1.1

time dimension - often omitted

date, location, state of nature

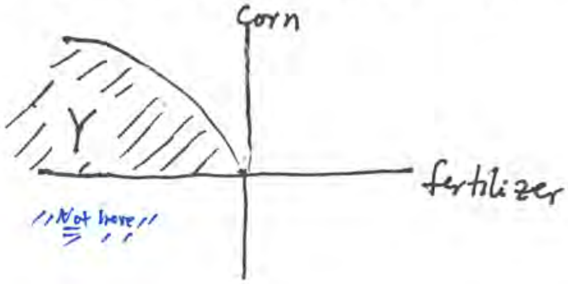
(1.2)

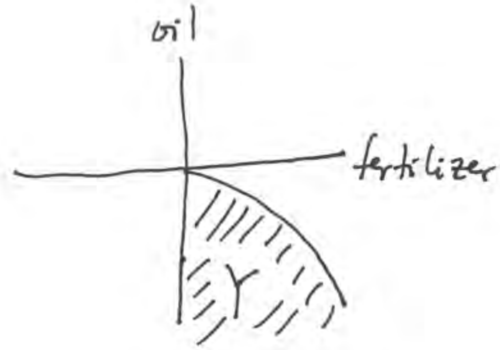
good j $\quad y_j^i$ as input

y_j^o as output

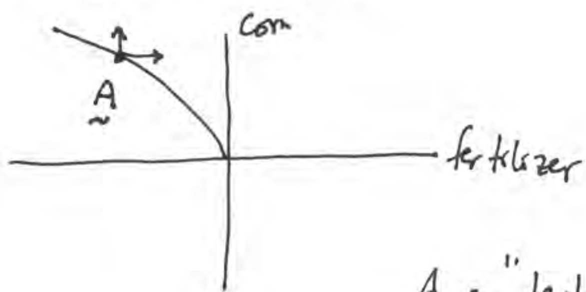
$y_j = y_j^o - y_j^i$ net output

Production Possibilities Set Y





(2) Transformation Function



(1) Arrows point in the direction of $\underset{\sim}{x} \geq \underset{\sim}{A}$.

$\underset{\sim}{A}$ is "technologically efficient" if $A \in Y$ and $\nexists$ an $x \in Y$ s.t. $\underset{\sim}{x} \geq \underset{\sim}{A}$ and $\underset{\sim}{x} \neq \underset{\sim}{A}$.

Transformation function $T: \mathbb{R}^n \to \mathbb{R}^1$ s.t.

a) $T(\underset{\sim}{y}) = 0$ iff $\underset{\sim}{y}$ is technologically efficient (OK for > 1 output)

b) $Y = \{ \underset{\sim}{y} : T(\underset{\sim}{y}) \leq 0 \}$

One Output Framework:

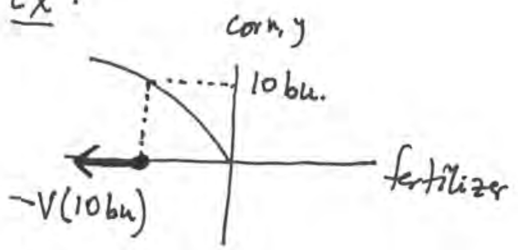
Production Function

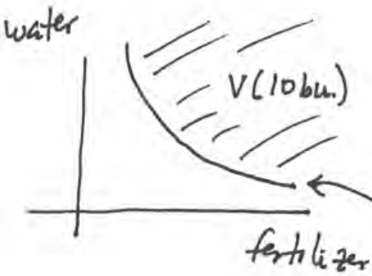
$$f(\underset{\sim}{x}) = \{ \max y : (y, -\underset{\sim}{x}) \in Y \}$$

↑scalar

Input Requirement Set $V(y) = \{ \underset{\sim}{x} : f(\underset{\sim}{x}) \geq y \}$, the upper contour set of $f(\underset{\sim}{x})$. $V(y): \mathbb{R}^1 \to \mathbb{R}^n$

Ex:





