(10.1) $(\underset{\sim}{p}^0; m^0)$ vs $(\underset{\sim}{p}'; m')$

Δv only ordinal

need cardinal for: [establishing priorities when ∄ …]
aggregate across consumers

money metric utility function $m(\underset{\sim}{p}, \underset{\sim}{x}) = e(\underset{\sim}{p}, u(\underset{\sim}{x}))$

fix; then $\uparrow \underset{\sim}{x} \Rightarrow \uparrow m$, just like u.

m is a monotonically $\uparrow$ transformation of u

money metric indirect utility function $\mu(\overset{P_1}{\underset{\sim}{p}}; \overset{P_2}{\underset{\sim}{q}}, m) \equiv e(\overset{P_1}{\underset{\sim}{p}}, v(\overset{P_2}{\underset{\sim}{q}}, m))$

fix

"\# needed at prices $\overset{P_1}{\underset{\sim}{p}}$ to be
as well off as you are at $(\overset{P_2}{\underset{\sim}{q}}, m)$"

If freeze $\overset{P_1}{\underset{\sim}{p}}$, this is just a transformation of v

All arguments observable.

Turn Utility Change

ordinal → $v(\underset{\sim}{p}', m') - v(\underset{\sim}{p}^0, m^0)$

into

cardinal → $e(\overset{\hat{p}}{\underset{\sim}{q}}, v(\underset{\sim}{p}', m')) - e(\overset{\hat{p}}{\underset{\sim}{q}}, v(\underset{\sim}{p}^0, m^0))$

→ Interpretation: $ this is −\$15.
Then
$e(q, v(\underset{\sim}{p}', m')) + \$15 = e(q, v(\underset{\sim}{p}^0, m^0))$

$(\hat{P}_1 = p^0)$?

		gain	loss	
$\hat{P}$ in Base Year: EV	If we didn't make the Δ …	WTA	WTP	≠ ∴ problems w/ social decision making
$\hat{P}$ in Final ~~[illegible]~~ Year: CV $(\hat{P}_1 = P')$	If we made the Δ …	WTP	WTA	

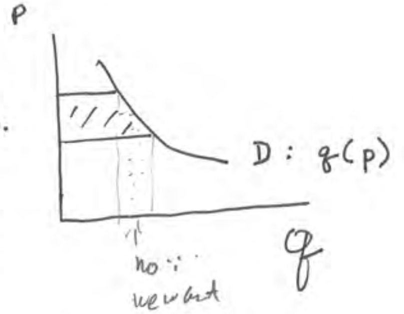
Fig. 10.1

EV better for hypothetical, "what-if" questions ∵ base prices are known
maybe

(10.2)

ΔC.S.

Varian omits



no ∵ we want Δ net C.S., see

Net Consumer Surplus = Consumer Surplus

Gross Benefit, Gross Consumer Surplus

(10.3) quasilinear utility $U(\underset{\sim}{x}) = x_0 + u(x_1, .., x_K)$

p.164 method: $u'(x_1) = p_1$ w/ $k = 1$, x_0 the numeraire

good of interest   No income effect!

D for x_1 independent of income... $\forall m$?? No: p.165

(10.4) CV = EV = CS w/ quasilinear utility

(10.5)

$$EV = e(p^0, v(p', m)) - \underbrace{e(p^0, v(p^0, m))}_{\equiv m}$$ if only one price changes (no income Δ)

Nice copy next.

$$\overbrace{= e(p', v(p', m))}^{\equiv m} \quad = e(p^0, u') - e(p', u')$$

$$= \int_{p'}^{p^0} h(p, u')\,dp$$

$\because h = \partial e / \partial p$

EV: base year prices, final " utility

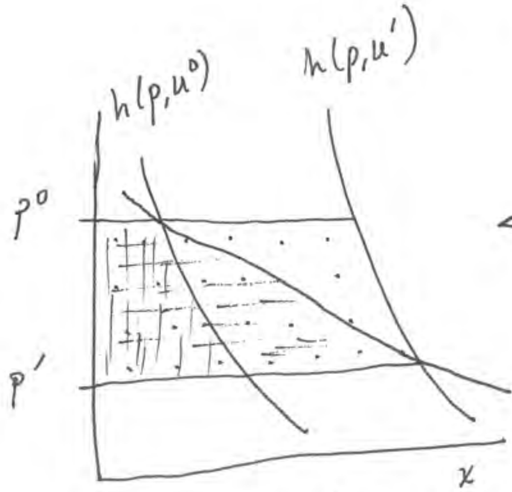
$$CV = \underbrace{e(p', v(p', m))}_{\equiv m} - e(p', v(p^0, m))$$

