Problem Set on Positive General Equilibrium

Ch. 17: 4

6 → Change the "y" in the v_1 equation to y_1 and
" " " " " v_2 " " y_2.
They stand for income (for which Varian usually uses "m").

10

11

Ch. 18: 1

2

Answers to the Problem Set On
Positive General Equilibrium

(17,4) $u_A = a \ln x_{1A} + (1-a) \ln x_{2A}$ note slightly different notation than Varian's

$\omega_A = (0,1)$

Person A: $\max u_A$ s.t. $p_1 x_{1A} + p_2 x_{2A} = p_1 0 + p_2 1$

$$\mathcal{L} = a \ln x_{1A} + (1-a) \ln x_{2A} + \lambda \left[p_2 - p_1 x_{1A} - p_2 x_{2A} \right]$$

FOC $0 = \partial\mathcal{L}/\partial\lambda = p_2 - p_1 x_{1A} - p_2 x_{2A}$ (1)

$$\left.\begin{aligned} 0 &= \partial\mathcal{L}/\partial x_{1A} = \frac{a}{x_{1A}} - \lambda p_1 \\ 0 &= \partial\mathcal{L}/\partial x_{2A} = \frac{1-a}{x_{2A}} - \lambda p_2 \end{aligned}\right\} \quad \frac{p_1}{p_2} = \frac{a}{x_{1A}} \frac{x_{2A}}{1-a} \quad (2)$$

(1) $\Rightarrow$ $p_2 = p_1 x_{1A} + p_2 x_{2A}$

$1 = \frac{p_1}{p_2} x_{1A} + x_{2A}$; then (2) $\Rightarrow$

$$1 = \frac{a}{1-a} x_{2A} + x_{2A} = x_{2A}\left(\frac{a}{1-a} + 1\right) = x_{2A} \frac{a + (1-a)}{1-a} = x_{2A} \frac{1}{1-a}$$

$\Rightarrow$ $x_{2A} = 1 - a$ (3)

(2) $\Rightarrow$ $x_{1A} = \frac{p_2}{p_1} \frac{a}{1-a} x_{2A} = \frac{p_2}{p_1} \frac{a}{1-a}(1-a) = \frac{p_2}{p_1} a$. (4)

$\underset{\sim}{\omega}_A + \underset{\sim}{\omega}_B = (1,1)$, so

(3) $\Rightarrow$ $x_{2B} = 1 - x_{2A} = 1 - (1-a) = a$

(4) $\Rightarrow$ $x_{1B} = 1 - x_{1A} = 1 - \frac{p_2}{p_1} a = \frac{p_1 - p_2 a}{p_1}$.

Person B: $x_{1B} = x_{2B}$ else he's spending money buying extra increments of a commodity when those increments add nothing to utility

So $1-\frac{p_2}{p_1}a=a$

$$1=a+\frac{p_2}{p_1}a$$

$$\frac{1-a}{a}=\frac{p_2}{p_1}$$

$$x_{1A}=\frac{p_2}{p_1}a=\frac{1-a}{a}a=1-a$$

$$x_{2A}=1-a$$

$$x_{1B}=1-\frac{p_2}{p_1}a=a$$

$$x_{2B}=a$$

answers

17.6

$$v_1 = \ln y_1 - a \ln p_1 - (1-a)\ln p_2$$

Roy's Identity $x_{11} = -\dfrac{\partial v_1/\partial p_1}{\partial v_1/\partial y_1} = -\dfrac{-a/p_1}{1/y_1} = \dfrac{a y_1}{p_1}$

person

good

$$x_{12} = -\frac{\partial v_1/\partial p_2}{\partial v_1/\partial y} = -\frac{-(1-a)/p_2}{1/y_1} = \frac{(1-a)y_1}{p_2}$$

$$v_2 = \ln y_2 - b\ln p_1 - (1-b)\ln p_2$$

So by symmetry

$$x_{21} = \frac{b y_2}{p_1}$$

$$x_{22} = \frac{(1-b)y_2}{p_2}$$

Also, $\underset{\sim}{\omega}_1 = (1,1)$ so $y_1 = p_1(1) + p_2(1) = p_1 + p_2$

$\underset{\sim}{\omega}_2 = (1,1)$ so $y_2 = p_1(1) + p_2(1) = p_1 + p_2$.