Isoquant $Q(y) = \{ \underset{\sim}{x} : \underset{\sim}{x} \in V(y)$ and

unintuitive → $\underset{\sim}{x} \notin V(y')$ for $y' > y \}$. Example: Q(10bu.)

intuitive ↘ ⇒ $Q(y)$, the contour line of the prodⁿ fnctⁿ. ⇐

Arbitrary # of Outputs	One Output
Prodⁿ Poss. Set, Y	Input Requirement Set, V
Transformation Function, T	Production Function, f
input-output vectors $\underset{\sim}{y}$	output y inputs $\underset{\sim}{x}$
inputs ⊖ outputs ⊕	inputs ⊕ outputs ⊕

→ No aggregation

Technology 100% known : no novelty, no entrepreneurs! → Like consumer theory's completeness assumption.

risk OK
uncertainty not OK } Frank Knight

Success of D-Day Invasion : "probabilities" are uncertainty, not risk, even now

→ Capital : non-financial ("machines") : OK if not aggregated, but it has to be aggregated (via value)

financial : from savings, then need time, & probably need money (a socially accepted claim on unspecified goods) or other contracts. ← firm has no budget constraint?

from credit creation by merchants

"Technological Progress" : more output from the same inputs ???

G-R : flow / fund

a recipe = only a list of ingredients

(1.3) Activity analysis

$$
\begin{array}{lccc}
 & \text{output} & \text{input } 1 & \text{input } 2 \\
Y = \{ & (1, & -1, & -2) \\
 & (1, & -2, & -1) \ \}
\end{array}
$$

technique A

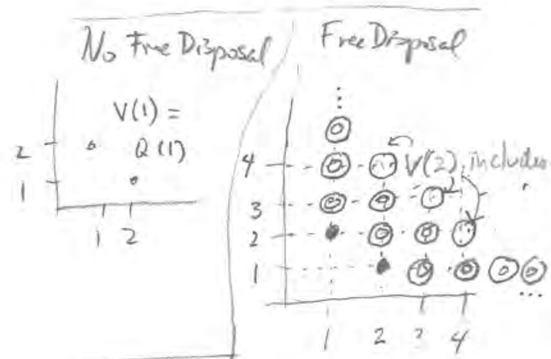
technique B

$V(1) \ni \{ \ (1, 2),$ technique A

$(2, 1) \ \}$ technique B

$\ni$ "Contains"

	inputs
2 of technique A ⇒ output = 2	(2, 4)
2 " " B ⇒ " 2	(4, 2)
1 " " A, 1 " " B ⇒ " 2	(1, 2) + (2, 1) = (3, 3)



V(1) : all these points & more

Q(1): ● and ◎ ... only, not o

(History of steel technology.)

Fig. 1.1, Leontief technology

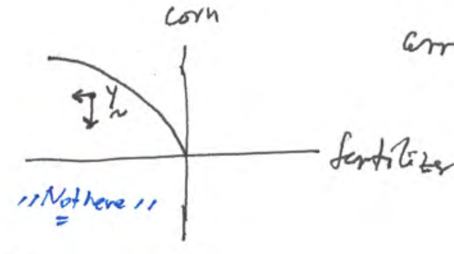
14

monotonic technologies

$$\underset{\sim}{x}' \geq \underset{\sim}{x} \Rightarrow$$
$$f(\underset{\sim}{x}') \geq f(\underset{\sim}{x}).$$

$\Rightarrow$ Fig. 1.3

Production sets:



arrows: $\overset{y'}{\cancel{\underset{\sim}{x}}} \leq \underset{\sim}{y}$

$$\left.\begin{array}{l} \underset{\sim}{y} \in Y \\ y'\cancel{\underset{\sim}{x}} \leq \underset{\sim}{y} \end{array}\right\} \Rightarrow \overset{y'}{\cancel{\underset{\sim}{x}}} \in Y$$

(1.5)

Convex technologies

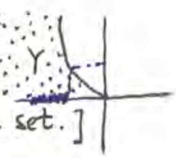
$$\left.\begin{array}{l} \underset{\sim}{x} \in V(y) \\ \underset{\sim}{x}' \in V(y) \end{array}\right\} \Rightarrow t\,\underset{\sim}{x} + (1-t)\,\underset{\sim}{x}' \in V(y). \text{ i.e., } V(y) \text{ is a convex set.}$$

for $t \in [0,1]$

$\Rightarrow$ Fig. 1.4 $\Leftarrow$

<u>Prop.</u> Y is a convex set $\Rightarrow V(y)$ is a convex set.