$$\overbrace{e(p^0, v(p^0, m))}^{\equiv m} \quad = e(p^0, u^0) - e(p', u^0)$$

$$= \int_{p'}^{p^0} h(p, u^0)\,dp$$

CV: final year p's, initial year u

↓p leads to



$x(p,m)$ if x is normal

{ EV: dots
CV: ||||
CS: ≡ }

δL_p

$\frac{\partial h}{\partial p}$ less negative than $\frac{\partial x}{\partial p}$

$$\frac{\partial h}{\partial p} = \frac{\partial x}{\partial p} + \oplus$$

RHS less negative (closer to zero) than LHS ⇒

⇒ Hicksian less steeply sloped ⇒ now rotate axes around 45°

h: no shift in curve ~~instead of~~ just Δ ~~movement along curve~~ ∴ curve ~~u changes when p changes~~

x: no shift in curve as p↓

10.5

$$EV = e(p^0, v(p', m)) - \underbrace{e(p^0, v(p^0, m))}_{\equiv m} \quad \text{if only one price changes (no income change)}$$

base year prices

$$\overbrace{}^{\equiv m}$$

$$= e(p', v(p', m))$$

$$= e(p^0, u') - e(p', u') = \int_{p'}^{p^0} h(p, u')\, dp \quad \because h = \frac{\partial e}{\partial p}$$

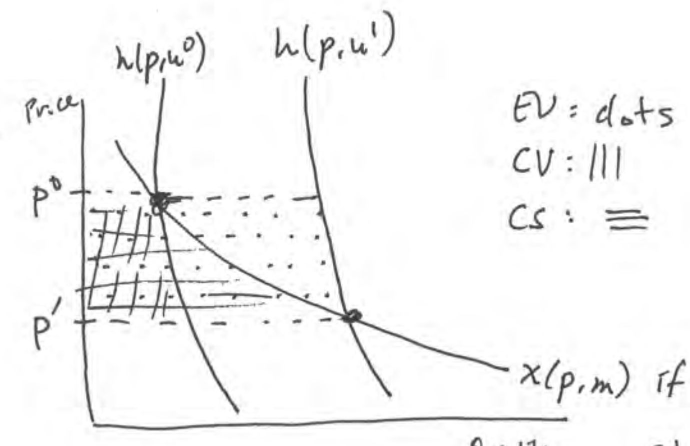
final year prices

$$CV = \underbrace{e(p', v(p', m))}_{\equiv m} - e(p', v(p^0, m)) = e(p^0, u^0) - e(p', u^0)$$

$$\overbrace{e(p^0, v(p^0, m))}^{\equiv m}$$

$$= \int_{p'}^{p^0} h(p, u^0)\, dp$$

$\downarrow p$ leads to



EV : dots

CV : |||

CS : $\equiv$

x : no shift as p changes

$$\frac{\partial h}{\partial p} = \frac{\partial x}{\partial p} + \oplus \quad x\frac{\partial x}{\partial m} \text{ normal (Slutsky)}$$

RHS less negative (closer to zero) than LHS $\Rightarrow$

Hicksian less steeply sloped $\Rightarrow$

how to take axes

10.6 Aggregation

p169 "aggregate CS is the appropriate welfare measure for quasilinear utility"

(13.6) study x w/ $U = u(x) + y$ quasilinear utility

$\max_{x,y} U$ s.t. $P_y y + P_x x = m$

① y from B.C.

