{\sim}{x}_n)$

feasible allocation $\sum_{i=1}^{n} \underset{\sim}{x}_i \leq \sum_{i=1}^{n} \underset{\sim}{\omega}_i$ (persons i)

Edgeworth Box

Do non-price §17.3 first.

(17.2)

Consumers take $\underset{\sim}{p}$ as given and $\max_{\underset{\sim}{x}} u(\underset{\sim}{x})$ s.t. $\underset{\sim}{p} \cdot \underset{\sim}{x}_i \leq \underset{\sim}{p} \cdot \underset{\sim}{\omega}_i$ ← good notation, better than Varian's

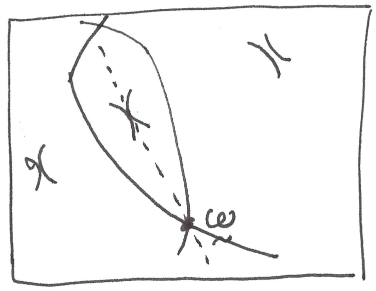
Now do price §17.3.

excess S OK in equ. under some conditions (an undesired good):

$$\sum_{i=1}^{n} \underset{\sim}{x}_i(\underset{\sim}{p}^*, \underset{\sim}{\omega}_i) \leq \sum_i \underset{\sim}{\omega}_i$$ "Walrasian Equilibrium"

(so $D \leq S$)

17.3



contract curve
offer curves - nah

- negotiation allowed (naive negotiators)
- " not "

(17.4)

$$D - S \leq 0$$

$$\sum_i \underset{\sim}{x}_i(\underset{\sim}{p}, \underset{\sim}{\omega}_i) - \sum_i \underset{\sim}{\omega}_i \leq \underset{\sim}{0} \quad \text{(persons } i\text{)}$$

$$\underbrace{\sum_i \left[\underset{\sim}{x}_i(\underset{\sim}{p}, \underset{\sim}{\omega}_i) - \underset{\sim}{\omega}_i \right]}_{} \leq \underset{\sim}{0}$$

$\underset{\sim}{z}(\underset{\sim}{p}, \underset{\sim}{\omega})$ or just $\underset{\sim}{z}(\underset{\sim}{p})$, the aggregate excess demand function

↑ a matrix, actually

§17.2 Wal. Eq.: $\sum x_i \leq \sum \omega_i \Leftrightarrow \underset{\sim}{z} \leq \underset{\sim}{0}$.

$\underset{\sim}{z}(\underset{\sim}{p})$ is homogeneous of degree zero in $\underset{\sim}{p}$

" " continuous if the individual demand functions are

② $\underset{\sim}{p} \cdot \underset{\sim}{z}(\underset{\sim}{p}) \equiv 0$ Walras' Law, the value of excess demand $= 0$.

③ ④→ So if $z_j < 0$ (excess S), $p_j = 0$ (a free good). (Since $p_j \geq 0$)*

Contrapositive: If $p_j > 0$ then $z_j = 0$.

① Proof of Walras' Law.

$$\underset{\sim}{z} = \sum_i \left[\underset{\sim}{x}_i - \underset{\sim}{\omega}_i \right] \leftarrow \text{consumers } i$$

$$\underset{\sim}{p} \cdot \underset{\sim}{z} = \underset{\sim}{p} \cdot \sum_i \left[\underset{\sim}{x}_i - \underset{\sim}{\omega}_i \right] = \sum_i \left[\underset{\sim}{p} \cdot \underset{\sim}{x}_i - \underset{\sim}{p} \cdot \underset{\sim}{\omega}_i \right]$$

$= 0$ from the budget constraint of each consumer i assuming nonsatiation. (If allow satiation, see problem 17.10.) ← monotonicity

Claim: $p_k z_k = 0 \; \forall k$.

Corollary: If $z_k < 0$ then $p_k = 0$.

Contrapositive: If $p_k = 0$ then $z_k = 0$.

*Proof of Free Goods result:

$0 = \underset{\sim}{p} \cdot \underset{\sim}{z} = \sum_k p_k z_k = 0$ commodities k not consumers i (Walras' Law)

$p_k \geq 0$, $z_k \leq 0$ in equ.; $p_k z_k \leq 0$ in equ.

