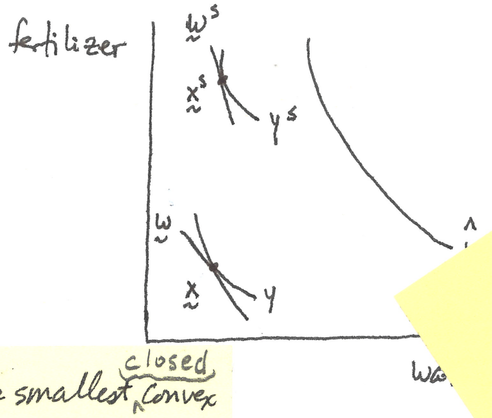
6.1

$VO(\hat{y})$



(or as expensive as)

Consid[...]ll the x's more expensive than x^s @ w^s,

——— x @ w,

[...]ll $y^t \le \hat{y}$.

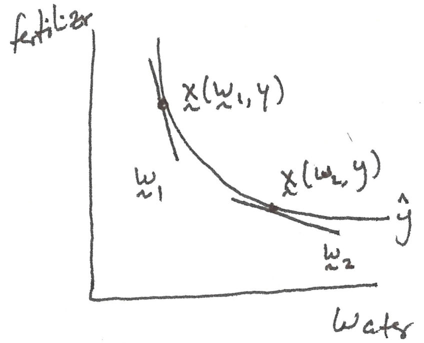
[...] the intersection.

Omit.
But give the intuition (cf. §9.6): firm'll never be where $V(y)$ differs from $V^*(y)$.

V^* is the smallest closed convex monotonic set containing V. (It's the intersection of all convex monotonic sets containing V.) [cf convex hull]

Include if want to do bottom of p. 6.1(2) formally.

$V^*(\hat{y})$



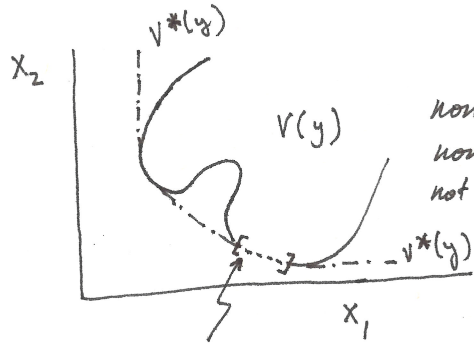
$w_i \ge 0$

For each w_i, locate $x(w_i, y)$.

Consider all the x's more expensive than (or as expensive as) $x(w_i, y)$.

Take the intersection.

$$V^*(y) \triangleq \{x : w\cdot x \ge \overbrace{w\cdot x(w,y)}^{c(w,y)} \ \forall w \ge 0\}$$



Prop. $V^*(\ldots) \supseteq V(y)$.

Proof.

If not th[...] $\hat{x} \notin V^*(y)$.

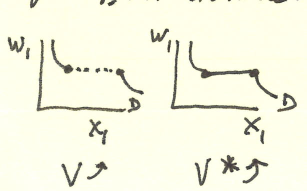
The latte[r] [...] $w_1 \ge 0$ such

that [...] [s]uppose input prices are w_1.

Then by using $\hat{x}$ [...] produce y (since $\hat{x} \in V(y)$) at a lower [ex]pense than the cost-minimizing bundle $x(w, y)$, which is impossible. ∎

Omit

"The firm has V instead of V^*" is not refutable.



<u>Prop.</u> If $V(y)$ is regular, convex, an… …nic then $V^*(y) = V(y)$.

<u>Proof.</u> $V^*(y) \supseteq V(y)$ (… So we only have to show that $V^*(y) \subseteq V(y$ …

Omit

I claim that if $\mathbf{x}$ … $\in V(y)$; for suppose on the contrary that $\mathbf{x} \notin V(y)$. … the sets $\{\mathbf{x}\}$ and $\{V(y)\}$ are nonempty (since $V(y)$ is regular), disjoint, and convex, so there is a hyperplane separating them; in other words, $\exists\, \mathbf{w}_0 \geq \mathbf{0}$ s.t. $\mathbf{w}_0 \cdot \mathbf{x} < \mathbf{w}_0 \cdot \mathbf{z} \;\; \forall\, \mathbf{z} \in V(y)$.