Substitute these in for y_1 and y_2. Then

Good 1 supply = demand $\Rightarrow$ $1 + 1 = \dfrac{a(p_1+p_2)}{p_1} + \dfrac{b(p_1+p_2)}{p_1}$

$$2 = a + \frac{p_2}{p_1}a + b + \frac{p_2}{p_1}b$$

$$2 - a - b = \frac{p_2}{p_1}(a+b)$$

$$\frac{p_2}{p_1} = \frac{2-(a+b)}{a+b} = \frac{2}{a+b} - 1.$$

This is the equilibrium price ratio. The other market will have to clear since this one does.

$$\frac{p_2}{p_1} = \frac{2}{a+b} - 1 \quad . \text{ Optional:}$$

$$x_{11} = \frac{a y_1}{p_1} = \frac{a(p_1+p_2)}{p_1} = a + a\frac{p_2}{p_1} = a + a\left(\frac{2}{a+b} - 1\right) = \frac{2a}{a+b}$$

$$x_{12} = \frac{(1-a)y_1}{p_2} = \frac{(1-a)(p_1+p_2)}{p_2} = (1-a)\frac{p_1}{p_2} + 1-a$$

$$= (1-a)\left(\frac{2}{a+b} - \frac{a+b}{a+b}\right)^{-1} + 1-a = (1-a)\frac{a+b}{2-(a+b)} + 1-a$$

$$= (1-a)\left[\frac{a+b}{2-(a+b)} + 1\right] = (1-a)\frac{a+b+2-(a+b)}{2-(a+b)}$$

$$= \frac{2}{2-(a+b)}(1-a)$$

$$x_{21} = \frac{b y_2}{p_1} = \frac{b(p_1+p_2)}{p_1} = b + b\frac{p_2}{p_1} = b + b\left(\frac{2}{a+b} - 1\right) = \frac{2b}{a+b}$$

$$x_{22} = \frac{(1-b)y_2}{p_2} = \frac{(1-b)(p_1+p_2)}{p_2} = (1-b)\frac{p_1}{p_2} + 1-b$$

$$= (1-b)\left(\frac{p_1}{p_2} + 1\right) = (1-b)\left[\left(\frac{2}{a+b} - 1\right)^{-1} + 1\right]$$

$$= (1-b)\left[\left(\frac{2}{a+b} - \frac{a+b}{a+b}\right)^{-1} + 1\right] = (1-b)\left(\frac{a+b}{2-(a+b)} + 1\right)$$

$$= \left(\frac{a+b}{2-(a+b)} + \frac{2-(a+b)}{2-(a+b)}\right)(1-b) = \frac{2}{2-(a+b)}(1-b).$$

You can verify that $x_{11} + x_{21} = 2$ and

$x_{12} + x_{22} = 2$ so both markets clear.

17.10

$\underset{\sim}{p} \cdot \underset{\sim}{z} \leq 0 \ \forall \underset{\sim}{p} \geq 0$ (some printings have $\forall \underset{\sim}{p} \leq 0$, which is a misprint)

p.321, second-to-last equation

$$\underbrace{\underbrace{\left(\sum_{j=1}^{k} \max(0, z_j)\right)}_{\substack{z_j<0 \Rightarrow \text{term is } 0 \\ z_j=0 \Rightarrow \text{term is } 0 \\ z_j>0 \Rightarrow \text{term is } \oplus}} \underbrace{\sum_{i=1}^{k} p_i^* z_i}_{\leq 0}}_{0 \text{ or } \ominus} = \sum_{i=1}^{k} z_i \max(0, z_i)$$

If it's 0, this is the same as the earlier proof.

If it's $\ominus$, then $\sum_{i=1}^{k} \underbrace{z_i \max(0, z_i)}_{\substack{z_i<0 \Rightarrow \text{this is } 0 \\ z_i=0 \Rightarrow \text{this is } 0 \\ z_i>0 \Rightarrow \text{this is } z_i^2}} < 0$

this can't add up to something < 0, so this is impossible, so we're left with this just as before.