[Note: the converse is false. Here $V(y)$ is a convex set but Y is not a convex set.]



<u>Proof.</u>

$$\left.\begin{array}{l} (y,-\underset{\sim}{x}) \in Y \\ (y,-\underset{\sim}{x}') \in Y \end{array}\right\} \Rightarrow t\,(y,-\underset{\sim}{x}) + (1-t)(y,-\underset{\sim}{x}') \in Y \Rightarrow \quad t \in [0,1]$$

$$\Updownarrow$$

$$\underbrace{\qquad\qquad}_{\begin{array}{l}\underset{\sim}{x} \in V(y) \\ \underset{\sim}{x}' \in V(y)\end{array}}$$

$$(ty, -t\,\underset{\sim}{x}) + ((1-t)y, -(1-t)\,\underset{\sim}{x}') \in Y \Rightarrow$$

$$(y, -t\,\underset{\sim}{x} - (1-t)\,\underset{\sim}{x}') \in Y \Rightarrow$$

$$t\,\underset{\sim}{x} + (1-t)\,\underset{\sim}{x}' \in V(y). \blacksquare$$

<u>Prop.</u> $V(y)$ convex set iff production fnctn. $f(\underset{\sim}{x})$ is quasiconcave.

<u>Proof.</u> $V(y)$ is f's upper contour set. ▮

SUMMARY:	any # of outputs	only one output
set	Y convex $\Rightarrow$	$V(y)$ convex $\Updownarrow$
function	T	$f(\underset{\sim}{x})$ quasiconcave

(1.6) Regular technology.

$V(y)$ is closed and nonempty $\forall\ y \geq 0$.

closure: $\underset{\sim}{x}^i \in V(y)$

$\downarrow$

$\underset{\sim}{x}$ then

$\underset{\sim}{x} \in V(y)$.

(1.7)

Smoothing an isoquant

representation using a few unknown parameters ; then may be able to more easily apply calculus to investigate firm behavior

Philosophical criticism :
(methodological)

1) prod^n possibilities may be unknown

2) may be possible to use only 1 process, in short-run & in medium-run

similar

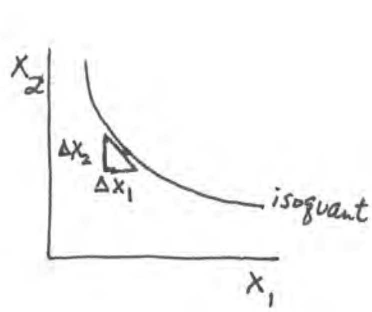
3) the only known alternative production methods might be now inefficient (so there's only 1 process currently understood & not inefficient)

Why investigate ineff. portions of Y ?

Neoclassical economics can be done w/ activity analysis.

(Mirowski B on §5.6.)

1.8



$$\text{Slope of isoquant} = \left.\frac{dx_2}{dx_1}\right|_{\substack{\text{output}\\ \text{fixed}}} .$$

$y_0 = f(x_1, x_2)$. Take the ~~total~~ differential:

$$\underbrace{dy_0}_{=0} = \frac{\partial f}{\partial x_1}\, dx_1 + \frac{\partial f}{\partial x_2}\, dx_2$$

so

$$\frac{dx_2}{dx_1} = -\frac{\partial f/\partial x_1}{\partial f/\partial x_2} .$$