"<" ∵ monotonicity of $V(y)$ & $\mathbf{w}_0 \geq 0$; $\mathbf{w}_0 \cdot \mathbf{z}$ could be a very large + number

Let $\mathbf{z}_0$ be a point in $V(y)$ which mi… Then $\mathbf{w}_0 \cdot \mathbf{x} < \mathbf{w}_0 \cdot \mathbf{z}_0 = c(\mathbf{w}_0, y)$. Bu… $V^*(y)$ by the defn. of $V^*(y)$; this contradiction est…

Could include ↓

<u>Prop.</u> Let $c^*(\mathbf{w}, y) = \min\limits_{\mathbf{x}} \mathbf{w} \cdot \mathbf{x}$ s.t. $\mathbf{x} \in V^*(y)$. Then $c^*(\mathbf{w}, y) = c(\mathbf{w}, y)$.

<u>Proof.</u>
Since $V^*(y) \supseteq V(y)$, the condition "$\mathbf{x} \in V^*(y)$" is less restrictive than the condition "$\mathbf{x} \in V(y)$". Hence $c^*(\mathbf{w}, y) \leq c(\mathbf{w}, y)$.

Suppose by way of contradiction that $c^*(\mathbf{w}', y) < c(\mathbf{w}', y)$ for some $\mathbf{w}' \geq 0$. Let $\mathbf{x}'$ be the bundle in $V^*(y)$ which minimizes cost* at prices $\mathbf{w}'$ and output y. Then $\mathbf{w}' \cdot \mathbf{x}' = c^*(\mathbf{w}', y)$, and by hypothesis, $\mathbf{w}' \cdot \mathbf{x}' < c(\mathbf{w}', y)$. However, since $\mathbf{x}' \in V^*(y)$, we have by $\underbrace{\text{definition}}_{\text{of } V^*}$ that $\mathbf{w}' \cdot \mathbf{x}' \geq c(\mathbf{w}', y)$. This contradiction establishes the proof. ∎

6.2 omit

(6.3)

Skip the rest starting w/ *

Spring 2004

* D OK iff $\nabla_w^2 x$ is OK.

Well, do the 1 ¶ before the example of p 87. -Fall 2005

Note example on p. 87.

cf. §5.1

<u>Prop</u>. If $V(y)$ is regular, convex, and monotonic, then:

"$c(\underset{\sim}{w}, y)$ can be written as $y\, c(\underset{\sim}{w})$" $\Rightarrow$ "$V(y)$ has constant returns to scale."

<u>Proof.</u> Given these assump[illegible] that $V(y) = V^*(y)$. Therefore

$$V(y) = \{ \underset{\sim}{x} \text{ [illegible] } \geq \underset{\sim}{0} \}.$$

Skip the proof.

-Winter 1996

Let $t\underset{\sim}{x}_0 = \underset{\sim}{x}_1$. We wis[illegible] $V(y)$ then $\underset{\sim}{x}_1 = t\underset{\sim}{x}_0 \in V(ty)$.

Since $\underset{\sim}{x}_0 \in V(y)$, $\underset{\sim}{w} \cdot \underset{\sim}{x}_0 \geq y\, c(\underset{\sim}{w}) \; \forall\, \underset{\sim}{w} \geq \underset{\sim}{0}$. Multiplying by t,

$$t\,\underset{\sim}{w} \cdot \underset{\sim}{x}_0 \geq t y\, c(\underset{\sim}{w}) \; \forall\, \underset{\sim}{w} \geq \underset{\sim}{0} \quad \text{and}$$

$$\underset{\sim}{w} \cdot \underset{\sim}{x}_1 \geq t y\, c(\underset{\sim}{w}) \; \forall\, \underset{\sim}{w} \geq \underset{\sim}{0}.$$

Now by definition

$$V(ty) = \{ \underset{\sim}{x} : \underset{\sim}{w} \cdot \underset{\sim}{x} \geq t y\, c(\underset{\sim}{w}) \; \forall\, \underset{\sim}{w} \geq \underset{\sim}{0} \}.$$

Comparing the last two equations shows that $\underset{\sim}{x}_1 \in V(ty)$. ∎

<u>Prop.</u> Elasticity of scale $e(\underset{\sim}{x}) = \dfrac{AC(y)}{MC(y)}$.

Skip the proof.

