(4.1)

costminimization: useful both for competitive and for non-competitive firms

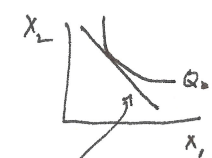
$c(w, \hat{y}) = \min \underset{\sim}{w} \cdot \underset{\sim}{x}$ s.t. $f(\underset{\sim}{x}) = \hat{y}$. $\mathcal{L} = \underset{\sim}{w} \cdot \underset{\sim}{x} - \lambda [f(\underset{\sim}{x}) - \hat{y}]$.

F.O.C. $\underset{\sim}{w} = \lambda \nabla_{\underset{\sim}{x}} f$ or $\dfrac{w_i}{w_j} = \dfrac{\partial f / \partial x_i}{\partial f / \partial x_j}$

TRS

If $\dfrac{w_i}{w_j} = \dfrac{2}{1}$ and $\dfrac{1}{1} = \dfrac{\partial f / \partial x_i}{\partial f / \partial x_j}$ then

$\left. \begin{array}{l} i \downarrow \text{ by } 1 \\ j \uparrow \text{ by } 1 \end{array} \right\} \left\{ \begin{array}{l} f \text{ unchanged} \\ \text{cost} \downarrow \text{ by } \$1 \end{array} \right.$



Cost equation (slope = $-P_1/P_2$)

S.O.C. $\cancel{\underset{\sim}{\Delta x}^T \nabla^2 f \underset{\sim}{\Delta x} \le 0 \ \forall \underset{\sim}{\Delta x} \text{ satisfying } \underset{\sim}{w} \cdot \underset{\sim}{\Delta x} = 0.}$ (over →) (to next section)

~~not $\mathcal{L}$; don't emphasize~~

... Why the firm has no budget constraint ...

(4.2)

§7.2 $m = 1$ constraint

$n = 2, 3,$ etc. factors

Start w/ the principal minor of $\nabla^2 \mathcal{L}$ of order $2m+1 = 3$.

$$\left[\begin{array}{c|c} 0 & \\ \hline & \end{array}\right]$$

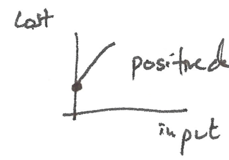
For a cost minimum: all determinants following should have the sign of $(-1)^m = -1 < 0$.

(4.3)

Difficulties w/ cost-minimization: cf §2.2 on difficulties w/ π-max

1) prodⁿ fnctⁿ may not be differentiable (e.g. Leontief)

2) $\lambda \frac{\partial f}{\partial x_i} - w_i \leq 0$ if $x_i^* = 0$



positive derivative OK if @ boundary ("corner")

— " — $= 0$ if $x_i^* > 0$.

$x_i^* \geq 0,\ w_i - \lambda^* \frac{\partial f^*}{\partial x_i} \geq 0,$

$x_i^* \left(w_i - \lambda^* \frac{\partial f}{\partial x_i}^*\right) = 0.$

3) existence — not a problem

$\underset{\sim}{w} \cdot \underset{\sim}{x}$, $V(y)$

continuous closed

bounded? Pick an arbitrary $\underset{\sim}{x}' \in V(y)$.

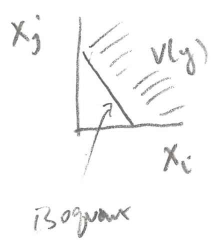
Instead of considering all $\underset{\sim}{x} \in V(y)$, only consider

$\{ \underset{\sim}{x} \in V(y) : \underset{\sim}{w} \cdot \underset{\sim}{x} \leq \underset{\sim}{w} \cdot \underset{\sim}{x}' \}$.