17.11

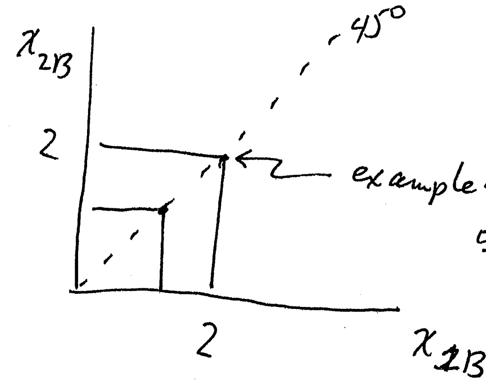
$u_A = x_{1A} + x_{2A} \longrightarrow$

$u_B = \max(x_{1B}, x_{2B}) \longrightarrow$

$\underset{\sim}{\omega}_A = \underset{\sim}{\omega}_B = (\tfrac{1}{2}, \tfrac{1}{2})$

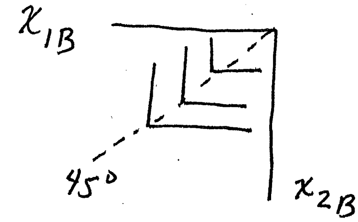
$\underset{\sim}{\omega}_A + \underset{\sim}{\omega}_B = (1, 1)$

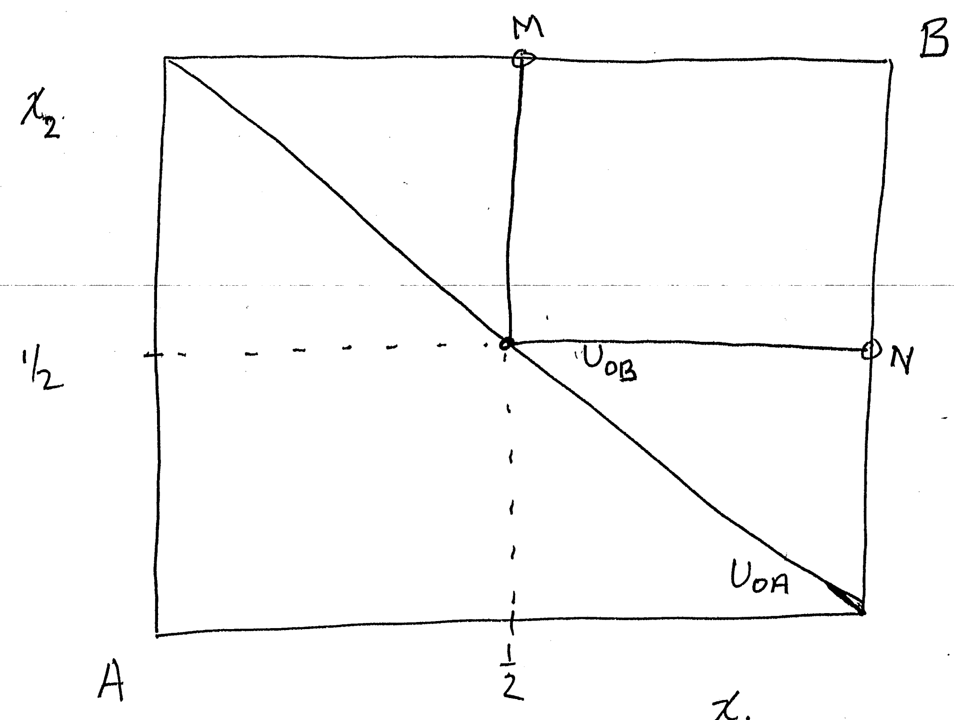
Indifference Curves



example: at (2,2) if ↓ x_{1B} or ↓ x_{2B}, utility stays the same

rotate axes:





Moving to M or N would keep B's utility the same while ↑ A's utility.