$\Leftrightarrow \max_x u(x) + m - P_x x \Rightarrow$ and $p = u'(x)$

$\Rightarrow x^D$ not a function of income (over some range)

$\pi = px - c(x)$

$\max \pi \Rightarrow p = c'(x)$

$u'(x) = c'(x)$ at equilibrium $\because$ $u'(x(p)) = p = c'(x)$

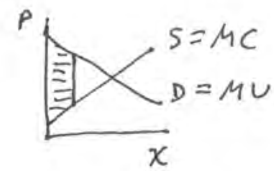
demand ↑ ↑ supply

(13.7) max total utility: $\max_{x,y} u(x) + y$ s.t. $\omega = y + c(x)$

ω: endowment of y; y: consumption of y; $c(x)$: amount of y needed to make x

$$\Leftrightarrow \max_x \; u(x) - c(x) + \omega \quad \Rightarrow u'(x) = c'(x) \text{ and}$$

$$= \max_x \int_0^x \left[u'(\hat{x}) - c'(\hat{x}) \right] d\hat{x} \quad \text{if } u(0) = c(0) = 0$$

$= \max_x$ 

$\Rightarrow x^*$ is equ. value of x. So comp. equ. maximizes total quasilinear utility.

Consumer's surplus $u(x) - px$

producer's surplus $px - c(x)$

$$\max_x (CS + PS) = \max_x \left[u(x) - px + px - c(x) \right]$$

$$= \max_x \; u(x) - c(x)$$ So max. total utility = max. total surplus

(13.8) w/ quasilinear u,

equ. maximizes Σ utilities.

this is only one welfare measure

(13.9)

Pareto efficiency : no way to make anyone better off w/o making someone else worse off — §17.6 strongly Pareto efficient

: no way to make everyone better off. — §17.6 weakly Pareto efficient

¬ weak ⇒ ¬ strong
trivial

¬ strong ⇒ ¬ weak
(use divisibility)

for quasilinear U
functions . . .

13.10 omit

(17.6)

See §13.9 on strong & weak Pareto efficiency.

strong ⇒ weak (if can't make anyone better off, you
" " everyone " ")

weak ⇒ strong (if you can make someone better off w/o making anyone else worse off, make the better-off person a bit less better off and redistribute the goods to everyone else) [Proof: ~strong ⇒ ~weak]

weak P.E. is the more convenient notion mathematically

P.E. is normatively weak (may even not be necessary for social optimality)
→ (almost surely it's not sufficient " " ")

B

A

max u_B st u_A fixed

$\max\ \alpha u_A + \beta u_B$ st $\alpha + \beta = 1$

⇒ equality of A & B's MRS

Contract curve = Pareto set

Walrasian Equ: 1) feasible $\sum_{i=1}^{n} \tilde{x}_i = \sum_{i=1}^{n} \tilde{\omega}_i$

2) optimal for each consumer:

$$\tilde{x}_i' \succ_i \tilde{x}_i \Rightarrow \underbrace{\tilde{p} \cdot \tilde{x}_i'}_{\text{expenditures}} > \underbrace{\tilde{p} \cdot \tilde{\omega}_i}_{\text{income}} = \tilde{p} \cdot \tilde{x}_i$$

First Theorem of Welfare Economics.

$$(\underset{\sim}{x}, \underset{\sim}{p}) \text{ W.E.} \Rightarrow \underset{\sim}{x} \text{ is P.E.}$$

"Walrasian Equilibrium" not "Welfare Economics"

Proof. Suppose not; then let the feasible alternative to $\underset{\sim}{x}$ that makes everyone better off be $\underset{\sim}{x}'$. Then $\underset{\sim}{x}_i' \succ \underset{\sim}{x}_i \ \forall i$. Then

$\underset{\sim}{x}$ is W.E. $\Rightarrow \underset{\sim}{p} \cdot \underset{\sim}{x}_i' > \underset{\sim}{p} \cdot \underset{\sim}{\omega}_i \ \forall i$ (otherwise everyone would buy $\underset{\sim}{x}_i'$ in W.E. instead of buying $\underset{\sim}{x}_i$)

$$\sum_i \underset{\sim}{p} \cdot \underset{\sim}{x}_i' > \sum_i \underset{\sim}{p} \cdot \underset{\sim}{\omega}_i$$

