2.0

$\pi = TR - TC$

$TR \to p \cdot q$

$TC \to$ input prices × quantities of inputs   incl. opportunity costs

price-taking behavior ; competitive firm

competitive behavior as an ↗ irrational expectation

(2.1)

$$\pi^\circ(\underset{\sim}{y}) = \underset{\sim}{p} \cdot \underset{\sim}{y}$$ ← given

"direct profit function" (?) ; "profit equation" (?) "unoptimized profit function"?

∵ inputs are negative components of $\underset{\sim}{y}$

$$\max_{\underset{\sim}{y}} \pi^\circ(\underset{\sim}{y}) \text{ s.t. } \underset{\sim}{y} \in Y.$$

Profit function $\pi(\underset{\sim}{p}) = \max_{\underset{\sim}{y}} \underset{\sim}{p} \cdot \underset{\sim}{y}$ s.t. $\underset{\sim}{y} \in Y$

$$\left[\qquad = \max_{\underset{\sim}{y}} \pi^\circ(\underset{\sim}{y}) \text{ s.t. } \underset{\sim}{y} \in Y. \right]$$

If the firm produces only one output:

$$\pi^\circ(\underset{\sim}{x}) = p\, f(\underset{\sim}{x}) - \underset{\sim}{w} \cdot \underset{\sim}{x}$$

$\underset{\sim}{x}$: inputs (nonnegative); p: scalar; $\underset{\sim}{w}$: input prices

$$\pi(p, \underset{\sim}{w}) = \max_{\underset{\sim}{x}} \pi^\circ(\underset{\sim}{x})$$

"Cost equation" $c^\circ(\underset{\sim}{x}) = \underset{\sim}{w} \cdot \underset{\sim}{x}$

cost function $c(\underset{\sim}{w}, y) = \min_{\underset{\sim}{x}} \underset{\sim}{w} \cdot \underset{\sim}{x}$ s.t. $\underset{\sim}{x} \in V(y)$

$$\left[\qquad = \min_{\underset{\sim}{x}} c^\circ(\underset{\sim}{x}) \text{ s.t. } \underset{\sim}{x} \in V(y) \right]$$

factors not ~~morally~~ morally due this

Rules out U-shaped AC curves

$$\max p f(\underset{\sim}{x}) - \underset{\sim}{w} \cdot \underset{\sim}{x} \Rightarrow \underbrace{p \nabla_{\underset{\sim}{x}} f(\underset{\sim}{x})}_{\text{value of Marginal Product}} - \underbrace{\underset{\sim}{w}}_{\text{factor price}} = 0 \quad \text{FOC.}$$

(no constraints needed)

SOC: $\nabla^2 f(\underset{\sim}{x})$ negative semidefinite. (It's obviously symmetric.)

Minimum wages?
- Efficiency wages
- Macro effects

(2.2)

factor demand function $\underset{\sim}{x}(p, \underset{\sim}{w})$ (π-max) or $\underset{\sim}{x}(\underset{\sim}{w}, y)$ (cost-min)

supply function $y(p, \underset{\sim}{w}) = f(\underset{\sim}{x}(p, \underset{\sim}{w}))$ (π-max) ~~or $f(\underset{\sim}{x}(w, y))$~~ (cost-min, y-fixed)

1) f nondifferentiable

2) corner solutions }
interior " }

$$\frac{\partial \pi^{o*}}{\partial x_i} \le 0 \text{ if } x_i^* = 0$$

$$'' = 0 \text{ if } x_i^* > 0.$$

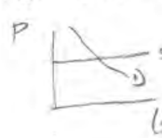
(K-T p. 503) $\Updownarrow$

$$\frac{\partial \pi^o}{\partial x_i} \le 0, \; x_i \ge 0, \; \frac{\partial \pi^o}{\partial x_i} \cdot x_i = 0.$$

3) f CRS; then if $\exists$ some $\hat{\underset{\sim}{x}}$ s.t. profits $p f(\hat{\underset{\sim}{x}}) - \underset{\sim}{w} \cdot \hat{\underset{\sim}{x}} = \hat{\pi} > 0$,

then using $t\hat{\underset{\sim}{x}}$ for $t > 1$ will yield profits $t\hat{\pi} > \hat{\pi}$.