(18.1) a) land "n" apples "a" Constant returns to scale in production of a & b

labor "l" bandannas "b" N identical people

Vector: n, l, a, b

$$\underset{\sim}{\omega}_i = (10, 15, 0, 0) \quad \forall i = 1, \dots, N$$

$$u_i = c \ln a + (1-c) \ln b \quad \text{with } c \in (0,1)$$

production $\begin{cases} \text{apples}: (-1, -1, 1, 0) \\ \text{bandannas}: (0, -1, 1, 0) \end{cases}$

$$p_l \equiv 1$$

No profits returned to consumers since $\pi = 0$ by constant returns to scale

Utility maximization:

$$\mathcal{L}_i = c \ln a + (1-c) \ln b + \lambda \left(10 p_n + 15 - p_a a - p_b b\right)$$

FOC $\quad 0 = 10 p_n + 15 - p_a a - p_b b$

$$\left.\begin{aligned} 0 &= \frac{c}{a} - \lambda p_a \\ 0 &= \frac{1-c}{b} - \lambda p_b \end{aligned}\right\} \frac{p_a}{p_b} = \frac{c}{a} \frac{b}{1-c} = \frac{c}{1-c} \frac{b}{a}$$

$$\Rightarrow b = \frac{p_a}{p_b} \frac{1-c}{c} a \text{ ; into budget constraint} \Rightarrow$$

$$10 p_n + 15 = p_a a + p_b \frac{p_a}{p_b} \frac{1-c}{c} a$$

$$= p_a a + p_a \frac{1-c}{c} a = p_a a \left(1 + \frac{1-c}{c}\right) = p_a a \frac{c+1-c}{c} = \frac{p_a a}{c}$$

$$\Rightarrow a = \frac{c}{p_a}(10 p_n + 15),$$

$$\text{agg. Demand for } a = N \frac{c}{p_a}(10 p_n + 15).$$

$$b = \frac{p_a}{p_b} \frac{1-c}{c} \cdot \frac{c}{p_a}(10 p_n + 15) = \frac{1-c}{p_b}(10 p_n + 15)$$

$$\text{agg. Demand for } b = N \frac{1-c}{p_b}(10 p_n + 15)$$

Apple Firm : $0 = \pi_a = p_a a_j - p_n n_a - 1\, l_a$

apples made by one firm

land used to make apples of one firm

labor used to make apples of one firm

Due to technology, efficiency requires $a_j = n_a$
$a_j = l_a$ so

$$0 = p_a a_j - p_n a_j - a_j = a_j(p_a - p_n - 1)$$

$$\Rightarrow p_a - p_n - 1 = 0 \Rightarrow p_a - p_n = 1 \text{ or } p_a = 1 + p_n. \qquad (1)$$

Bandanna Firm : $0 = \pi_b = p_b b_j - 1\, l_b$

technological efficiency : $l_b = b_j$ so

$$0 = p_b b_j - b_j = b_j(p_b - 1) \Rightarrow \boxed{p_b = 1.}$$

n clears : supply $= 10N$

indiv. demand for n : $n_a + \underbrace{n_b}_{0} = n_a = a_j$. If N_j = number of apple firms then

agg. demand for n : $N_j a_j$

$S = D \Rightarrow 10N = N_j a_j$

l clears : supply $= 15N$

If $\hat{N}_j$ = number of bandanna firms then aggregate D for l is $N_j l_a + \hat{N}_j l_b = N_j a_j + \hat{N}_j l_b = 10N + \hat{N}_j l_b$. Agg. S = Agg. D $\Rightarrow 15N = 10N + \hat{N}_j l_b \Rightarrow \hat{N}_j l_b = 5N$, the number of bandannas in aggregate.

a clears : Supply $= a_j N_j = 10N$ (So each person consumes $5N \div N = 5$ bandannas.)

indiv. D $= \frac{c}{p_a}(10p_n + 15)$

$= \frac{c}{1+p_n}(10p_n + 15)$ due to (1) above; agg. D is N times this, so agg. S = agg. D $\Rightarrow$ $10N = \frac{c}{1+p_n}(10p_n + 15)N$. Divide both sides by $10N$:

