

10.1

$(\tilde{p}^0, \tilde{m}^0)$ vs (\tilde{p}', \tilde{m}')

Δv only ordinal

need cardinal for: [establishing priorities when ...]

aggregate across consumers

money metric utility function $m(\tilde{p}, \tilde{x}) = e(\tilde{p}, u(\tilde{x}))$

↑

fix; then $\uparrow \tilde{x} \Rightarrow \uparrow m$, just like u .

m is a monotonically ↑ transformation of u

money metric indirect utility function $\mu(\tilde{p}; \tilde{q}, \tilde{m}) \equiv e(\tilde{p}, v(\tilde{q}, \tilde{m}))$

"\$ needed at prices \tilde{p} to be as well off as you are at (\tilde{q}, \tilde{m}) "

If freeze \tilde{p} , this is just a transformation of v

All arguments observable.

Turn Utility Change

$$v(\tilde{p}', \tilde{m}') - v(\tilde{p}^0, \tilde{m}^0)$$

ordinal →

into

$$e(\tilde{q}, v(\tilde{p}', \tilde{m}')) - e(\tilde{q}, v(\tilde{p}^0, \tilde{m}^0))$$

cardinal →

$(\tilde{p}_1 = \tilde{p}_1^0)$

Pin Base Year: EV

If we didn't make the Δ ...

gain loss

WTA WTP

∴ problems of social decision making

Pin Final Year: CV

$(\tilde{p}_1 = \tilde{p}_1')$

If we made the Δ ...

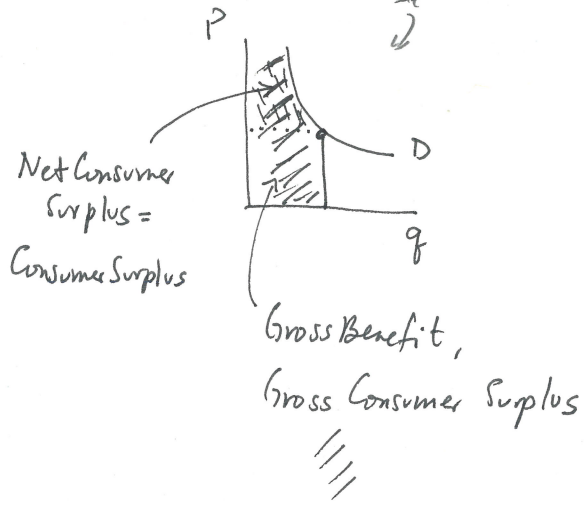
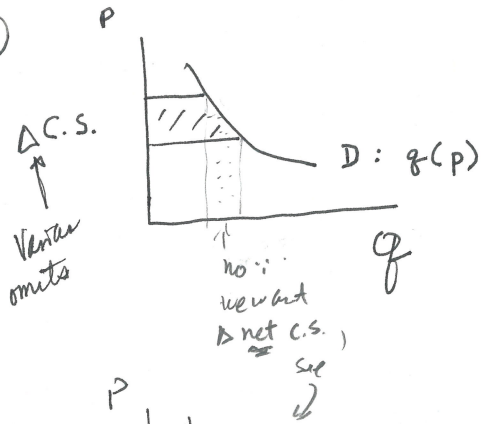
WTP WTA

Fig. 10.1

EV better for hypothetical, "what-if" questions ∴ base prices are known

↑ may be

10.2



10.3 quasilinear utility $W(x) = x_0 + u(x_1, \dots, x_k)$

p.164 method: $u'(x_k) = p_k$, w/ $k=1$, x_0 the numeraire

↑ good of interest No income effect!

D for x_k independent of income... $\forall m$?? No: p.165

10.4 $CV = EV = CS$ w/ quasilinear utility

10.5

$EV = e(p^0, v(p', m)) - e(p^0, v(p^0, m))$ if only one price changes (no income Δ)

$\underbrace{\quad}_{\equiv m} = e(p', v(p', m)) = e(p^0, u') - e(p', u')$

$= \int_{p'}^{p^0} h(p, u') dp$

$\because h = \partial e / \partial p$

EV: base year prices, final " utility

Nice copy next.