But if any of the terms were < 0 then the sum couldn't be $= 0$.

So $p_k z_k = 0 \; \forall k$. Thus $z_k < 0 \Rightarrow p_k = 0$. ■

sum of individuals' z_{ij}

<u>Market Clearing</u> $0 \equiv \underset{\sim}{p} \cdot \underset{\sim}{z} = \sum_{k=1}^{K} p_k z_k$ (commodities k)

$= \underbrace{\sum_{k=1}^{K-1} p_k z_k}$ $+ p_K z_K$

if this $=0$, then this $=0$, and if $p_K > 0$ then $z_K = 0$.

If $z_k = 0 \; \forall k$ between 1 and $K-1$, & if $p_K > 0$, then $z_K = 0$.

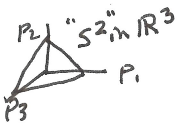
<u>Equality of D&S.</u> If all goods are desirable (i.e., if $p_k = 0$ then $z_k > 0$ $\forall k$). Then $\underset{\sim}{z}(\underset{\sim}{p}^*) = \underset{\sim}{0}$ for all Walrasian equilibrium price vectors $\underset{\sim}{p}^*$.

<u>Proof</u>. If not then $z^*_k < 0$ for some k. By the "free goods" result, this implies $p^*_k = 0$. But that by the desirability assumption implies $z^*_k > 0$, a contradiction. ∎

(17.5)

relative prices; numéraire

normalize prices so that $\sum_j p_j = 1$

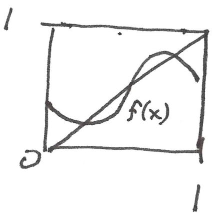
Fig. 17.3 simplex 

simplex, not supply!

Brouwer Fixed-Point Theorem $\left.\begin{array}{l} f : S^{k-1} \to S^{k-1} \\ f \text{ continuous} \end{array}\right\} \Rightarrow \exists\, \underset{\sim}{x} \text{ s.t. } f(\hat{x}) = \underset{\sim}{x}.$

do this after the intuition at the bottom of this page

Proof for ~~k=2~~ k=2.



Define $g(x) = f(x) - x$

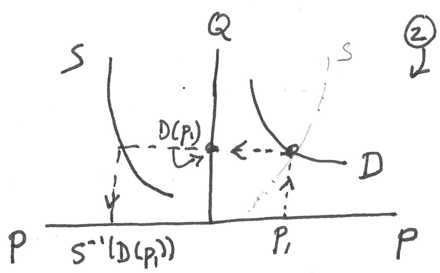
$g(0) \geq 0$

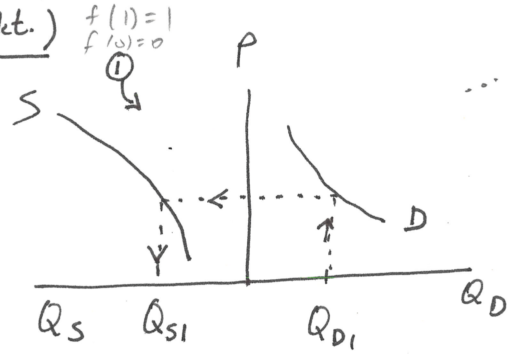
$g(1) \leq 0$

Int. Value Thm. $\Rightarrow \exists\, x$ s.t. $g(x) = 0$. ∎

Intuition for fixed points (in one mkt.)

$f(x) = x^2$
$f(1) = 1$
$f(0) = 0$





... Or could do  in either graph.

$$S^{-1}(D(p_1)) = p_1 \,?$$

$$\| \qquad S(D^{-1}(Q_{D1})) = Q_{D1} \,?$$

$D(p) = 10 - p \Rightarrow p = 10 - D$ so $D^{-1}(Q) = 10 - Q$

$S(p) = 3p + 1 \Rightarrow S - 1 = 3p \Rightarrow p = \frac{1}{3}S - \frac{1}{3}$ so $S^{-1}(Q) = \frac{1}{3}Q - \frac{1}{3}$.