$\underset{\sim}{p} \cdot \sum_i \underset{\sim}{x}_i' > \underset{\sim}{p} \cdot \sum_i \underset{\sim}{\omega}_i$. But feasibility implies that $\sum_i \underset{\sim}{x}_i' = \sum_i \underset{\sim}{\omega}_i$, so

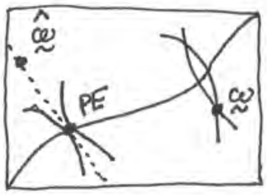
$\parallel$

$\underset{\sim}{p} \cdot \sum_i \underset{\sim}{\omega}_i$, a contradiction. ∎

(The W.E. may be quite unfair.)

17.7

Every PE allocation can be achieved by a suitable reallocation of endowments and, afterwards, the market mechanism. (Sort of PE $\Rightarrow$ WE)



PE is not a WE from $\underset{\sim}{\omega}$. But if we make confiscatory transfers from $\underset{\sim}{\omega}$ to $\hat{\underset{\sim}{\omega}}$, then PE is a WE from $\hat{\underset{\sim}{\omega}}$. Of course if you can get from $\underset{\sim}{\omega}$ to $\hat{\underset{\sim}{\omega}}$, you could also have gone from $\underset{\sim}{\omega}$ to PE directly.

<u>Second Theorem of Welfare Economics.</u> Suppose that preferences are nonsatiated and that: a) $\underset{\sim}{x}^{PE}$ is a Pareto efficient allocation.

b) a WE exists from initial endowments $\underset{\sim}{\omega}_i = \underset{\sim}{x}_i^{PE}$ $\forall i$.

c) this WE is $(\underset{\sim}{p}^{WE}, \underset{\sim}{x}^{CE})$.

Then $(\underset{\sim}{p}^{CE}, \underset{\sim}{x}^{PE})$ is a Walrasian ("competive") equilibrium.

[similar to Varian p.329 not p.326]

<u>Proof.</u>

$\underset{\sim}{\omega}_i = \underset{\sim}{x}_i^{PE} \Rightarrow \underset{\sim}{x}_i^{PE}$ is in i's affordable set $\Rightarrow \underset{\sim}{x}_i^{CE} \succsim_i \underset{\sim}{x}_i^{PE}$ $\forall i$. But since $\underset{\sim}{x}^{PE}$ is PE, it can't be true that $\underset{\sim}{x}_i^{CE} \succ_i \underset{\sim}{x}_i^{PE}$ for any i. So $\underset{\sim}{x}_i^{CE} \sim_i \underset{\sim}{x}_i^{PE}$ $\forall i$. So since $\underset{\sim}{x}_i^{CE}$ is utility-maximizing at $\underset{\sim}{p}^{CE}$ given $\underset{\sim}{\omega}_i$, then $\underset{\sim}{x}_i^{PE}$ must also be " " " " ". Hence $(\underset{\sim}{p}^{CE}, \underset{\sim}{x}^{PE})$ is a competitive equilibrium. ∎

17.8 omit

(17.9) social welfare function

$$W(u_1, \dots, u_n) : \mathbb{R}^n \to \mathbb{R}^1$$

& W is increasing in each u_i.

Or maybe: $W(\underset{\sim}{x}_1, \underset{\sim}{x}_2, \dots, \underset{\sim}{x}_n) : \mathbb{R}^k \times \mathbb{R}^n \to \mathbb{R}^1$

Prop. $\underset{\sim}{x}^*$ maximizes $W^{(u_1,\dots,u_n)} \Rightarrow \underset{\sim}{x}^*$ is P.E.