$$1 = \frac{c}{1+p_n}\left(p_n + \frac{3}{2}\right)$$

$$1 + p_n = cp_n + \frac{3}{2}c$$

$$(1-c)p_n = \frac{3c}{2} - 1$$

$$\boxed{p_n = \frac{\frac{3c}{2} - 1}{1-c}}$$

From (1), $p_a = 1 + p_n = 1 + \frac{\frac{3c}{2}-1}{1-c} = \frac{1 - c + \frac{3}{2}c - 1}{1-c} = \frac{\frac{1}{2}c}{1-c} = \boxed{\frac{1}{2}\frac{c}{1-c}.}$

b will clear when the other three markets clear; you can verify this if you wish, but it's optional:

$$\overset{S}{b} \overset{?}{=} \overset{D}{\frac{1-c}{p_b}(10p_n + 15)}$$ but $b = 5$, $p_b = 1$, p_n is above, so

$$5 \overset{?}{=} (1-c)\left(10\frac{\frac{3}{2}c - 1}{1-c} + 15\right)$$

$$\overset{?}{=} 10\left(\frac{3}{2}c - 1\right) + 15(1-c) = 15c - 10 + 15 - 15c$$

$$= 5 \text{ OK, verified.}$$

This equilibrium is OK as long as $p_a > 0$ and $p_n > 0$.

$$\frac{1}{2}\frac{c}{1-c} > 0$$

$$\frac{c}{1-c} > 0$$

either $c > 0$ and $1 - c > 0 \Leftrightarrow 1 > c$, giving $\boxed{0 < c < 1}$

or $c < 0$ and $1 - c < 0$, $1 < c$: incompatible

$p_a - 1 > 0$ from (1)

$$\frac{1}{2}\frac{c}{1-c} - 1 > 0$$

$\frac{c}{1-c} > 2$. We know from $p_a > 0$ that $0 < c < 1$ so $1 - c > 0$:

$$c > 2(1-c) = 2 - 2c$$

$$3c > 2$$

$$\boxed{c > 2/3}$$

Combining these two requirements means the equilibrium is valid when

$$\frac{2}{3} < c < 1.$$

From the utility function, for increases in a and b to increase utility, we need

$$c > 0 \text{ and } c < 1.$$

We're lacking an equilibrium for $0 < c < \frac{2}{3}$.

⊗ Let's try the land market not clearing in equilibrium, so that $p_n = 0$ and it has excess supply. Then $p_a = 1$ from (1); $p_b = 1$ is unaffected by this change. So those are all the prices.

indiv. a demanded $= \frac{c}{p_a}(10p_n + 15) = 15c$,

aggregate a demanded $= 15cN$.

This must equal aggregate a supplied, so

$$N_j n_a = N_j l_a = 15cN.$$

indiv. b demanded $= \frac{1-c}{p_b}(10p_n + 15) = 15(1-c)$

aggregate b demanded $= 15N(1-c)$

This must equal aggregate b supplied, so

$$\hat{N}_j l_b = 15N(1-c).$$

l clears: agg. supply is $15N$, agg. D is $N_j l_a + \hat{N}_j l_b = 15cN + 15N(1-c)$,

so $S = D \Rightarrow 15N = 15cN + 15N(1-c)$

$$= 15cN + 15N - 15Nc$$

$$15N = 15N \quad \text{OK}.$$

n should have excess S: its supply is $10N$, its demand is

$$\underset{0}{N_j n_a + N n_b} = N_j n_a = N \cdot 15c, \text{ so } S > D \Rightarrow 10N > 15cN$$

$$1 > \tfrac{3}{2}c$$

$$\tfrac{2}{3} > c.$$

So this is the equilibrium for $c < 2/3$. (We still need $c > 0$.)

Why, at point $\otimes$, did we try the land market not clearing? How about the other markets not clearing?

- If n, l, and a markets clear then so does b, as we verified
- If a did not clear then $p_a = 0$. Also,

$$0 = \pi_a = \underset{\substack{\uparrow \\ 0}}{p_a} a - p_n n_a - 1\, l_a$$

which can only be satisfied if $a = n_a = l_a = 0$. However, aggregate demand for a is $N \frac{c}{p_a}(10p_n + 15)$, which is ∞, not 0, if $p_a = 0$. So this doesn't work.

- If l did not clear then $p_l = 0$ instead of $p_l = 1$ as we assumed before.

The budget constraint would be $10p_n - p_a a - p_b b = 0$.

