8.1

income expansion path

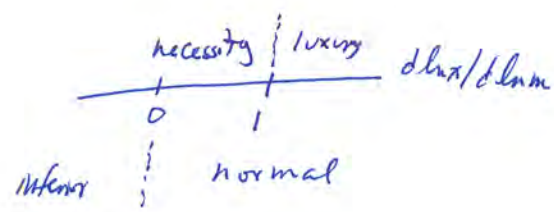
Engel curve

$$\begin{cases} \text{luxury} & \dfrac{d \ln x}{d \ln I} > 1 \quad \text{or} \quad \dfrac{dx/x}{dI/I} > 1 \iff \dfrac{I}{x}\dfrac{dx}{dI} > 1 \\ \text{necessity} & \quad " \quad < \quad " \end{cases}$$

$\boxed{"I" \equiv "m"}$

$$\begin{cases} \text{inferior} & \dfrac{dx}{dI} < 0 \\ \text{normal} & \dfrac{dx}{dI} > 0 \end{cases}$$

Fig. 8.1



price offer curve Fig. 8.2

Giffen good - Ray Battalio, TAMU mts

(8.2)

review income and substitution effects of a price change from intermediate micro

<u>Slutsky Equation.</u>

$$\frac{\partial x_j(\underset{\sim}{p}, m)}{\partial p_i} = \frac{\partial h_j(\underset{\sim}{p}, \overset{\text{utility}}{v(\underset{\sim}{p}, m)})}{\partial p_i} - \frac{\partial x_j(\underset{\sim}{p}, m)}{\partial m} x_i(\underset{\sim}{p}, m).$$

<u>Proof.</u>

Let $\underset{\sim}{x}^*$ maximize utility at $(\underset{\sim}{p}, m)$.

Let $u^* = u(\underset{\sim}{x}^*) = v(\underset{\sim}{p}, m)$.

p.106, (4): $h_j(\underset{\sim}{p}, u^*) = x_j(\underset{\sim}{p}, \overset{m}{e(\underset{\sim}{p}, u^*)})$. (Switch LHS & RHS.)

$u^* \to v(\underset{\sim}{p}, m)$

Differentiate w.r.t. p_i:

$$\underbrace{\frac{\partial h_j(\underset{\sim}{p}, u^*)}{\partial p_i}}_{= \frac{\partial x_j(\underset{\sim}{p}, e(\underset{\sim}{p}, u^*))}{\partial p_i}} = \frac{\partial x_j(\underset{\sim}{p}, m)}{\partial p_i} + \frac{\partial x_j(\underset{\sim}{p}, m)}{\partial m} \frac{\partial e(\underset{\sim}{p}, u^*)}{\partial p_i}$$

" $\frac{\partial x_j}{\partial e} \frac{\partial e}{\partial p_i}$ "

$h_i(\underset{\sim}{p}, u^*)$ §7.3 Shephard's Lemma

$= x_i$. ∎

$$\nabla_{\underset{\sim}{p}} \underset{\approx}{x}(\underset{\sim}{p}, m) = \nabla_{\underset{\sim}{p}} \underset{\approx}{h}(\underset{\sim}{p}, u) - \nabla_m \underset{\approx}{x}(\underset{\sim}{p}, m)\, \underset{\sim}{h}$$

a Jacobian

$$\nabla_{\underset{\sim}{p}} \underset{\approx}{x} = \nabla_{\underset{\sim}{p}} \underset{\sim}{h} - \overbrace{\nabla_m \underset{\sim}{x}}^{n\times 1}\, \overbrace{\underset{\sim}{h}}^{1\times n}$$

or $\underset{\sim}{x}$

substitution effect

income effect

(8.3) $\nabla_{\underset{\sim}{p}} \underset{\sim}{h}$ is the Slutsky, or substitution, matrix

follow this order

$\nabla_{\underset{\sim}{p}} \underset{\sim}{h}$

- is negative semidefinite (3)

