H.W. Set 1 (Consumption)

1. Assume a consumer's utility function is:

$$U(x_1,x_2) = x_1^a x_2^{1-a} \quad . \qquad (0<a<1)$$

(a) Derive the uncompensated (Marshallian) demand functions and indirect utility function. Verify Roy's identity.

(b) Verify the symmetry restriction of the Slutsky substitution matrix.

(c) Derive the expenditure function and compensated demand functions. Verify Shephard's Lemma.

(d) The <u>monotonicity condition</u> is

$$\frac{\partial h_i}{\partial p_i} < 0 \;\forall i \; . \qquad \text{(Where } h_i \text{ is the Hicksian, or compensated, demand function.)}$$

Is it satisfied also?

2. Varian, problem 7.4. Part (c) is much too hard for an exam question.

3. Varian, problem 8.11.

4. Varian, problem 8.14.

5. Varian, problem 8.16.

Q. 6. "Commodities i and j are gross substitutes or gross complements according to whether the total effect $\partial q_i/\partial p_j$ is positive or negative" (J. M. Henderson and R. E. Quandt, Microeceonomic Theory, third edition, p. 32).

Let commodities 1 and 2 be food and drink respectively and let the consumer's utility function take the following form:

$$u(q_1,q_2) = \log q_1 + 2(q_2)^{\frac{1}{2}} \quad .$$

assume this is natural log

Assume the consumer allocates a fixed total expenditure, y, on food and drink to maximize $U(q_1,q_2)$ where the prices of food and drink, p_1 and p_2, are independent of the consumer's purchases.

With the utility function defined above, first derive the expression for the effect of an increase in the price of food on the consumption of drink and determine whether food and drink are gross complements according to the definition given by Henderson and Quandt. Second, consider the effect of an increase in the price of drink on the consumption of food and again state whether food and drink are gross complements. You should find that, in one case, food and drink appear to be gross substitutes while, in the other case, food and drink are independent in consumption. Explain why this contradiction is fully consistent with the constrained maximization of any well-behaved utility function.

(Helpful hint: do not try to obtain the demand functions in explicit form. Write each demand function in implicit form, then differentiate with respect to q_i and to p_j, and then form $\partial q_i/\partial p_j$.)

Answers to H.W. Set 1

1. a. $\underset{x_1, x_2}{Max} \; x_1^a x_2^{1-a}$ s. t. $p_1 x_1 + p_2 x_2 = m$

$$\mathcal{L} = x_1^a x_2^{1-a} + \lambda (m - p_1 x_1 - p_2 x_2)$$

$$\left.\begin{aligned} \frac{\partial \mathcal{L}}{\partial x_1} &= 0 = a \frac{U}{x_1} - \lambda p_1 \\ \frac{\partial \mathcal{L}}{\partial x_2} &= 0 = (1-a) \frac{U}{x_2} - \lambda p_2 \end{aligned}\right\} \quad \frac{p_1}{p_2} = \frac{x_2}{x_1} \frac{a}{1-a} \text{ or}$$

$$x_1 = \frac{a}{1-a} \frac{p_2}{p_1} x_2$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 = m - p_1 x_1 - p_2 x_2$$

$$m = p_1 \left(\frac{a}{1-a} \frac{p_2}{p_1} x_2 \right) + p_2 x_2$$

$$= \frac{a}{1-a} p_2 x_2 + p_2 x_2 = \frac{1}{1-a} p_2 x_2 ,$$

$$\boxed{\begin{aligned} x_2^* &= \frac{1-a}{p_2} m \\ x_1^* &= \frac{a}{p_1} m . \end{aligned}}$$

Indirect: $v(p, m) = \left(\frac{1-a}{p_2} m \right)^{1-a} \left(\frac{a}{p_1} m \right)^a \Rightarrow$

$$\boxed{v(p, m) = \left(\frac{1-a}{p_2} \right)^{1-a} \left(\frac{a}{p_1} \right)^a m} .$$

Roy's Identity: $x_i^* = - \frac{\partial v / \partial p_i}{\partial v / \partial m}$.

We have $\frac{\partial v}{\partial p_1} = \frac{\partial}{\partial p_1} \left(\frac{1-a}{p_2} \right)^{1-a} a^a (p_1)^{-a} m$

$$= (-a)\left(\frac{1-a}{p_2}\right)^{1-a} a^{a}\, p_1^{-a-1}\, m .$$

$$\frac{\partial v}{\partial m} = \left(\frac{1-a}{p_2}\right)^{1-a}\left(\frac{a}{p_1}\right)^{a} \Rightarrow$$

$$\frac{\partial v/\partial p_1}{\partial v/\partial m} = (-a)\frac{m}{p_1} = \frac{-am}{p_1} = -x_1^* \quad \text{ok.}$$