$D = S \Rightarrow 10 - p = 3p + 1 \Leftrightarrow 9 = 4p \Leftrightarrow p = 9/4$

$S^{-1}(D(p_1)) = p_1 \Leftrightarrow \frac{1}{3}D - \frac{1}{3} = p_1 \Leftrightarrow \frac{1}{3}(10 - p_1) - \frac{1}{3} = p_1 \Leftrightarrow 10 - p_1 - 1 = 3p_1$ OK

$S(D^{-1}(Q_1)) = Q \Leftrightarrow$ ~~$\frac{1}{3}D^{-1} - \frac{1}{3} = Q \Leftrightarrow \frac{1}{3}(10 - Q) - \frac{1}{3} = Q \Leftrightarrow 10 - Q - 1 = 3Q \Leftrightarrow 9 = 4Q$~~ ~~$9/4 = Q$~~

$3(D^{-1})+1 = Q_1$

$3(10-Q_1)+1 = Q_1$

$30-3Q_1+1 = Q_1$

$31 = 4Q_1$

$\frac{31}{4} = Q_1$ and $D(\frac{9}{4}) = 10 - \frac{9}{4} = \frac{40-9}{4} = \frac{31}{4}$

$S(\frac{9}{4}) = 3(\frac{9}{4})+1 = \frac{27}{4}+\frac{4}{4} = \frac{31}{4}$ ok.

Main Existence Proof.

$\underset{\sim}{z} : S^{k-1} \to R^k$ continuous, $\underset{\sim}{p} \cdot \underset{\sim}{z} \equiv 0$ $\Rightarrow \exists\ \underset{\sim}{p}^* \in S^{k-1}$ s.t. $\underset{\sim}{z}(\underset{\sim}{p}^*) \leq 0$.

$\underset{\sim}{p} \cdot \underset{\sim}{z} \equiv 0 \leftrightarrow$ "$\underset{\sim}{p} \cdot \underset{\sim}{z}(\underset{\sim}{p}) = 0\ \forall\ \underset{\sim}{p} \in S^{k-1}$"

$\underset{\sim}{g} : S^{k-1} \to S^{k-1}$

$$g_i(\underset{\sim}{p}) = \frac{p_i + \max(0, z_i(p))}{1 + \sum_{j=1}^{k} \max(0, z_j(\underset{\sim}{p}))}$$

i: commodities
j: "

- continuous
- is $S^{k-1} \to S^{k-1}$ — it's a way to adjust the $\underset{\sim}{p}$ vector (To prove that the range is S^{k-1}, show that $1 = \sum_{i=1}^{k} g_i$ by doing the calculation.)
- $\therefore \exists$ a fixed point

$\underset{\sim}{p}^* = \underset{\sim}{g}(\underset{\sim}{p}^*) \Rightarrow p_i^* = g(\underset{\wedge}{p}^*)_i$

$$p_i^* = \frac{p_i^* + \max(0, z_i(\underset{\sim}{p}^*))}{1 + \sum_{j=1}^{k} \max(0, z_j(\underset{\sim}{p}^*))}.$$ Crossmultiply:

$$p_i^* + p_i^* \sum_{j=1}^{k} \max(0, z_j(\underset{\sim}{p}^*)) = p_i^* + \max(0, z_i(\underset{\sim}{p}^*)).$$

Cancel, then Multiply by z_i^*: [i.e., $z_i(\underset{\sim}{p}^*)$]

$$z_i^* p_i^* \sum_{j=1}^{k} \max(0, z_j^*) = z_i^* \max(0, z_i^*).$$ Sum over commodities i:

does not depend on i

$$\underbrace{\sum_i z_i^* p_i^*}_{\equiv 0 \text{ by Walras' Law}} \overbrace{\sum_{j=1}^{k} \max(0, z_j^*)} = \sum_i z_i^* \max(0, z_i^*)$$

$$\Rightarrow \sum_i \underbrace{z_i \max(0, z_i)} = 0.$$

either $0 \geq z_i(\underset{\sim}{p}^*) \Rightarrow \cdots$
or $0 < z_i(\underset{\sim}{p}^*) \Rightarrow \cdots$

→ either 0 or $z_i^2 > 0$

0 ↓ OK

$z_i^2 > 0$ ↓ can't be this because then the sum couldn't be zero.