Shephard's Lemma $\quad \underset{\sim}{h} = \nabla_{\underset{\sim}{p}}\, e(\underset{\sim}{p}, \bar{u})$ (1)

$$\nabla_{\underset{\sim}{p}} \underset{\sim}{h} = \nabla^2_{\underset{\sim}{p}}\, e(\underset{\sim}{p}, \bar{u})$$

concave from §7.3 (2)

- is symmetric (4)

Hessians are symmetric

$$h_j = \frac{\partial e}{\partial p_j} \qquad\Bigg|\qquad h_i = \frac{\partial e}{\partial p_i}$$

$$\frac{\partial h_j}{\partial p_i} = \frac{\partial^2 e}{\partial p_i \partial p_j} \qquad\Bigg|\qquad \frac{\partial h_i}{\partial p_j} = \frac{\partial^2 e}{\partial p_j \partial p_i}$$

$$\Rightarrow \frac{\partial h_j}{\partial p_i} = \frac{\partial h_i}{\partial p_j}$$

(5)

- has non-positive diagonal terms $\boxed{\dfrac{\partial h_i}{\partial p_i} \le 0}$ "Hicksian demand curves slope downwards"

bizarre source, intuitive source

Since $\nabla_{\underset{\sim}{p}} \underset{\sim}{h} = \nabla_{\underset{\sim}{p}} \underset{\sim}{x} + \nabla_m \underset{\sim}{x}\, \underset{\sim}{x}$,

these predictions are observable and testable.

(8.4) omit. In its place, do pp. 404 ff of Chiang, 3rd edition.

If $\exists$ j endogenous variables, $\exists$ j equations. & $\exists$ 'k' exogenous variables.
call these $\underset{\sim}{x}$ — call these $\underset{\sim}{y}$

$$\left.\begin{array}{l} f_1(\underset{\sim}{x},\underset{\sim}{y})=0 \\ f_2(\underset{\sim}{x},\underset{\sim}{y})=0 \\ \vdots \\ f_j(\underset{\sim}{x},\underset{\sim}{y})=0 \end{array}\right\} \Rightarrow \underset{j\times j}{\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_j} \\ \vdots & & \vdots \\ \frac{\partial f_j}{\partial x_1} & \cdots & \frac{\partial f_j}{\partial x_j} \end{bmatrix}} \underset{j\times 1}{\begin{bmatrix} dx_1 \\ \vdots \\ dx_j \end{bmatrix}} + \underset{j\times k}{\begin{bmatrix} \frac{\partial f_1}{\partial y_1} & \cdots & \frac{\partial f_1}{\partial y_k} \\ \vdots & & \vdots \\ \frac{\partial f_j}{\partial y_1} & \cdots & \frac{\partial f_j}{\partial y_k} \end{bmatrix}} \underset{k\times 1}{\begin{bmatrix} dy_1 \\ \vdots \\ dy_k \end{bmatrix}} = \underset{\sim}{0}$$

$$\text{or } \underset{\sim}{f}(\underset{\sim}{x},\underset{\sim}{y}) = \underset{\sim}{0} \quad \Rightarrow \quad (\nabla_{\underset{\sim}{x}} \underset{\sim}{f})\, d\underset{\sim}{x} + (\nabla_{\underset{\sim}{y}} \underset{\sim}{f})\, d\underset{\sim}{y} = \underset{\sim}{0}.$$

Notice that usually in economics, $\underset{\sim}{f}(\underset{\sim}{x},\underset{\sim}{y})=\underset{\sim}{0}$ is $\nabla_{\underset{\sim}{x}} \mathcal{L}(\underset{\sim}{x},\underset{\sim}{y}) = \underset{\sim}{0}$ (where $\underset{\sim}{x}$ is all the endogenous variables, including the λ's). Then one has $(\nabla^2_{\underset{\sim}{x}} \mathcal{L})\, d\underset{\sim}{x} + (\nabla_{\underset{\sim}{y}} \nabla_{\underset{\sim}{x}} \mathcal{L})\, d\underset{\sim}{y} = \underset{\sim}{0}$.