-Winter 1997

<u>Proof.</u> $$e(\underset{\sim}{x}) \triangleq \frac{t}{f(\underset{\sim}{x})} \left. \frac{d f(t\underset{\sim}{x})}{dt} \right|_{t=1}.$$

$$\frac{d f(t\underset{\sim}{x})}{dt} = \frac{d f(t\underset{\sim}{x})}{d(t\underset{\sim}{x})} \frac{d(t\underset{\sim}{x})}{dt} = \frac{d f(\underset{\sim}{x})}{d\underset{\sim}{x}} \cdot \underset{\sim}{x} = \nabla f(\underset{\sim}{x}) \cdot \underset{\sim}{x}. \text{ So}$$

$$e(\underset{\sim}{x}) = \frac{t}{f(\underset{\sim}{x})} \nabla f(\underset{\sim}{x}) \cdot \underset{\sim}{x} \Big|_{t=1}$$

$$= \frac{t}{f(\underset{\sim}{x})} \sum_{i=1}^{n} \frac{\partial f(\underset{\sim}{x})}{\partial x_i} x_i \Big|_{t=1} = \frac{1}{f(\underset{\sim}{x})} \sum_{i=1}^{n} \frac{\partial f(\underset{\sim}{x})}{\partial x_i} x_i .$$

F.O.C. of $\min_{\underset{\sim}{x}} \underset{\sim}{w} \cdot \underset{\sim}{x}$ s.t. $f(\underset{\sim}{x}) = y$, with $\mathcal{L} = \underset{\sim}{w} \cdot \underset{\sim}{x} + \lambda(y - f(\underset{\sim}{x}))$, is $0 = w_i - \lambda \frac{\partial f}{\partial x_i}$,

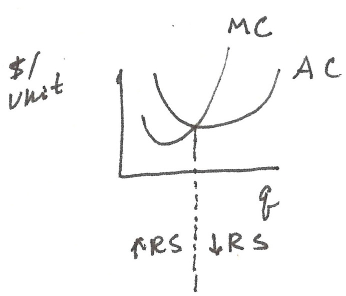
so $\partial f / \partial x_i = w_i / \lambda$. From my §5.4, $MC = \lambda$. Hence $\partial f / \partial x_i = w_i / MC$. Finally,

$$e(\underset{\sim}{x}) = \frac{1}{f(\underset{\sim}{x})} \sum_{i=1}^{n} \frac{w_i x_i}{MC} = \frac{1}{y} \sum_{i=1}^{n} \frac{w_i x_i}{MC}$$

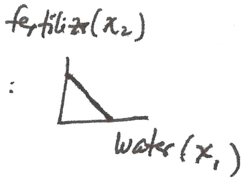
$$= \frac{1}{y} \frac{1}{MC} \sum_{i=1}^{n} w_i x_i = \frac{1}{y} \frac{1}{MC} (TC)$$

TC = total cost
MC = marginal cost
AC = average cost

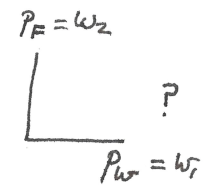
$$= \frac{TC/y}{MC} = \frac{AC}{MC} . \blacksquare$$



Slope of isocost curve :



but what about



$$\Rightarrow \boxed{\frac{dx_2}{dx_1} = \frac{-w_1}{w_2}}$$

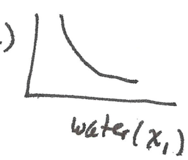
$c(\underset{\sim}{w}) = \text{const.}$ (Supress the y in $c(\underset{\sim}{w}, y)$.)

$$\frac{\partial c}{\partial w_1} dw_1 + \frac{\partial c}{\partial w_2} dw_2 = 0$$

$$\Rightarrow \boxed{\left.\frac{dw_2}{dw_1}\right|_{cost} = \frac{-\partial c/\partial w_1}{\partial c/\partial w_2} = \frac{-x_1}{x_2}}$$

(last equality $\because$ Shephard's Lemma)

Slope of isoquant : as usual, in fertilizer (x_2)



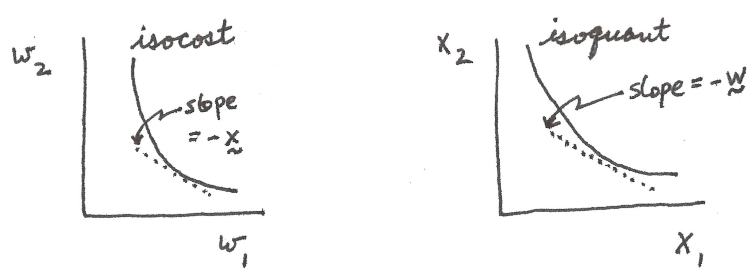
space.