So $\max(0, z_i) = 0 \ \forall i \Rightarrow z_i \leq 0 \ \forall i$. ∎

Read p. 322.

n consumers
m firms
K commodities

Y_j production possibilities set for firm j

$\tilde{y}_j$ net output vector (negative entries denote inputs, positive " " outputs)

$\tilde{p} \cdot \tilde{y}_j$ profit

↑ fixed (perfect competition)

$Y = \{\tilde{y} : \tilde{y} = \sum_j \tilde{y}_j\}$ (contra Varian) ← no externalities!
(Jehle & Reny p. 207)

	Firm #1	Firm #2
wood	−	
chemicals	−	
paper	+	
fishing boats		−
fish		+

(separate)

Prop. $\tilde{y}$ maximizes aggregate profit iff $\tilde{y}_j$ maximizes firm j's profit $\forall j$.

Proof. ⇒ ("if") (sufficiency). By contradiction. Suppose $\tilde{y}$ maximizes aggregate π but that firm k could have made higher profits with a different plan. Then letting k adopt this different plan and having all other firms use their previous plan yields higher aggregate π.

~~Necessity ("only if")~~

Conversely, let $\tilde{y}_j$ maximize individual π_j but $\tilde{y} = \sum \tilde{y}_j$ not maximize aggregate π. Let $\tilde{y}' = \sum_{\text{firms}} \tilde{y}_j'$ maximize aggregate π.

~~Sufficiency (if)~~ Necessity ⇐ (only if). By contradiction.

Then

$$\tilde{p} \cdot \tilde{y}' = \sum_j \underbrace{\tilde{p} \cdot \tilde{y}_j'}_{\text{indiv. } \pi} = \underbrace{\tilde{p} \cdot \sum_j \tilde{y}_j' > \tilde{p} \cdot \sum_j \tilde{y}_j}_{\text{aggregate profits}} = \sum_j \underbrace{\tilde{p} \cdot \tilde{y}_j}_{\text{indiv. } \pi}.$$

firms $\pi_1' + \pi_2' + \cdots + \pi_m'$ ($\tilde{p} \cdot \tilde{y}'$) ... $\pi_1 + \pi_2 + \cdots + \pi_m$

So $\tilde{y}_j$ can't maximize ~~π~~ for every j, as claimed. ∎

So can analyze firms individually or in the aggregate.

(18.2)

Aggregate net S function well-behaved?

Y_j strictly convex : OK

nonconvex : multiple optimizing $\tilde{y}_j$'s may exist

CRS : supply correspondence not
" function

(→ further mathematical complications)

(18.3)

income $\underset{\sim}{p} \cdot \underset{\sim}{\omega}_i$

↑ but supply of labor isn't fixed

$\bar{L}$ time

ℓ labor

L leisure

budget constraint $p \cdot \text{cons.} = p \cdot \text{endowment}$ ~~of cons.~~ $+ \text{wage}(\bar{L} - L) + \text{profit}$

$p \cdot \text{cons.} + \text{wage} \cdot \text{leisure} = p \cdot \text{end.} + w \cdot \text{time} + \text{profit}$

$\underset{\sim}{p} \cdot \underset{\sim}{x} \le \underset{\sim}{p} \cdot \underset{\sim}{\omega} + \text{profit}$

extra constraint: $\bar{L} \le 24$ hrs

wage income =

wage ($\bar{L}$ - leisure)

= wage * $\bar{L}$ - wage * leisure

($\bar{L}$ ↑ endowment; wage * leisure → like consumption)

profit distribution

$$\underset{\sim}{p} \cdot \underset{\sim}{x}_i = \underset{\sim}{p} \cdot \underset{\sim}{\omega}_i + \sum_{\text{firms } j} T_{ij} \, \underset{\sim}{p} \cdot \underset{\sim}{y}_j$$

consumer i

T_{ij} → consumer i's share of firm j's profits

(18.4)

aggregate demand $\underset{\sim}{X} = \Sigma \underset{\sim}{x}_i$ consumers i

" supply $\underset{\sim}{Y}(p) + \underset{\sim}{\omega}$ where $\underset{\sim}{\omega} = \sum_i \underset{\sim}{\omega}_i$ consumers i $\quad \Sigma \underset{\sim}{y}_j \neq \underset{\sim}{Y}(\underset{\sim}{p})$ isn't $\underset{\sim}{Y}$ (supply, Prod Poss Set, p.339)

$\Sigma \underset{\sim}{y}_j + \Sigma \underset{\sim}{\omega}_i =$ output - input + input, sort of or produced outputs + unproduced outputs p.343 ℓ2-3

" excess demand $\underset{\sim}{z} = \underset{\sim}{X} - \underset{\sim}{Y}(p) - \underset{\sim}{\omega}$

Walras' Law. $\underset{\sim}{p} \cdot \underset{\sim}{z} = 0$.