$(\therefore \text{CRS} \Rightarrow \pi \equiv 0)$

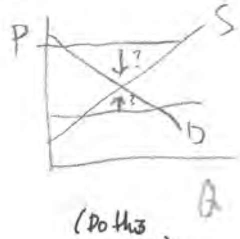


no "scissors" ("Both D & S" needed)

4) nonuniqueness

CRS
~~xxx~~
competition

If $\pi(\hat{\underset{\sim}{x}}) = 0$ then
$\pi(t\hat{\underset{\sim}{x}}) = 0 \; \forall t > 0$.



(Do this now.)
(Joan Robinson)

$f = L^{\alpha} K^{1-\alpha}$

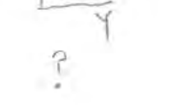
$\pi = p L^{\alpha} K^{1-\alpha} - wL - rK$

$0 = \pi_L' = \alpha p \frac{f}{L^*} - w \Rightarrow wL^* = \alpha pf$

$0 = \pi_K' = (1-\alpha) \frac{pf}{K^*} - r \Rightarrow rK^* = (1-\alpha) pf$

$\Rightarrow \pi = pf - wL^* - rK^*$
$= pf - \alpha pf - (1-\alpha) pf$
$= 0.$

Macro



?

(2.3)

$x_i(p, \underset{\sim}{w})$ homogeneous of degree zero $\because$ $\arg\max\, t p f(\underset{\sim}{x}) - t \underset{\sim}{w} \cdot x$

$= \arg\max\ p f(\underset{\sim}{x}) - \underset{\sim}{w} \cdot \underset{\sim}{x}$.

comparative statics; sensitivity analysis

(2,4)

Normalize $p=1$.

FOC: $\nabla_{\underset{\sim}{x}} f(\underset{\sim}{x}) = \underset{\sim}{w}$. Differentiate wrt $\underset{\sim}{w}$:

$$\nabla^2_{\underset{\sim}{x}\underset{\sim}{x}} f \cdot \nabla_{\underset{\sim}{w}} \underset{\sim}{x} = I$$

$$\nabla_{\underset{\sim}{w}} \underset{\sim}{x} = \left(\nabla^2_{\underset{\sim}{x}\underset{\sim}{x}} f\right)^{-1}.$$

$\nabla^2 f$ negative definite symmetric $\Rightarrow$

$(\nabla^2 f)^{-1}$ " " " *.

Well, this'll be superseded by § 3.4, so just skip it. At most, do p.34 ¶2

A neg. def. symm $\Rightarrow$ A^{-1} " " ".

Winter '96

$$\boxed{\frac{\partial x_i}{\partial w_i} < 0} \quad (\text{cf. p. } 123\ (3),\ \S 8.3)$$

$\partial h_i/\partial p_i \le 0$

Varian: differentiate $\underset{\sim}{x}(\underset{\sim}{w})$ to get $d\underset{\sim}{x} = \nabla_{\underset{\sim}{w}} \underset{\sim}{x} \cdot d\underset{\sim}{w} \Rightarrow$

$$\boxed{d\underset{\sim}{w} \cdot d\underset{\sim}{x}} = d\underset{\sim}{w} \cdot \underbrace{\nabla_{\underset{\sim}{w}} \underset{\sim}{x}}_{= d\underset{\sim}{w} \cdot (\nabla^2_{\underset{\sim}{x}} f)^{-1} \cdot d\underset{\sim}{w}} \cdot d\underset{\sim}{w} \ \boxed{\le 0}$$

Contradiction:

p.34 ℓ-8: $(\nabla^2 f)^{-1}$ negative definite

p.34 ℓ-4: " negative semidefinite!