& $d\underset{\sim}{y} = \underset{\sim}{e}_i\, dy_i$ (only one exogenous variable changes). Then

$$\begin{bmatrix} \partial f_1/\partial x_1 & \cdots & \partial f_1/\partial x_j \\ \vdots & & \vdots \\ \partial f_j/\partial x_1 & \cdots & \partial f_j/\partial x_j \end{bmatrix} \begin{bmatrix} dx_1 \\ \vdots \\ dx_j \end{bmatrix} + \begin{bmatrix} \partial f_1/\partial y_i \\ \vdots \\ \partial f_j/\partial y_i \end{bmatrix} dy_i = 0$$

$$\begin{bmatrix} \partial f_1/\partial x_1 & \cdots & \partial f_1/\partial x_j \\ \vdots & & \vdots \\ \partial f_j/\partial x_1 & \cdots & \partial f_j/\partial x_j \end{bmatrix} \begin{bmatrix} dx_1/dy_i \\ \vdots \\ dx_j/dy_i \end{bmatrix} = - \begin{bmatrix} \partial f_1/\partial y_i \\ \vdots \\ \partial f_j/\partial y_i \end{bmatrix}$$

$$\text{Cramer's Rule} \Rightarrow \begin{bmatrix} dx_1/dy_i \\ \vdots \\ dx_j/dy_i \end{bmatrix} = \begin{array}{c} \hspace{4em} i^{\text{th}} \text{ column} \\ \begin{bmatrix} \partial f_1/\partial x_1 & \cdots & -\partial f_1/\partial y_i & \cdots & \partial f_1/\partial x_j \\ \vdots & & \vdots & & \vdots \\ \partial f_j/\partial x_1 & \cdots & -\partial f_j/\partial y_i & \cdots & \partial f_j/\partial x_j \end{bmatrix} \end{array} \div \left| \nabla_{\underset{\sim}{x}} \underset{\sim}{f} \right|.$$

In economics, often $\left| \nabla_{\underset{\sim}{x}} \underset{\sim}{f} \right| = \left| \nabla^2_{\underset{\sim}{x}} \mathcal{L} \right|$, which second-order conditions help to sign.

8.5 omit

(8.6)

Usually, $v = v(\mathbf{p}, m)$.

Consider instead $v(\mathbf{p}) = \max_{\mathbf{x}} u(\mathbf{x})$ s.t. $\mathbf{p} \cdot \mathbf{x} = 1$.

(We've divided $\mathbf{p} \cdot \mathbf{x} = m$ by m.)

<u>Prop.</u> $u(\mathbf{x}^*) = \min_{\mathbf{p}} v(\mathbf{p})$ s.t. $\mathbf{p} \cdot \mathbf{x}^* = 1$.

<u>Proof.</u> Let $\mathbf{x}^*$ be demanded at $\mathbf{p}^*$. Then $\mathbf{x}^*$ is affordable at $\mathbf{p}^*$: $\mathbf{x}^* \cdot \mathbf{p}^* = 1$.

Let $\mathbf{p}'$ be any other price vector that satisfies the ~~budget~~ problem's constraint: $\mathbf{x}^* \cdot \mathbf{p}' = 1$.

$\mathbf{x}^*$ is ~~feasible~~ affordable at $\mathbf{p}'$. At $\mathbf{p}'$, $\mathbf{x}^*$ might max utility or it might not: ~~you might pick $\mathbf{x}^*$, but you might not~~.