Proof. $\underset{\sim}{p} \cdot \underset{\sim}{z} = \underset{\sim}{p} \cdot (\underset{\sim}{X} - \underset{\sim}{Y} - \underset{\sim}{\omega})$

$= \underset{\sim}{p} \cdot (\Sigma \underset{\sim}{x}_i - \Sigma \underset{\sim}{y}_j - \Sigma \underset{\sim}{\omega}_i)$ consumers i, firms j

$= \Sigma \underset{\sim}{p} \cdot \underset{\sim}{x}_i - \Sigma \underset{\sim}{p} \cdot \underset{\sim}{y}_j - \Sigma \underset{\sim}{p} \cdot \underset{\sim}{\omega}_i$

$\underset{\sim}{p} \cdot \underset{\sim}{\omega}_i + \sum_j T_{ij} \underset{\sim}{p} \cdot \underset{\sim}{y}_j$ consumer's budget constraints

$= \sum_i \underset{\sim}{p} \cdot \underset{\sim}{\omega}_i + \sum_i \sum_j T_{ij} \underset{\sim}{p} \cdot \underset{\sim}{y}_j - \sum_j \underset{\sim}{p} \cdot \underset{\sim}{y}_j - \sum_i \underset{\sim}{p} \cdot \underset{\sim}{\omega}_i$

$\sum_j \sum_i T_{ij} \underset{\sim}{p} \cdot \underset{\sim}{y}_j$

$\sum_j \underset{\sim}{p} \cdot \underset{\sim}{y}_j \sum_i T_{ij}$

$\sum_j \underset{\sim}{p} \cdot \underset{\sim}{y}_j \quad (1)$

$= 0$. ∎

Nice copy next.

18.4 Walras' Law: $\underset{\sim}{p} \cdot \underset{\sim}{z} = 0$.

Proof. $\underset{\sim}{p} \cdot \underset{\sim}{z} = \underset{\sim}{p} \cdot (\underset{\sim}{X} - \underset{\sim}{Y} - \underset{\sim}{\omega})$

$$= \underset{\sim}{p} \cdot \left(\sum \underset{\sim}{x}_i - \sum \underset{\sim}{y}_j - \sum \underset{\sim}{\omega}_i \right)$$ consumers i, firms j

$$= \underbrace{\sum \underbrace{\underset{\sim}{p} \cdot \underset{\sim}{x}_i}_{\underset{\sim}{p} \cdot \underset{\sim}{\omega}_i + \sum_j T_{ij} \underset{\sim}{p} \cdot \underset{\sim}{y}_j} - \sum \underset{\sim}{p} \cdot \underset{\sim}{y}_j - \sum \underset{\sim}{p} \cdot \underset{\sim}{\omega}_i}$$

consumers' budget constraints

$$= \sum_i \underset{\sim}{p} \cdot \underset{\sim}{\omega}_i + \underbrace{\sum_i \sum_j T_{ij} \underset{\sim}{p} \cdot \underset{\sim}{y}_j} - \sum_j \underset{\sim}{p} \cdot \underset{\sim}{y}_j - \sum_i \underset{\sim}{p} \cdot \underset{\sim}{\omega}_i$$

$$\sum_j \sum_i T_{ij} \underset{\sim}{p} \cdot \underset{\sim}{y}_j$$

$$\sum_j \underset{\sim}{p} \cdot \underset{\sim}{y}_j \sum_i T_{ij}$$

$$\sum_j \underset{\sim}{p} \cdot \underset{\sim}{y}_j \, (1)$$

$= 0$. ∎

(18.5)

read p. 344

Assumption 6 implies there are no fixed inputs, so it rules out the "short run."