$f(\underset{\sim}{x}) = y$ (const.)

$$\frac{\partial f}{\partial x_1} dx_1 + \frac{\partial f}{\partial x_2} dx_2 = 0$$

$$\Rightarrow \frac{dx_2}{dx_1} = -\frac{\partial f/\partial x_1}{\partial f/\partial x_2} \overset{\text{F.O.C.}}{=} \frac{-w_1}{w_2}$$

So :



(Fig. 6.2)

As $w_1 \uparrow$, $x_1 \downarrow$
$w_2 \downarrow$, $x_2 \uparrow$ $\Big\}$ $\frac{-x_1 \downarrow}{x_2 \uparrow}$ gets closer to zero

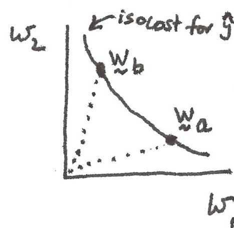
$\updownarrow$ slope

$\frac{-w_1}{w_2}$ gets farther from zero

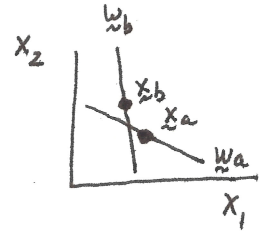
The rest of this is not very important.
~~Go to p.3 Fig. 6.5 & 6.6~~.
LH & RH diagrams on p. 6.4(3) are useful.

Indeed, skip all of this except for LH & RH fig. on p.92.

Isocost rather flat:

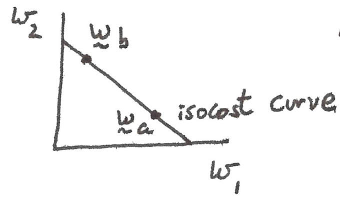


slope (i.e., $\frac{x_1}{x_2}$) changes little when $\frac{w_1}{w_2}$ changes a lot $\Rightarrow$

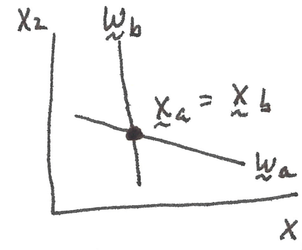


So isoquant for $\hat{y}$ must be sharply curved

In the extreme case:

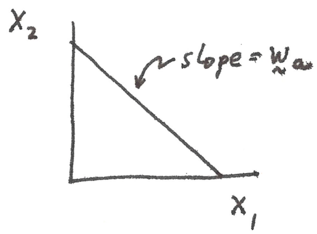


slope ($= -x_1/x_2$) doesn't change when $\underset{\sim}{w}$ changes a lot $\Rightarrow$



$\therefore$ Leontief.

Isoquant flat:



At $\underset{\sim}{w}_a$, it's possible for $x_1 > 0,\ x_2 = 0 \Rightarrow \frac{x_1}{x_2} = \infty$;

it's also possible that $x_1 = 0,\ x_2 > 0 \Rightarrow \frac{x_1}{x_2} = 0$.

At $\underset{\sim}{w}_a + \varepsilon w_1$ (for $\varepsilon > 0$), go to $x_1 = 0,\ x_2 > 0$

$\Rightarrow x_1/x_2 = 0$.

At $\underset{\sim}{w}_a + \varepsilon w_2$ (for $\varepsilon > 0$), go to $x_1 > 0,\ x_2 = 0 \Rightarrow x_1/x_2 = \infty$.