So the utility enjoyed at $\mathbf{p}' \geq u(\mathbf{x}^*)$.

$$\underbrace{\text{the utility enjoyed at } \mathbf{p}'}_{v(\mathbf{p}')} \geq \underbrace{u(\mathbf{x}^*)}_{v(\mathbf{p}^*)}$$

So, over all the possible $\mathbf{p}'$ vectors that satisfy the budget constraint:

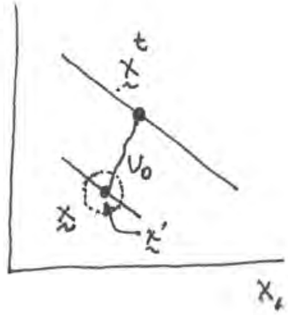
- $\mathbf{p}^*$ gives the smallest v
- $\min_{\mathbf{p}} v(\mathbf{p})$ s.t. $\mathbf{p} \cdot \mathbf{x}^* = 1$ $= v(\mathbf{p}^*)$. But at $\mathbf{p}^*$, $\mathbf{x}^*$ is demanded, so utility is $u(\mathbf{x}^*)$. ∎

(8.7)

$\underset{\sim}{p}^t \underset{\sim}{x}^t \geq \underset{\sim}{p}^t \underset{\sim}{x}$ $\underset{\sim}{x}$ is affordable, $\underset{\sim}{x}^t$ was chosen "$\underset{\sim}{x}^t$ directly revealed preferred to $\underset{\sim}{x}$"

$u(\underset{\sim}{x}^t) \geq u(\underset{\sim}{x})$

$\mathcal{S}$ $\underset{\sim}{p}^t \underset{\sim}{x}^t > \underset{\sim}{p}^t \underset{\sim}{x}$.



<u>Claim</u>: this implies $u(\underset{\sim}{x}^t) > u(\underset{\sim}{x})$.

<u>Proof</u>:

From above, $u(\underset{\sim}{x}^t) \geq u(\underset{\sim}{x})$.

$\mathcal{S}$ by way of contradiction that $u(\underset{\sim}{x}^t) = u(\underset{\sim}{x})$. (Indiff. curve U_0.)

By local nonsatiation, $\exists$ $\underset{\sim}{x}'$ which is also affordable ($\underset{\sim}{p}^t \underset{\sim}{x}^t \geq \underset{\sim}{p}^t \underset{\sim}{x}'$)

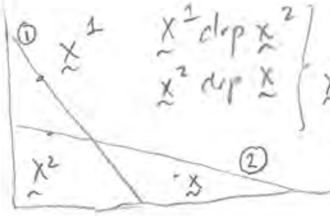
and for which $u(\underset{\sim}{x}') > u(\underset{\sim}{x}) = u(\underset{\sim}{x}^t)$.

This contradicts U. max of $\underset{\sim}{x}^t$.

Conclusion: $u(\underset{\sim}{x}^t) > u(\underset{\sim}{x})$.

{ "$\underset{\sim}{x}^t$ strictly directly revealed preferred to $\underset{\sim}{x}$."

WHY TALK ABOUT U?



$\underset{\sim}{x}^1$ d.r.p. $\underset{\sim}{x}^2$, $\underset{\sim}{x}^2$ d.r.p. $\underset{\sim}{x}$ } $\underset{\sim}{x}^1$ r.p. $\underset{\sim}{x}$

GARP: $\underset{\sim}{x}$ not s.d.r.p. $\underset{\sim}{x}^1$

$\underset{\sim}{x}^1$ d.r.p. $\underset{\sim}{x}^2$

$\underset{\sim}{x}^2$ " $\underset{\sim}{x}^3, \ldots,$

$\underset{\sim}{x}^n$ " $\underset{\sim}{x}$ $\Rightarrow$ $\underset{\sim}{x}^1$ r.p. $\underset{\sim}{x}$. "revealed preferred" $\Rightarrow u(\underset{\sim}{x}^1) \geq u(\underset{\sim}{x})$.

$\underset{\sim}{x}^t$ is revealed preferred to $\underset{\sim}{x}^s$ $\Rightarrow u(\underset{\sim}{x}^t) \geq u(\underset{\sim}{x}^s)$.

$\underset{\sim}{x}^s$ is strictly directly revealed preferred to $\underset{\sim}{x}^t$ $\Rightarrow u(\underset{\sim}{x}^s) > u(\underset{\sim}{x}^t)$.

These are incompatible $\Longleftrightarrow$ GARP.