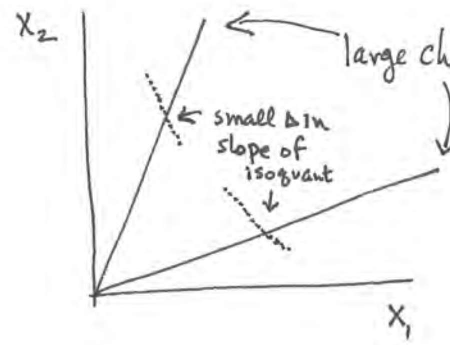
"Technical Rate of Substitution"

$X_2(x_1)$ is "explicit"

"naught" vs. "not"

(1.9)

elasticity of substitution



⇒ isoquant rather flat;
elasticity of substitution rather large

a measure of curvature, not of slope

$$\sigma = \frac{d \ln (x_2/x_1)}{d \ln |TRS|} = \frac{TRS}{x_2/x_1} \frac{d(x_2/x_1)}{d\, TRS} = \frac{\%\,\Delta (x_2/x_1)}{\%\,\Delta\, TRS}$$

Related to the second derivative of the isoquant?

1.10

Returns to Scale

constant: $f(t\underset{\sim}{x}) = t\, f(\underset{\sim}{x}) \quad \forall\, t \geq 0$ (i.e., f is homogeneous of degree 1)

increasing: " $>$ " $\forall\, t > 1$

decreasing: " $<$ " $\forall\, t > 1$.

\$ increasing returns to scale.
What if $t < 1$?

$t = 1.2$
$f(x) = 10$
$f(t\underset{\sim}{x}) = 15$
$\underset{\sim}{x} = (1,1)$
$1.2\underset{\sim}{x} = (1.2, 1.2)$

Now,
original $\underset{\sim}{x} = (1.2, 1.2)$
new $\underset{\sim}{x} = \frac{1}{1.2}\underset{\sim}{x} = t\underset{\sim}{x}$
$t = 1/1.2 \approx 0.8$
Output goes from 15 to 10.

What about not "$\forall\, t > 1$" — i.e., a local measure?

elasticity of scale $\quad \left. \dfrac{t}{f(t\underset{\sim}{x})} \dfrac{d f(t\underset{\sim}{x})}{dt} \right|_{t=1}$

$> 1 \Rightarrow$ locally increasing returns to scale
$= 1 \Rightarrow$ " constant —"—
$< 1 \Rightarrow$ " decreasing —"—

p. 16 "... it can always be assumed that decreasing returns to scale is due to the presence of some fixed input..."

(1.11)

Homogeneous

Homothetic

<u>Prop.</u>

If the production function is $g(h(\underset{\sim}{x}))$ where h is homogeneous of degree k and g is strictly monotonic.

If $\underset{\sim}{x}_1$ and $\underset{\sim}{x}_2$ lie on the same isoquant Q_0. Then $t\underset{\sim}{x}_1$ and $t\underset{\sim}{x}_2$ lie on the same isoquant Q', assuming $t > 0$.

<u>Proof.</u> $\underset{\sim}{x}_1, \underset{\sim}{x}_2$ lie on isoquant $Q_0 \Rightarrow g(h(\underset{\sim}{x}_1)) = Q_0$

$$g(h(\underset{\sim}{x}_2)) = Q_0.$$

$$\Downarrow$$

$$g \text{ is monotonic} \Rightarrow h(\underset{\sim}{x}_1) = h(\underset{\sim}{x}_2). \qquad (a)$$

$$g(h(t\underset{\sim}{x}_1)) = g(t^k h(\underset{\sim}{x}_1)) \qquad (b)$$

$$g(h(t\underset{\sim}{x}_2)) = g(t^k h(\underset{\sim}{x}_2))$$

$$= g(t^k h(\underset{\sim}{x}_1)) \text{ from above } \overset{(a)}{;} \text{ so}$$

$$= g(h(t\underset{\sim}{x}_1)) \text{ from } (b). \blacksquare$$

(Obvious from S&H's defn. of homotheticity.)

$$\left.\begin{array}{l} f(\underset{\sim}{x}) = f(\underset{\sim}{y}) \\ t > 0 \end{array}\right\} \Rightarrow f(t\underset{\sim}{x}) = f(t\underset{\sim}{y})$$