$CV = e(p', v(p', m)) - e(p', v(p^0, m))$

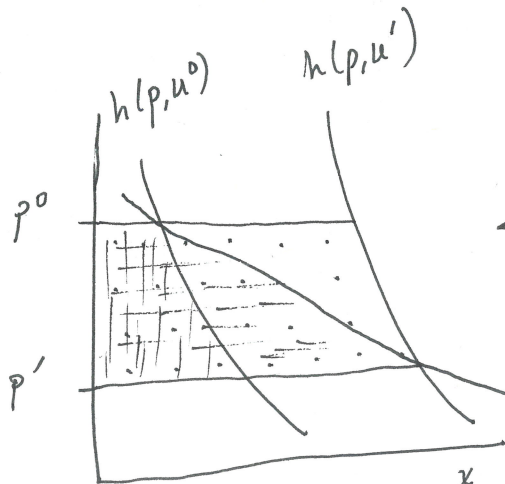
$\underbrace{\quad}_{\equiv m} = e(p^0, v(p^0, m))$

$= e(p^0, u^0) - e(p', u^0)$

$= \int_{p'}^{p^0} h(p, u^0) dp$

CV: final year p's, initial year u

↓ P leads to



EV: dots
CV: |||
CS: ≡

$x(p, m)$ if x is normal

$\frac{\partial h}{\partial p} = \frac{\partial x}{\partial p} + \oplus$

$\frac{\partial h}{\partial p}$ less negative than $\frac{\partial x}{\partial p}$

RHS less negative (closer to zero) than LHS \Rightarrow

Hicksian less steeply sloped \Rightarrow now rotate axes around 45°

h : no shift in curve instead of just a movement along curve \therefore curves

h changes w/ p changes

x : no shift in curve as $p \downarrow$

10.5

$$EV = e(p^0, v(p', m)) - e(p^0, v(p^0, m))$$

↑
base year prices

≡ m

if only one price changes
(no income change)

$$= e(p', v(p', m))$$

$$= e(p^0, u') - e(p', u') = \int_{p'}^{p^0} h(p, u') dp \quad \because h = \frac{\partial e}{\partial p}$$

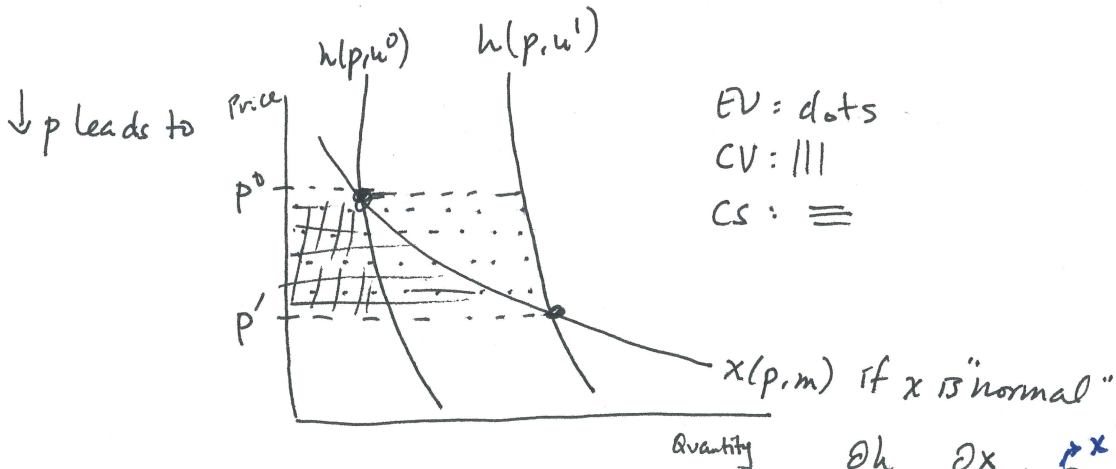
↓
final year prices

$$CV = e(p', v(p', m)) - e(p', v(p^0, m)) = e(p^0, u^0) - e(p', u^0)$$

≡ m

e(p^0, v(p^0, m))

$$= \int_{p'}^{p^0} h(p, u^0) dp$$



x: no shift as p changes

$$\frac{\partial h}{\partial p} = \frac{\partial x}{\partial p} + \left(\frac{\partial x}{\partial m} \right) \frac{\partial m}{\partial p}$$

normal (Slutsky)

RHS less negative (closer to zero) than LHS \Rightarrow
 Hicksian less steeply sloped \Rightarrow
 how to take axes

10.6

Aggregation

p169 "aggregate CS is the appropriate welfare measure for quasi-linear utility"

13.6

study x w/ $U = u(x) + y$ quasilinear utility

$\Rightarrow x^p$ not a function of income (over some range) and $p = u'(x)$

$\max_{x,y} U \text{ s.t. } p_y y + p_x x = m$
 $\Leftrightarrow \max_x u(x) + m - p_x x \Rightarrow$ (1) y from B.C.