Proof. If $\underset{\sim}{x}^*$ were not PE then $\exists\ \underset{\sim}{x}^{**}$ making u_i larger or the same as $\overbrace{\text{under}}$ $\underset{\sim}{x}^*$ $\forall i$; then $\underset{\sim}{x}^{**}$ would maximize W, instead of $\underset{\sim}{x}^*$ doing so. ∎

p334: PE $\Rightarrow$ maximizes $\underset{\text{a linear}}{\wedge}$ W for some choice of weights

21.1

A $\overbrace{\text{group of agents } S}^{\text{"Coalition" } \geq 1 \text{ person}}$ is said to <u>improve upon</u> a given allocation $\underset{\sim}{x}$ $\rightarrow (\underset{\sim}{x}_1, \underset{\sim}{x}_2, \ldots, \underset{\sim}{x}_n)$

if there is some allocation $\underset{\sim}{x}'$ which satisfies

$$\sum_{i \in S} \underset{\sim}{x}_i' = \sum_{i \in S} \underset{\sim}{\omega}_i \quad \text{and}$$

$$\underset{\sim}{x}_i' \succ_i \underset{\sim}{x}_i \quad \forall i \in S.$$

k commodities

people in the coalition (not nec. the whole economy)

An allocation is said to be in the <u>core</u> of an economy if it cannot be improved upon by any coalition. "coalition-proof"

<u>Prop</u>. $\underset{\sim}{x}$ in the core $\Rightarrow \underset{\sim}{x}$ is Pareto efficient. <u>Proof</u>: Consider the grand coalition. //

Fig. 21.1: PE $\not\Rightarrow$ core

<u>Prop</u>. If $(\underset{\sim}{x}^{CE}, \underset{\sim}{p}^{CE})$ is a Walrasian equilibrium with initial endowments $\underset{\sim}{\omega}_i$, then $\underset{\sim}{x}^{CE}$ is in the core.

<u>Proof</u>. If not; then $\exists$ some coalition S and some allocation $\underset{\sim}{x}'$ s.t. all agents in S prefer $\underset{\sim}{x}'$ to $\underset{\sim}{x}^{CE}$ and $\sum_{i \in S} \underset{\sim}{x}_i' = \sum_{i \in S} \underset{\sim}{\omega}_i$. (*)

Since $\underset{\sim}{x}_i' \succ_i \underset{\sim}{x}_i^{CE} \; \forall i \in S$, $\underset{\sim}{x}'$ must not be affordable for i in S:

$$\underset{\sim}{p}^{CE} \cdot \underset{\sim}{x}_i' > \underset{\sim}{p}^{CE} \cdot \underset{\sim}{\omega}_i \quad \forall i \in S.$$

$$\therefore \quad \underset{\sim}{p}^{CE} \cdot \sum_{i \in S} \underset{\sim}{x}_i' > \underset{\sim}{p}^{CE} \cdot \sum_{i \in S} \underset{\sim}{\omega}_i \quad \text{which contradicts } (*). //$$

type: preferences & endowment

	Type A	Type B	Type C
r-replica	2	1	4
r-core	4	2	8 its 2-replica

- Equal treatment in the Core   convex preferences ⇒ prefer mixtures to extremes

- If $\underset{\sim}{y}$ is not a Walrasian equilibrium then there is some replication of the economy in which $\underset{\sim}{y}$ is not in the r-core.

21.4

Read & criticize this section.

(18.6) $(\underset{\sim}{x}, \underset{\sim}{y})$ feasible if:

x_i demand by consumer i

y_j input-output vector of firm j

(from §18.4)

$$\underbrace{\sum_{i=1}^{n} \underset{\sim}{x}_i}_{\text{over consumers}} = \sum_{i=1}^{n} \underset{\sim}{\omega}_i + \underbrace{\sum_{j=1}^{m} \underset{\sim}{y}_j}_{\text{over firms}}$$

$4 = 1 + (9-6)$ example

First Theorem of Welfare Economics

$$(\underset{\sim}{x}, \underset{\sim}{y}, \underset{\sim}{p}) \text{ WE} \Rightarrow (\underset{\sim}{x}, \underset{\sim}{y}) \text{ PE}.$$

Proof. If not; let $(\underset{\sim}{x}', \underset{\sim}{y}')$ be a Pareto dominating allocation. In W.E.