(8) $Y \cap (-Y) = \{\underset{\sim}{0}\}$ (not $\subset$, as Varian has) (−Y is not the complement of Y)

(9) $\mathbb{R}^- \subset Y$ (better than Varian) (free disposal)

(7) $Y = \{\underset{\sim}{y} : \underset{\sim}{y} = \sum_j \underset{\sim}{y}_j\}$

(18.8) Robinson Crusoe Economy. Labor/leisure tradeoff. Criticisms:

↑ assumed good

- work hours are always a choice
- leisure is always better than working
- labor → consumption w/ no natural resources

Fig. 18.1: the CRS case. \$

$m(\text{labor}) = 3\,\text{labor}.$

↑ mangoes

Firm: $\pi = p_m\, m(\text{labor}) - w \cdot \text{labor}$

max over labor ⇒

$$0 = p_m m' - w$$

⇒ $m' = w/p_m$. Since m' is a constant (3), this determines the real wage, $3 = w/p_m$. So this equilibrium price has been determined w/o knowing demand.

Could choose $p_m \equiv 1 \Rightarrow 3 = w/1 \Rightarrow w = 3$; or

$w \equiv 1 \Rightarrow 3 = 1/p_m \Rightarrow p_m = \frac{1}{3}$; or

$p_m + w \equiv 1 \Rightarrow p_m + 3p_m = 1 \Rightarrow p_m = \frac{1}{4} \Rightarrow w = \frac{3}{4}$.

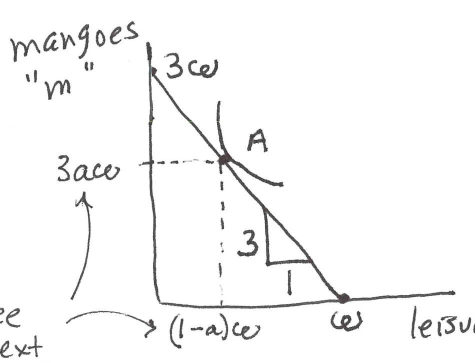
We'll choose the second. $\pi^* = \frac{1}{3} \cdot 3\,\text{labor} - 1 \cdot \text{labor} \equiv 0.$

Consumer: Budget constraint is $w(\omega - \text{leisure}) = p_m m$ (no profit income) ← or $w\omega = p_m m + w\,\text{leisure}$

or $w\omega = p_m m + w\,\text{leisure}$ ⇒ $m = \frac{-w}{p_m}\text{leisure} + \frac{w\omega}{p_m}$

or $m = \frac{w}{p_m}(\omega - \text{leisure})$

$= -\frac{w}{p_m}\text{leisure} + \frac{w}{p_m}\omega$. (It's best not to substitute prices in at this stage so you can get the labor supply & mango demand curves.)



see next page

Note: ω = leisure + labor

Suppose $u = a \ln m + (1-a)\ln(\text{leisure})$. Maximizing u s.t. the B.C. yields $\mathcal{L} = a \ln m + (1-a)\ln(\text{leisure}) + \lambda\left[-\frac{w}{p_m}\text{leisure} + \frac{w}{p_m}\omega - m\right]$.

$0 = \partial\mathcal{L}/\partial m = \frac{a}{m} - \lambda \Rightarrow \lambda = \frac{a}{m}$ and

$$0 = \partial \mathcal{L}/\partial \text{leisure} = \frac{1-a}{\text{leisure}} - \frac{\lambda w}{p_m}$$

$$= \frac{1-a}{\text{leisure}} - \frac{a}{m}\frac{w}{p_m} \Rightarrow \frac{aw}{m p_m} = \frac{1-a}{\text{leisure}} \Rightarrow \frac{aw}{1-a}\frac{\text{leisure}}{p_m} = m.$$

Substituting into the BC,

$$\frac{aw}{1-a}\frac{\text{leisure}}{p_m} = \frac{-w}{p_m}\text{leisure} + \frac{w}{p_m}\omega$$

$$\left(\frac{aw}{1-a}\frac{1}{p_m} + \frac{w}{p_m}\right)\text{leisure} = \frac{w\omega}{p_m}$$

$$\left(\frac{aw}{(1-a)p_m} + \frac{(1-a)w}{(1-a)p_m}\right)\text{leisure} = \frac{w\omega}{p_m}$$

$$\left[\frac{a}{(1-a)p_m} + \frac{1-a}{(1-a)p_m}\right]\text{leisure} = \frac{\omega}{p_m}$$

$$\frac{1}{(1-a)p_m}\text{leisure} = \frac{\omega}{p_m} \Rightarrow \frac{1}{1-a}\text{leisure} = \omega \Rightarrow$$

$\text{leisure}^* = (1-a)\,\omega$, the demand curve for leisure.