$u'(x) = c'(x)$ at equilibrium $\because u'(x(p)) = p = c'(x)$

$\pi = px - c(x)$
 $\max \pi \Rightarrow p = c'(x)$

demand \uparrow supply \uparrow

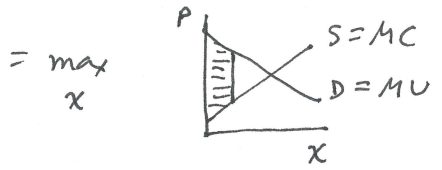
13.7

max total utility: $\max_{x,y} u(x) + y \text{ s.t. } ce = y + c(x)$

\uparrow Endowment of y \uparrow Consumption of y \uparrow amount of y needed to make x

$\Leftrightarrow \max_x u(x) - c(x) + ce \Rightarrow u'(x) = c'(x) \text{ and}$

$= \max_x \int_0^x [u'(\hat{x}) - c'(\hat{x})] d\hat{x} \text{ if } u(0) = c(0) = 0$



$\Rightarrow x^*$ is equ. value of x . So comp. equ. maximizes total quasilinear utility.

Consumer's surplus $u(x) - px$
 producer's surplus $px - c(x)$

$\max_x (CS + PS) = \max_x [u(x) - px + px - c(x)]$
 $= \max_x u(x) - c(x) \text{ so max. total utility} = \text{max. total surplus}$

13.8

w/ quasilinear u ,
 equ. maximizes Σ utilities.
 this is only one welfare measure

13.9

Pareto efficiency: no way to make anyone better off who making someone else worse off

§17.6 strongly Pareto efficient

: no way to make everyone better off.

§17.6 weakly Pareto efficient

\sim weak \Rightarrow \sim strong
trivial

\sim strong \Rightarrow \sim weak
(use divisibility)

for quasilinear \vee
separable...

13.10 omst

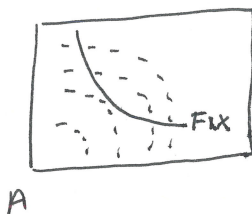
17.6

See § 13.9 on strong & weak Pareto efficiency.

strong \Rightarrow weak (it can't make anyone better off, you
" " everyone " ")

weak \Rightarrow strong (if you can make someone better off w/o making
anyone else worse off, make the better-off person
a bit less better off and redistribute the goods to
everyone else) [Proof: \neg strong \Rightarrow \neg weak]

Weak P.E. is the more convenient notion mathematically
P.E. is normatively weak (may even not be necessary for social optimality)
B \rightarrow (almost surely it's not sufficient " " ") \Downarrow



$\max u_B$ st u_A fixed

$\max \alpha u_A + \beta u_B$ st $\alpha + \beta = 1$

\Rightarrow equality of A & B's MRS

Contract curve = Pareto set

Walrasian Equ: 1) feasible $\sum_{i=1}^n x_i = \sum_{i=1}^n \omega_i$

2) optimal for each consumer:

$\tilde{x}'_i \succ_i \tilde{x}_i \Rightarrow \underbrace{p \cdot \tilde{x}'_i}_{\text{expenditures}} > \underbrace{p \cdot \omega_i}_{\text{income}} = p \cdot \tilde{x}_i$

First Theorem of Welfare Economics.

$$(\underline{x}, \underline{p}) \text{ W.E.} \Rightarrow \underline{x} \text{ is P.E.}$$

↑ "Walrasian Equilibrium" not "Welfare Economics"