(8.8) $(\underset{\sim}{p}^i, \underset{\sim}{x}^i)$, $i = 1, 2, \ldots$ are observations.

Afriat's Theorem.

$(1) \Rightarrow (2) \Rightarrow (3) \Rightarrow (4) \Rightarrow (1)$. In particular, $(1) \Rightarrow (4)$!

§8.7 ? below trivial

(3) ⇒ (4). Using my notation (replacing Varian's u^i by α^i), it suffices to show that if $\exists\ (\alpha^i, \lambda^i)\ \forall i \ni$

$$\alpha^s \le \alpha^t + \lambda^t \underset{\sim}{p}^t$$

then the function defined by

$$\hat{u}(\underset{\sim}{x}) = \min_i \{\alpha^i + \lambda$$

is a utility function (i.e., is loc ... cave, and monotonic) which rational ...

$\alpha^i \geq 0$ and $\lambda^i \geq 0\ \forall i$.

M-C, W, G

p 92

Proof. Part 1: $\hat{u}(\underset{\sim}{x})$ rationalizes the data. Part (2): $\hat{u}(\underset{\sim}{x})$ is locally nonsatiated, continuous, and monotonic. Part (3): $\hat{u}(\underset{\sim}{x})$ is concave.

Part 1: Need to show that at $\underset{\sim}{p}^s$, this utility function reaches a constrained maximum at $\underset{\sim}{x}^s$.

(p^0, w):

$$\tilde{h}(p, u^0) = x(p^0, w) + S(p^0, w)(p - p^0).$$

In particular, since only the price of good 1 is changing, we have

$$\tilde{h}_1(p_1, \bar{p}_{-1}, u^0) = x_1(p_1^0, \bar{p}_{-1}, w) + s_{11}(p_1^0, \bar{p}_{-1}, w)(p_1 - p_1^0),$$

where

$$s_{11}(p_1^0, \bar{p}_{-1}, w) = \frac{\partial x_1(p^0, w)}{\partial p_1} + \frac{\partial x_1(p^0, w)}{\partial w} x_1(p^0, w).$$

When $(p^1 - p^0)$ is small, this procedure provides a better approximation to the true compensating variation than does the area variation measure. However, if $(p^1 - p^0)$ is large, we cannot tell which is the better approximation. It is entirely possible for the area variation measure to be superior. After all, its use guarantees some sensitivity of the approximation to demand behavior away from p^0, whereas the use of $\tilde{h}(p, u^0)$ does not.

3.J The Strong Axiom of Revealed Preference

We have seen that in the context of consumer demand theory, consumer choice may satisfy the weak axiom but not be capable of being generated by a rational preference relation (see Sections 2.F and 3.G). We could therefore ask: Can we find a necessary and sufficient consistency condition on consumer demand behavior that is in the same style as the WA but that does imply that demand behavior can be rationalized by preferences? The answer is "yes", and it was provided by Houthakker (1950) in the form of the *strong axiom of revealed preference* (*SA*), a kind of recursive closure of the weak axiom.[30]

Definition 3.J.1: The market demand function $x(p, w)$ satisfies the *strong axiom of revealed preference* (the SA) if for any list

$$(p^1, w^1), \ldots, (p^N, w^N)$$

SARP

with $x(p^{n+1}, w^{n+1}) \neq x(p^n, w^n)$ for all $n \leq N - 1$, we have $p^N \cdot x(p^1, w^1) > w^N$ whenever $p^n \cdot x(p^{n+1}, w^{n+1}) \leq w^n$ for all $n \leq N - 1$.

In words, if $x(p^1, w^1)$ is *directly or indirectly revealed preferred* to $x(p^N, w^N)$, then $x(p^N, w^N)$ cannot be (directly) revealed preferred to $x(p^1, w^1)$ [so $x(p^1, w^1)$ cannot be affordable at (p^N, w^N)]. For example, the SA was violated in Example 2.F.1. It is clear that the SA is satisfied if demand originates in rational preferences. The converse is a deeper result. It is stated in Proposition 3.J.1; the proof, which is advanced, is presented in small type.