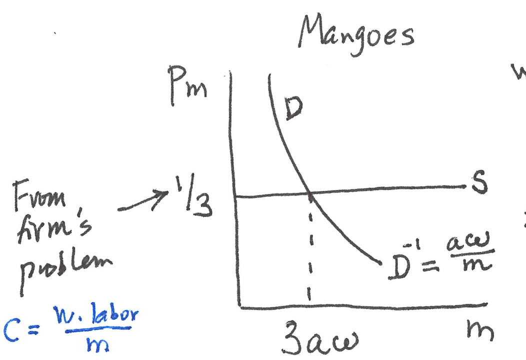
The corresponding supply curve for labor is

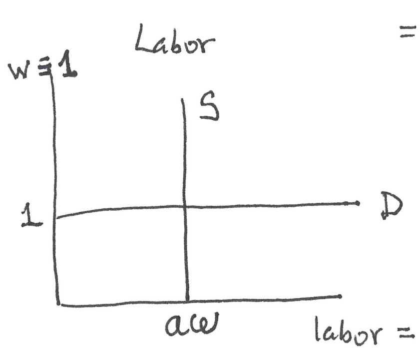
$\omega - \text{leisure} = \omega - (1-a)\omega = a\omega.$

Also, the demand for mangoes is

$$m = \frac{aw}{1-a}\frac{(1-a)\omega}{p_m} = a\omega\frac{w}{p_m} = a\omega\frac{1}{1/3}$$

(w → $= 1$; p_m → $= \frac{1}{3}$ in equ. but leave unspecified for now)





labor = ω − leisure

$AC = \frac{w \cdot \text{labor}}{m} = \frac{w(m/3)}{m} = \frac{w}{3} = \frac{1}{3}.$

↳ either from the prod^n fnct^n or from $D_{mangoes} = a\omega\frac{1}{1/3} = 3a\omega.$

Fig. 18.2 ↓RS case.

Firm $\pi = p_m\, m(\text{labor}) - w \cdot \text{labor}$

max over labor ⇒

$0 = p_m\, m' - w$

⇒ $m' = w/p_m$.

} As in CRS case.

Since $m'(\text{labor}) = w/p_m$, the demand for labor is $\text{labor} = (m')^{-1}(w/p_m)$, and the supply of mangoes is $m\left[(m')^{-1}(w/p_m)\right]$. Also,

$$\pi = p_m\, m\left[(m')^{-1}(w/p_m)\right] - w \cdot (m')^{-1}(w/p_m).$$

Consumer. Budget Constraint $w(\omega - \text{leisure}) + \pi = p_m m$

$$\Leftrightarrow \frac{w}{p_m}\omega - \frac{w}{p_m}\text{leisure} + \frac{\pi}{p_m} = m \quad \rightarrow m = \frac{-w}{p_m}\text{leisure} + \frac{w\omega + \pi}{p_m}$$

$$\Leftrightarrow \frac{-w}{p_m}\text{leisure} + \frac{w}{p_m}\omega - \left(m - \frac{\pi}{p_m}\right) = 0$$

which is the same as the budget constraint in the CRS case except that "$m - \frac{\pi}{p_m}$" here replaces "m" there. One can check that the $0 = \partial\mathcal{L}/\partial m$ and $0 = \partial\mathcal{L}/\partial\,\text{leisure}$ F.O.C.'s are the same as in the CRS case. Substituting into the B.C.,

$$\frac{-w}{p_m}\text{leisure} + \frac{w}{p_m}\omega - \underbrace{\frac{aw}{1-a}\frac{\text{leisure}}{p_m}}_{\text{same as the CRS case's "m"}} + \frac{\pi}{p_m} = 0$$

$$\Rightarrow \frac{w\,\omega}{p_m} + \frac{\pi}{p_m} = \text{leisure}\left[\frac{w}{p_m} + \frac{1}{p_m}\frac{a w}{1-a}\right]$$

$$= \text{leisure}\left[\frac{w(1-a) + a w}{p_m(1-a)}\right] = \text{leisure}\ \frac{w}{p_m(1-a)}$$