Proof. Suppose not; then let the feasible alternative to \underline{x} that makes everyone better off be \underline{x}' . Then $\underline{x}'_i \succ \underline{x}_i \forall i$. Then

$$\underline{x} \text{ is W.E.} \Rightarrow \underline{p} \cdot \underline{x}'_i > \underline{p} \cdot \underline{\omega}_i \forall i \text{ (otherwise everyone would buy } \underline{x}'_i \text{ in W.E. instead of buying } \underline{x}_i \text{)}$$

$$\sum_i \underline{p} \cdot \underline{x}'_i > \sum_i \underline{p} \cdot \underline{\omega}_i$$

$$\underline{p} \cdot \sum_i \underline{x}'_i > \underline{p} \cdot \sum_i \underline{\omega}_i. \text{ But feasibility implies that}$$

$$\sum_i \underline{x}'_i = \sum_i \underline{\omega}_i, \text{ so}$$

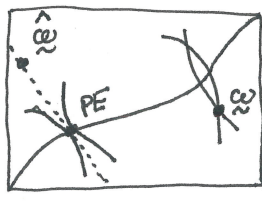
$$\parallel$$

$$\underline{p} \cdot \sum_i \underline{\omega}_i, \text{ a contradiction. } \blacksquare$$

(The W.E. may be quite unfair.)

17.7

Every PE allocation can be achieved by a suitable reallocation of endowments and, afterwards, the market mechanism. (Sort of PE \Rightarrow WE)



PE is not a WE from \underline{w} . But if we make confiscatory transfers from \underline{w} to $\hat{\underline{w}}$, then PE is a WE from $\hat{\underline{w}}$. Of course if you can get from \underline{w} to $\hat{\underline{w}}$, you could also have gone from \underline{w} to PE directly.

Second Theorem of Welfare Economics. Suppose that preferences are nonsatiated

- and that :
- a) \underline{x}^{PE} is a Pareto efficient allocation.
 - b) a WE exists from initial endowments $\underline{w}_i = \underline{x}_i^{PE} \forall i$.
 - c) this WE is $(\underline{p}^{WE}, \underline{x}^{CE})$.

Then $(\underline{p}^{CE}, \underline{x}^{PE})$ is a Walrasian ("competitive") equilibrium.

[similar to Varian p.329 not p.326]

Proof.

$\underline{w}_i = \underline{x}_i^{PE} \Rightarrow \underline{x}_i^{PE}$ is in i 's affordable set $\Rightarrow \underline{x}_i^{CE} \succsim_i \underline{x}_i^{PE} \forall i$. But since \underline{x}_i^{PE} is PE, it can't be true that $\underline{x}_i^{CE} \succ_i \underline{x}_i^{PE}$ for any i . So

$\underline{x}_i^{CE} \sim_i \underline{x}_i^{PE} \forall i$. So since \underline{x}_i^{CE} is utility-maximizing at \underline{p}^{CE} given \underline{w}_i , then \underline{x}_i^{PE} must also be " " " ". Hence

$(\underline{p}^{CE}, \underline{x}^{PE})$ is a competitive equilibrium. ■

(17.8) omit

17.9

social welfare function

$$W(u_1, \dots, u_n) : \mathbb{R}^n \rightarrow \mathbb{R}^1$$

§ W is increasing in each u_i .

Or maybe: $W(x_1, x_2, \dots, x_n) :$
 $\mathbb{R}^k \times \mathbb{R}^n \rightarrow \mathbb{R}^1$

Prop. \underline{x}^* maximizes $W^{(u_1, \dots, u_n)} \Rightarrow \underline{x}^*$ is P.E.

Proof. If \underline{x}^* were not PE then $\exists \underline{x}^{**}$ making u_i larger or the same ^{under} as \underline{x}^*
 $\forall i$; then \underline{x}^{**} would maximize W , instead of \underline{x}^* doing so. \square

p334: PE \Rightarrow maximizes W for some choice of weights
 \uparrow
a linear

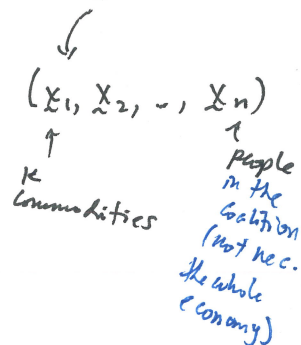
21.1

"Coalition" \Rightarrow 1 person

A group of agents S is said to improve upon a given allocation \underline{x} if there is some allocation \underline{x}' which satisfies

$$\sum_{i \in S} \underline{x}'_i = \sum_{i \in S} \omega_i \quad \text{and}$$

$$\underline{x}'_i \succ_i \underline{x}_i \quad \forall i \in S.$$



An allocation is said to be in the core of an economy if it cannot be improved upon by any coalition. "coalition-proof"

Prop. \underline{x} in the core $\Rightarrow \underline{x}$ is Pareto efficient. Proof: Consider the grand coalition. ||

Fig. 21.1: PE $\not\Rightarrow$ core

Prop. If $(\underline{x}^{CE}, \underline{p}^{CE})$ is a Walrasian equilibrium with initial endowments ω_i , then \underline{x}^{CE} is in the core.