$$\frac{\partial v}{\partial p_2} = \frac{\partial}{\partial p_2}(1-a)^{1-a}\, p_2^{a-1}\left(\frac{a}{p_1}\right)^{a} m = (a-1)(1-a)^{1-a}\, p_2^{a-2}\left(\frac{a}{p_1}\right)^{a} m,$$

so

$$\frac{\partial v/\partial p_2}{\partial v/\partial m} = (a-1)\frac{1}{p_2} m = -\frac{(1-a)m}{p_2} = -x_2^*.$$

b. The Slutsky substitution matrix is $\left[\frac{\partial x_r}{\partial p_j} + x_j \frac{\partial x_r}{\partial m}\right]$:

$$\begin{bmatrix} \frac{\partial x_1}{\partial p_1} + x_1\frac{\partial x_1}{\partial m} & \frac{\partial x_1}{\partial p_2} + x_2\frac{\partial x_1}{\partial m} \\ \frac{\partial x_2}{\partial p_1} + x_1\frac{\partial x_2}{\partial m} & \frac{\partial x_2}{\partial p_2} + x_2\frac{\partial x_2}{\partial m} \end{bmatrix} = \begin{bmatrix} \frac{-am}{p_1^2} + \frac{am}{p_1}\frac{a}{p_1} & 0 + \frac{(1-a)m}{p_2}\frac{a}{p_1} \\ 0 + \frac{am}{p_1}\frac{1-a}{p_2} & -\left(\frac{1-a}{p_2^2}\right)m + \frac{(1-a)m}{p_2}\frac{1-a}{p_2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{am(a-1)}{p_1^2} & \frac{a(1-a)m}{p_1 p_2} \\ \frac{a(1-a)m}{p_1 p_2} & \frac{(a-1)m(a)}{p_2^2} \end{bmatrix}$$

which is symmetric

$\leftarrow$ Sign pattern is $\begin{bmatrix} - & + \\ + & - \end{bmatrix}$.

You can also verify the other properties which this matrix is supposed to have:

1) be negative semidefinite
2) be symmetric [already done]
3) have nonpositive diagonal terms [see bottom of previous page].

To verify negative semidefiniteness:

$$\begin{bmatrix} \delta_1 & \delta_2 \end{bmatrix} \begin{bmatrix} \dfrac{am(a-1)}{p_1^2} & \dfrac{a(1-a)m}{p_1 p_2} \\ \dfrac{a(1-a)m}{p_1 p_2} & \dfrac{a(a-1)m}{p_2^2} \end{bmatrix} \begin{bmatrix} \delta_1 \\ \delta_2 \end{bmatrix} = \begin{bmatrix} \delta_1 & \delta_2 \end{bmatrix} \begin{bmatrix} \dfrac{am(a-1)}{p_1^2}\delta_1 + \dfrac{a(1-a)m}{p_1 p_2}\delta_2 \\ \dfrac{a(1-a)m}{p_1 p_2}\delta_1 + \dfrac{a(a-1)m}{p_2^2}\delta_2 \end{bmatrix}$$

$$= \frac{am(a-1)}{p_1^2}\delta_1^2 + \frac{a(1-a)m}{p_1 p_2}\delta_1\delta_2 + \frac{a(1-a)m}{p_1 p_2}\delta_1\delta_2 + \frac{a(a-1)m}{p_2^2}\delta_2^2$$

$$= am(a-1)\left[\frac{\delta_1^2}{p_1^2} - 2\frac{\delta_1\delta_2}{p_1 p_2} + \frac{\delta_2^2}{p_2^2}\right]$$

$$= am(a-1)\left(\frac{\delta_1}{p_1} - \frac{\delta_2}{p_2}\right)^2$$

≤ 0 for all δ_1, δ_2 since $a > 0$, $m > 0$, $a - 1 < 0$, $(\cdot)^2 > 0$.

(δ_1 and δ_2 cannot both be zero)

c) <u>The Easy Way</u>. (This is possible only if you have worked part (a) already.)

$$v(\underset{\sim}{p}, e(\underset{\sim}{p}, u)) \equiv u \quad (\text{Section 7.4, (2)})$$

Since $v(\underset{\sim}{p}, m) = \left(\frac{1-a}{p_2}\right)^{1-a}\left(\frac{a}{p_1}\right)^a m$,

$$v(\underset{\sim}{p}, e(\underset{\sim}{p}, u)) = \left(\frac{1-a}{p_2}\right)^{1-a}\left(\frac{a}{p_1}\right)^a e(\underset{\sim}{p}, u) = u$$

and $e(\underset{\sim}{p}, u) = \left(\frac{p_2}{1-a}\right)^{1-a}\left(\frac{p_1}{a}\right)^a u.$