Therefore:

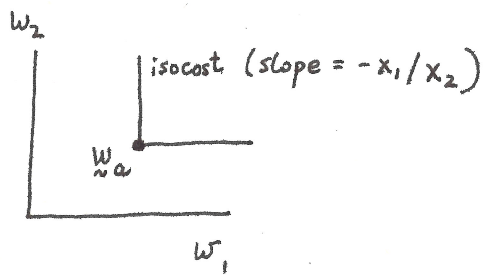
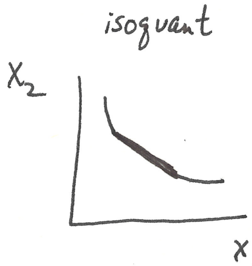
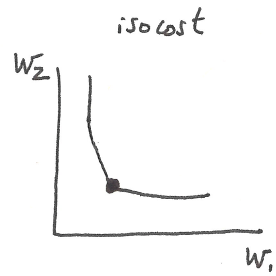


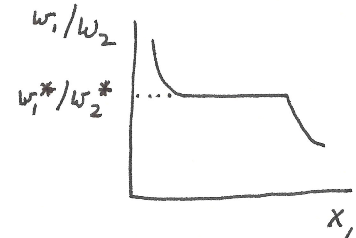
Fig. 6.4:



⇒



⇒



At a constant slope $-w_2^*/w_1^*$, $\exists$ a range of x_1/x_2 (so a range of slopes for the isocost curve)

Demand for x_1

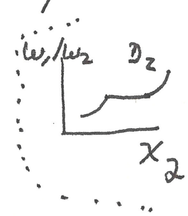
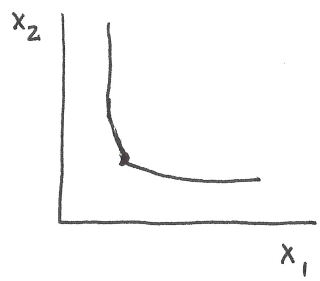


Fig. 6.5:



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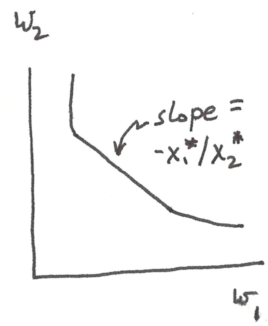

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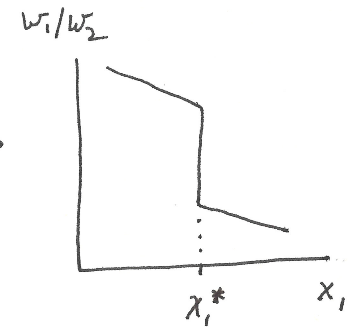
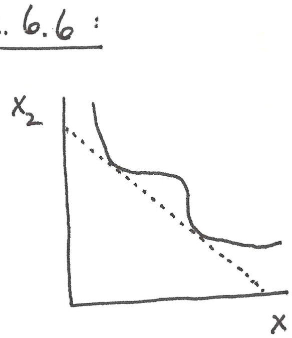


At (x_1^*, x_2^*), $\exists$ a range of w_1/w_2.

↳ constant slope of isocost curve

Fig. 6.6:



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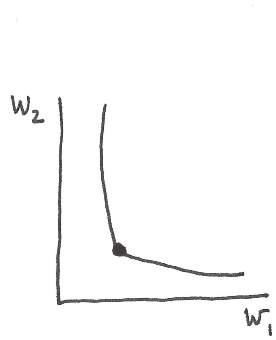
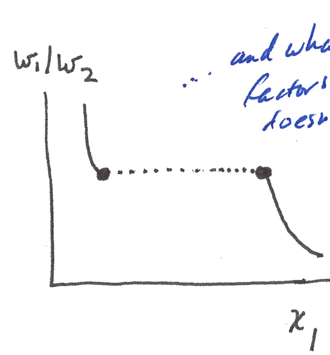


⇒



... and what if factor supply doesn't grow?
cf § 13.2

At slope $-w_2^*/w_1^*$ $\exists$ two values for x_1/x_2
(so $\exists$ two slopes of the isocost curve)

(6.5)

Uses of duality (i.e., uses of $c(\underset{\sim}{w}, y)$ and $\pi(\underset{\sim}{p})$ instead of $f(\underset{\sim}{x})$ or $V(y)$)

1) Easier proofs (example: p. 92, p. 43)

2) Can start directly w/ $c(\cdot)$ or $\pi(\cdot)$ instead of having to solve an optimization problem

3) We know all the general restrictions on $c(\cdot)$ and on $\pi(\cdot)$.

4) Convenient parametric representations of $c(\cdot)$ and $\pi(\cdot)$.

5) Econometrically estimate technology using exogenous variables ($\underset{\sim}{w}$ or $\underset{\sim}{p}$) instead of endogenous ones ($\underset{\sim}{x}$): may be better.