Proposition 3.J.1: If the Walrasian demand function $x(p, w)$ satisfies the strong axiom of revealed preference then there is a rational preference relation $\succsim$ that rationalizes $x(p, w)$, that is, such that for all (p, w), $x(p, w) \succ y$ for every $y \neq x(p, w)$ with $y \in B_{p,w}$.

main result

30. For an informal account of revealed preference theory after Samuelson, see Mas-Colell (1982).

Proof: We follow Richter (1966). His proof is based on set theory and differs markedly from the differential equations techniques used originally by Houthakker.[31]

Define a relation $\succ^1$ on commodity vectors by letting $x \succ^1 y$ whenever $x \neq y$ and we have $x = x(p, w)$ and $p \cdot y \leq w$ for some (p, w). The relation $\succ^1$ can be read as "directly revealed preferred to." From $\succ^1$ define a new relation $\succ^2$, to be read as "directly or indirectly revealed preferred to," by letting $x \succ^2 y$ whenever there is a chain $x^1 \succ^1 x^2 \succ^1, \ldots, \succ^1 x^N$ with $x^1 = x$ and $x^N = y$. Observe that, by construction, $\succ^2$ is transitive. According to the SA, $\succ^2$ is also irreflexive (i.e., $x \succ^2 x$ is impossible). A certain axiom of set theory (known as Zorn's lemma) tells us the following: *Every relation $\succ^2$ that is transitive and irreflexive* (called a *partial order*) *has a total extension $\succ^3$*, an irreflexive and transitive relation such that, first, $x \succ^2 y$ implies $x \succ^3 y$ and, second, whenever $x \neq y$, we have either $x \succ^3 y$ or $y \succ^3 x$. Finally, we can define $\succsim$ by letting $x \succsim y$ whenever $x = y$ or $x \succ^3 y$. It is not difficult now to verify that $\succsim$ is complete and transitive and that $x(p, w) \succ y$ whenever $p \cdot y \leq w$ and $y \neq x(p, w)$. ∎

The proof of Proposition 3.J.1 uses only the single-valuedness of $x(p, w)$. Provided choice is single-valued, the same result applies to the abstract theory of choice of Chapter 1. The fact that the budgets are competitive is immaterial.

In Exercise 3.J.1, you are asked to show that the WA is equivalent to the SA when $L = 2$. Hence, by Proposition 3.J.1, when $L = 2$ and demand satisfies the WA, we can always find a rationalizing preference relation, a result that we have already seen in Section 3.H. When $L > 2$, however, the SA is stronger than the WA. In fact, Proposition 3.J.1 tells us that a choice-based theory of demand founded on the strong axiom is essentially equivalent to the preference-based theory of demand presented in this chapter.

The strong axiom is therefore essentially equivalent both to the rational preference hypothesis and to the symmetry and negative semidefiniteness of the Slutsky matrix. We have seen that the weak axiom is essentially equivalent to the negative semidefiniteness of the Slutsky matrix. It is therefore natural to ask whether there is an assumption on preferences that is weaker than rationality and that leads to a theory of consumer demand equivalent to that based on the WA. Violations of the SA mean cycling choice, and violations of the symmetry of the Slutsky matrix generate path dependence in attempts to "integrate back" to preferences. This suggests preferences that may violate the transitivity axiom. See the appendix with W. Shafer in Kihlstrom, Mas-Colell, and Sonnenschein (1976) for further discussion of this point.

APPENDIX A: CONTINUITY AND DIFFERENTIABILITY PROPERTIES OF WALRASIAN DEMAND

In this appendix, we investigate the continuity and differentiability properties of the Walrasian demand correspondence $x(p, w)$. We assume that $x \gg 0$ for all $(p, w) \gg 0$ and $x \in x(p, w)$.