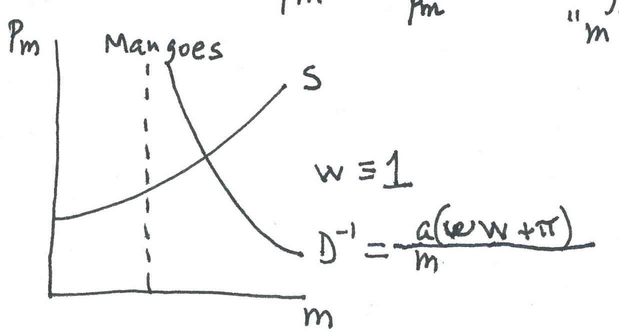
$$\frac{\omega}{p_m} + \frac{\pi}{w\,p_m} = \frac{\text{leisure}}{p_m(1-a)} \Rightarrow \text{leisure}^* = (1-a)\,\omega + \frac{(1-a)\cdot\pi}{w},$$

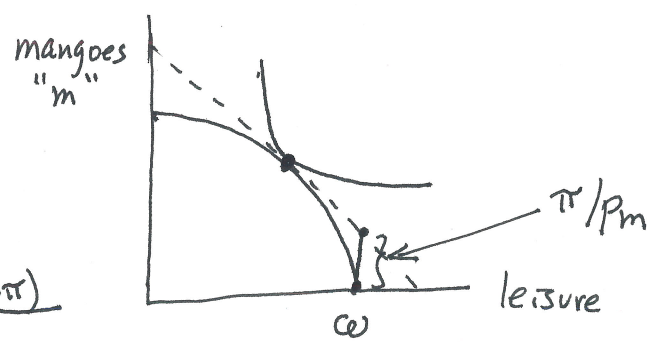
the demand for leisure. The corresponding supply of labor is

$\omega - \text{leisure} = a\omega - \dfrac{(1-a)\cdot\pi}{w}$. The demand for mangoes is

$$m = \frac{a w}{1-a}\frac{1}{p_m}\left[(1-a)\,\omega + \frac{(1-a)\cdot\pi}{w}\right]$$

$$= a w\frac{\omega}{p_m} + \frac{a\pi}{p_m}.$$





Note: If we take labor as the numeraire*, then equilibrium can be obtained by clearing either the mango market or the labor market.

* or mangoes as the numeraire, or use a simplex,

(18.9)

Nonsubstitution Theorem.

One-output notational framework! But >1 output: $\underset{\sim}{y}$

& $\underset{\sim}{w}$ is prices of inputs & outputs

(bizarre notation)

Only one nonproduced input to production, y_0
this input is indispensable to production
no joint production
CRS
$(\underset{\sim}{x}, \underset{\sim}{y}, \underset{\sim}{w})$ is the Walrasian equilibrium with $y_i > 0 \ \forall i$

$\Rightarrow$ $\underset{\sim}{w}$ is the unique solution to $w_i = c_i(\underset{\sim}{w})$

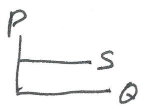
↑ unit cost function: how much it costs to produce one unit of output i at prices $\underset{\sim}{w}$

"price = Average Cost"

inputs — $\underset{\sim}{x}$

outputs — $\underset{\sim}{y}$

$(w_0, w_1, \ldots, w_n)$ prices of inputs & outputs — $\underset{\sim}{w}$



P determined by AC = MC not by D

Read top of p. 356.

Proof. CRS $\Rightarrow$ zero π $\Rightarrow$

So these are the premises you need to get this general equilibrium.

(18.10)

CRS: # firms indeterminate

↓RS: $\pi > 0$

if entry, # firms $\to \infty$

note this is also socially optimal & that smaller firms can "out-compete" larger ones