"the substitution matrix"

---

* Proof. (1) If A is symmetric then A^{-1} is symmetric: p34 ¶2 sentence 2

$A(A)^{-1} = I$ by defn. of inverse

$A(A^T$ $\because A = A^T$ (A is symmetric). Take the inverse of both sides:

$(A^T)A^{-1}$ $\leftarrow$. Take the transpose of both sides:

$(A^{-1})^T A$ Right-multiply by A^{-1}:

$(A^{-1})^T$ QED.

Take transpose:

$(A^{-1})^T A^T = I$.

$(A^{-1})^T A = I$ by symmetry of A;

postmultiply by A^{-1}:

$(A^{-1})^T = A^{-1}$. //

(2) If A is negative definite ~~symmetric~~ then

A^{-1} " " ~~"~~ . (over $\Rightarrow$)

2017: you need symmetry. 2019: yes!

2018: (2) "A negative definite" $\Rightarrow$ "A^{-1} negative definite"

$\leftarrow$ you ~~don't~~ do (2019) need symmetry

<u>Proof</u>. $\underset{\sim}{x}^T A \underset{\sim}{x} < 0 \quad \forall \underset{\sim}{x} \neq \underset{\sim}{0}$ by assumption.

Replace $\underset{\sim}{x}$ with $A^{-1}\underset{\sim}{y}$. (A^{-1} exists: A is negative definite, so all its eigenvalues are negative. Its determinant is the product of its eigenvalues, so its determinant isn't zero.)

$(A\underset{\sim}{x} = \underset{\sim}{y})$

$(A^{-1}\underset{\sim}{y})^T A A^{-1} \underset{\sim}{y} < 0$

$(A^{-1}\underset{\sim}{y})^T \underset{\sim}{y} < 0$. Transpose both sides: $\underset{\sim}{y}^T (A^{-1})^T \underset{\sim}{y} < 0$. But A is symmetric, so by (1) so is A^{-1}, so

$\underset{\sim}{y}^T A^{-1} \underset{\sim}{y} < 0$, which proves that A^{-1} is negative definite. //

(2.5) $(p^1, y^1), (p^2, y^2) \dots, (p^t, y^t), \dots, (p^T, y^T)$: observations

$p^t \cdot y^t \geq p^t \cdot y^s \quad \{\forall t, \forall s\} \in \{1, \dots, T\}$. <u>Weak Axiom of Profit Maximization</u>

Fig. 2.2. The solid line through each y^i is p^i.

Skip this.
-Winter 1996.

Observation t $\Rightarrow p^t(y^t - y^s) \geq 0$

Observation s : $\Rightarrow -p^s(y^t - y^s) \geq 0$

$$(p^t - p^s)(y^t - y^s) \geq 0$$

$$\Delta p \cdot \Delta y \geq 0. \quad (2.2)$$

$\forall (t,s) \in \{1, \dots, T\}$

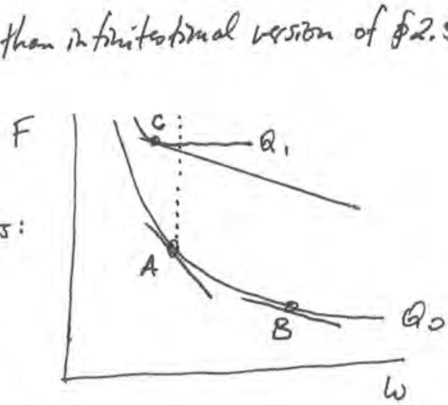
If $\Delta p = (1, 0, \dots, 0)$. Then $\Delta y_i \geq 0$.

i is output : output can't fall

i is input : input can't get more negative.

(2.2) stronger than infinitesimal version of §2.5.

Not obvious: rules out this:



$P_w \downarrow$

A→B substitution effect
B→C output effect

2.6 omit