31. Yet a third approach, based on linear programming techniques, was provided by Afriat (1967).

$$\hat{u}(\underset{\sim}{x}^s) = \min_i \{ \alpha^i + \lambda^i \underset{\sim}{p}^i (\underset{\sim}{x}^s - \underset{\sim}{x}^i) \}$$

If this min is achieved at $i = s$ then $u(\underset{\sim}{x}^s) = \alpha^s$

" " " " " " $i = t \neq s$ " RHS @ t was < RHS @ s $\Rightarrow$

$$\alpha^t + \lambda^t \underset{\sim}{p}^t (\underset{\sim}{x}^s - \underset{\sim}{x}^t) < \alpha^s \text{, violating (3).}$$

So : $\hat{u}(\underset{\sim}{x}^s) = \alpha^s$.

Now suppose $\underset{\sim}{p}^s \cdot \underset{\sim}{x} - \underset{\sim}{p}^s \cdot \underset{\sim}{x}^s \leq 0$. Then

$$\begin{aligned}
\hat{u}(\underset{\sim}{x}) &= \min_i \{ \alpha^i + \lambda^i \underset{\sim}{p}^i (\underset{\sim}{x} - \underset{\sim}{x}^i) \} \\
&\leq \alpha^s + \lambda^s \underset{\sim}{p}^s (\underset{\sim}{x} - \underset{\sim}{x}^s) \quad \because \text{defn. of "minimum"} \\
&\leq \alpha^s \quad \text{since by assumption } \underset{\sim}{p}^s \cdot \underset{\sim}{x} - \underset{\sim}{p}^s \cdot \underset{\sim}{x}^s \leq 0 \\
&= \hat{u}(\underset{\sim}{x}^s) \quad \text{as we found before.}
\end{aligned}$$

Conclusion : $\forall \underset{\sim}{x}$ s.t. $\underset{\sim}{p}^s \cdot \underset{\sim}{x} - \underset{\sim}{p}^s \cdot \underset{\sim}{x}^s \leq 0$ (which is to say, $\forall \underset{\sim}{x}$ which are affordable at prices $\underset{\sim}{p}^s$), $\hat{u}(\underset{\sim}{x}^s) \geq \hat{u}(\underset{\sim}{x})$. So $\hat{u}(\underset{\sim}{x})$ rationalizes the observed choices.

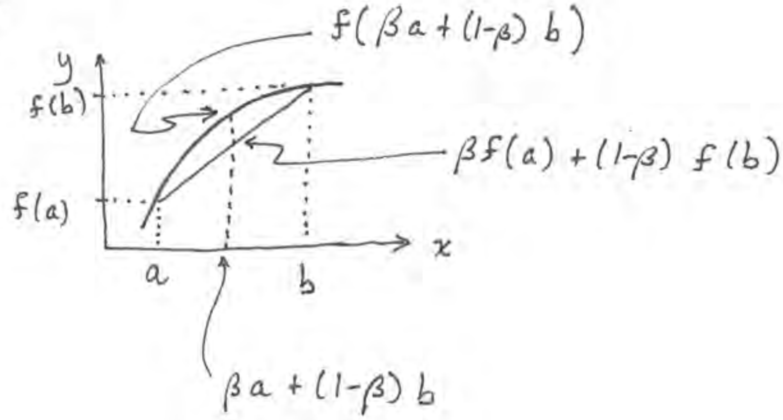
Part 2 : $\hat{u}(\underset{\sim}{x})$ is locally nonsatiated, continuous, and monotonic.

Take two observations ('93 and '94) and two commodities (apples & bananas).

$$\hat{u}(\underset{\sim}{x}) = \min_{\{'93,'94\}} \left\{ \alpha^{93} + \lambda^{93} \left(p_a^{93}, p_b^{93}\right) \cdot \begin{pmatrix} x_a - x_a^{93} \\ x_b - x_b^{93} \end{pmatrix}, \right.$$

$$\left. \alpha^{94} + \lambda^{94} \left(p_a^{94}, p_b^{94}\right) \cdot \begin{pmatrix} x_a - x_a^{94} \\ x_b - x_b^{94} \end{pmatrix} \right\}.$$

Part 3. For concavity:



$$f(\beta a + (1-\beta) b) > \beta f(a) + (1-\beta) f(b).$$

So we wish to show that

$$\hat{u}\left(\beta \underset{\sim}{a} + (1-\beta) \underset{\sim}{b}\right) \geq \beta \hat{u}(\underset{\sim}{a}) + (1-\beta) \hat{u}(\underset{\sim}{b})$$

where both $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are vectors of commodities. (For example, $\underset{\sim}{a}$ might be (2 bananas, 1 sweater, 2 ski vacations); the notation is different from Part 2's.)