Proof. \not not; then \exists some coalition S and some allocation \underline{x}' s.t. all agents in S prefer \underline{x}' to \underline{x}^{CE} and $\sum_{i \in S} \underline{x}'_i = \sum_{i \in S} \omega_i$. (*)

Since $\underline{x}'_i \succ_i \underline{x}^{CE}_i \quad \forall i \in S$, \underline{x}' must not be affordable for $\overset{i \text{ in}}{S}$:

$$\underline{p}^{CE} \cdot \underline{x}'_i > \underline{p}^{CE} \cdot \omega_i \quad \forall i \in S.$$

$\therefore \underline{p}^{CE} \cdot \sum_{i \in S} \underline{x}'_i > \underline{p}^{CE} \cdot \sum_{i \in S} \omega_i$ which contradicts (*). ||

type: preferences & endowment

r-replica
r-core

Type A	Type B	Type C
2	1	4
4	2	8 its 2-replica

- Equal treatment in the Core convex preferences \Rightarrow prefer mixtures to extremes
- If y is not a Walrasian equilibrium then there is some replication of the economy in which y is not in the r-core.

21.4

Read & criticize this section.

18.6

$(\underline{x}, \underline{y})$ feasible if:

$$\sum_{i=1}^n \underline{x}_i = \sum_{i=1}^n \underline{\omega}_i + \sum_{j=1}^m \underline{y}_j$$

↑
↑
↑

 over consumers + (9-6) example over firms

x_i demand by consumer i
 Y_j input-output vector of firm j
 (from 18.4)

First Theorem of Welfare Economics

$(\underline{x}, \underline{y}, p)$ WE $\Rightarrow (\underline{x}, \underline{y})$ PE.

Proof. \nexists not; let $(\underline{x}', \underline{y}')$ be a Pareto dominating allocation. In W.E.

Consumers are maximizing utility, so \underline{x}' must be unaffordable:

$$p \cdot \underline{x}'_i > p \cdot \underline{\omega}_i + \sum_{j=1}^m T_{ij} p \cdot \underline{y}_j$$

Sum over consumers:

$$\sum_{i=1}^n p \cdot \underline{x}'_i > \sum_{i=1}^n p \cdot \underline{\omega}_i + \underbrace{\sum_{i=1}^n \sum_{j=1}^m T_{ij} p \cdot \underline{y}_j}_{\sum_{j=1}^m \left[p \cdot \underline{y}_j \underbrace{\sum_{i=1}^n T_{ij}}_{=1} \right]}$$

$$\sum_{i=1}^n p \cdot \underline{x}'_i > \sum_{i=1}^n p \cdot \underline{\omega}_i + \sum_{j=1}^m p \cdot \underline{y}_j \quad \text{From feasibility:}$$

$$p \cdot \left[\sum_{i=1}^n \underline{\omega}_i + \sum_{j=1}^m \underline{y}'_j \right] > \sum_{i=1}^n p \cdot \underline{\omega}_i + \sum_{j=1}^m p \cdot \underline{y}_j$$

$$\sum_{j=1}^m p \cdot \underline{y}'_j > \sum_{j=1}^m p \cdot \underline{y}_j$$

which contradicts y being profit-maximizing. \blacksquare

Second Theorem of Welfare Economics. (PE \Rightarrow WE)

Suppose that preferences are locally non-satiated and that

(a) $(\tilde{x}^{PE}, \tilde{y}^{PE})$ is a Pareto efficient allocation

(b) a competitive equilibrium exists from initial endowments

$$\omega_i = \tilde{x}_i^{PE} \text{ with profit shares } T_{ij} = 0 \forall i \neq j.$$