Aggregate D for $a = N \frac{c}{p_a} 10p_n$

— " — $b = N \frac{1-c}{p_b} 10p_n$

Apple firms: $0 = \pi_a = p_a a_j - p_n n_a = p_a a_j - p_n a_j$ by technological efficiency

$= a_j(p_a - p_n) \Rightarrow p_a = p_n$.

Bandanna firms: $0 = \pi_b = p_b b_j$. Agg. S of $b = b_j \hat{N}_j$, Agg. D of $b = N \frac{1-c}{p_b} 10p_n$.

Set agg. S $\geq$ agg. D and multiply both sides by p_b:

$$\underbrace{p_b b_j}_{=0} \hat{N}_j \geq p_b N \frac{1-c}{p_b} 10 p_n \iff 0 \geq N(1-c)\, 10 p_n \Rightarrow p_n = 0 \text{ and,}$$

from the apple firm analysis, $p_a = p_n = 0$. This would make $p_b = 0$ necessary to satisfy the budget constraint; demands are no longer well-defined. This case is not interesting.

b) When $\frac{2}{3} < c < 1$, small changes or big changes in the endowment of anything will affect equilibrium prices and quantities. But for $0 < c < \frac{2}{3}$, land is in excess supply, so small changes in its endowment will not affect equilibrium prices or quantities.

c) see (b).

(18.2)

$$\overset{\text{guns}}{g} = 2\overset{\text{oil}}{x} \qquad \text{Firm 2: } \overset{\text{butter}}{b} = 3\overset{\text{oil}}{x}$$

Firm 1: $g = 2x$ Firm 2: $b = 3x$

$$\omega_1 = \omega_2 = (\underset{g}{0}, \underset{b}{0}, \underset{x}{10})$$

Consumer 1: $\max u_1$ s.t. $p_g g_1 + p_b b_1 = p_x 10 + \pi_1$

$$\mathcal{L} = g_1^{0.4} b_1^{0.6} + \lambda (p_x 10 + \pi_1 - p_g g_1 - p_b b_1)$$

$$0 = p_x 10 + \pi_1 - p_g g_1 - p_b b_1$$

$$\left.\begin{aligned} 0 &= 0.4 \frac{g_1^{0.4} b_1^{0.6}}{g_1} - \lambda p_g \\ 0 &= 0.6 \frac{g_1^{0.4} b_1^{0.6}}{b_1} - \lambda p_b \end{aligned}\right\} \frac{0.4}{0.6} \frac{b_1}{g_1} = \frac{p_g}{p_b} \Rightarrow g_1 = \frac{2}{3} b_1 \frac{p_b}{p_g}.$$

Let $p_g + p_b + p_x = 1$. Substitute into the B. C. :

$$\pi_1 + p_x 10 = p_g g_1 + p_b b_1$$

$$= p_g \frac{2}{3} b_1 \frac{p_b}{p_g} + p_b b_1 = b_1 \left[\frac{2}{3} p_b + p_b\right] = \frac{5}{3} b_1 p_b$$

$$\Rightarrow b_1 = \frac{3}{5} \frac{1}{p_b} (10 p_x + \pi_1)$$

$$= \frac{3}{5 p_b} 10 (1 - p_g - p_b) + \frac{3\pi_1}{5 p_b}$$

$$= \frac{6}{p_b} (1 - p_g - p_b) + \frac{3\pi_1}{5 p_b}. \qquad (1)$$

$$g_1 = \frac{2}{3} \frac{p_b}{p_g} \left[\frac{6}{p_b} (1 - p_g - p_b) + \frac{3\pi_1}{5 p_b}\right]$$

$$= \frac{4}{p_g} (1 - p_g - p_b) + \frac{2\pi_1}{5 p_g}. \qquad (2)$$

Consumer 2.