Consumers are maximizing utility, so $\underset{\sim}{x}'$ must be unaffordable:

$$\underset{\sim}{p} \cdot \underset{\sim}{x}_i' > \underset{\sim}{p} \cdot \underset{\sim}{\omega}_i + \sum_{j=1}^{m} T_{ij}\, \underset{\sim}{p} \cdot \underset{\sim}{y}_j .$$

Sum over consumers:

$$\sum_{i=1}^{n} \underset{\sim}{p} \cdot \underset{\sim}{x}_i' > \sum_{i=1}^{n} \underset{\sim}{p} \cdot \underset{\sim}{\omega}_i + \underbrace{\sum_{i=1}^{n} \sum_{j=1}^{m} T_{ij}\, \underset{\sim}{p} \cdot \underset{\sim}{y}_j}_{\sum_{j=1}^{m} \left[\underset{\sim}{p} \cdot \underset{\sim}{y}_j \underbrace{\sum_{i=1}^{n} T_{ij}}_{=1} \right]}$$

$$\sum_{i=1}^{n} \underset{\sim}{p} \cdot \underset{\sim}{x}_i' > \sum_{i=1}^{n} \underset{\sim}{p} \cdot \underset{\sim}{\omega}_i + \sum_{j=1}^{m} \underset{\sim}{p} \cdot \underset{\sim}{y}_j . \text{ From feasibility:}$$

$$\underset{\sim}{p} \cdot \left[\sum_{i=1}^{n} \underset{\sim}{\omega}_i + \sum_{j=1}^{m} \underset{\sim}{y}_j' \right] > \sum_{i=1}^{n} \underset{\sim}{p} \cdot \underset{\sim}{\omega}_i + \sum_{j=1}^{m} \underset{\sim}{p} \cdot \underset{\sim}{y}_j$$

$$\sum_{j=1}^{m} \underset{\sim}{p} \cdot \underset{\sim}{y}_j' > \sum_{j=1}^{m} \underset{\sim}{p} \cdot \underset{\sim}{y}_j$$

which contradicts $\underset{\sim}{y}$ being profit-maximizing. ∎

Second theorem of Welfare Economics. $(PE \Rightarrow WE)$

Suppose that preferences are locally nonsatiated and that

(a) $(\underset{\sim}{x}^{PE}, \underset{\sim}{y}^{PE})$ is a Pareto efficient allocation

(b) a competitive equilibrium exists from initial endowments
$\underset{\sim}{\omega}_i = \underset{\sim}{x}_i^{PE}$ with profit shares $T_{ij} = 0 \; \forall i \, \& \, j$.

[Note: hence profits are thrown away. The existence proof will still go through, however. From p. 348,

$$\underset{\sim}{p} \cdot \underset{\sim}{z} = \underset{\sim}{p} \cdot \underset{\sim}{X} - \underset{\sim}{p} \cdot \underset{\sim}{\omega} - \underset{\sim}{p} \cdot \underset{\sim}{Y}$$

but the budget constraint is, for person i, $\underset{\sim}{p} \cdot \underset{\sim}{x}_i = \underset{\sim}{p} \cdot \underset{\sim}{\omega}_i + \sum_{j=1}^{m} T_{ij} \underset{\sim}{p} \cdot \underset{\sim}{y}_j$, &with $T_{ij} \equiv 0, \ldots$

$$= \underset{\sim}{p} \cdot \underset{\sim}{X} - \underset{\sim}{p} \cdot \underset{\sim}{X} - \underset{\sim}{p} \cdot \underset{\sim}{Y} = -\underset{\sim}{p} \cdot \underset{\sim}{Y} \leq 0$$

since profits $\underset{\sim}{p} \cdot \underset{\sim}{Y}$ are thrown away (so D is smaller than it would otherwise be). But from problem 17.10, the existence proof goes through even when $\underset{\sim}{p} \cdot \underset{\sim}{z} \leq 0$.

(c) this competitive equilibrium is $(\underset{\sim}{p}^{CE}, \underset{\sim}{x}^{CE}, \underset{\sim}{y}^{CE})$.

Then $(\underset{\sim}{p}^{CE}, \underset{\sim}{x}^{PE}, \underset{\sim}{y}^{PE})$ is a competitive equilibrium.

Proof.