Note: hence profits are thrown away. The existence proof will still go through, however. From p. 348,

$$\begin{aligned} p \cdot \tilde{z} &= p \cdot \tilde{x} - p \cdot \omega - p \cdot \tilde{y} \quad \text{but the budget constraint is, for person } i, \\ & \quad p \cdot \tilde{x}_i = p \cdot \omega_i + \sum_{j=1}^n T_{ij} p \cdot \tilde{y}_j \quad \text{with } T_{ij} = 0, \dots \\ &= p \cdot \tilde{x} - p \cdot \tilde{x} - p \cdot \tilde{y} = -p \cdot \tilde{y} \leq 0 \end{aligned}$$

since profits $p \cdot \tilde{y}$ are thrown away (so D is smaller than it would otherwise be). But from problem 17.10, the existence proof goes through even when $p \cdot \tilde{z} \leq 0$.

(c) this competitive equilibrium is $(\tilde{p}^{CE}, \tilde{x}^{CE}, \tilde{y}^{CE})$.

Then $(\tilde{p}^{CE}, \tilde{x}^{PE}, \tilde{y}^{PE})$ is a competitive equilibrium.

Proof.

$$\omega_i = \tilde{x}_i^{PE} \Rightarrow \tilde{x}_i^{PE} \text{ is in } i\text{'s affordable set} \Rightarrow \tilde{x}_i^{CE} \succsim_i \tilde{x}_i^{PE}$$

But since \tilde{x}_i^{PE} is PE, it must be that $\tilde{x}_i^{CE} \sim_i \tilde{x}_i^{PE}$.

So ~~if~~ since \tilde{x}_i^{CE} is utility-maximizing at \tilde{p}^{CE} given ω_i , then \tilde{x}_i^{PE} must also be " " " "

Next check all is OK with the firms:

Nonsatiation \Rightarrow budget constraints meet w/ equality \Rightarrow

$$p_{\sim}^{CE} \cdot x_{\sim i}^{CE} = p_{\sim}^{CE} \cdot x_{\sim i}^{PE} \quad \forall i$$

$$\sum_i p_{\sim}^{CE} \cdot x_{\sim i}^{CE} = \sum_i p_{\sim}^{CE} \cdot x_{\sim i}^{PE} \quad ; \text{ using feasibility as at the beginning of this section,}$$

$$p_{\sim}^{CE} \cdot \left(\sum_{i=1}^n \omega_{\sim i} + \sum_{j=1}^m y_{\sim j}^{CE} \right) = p_{\sim}^{CE} \cdot \left(\sum_{i=1}^n \omega_{\sim i} + \sum_{j=1}^m y_{\sim j}^{PE} \right)$$

$$p_{\sim}^{CE} \cdot \sum_{j=1}^m y_{\sim j}^{CE} = p_{\sim}^{CE} \cdot \sum_{j=1}^m y_{\sim j}^{PE}$$

Since the LHS maximizes aggregate profits, so does the RHS. By p. 339's result, the RHS maximizes each individual firm's profits. \blacksquare

18.7

$$\max W(u_1(x_1) + u_2(x_2) + \dots + u_n(x_n)) \text{ s.t. } T(\sum_{i=1}^n x_i) = 0.$$

↑ not +s, but minus!

$$\mathcal{L} = W(u_1(x_1) + u_2(x_2) + \dots + u_n(x_n)) - \lambda T(\sum_i x_i)$$

$i = 1, \dots, n$ consumers.

Socially optimal number of apples for person i :

$$0 = W' \frac{\partial u_i}{\partial \text{apples}_i} - \lambda \frac{\partial T}{\partial \text{apples}}$$

↑ dW/dU_i *

... do the F.O.C. for bananas for person i ...

$$\Rightarrow \frac{\frac{\partial u_i}{\partial \text{apples}_i}}{\frac{\partial u_i}{\partial \text{bananas}_i}} = \frac{\frac{\partial T}{\partial \text{apples}}}{\frac{\partial T}{\partial \text{bananas}}}$$

$$MRS = MRT$$

[solve for λ]

$$\lambda \frac{\partial T}{\partial \text{apples}} = W' \frac{\partial u_i}{\partial \text{apples}_i}$$
$$\frac{\lambda}{W' \frac{\partial u_i}{\partial \text{apples}_i}} = \frac{\partial T / \partial \text{apples}_i}{\partial T / \partial \text{apples}}$$

* really: $\frac{dW}{dU} \frac{\partial U}{\partial u_i} \frac{\partial u_i}{\partial \text{apples}_i}$