$\max u_2$ s.t. $p_g g_2 + p_b b_2 = p_x 10 + \pi_2$

$$\mathcal{L} = 10 + \tfrac{1}{2}\ln g_2 + \tfrac{1}{2}\ln b_2 + \lambda\left(p_x 10 + \pi_2 - p_g g_2 - p_b b_2\right)$$

$$0 = p_x 10 + \pi_2 - p_g g_2 - p_b b_2$$

$$\left.\begin{aligned} 0 &= \frac{1}{2g_2} - \lambda p_g \\ 0 &= \frac{1}{2b_2} - \lambda p_b \end{aligned}\right\} \quad \frac{\cancel{2} b_2}{\cancel{2} g_2} = \frac{p_g}{p_b} \Rightarrow b_2 = \frac{p_g}{p_b} g_2 .$$ Substitute into the first FOC:

$$0 = p_x 10 + \pi_2 - p_g g_2 - p_b \frac{p_g}{p_b} g_2$$

$$\Rightarrow p_x 10 + \pi_2 = 2 p_g g_2 \Rightarrow g_2 = \frac{5p_x}{p_g} + \frac{\pi_2}{2p_g}$$

$$= \frac{5(1-p_g-p_b)}{p_g} + \frac{\pi_2}{2p_g} \quad (3)$$

$$b_2 = \frac{5(1-p_g-p_b)}{p_b} + \frac{\pi_2}{2p_b} \quad (4).$$

Aggregate Demands

$$g^D = g_1 + g_2 = \frac{4(1-p_g-p_b)}{p_g} + \frac{2\pi_1}{5p_g} + \frac{5(1-p_g-p_b)}{p_g} + \frac{\pi_2}{2p_g}$$

$$= 9\,\frac{1-p_g-p_b}{p_g} + \frac{1}{p_g}\left(\frac{2\pi_1}{5} + \frac{\pi_2}{2}\right) \quad (5)$$

$$b^D = b_1 + b_2 = 6\,\frac{1-p_g-p_b}{p_b} + \frac{3\pi_1}{5p_b} + 5\,\frac{1-p_g-p_b}{p_b} + \frac{\pi_2}{2p_b}$$

$$= 11\,\frac{1-p_g-p_b}{p_b} + \frac{1}{p_b}\left(\frac{3\pi_1}{5} + \frac{\pi_2}{2}\right) \quad (6).$$

Firm 1: $\pi_1 = p_g g - p_x x_1$ where x_1 is the oil used by firm 1

$= p_g \cdot 2x_1 - p_x x_1 = (2p_g - p_x)x_1$. But firm 1's production function is linear in x, so it has constant returns to scale, so $\pi_1 = 0$, so $\underline{2p_g = p_x}$.

Firm 2: $\pi_2 = p_b b - p_x x_2$ where x_2 is the oil used by Firm 2.

$= p_b(3x_2) - p_x x_2 = (3p_b - p_x)x_2$. Firm 2's production function has CRS just like Firm 1's, so $\pi_2 = 0$, so $\underline{3p_b = p_x}$.

Prices. Since the prices were normalized to be on the simplex,

$1 = p_g + p_b + p_x$; from above on this page,

$= \frac{1}{2}p_x + \frac{1}{3}p_x + p_x = \left(\frac{3}{6} + \frac{2}{6} + \frac{6}{6}\right)p_x = \frac{11}{6}p_x \Rightarrow \boxed{p_x = \frac{6}{11} \quad p_g = \frac{3}{11} \quad p_b = \frac{2}{11}}$.

(5) $\Rightarrow g^D = 9\,\frac{6/11}{3/11} + \frac{1}{p_g}(0+0) = \underline{18}$. This must equal g^s in equilibrium,

so $18 = g^s = 2x_1 \Rightarrow \underline{x_1 = 9}$.

(6) $\Rightarrow b^D = 11\,\frac{6/11}{2/11} + \frac{1}{p_b}(0+0) = \underline{33}$. This must equal b^s in equilibrium,

so $33 = b^s = 3x_2 \Rightarrow \underline{x_2 = 11}$.

(2) $\Rightarrow \underline{g_1} = 4\,\frac{6/11}{3/11} + 0 = \underline{8}$ so $\underline{g_2} = g^D - g_1 = 18 - 8 = \underline{10}$.

(1) $\Rightarrow \underline{b_1} = 6\,\frac{6/11}{2/11} + 0 = \underline{18}$ so $b_2 = b^D - b_1 = 33 - 18 = \underline{15}$.