RHS:

$$\beta \hat{u}(\underset{\sim}{a}) \triangleq \beta \min_i \left\{\alpha^i + \lambda^i \underset{\sim}{p}^i (\underset{\sim}{a} - \underset{\sim}{x}^i)\right\} = \min_i \beta \left\{\alpha^i + \lambda^i \underset{\sim}{p}^i (\underset{\sim}{a} - \underset{\sim}{x}^i)\right\} \text{ and}$$

$$(1-\beta) \hat{u}(\underset{\sim}{b}) \triangleq (1-\beta) \min_i \left\{\alpha^i + \lambda^i \underset{\sim}{p}^i (\underset{\sim}{b} - \underset{\sim}{x}^i)\right\}$$

$$= \min_i (1-\beta) \left\{\alpha^i + \lambda^i \underset{\sim}{p}^i (\underset{\sim}{b} - \underset{\sim}{x}^i)\right\}.$$

LHS:

$$\hat{u}\left(\beta \underset{\sim}{a} + (1-\beta) \underset{\sim}{b}\right) \triangleq \min_i \left\{ \alpha^i + \lambda^i \underset{\sim}{p}^i \left(\beta \underset{\sim}{a} + (1-\beta) \underset{\sim}{b} - \underset{\sim}{x}^i \right) \right\}.$$

Since $\beta + (1-\beta) = 1$,

$$= \min_i \left\{ \overbrace{[\beta + (1-\beta)]}^{=1} \alpha^i + \lambda^i \underset{\sim}{p}^i \left(\beta \underset{\sim}{a} + (1-\beta) \underset{\sim}{b} - \overbrace{[\beta + (1-\beta)]}^{=1} \underset{\sim}{x}^i \right) \right\}$$

$$= \min_i \left\{ \beta \alpha^i + \lambda^i \underset{\sim}{p}^i (\beta \underset{\sim}{a} - \beta \underset{\sim}{x}^i) + (1-\beta)\alpha^i + \lambda^i \underset{\sim}{p}^i \left((1-\beta) \underset{\sim}{b} - (1-\beta) \underset{\sim}{x}^i \right) \right\}$$

$$= \min_i \left\{ \beta \left[\alpha^i + \lambda^i \underset{\sim}{p}^i (\underset{\sim}{a} - \underset{\sim}{x}^i) \right] + (1-\beta) \left[\alpha^i + \lambda^i \underset{\sim}{p}^i (\underset{\sim}{b} - \underset{\sim}{x}^i) \right] \right\}.$$

Let $\underset{\sim}{A} \triangleq \left\langle \beta \left[\alpha^i + \lambda^i \underset{\sim}{p}^i (\underset{\sim}{a} - \underset{\sim}{x}^i) \right] \right\rangle_{i=1}^{n}$

$\underset{\sim}{B} \triangleq \left\langle (1-\beta) \left[\alpha^i + \lambda^i \underset{\sim}{p}^i (\underset{\sim}{b} - \underset{\sim}{x}^i) \right] \right\rangle_{i=1}^{n}$.

Then we wish to show that

$$\min_i \{ \underset{\sim}{A} + \underset{\sim}{B} \} \geq (\min_i \underset{\sim}{A}) + (\min_i \underset{\sim}{B}).$$

This is obvious. ∎

example: $\underset{\sim}{A} = (2, 4)$ LHS: 6
$\underset{\sim}{B} = (4, 2)$ RHS: 4

8.9 omit

8.10 omit

8.11 omit