Upper limit on expenditures (like a budget constraint)

4) nonuniqueness (not a problem if $V(y)$ is convex)

well... need isoquants to be strictly convex



4.4

$$\left.\begin{array}{r} f(\underset{\sim}{x}) \equiv y \\ \lambda \nabla f(\underset{\sim}{x}) \equiv \underset{\sim}{w} \end{array}\right\} \text{F.O.C.'s}$$

This section does treat $d\lambda$ and dy as well as $d\underset{\sim}{x}$ and $d\underset{\sim}{w}$, but otherwise it's less useful than §5.6.

$$\nabla f \cdot d\underset{\sim}{x} = dy$$

$$\nabla f \cdot d\lambda + \lambda \nabla^2 f \cdot d\underset{\sim}{x} = d\underset{\sim}{w}$$

This section does F.O.C. ⇒ comp. stat. §5.6 uses duality to get comp. stat. So just do a 2-input example here.

Min $w_1 x_1 + w_2 x_2$ st. $x_1^{1/4} x_2^{3/4} \geq y$.

Find $\partial x_1 / \partial w_2$.

$$\underbrace{\begin{bmatrix} 0 & (\nabla f)^T \\ \nabla f & \lambda \nabla^2 f \end{bmatrix}}_{\nabla^2 \mathcal{L}} \begin{bmatrix} d\lambda \\ d\underset{\sim}{x} \end{bmatrix} = \begin{bmatrix} dy \\ d\underset{\sim}{w} \end{bmatrix}$$

Omit or do briefly. – Spring 2004

$$\Rightarrow \begin{bmatrix} d\lambda \\ d\underset{\sim}{x} \end{bmatrix} = [\nabla^2 \mathcal{L}]^{-1} \begin{bmatrix} dy \\ d\underset{\sim}{w} \end{bmatrix}$$

symmetric (both $\nabla^2 \mathcal{L}$ and its inverse)

Varian calls $\square$ in $d\underset{\sim}{x} = \square\, d\underset{\sim}{w}$ "the substitution matrix."

This is the $n \times n$ matrix in the lower right of $[\nabla^2 \mathcal{L}]^{-1}$.

(4.5) $(y^t, \underset{\sim}{w}^t, \underset{\sim}{x}^t)$, $t = 1, 2, \ldots, T$.

output ↑ ↑ input prices ↑ inputs

Skip.
-Winter 1996.

Weak Axiom of Cost Minimization:

$$\forall\, y^s \geq y^t, \quad \underset{\sim}{w}^t \underset{\sim}{x}^t \leq \underset{\sim}{w}^t \underset{\sim}{x}^s$$

↳ could have chosen $\underset{\sim}{x}^s$ at time t

time t prices

Suppose $y^t = y^s$ (??). Then

time t: $\underset{\sim}{w}^t \cdot \underset{\sim}{x}^t \leq \underset{\sim}{w}^t \cdot \underset{\sim}{x}^s$

time s: $\underset{\sim}{w}^s \cdot \underset{\sim}{x}^s \leq \underset{\sim}{w}^s \cdot \underset{\sim}{x}^t \iff -\underset{\sim}{w}^s \cdot \underset{\sim}{x}^t \leq -\underset{\sim}{w}^s \cdot \underset{\sim}{x}^s$

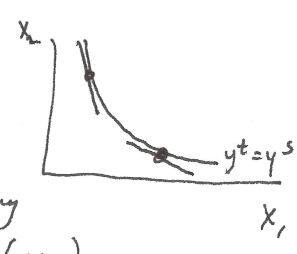
↓ add ↓

$$(\underset{\sim}{w}^t - \underset{\sim}{w}^s)\, \underset{\sim}{x}^t \leq (\underset{\sim}{w}^t - \underset{\sim}{w}^s) \cdot \underset{\sim}{x}^s$$

$$(\underset{\sim}{w}^t - \underset{\sim}{w}^s)(\underset{\sim}{x}^t - \underset{\sim}{x}^s) \leq 0$$

Compare w/ §2.5, WAPM, $\Delta \underset{\sim}{p} \cdot \Delta \underset{\sim}{y} \geq 0$ .... $\Delta \underset{\sim}{w} \cdot \Delta \underset{\sim}{x} \leq 0$.

But you knew this before:



(Except here you needn't put any conditions on the technology.)