$\underset{\sim}{\omega}_i = \underset{\sim}{x}_i^{PE} \Rightarrow \underset{\sim}{x}_i^{PE}$ is in i's affordable set $\Rightarrow \underset{\sim}{x}_i^{CE} \succsim_i \underset{\sim}{x}_i^{PE}$.

But since $\underset{\sim}{x}^{PE}$ is PE, it must be that $\underset{\sim}{x}_i^{CE} \sim_i \underset{\sim}{x}_i^{PE}$.

So since $\underset{\sim}{x}_i^{CE}$ is utility-maximizing at $\underset{\sim}{p}^{CE}$ given $\underset{\sim}{\omega}_i$, then $\underset{\sim}{x}_i^{PE}$ must also be ——— " ———.

Next check all is OK with the firms:

Nonsatiation $\Rightarrow$ budget constraints meet w/ equality $\Rightarrow$

$$\underset{\sim}{p}^{CE} \cdot \underset{\sim}{x}_i^{CE} = \underset{\sim}{p}^{CE} \cdot \underset{\sim}{x}_i^{PE} \quad \forall i$$

$$\sum_i \underset{\sim}{p}^{CE} \cdot \underset{\sim}{x}_i^{CE} = \sum_i \underset{\sim}{p}^{CE} \cdot \underset{\sim}{x}_i^{PE}$$; using feasibility as at the beginning of this section,

$$\underset{\sim}{p}^{CE} \cdot \left(\sum_{i=1}^{n} \underset{\sim}{\omega}_i + \sum_{j=1}^{m} \underset{\sim}{y}_j^{CE} \right) = \underset{\sim}{p}^{CE} \cdot \left(\sum_{i=1}^{n} \underset{\sim}{\omega}_i + \sum_{j=1}^{m} \underset{\sim}{y}_j^{PE} \right)$$

$$\underset{\sim}{p}^{CE} \cdot \sum_{j=1}^{m} \underset{\sim}{y}_j^{CE} = \underset{\sim}{p}^{CE} \cdot \sum_{j=1}^{m} \underset{\sim}{y}_j^{PE}$$

Since the LHS maximizes aggregate profits, so does the RHS. By p. 339's result, the RHS maximizes each individual firm's profits. ∎

18.7

$$\max \ W(u_1(\underset{\sim}{x}_1) + u_2(\underset{\sim}{x}_2) + \cdots + u_n(\underset{\sim}{x}_n)) \ \text{s.t.} \ T\left(\sum_{i=1}^{n} \underset{\sim}{x}_i\right) = 0.$$

not +'s, but commas!

$$\mathcal{L} = W\left(u_1(\underset{\sim}{x}_1) + u_2(\underset{\sim}{x}_2) + \cdots + u_n(\underset{\sim}{x}_n)\right) - \lambda T\left(\sum_i \underset{\sim}{x}_i\right)$$

$i = 1, \ldots, n$ consumers.

Socially optimal number of apples for person i:

$$0 = W' \frac{\partial u_i}{\partial \text{apples}_i} - \lambda \frac{\partial T}{\partial \text{apples}} \underbrace{\frac{\partial \text{apples}}{\partial \text{apples}_i}}_{=1}$$

$\uparrow \partial W / \partial u_i$*

$\boxed{\text{solve for } \lambda}$

$$\lambda \frac{\partial T}{\partial \text{apples}} = W' \frac{\partial U}{\partial u_i} \frac{\partial u_i}{\partial \text{apples}_i}$$

$$\frac{\lambda}{W' \frac{\partial U}{\partial u_i}} = \frac{\partial u_i / \partial \text{apples}_i}{\partial T / \partial \text{apples}}$$

... do the F.O.C. for bananas for person i ...

$$\Rightarrow \frac{\dfrac{\partial u_i}{\partial \text{apples}_i}}{\dfrac{\partial u_i}{\partial \text{bananas}_i}} = \frac{\dfrac{\partial T}{\partial \text{apples}}}{\dfrac{\partial T}{\partial \text{bananas}}}$$

$$MRS = MRT$$

* ~~really~~: $\cancel{\frac{dW}{dU}} \ \cancel{\frac{\partial U}{\partial u_i}} \ \cancel{\frac{\partial u_i}{\partial \text{apples}_i}}$