• Hicksian demand curves via Shephard's Lemma $h_i = \frac{\partial e}{\partial p_i}$:

$$h_1(\underset{\sim}{p}, u) = \frac{\partial e}{\partial p_1} = \frac{\partial}{\partial p_1}\left(\frac{p_2}{1-a}\right)^{1-a} p_1^a a^{-a} u = a\, p_1^{a-1}\left(\frac{p_2}{1-a}\right)^{1-a} a^{-a} u$$

note

$$= \left(\frac{a}{1-a}\frac{p_2}{p_1}\right)^{1-a} u$$

note: $\left(\frac{1}{1-a}\right)^{-1}\left(\frac{1}{1-a}\right)^{1-a} = \left(\frac{1}{1-a}\right)^{-a}$

$$h_2(\underset{\sim}{p}, u) = \frac{\partial e}{\partial p_2} = \left(\frac{1}{1-a}\right)^{1-a}\left(\frac{p_1}{a}\right)^a u \frac{\partial}{\partial p_2} p_2^{1-a} = (1-a)\, p_2^{-a}\left(\frac{1}{1-a}\right)^{1-a}\left(\frac{p_1}{a}\right)^a u$$

$$= \left(\frac{1-a}{a}\frac{p_1}{p_2}\right)^a u.$$

• Hicksian demand curves via Section 7.4 (if part (a) has already been worked):

$$h_i(\underset{\sim}{p}, u) = x_i(\underset{\sim}{p}, e(\underset{\sim}{p}, u))$$

from p.1   from $e(\underset{\sim}{p}, u)$ above

$$h_1(\underset{\sim}{p}, u) = x_1(\underset{\sim}{p}, e(\underset{\sim}{p}, u)) = \frac{a}{p_1} e(\underset{\sim}{p}, u) = \left(\frac{a}{1-a}\frac{p_2}{p_1}\right)^{1-a} u.$$

$$h_2(\underset{\sim}{p}, u) = x_2(\underset{\sim}{p}, e(\underset{\sim}{p}, u)) = \frac{1-a}{p_2} e(\underset{\sim}{p}, u) = \left(\frac{1-a}{a}\frac{p_1}{p_2}\right)^a u.$$ So Shephard's Lemma is verified – the two methods of finding the h's agree.

The Hard Way. (If you have not worked part (a) already, though, it's easier to do this than to work part (a) first.)

$e(\underset{\sim}{p}, u)$ is defined to be $\min\limits_{\underset{\sim}{x}} \underset{\sim}{p} \cdot \underset{\sim}{x}$ s.t. $U(\underset{\sim}{x}) \geq U_0$.

$$\mathcal{L} = p_1 x_1 + p_2 x_2 + \lambda (x_1^a x_2^{1-a} - u_0)$$

$$\frac{\partial \mathcal{L}}{\partial x_1} = 0 = p_1 + \lambda a \frac{U}{x_1}$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = 0 = p_2 + \lambda (1-a) \frac{U}{x_2}$$

$$\frac{p_1}{p_2} = \frac{a}{1-a} \frac{x_2}{x_1} \Rightarrow x_1 = \frac{a}{1-a} \frac{p_2}{p_1} x_2$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 = x_1^a x_2^{1-a} - u_0 \longrightarrow u_0 = \left[\frac{a}{1-a} \frac{p_2}{p_1} x_2 \right]^a x_2^{1-a} = \left(\frac{a}{1-a} \frac{p_2}{p_1} \right)^a x_2 \Rightarrow$$

$$x_2^* = \left(\frac{1-a}{a} \frac{p_1}{p_2} \right)^a u_0 = h_2(\underset{\sim}{p}, u_0)$$

$$x_1^* = \left(\frac{a}{1-a} \frac{p_2}{p_1} \right)^{1-a} u_0 = h_1(\underset{\sim}{p}, u_0)$$

the compensated demand functions (compare with bottom and middle of page 4). Then

$e(\underset{\sim}{p}, u_0) = \underset{\sim}{p} \cdot \underset{\sim}{x}^*$ where $\underset{\sim}{x}^*$ solves the above minimization problem

$$= p_1 x_1^* + p_2 x_2^* = p_1^a \left(\frac{a}{1-a} p_2 \right)^{1-a} u_0 + p_2^{1-a} \left(\frac{1-a}{a} p_1 \right)^a u_0$$

$$= p_1^a p_2^{1-a} \left[\left(\frac{a}{1-a} \right)^{1-a} + \left(\frac{1-a}{a} \right)^a \right] u_0$$ The term in brackets is

$$\underbrace{\frac{a^a}{a^a}}_{=1} \cdot \left(\frac{a}{1-a} \right)^{1-a} + \left(\frac{1-a}{a} \right)^a \cdot \underbrace{\frac{(1-a)^{1-a}}{(1-a)^{1-a}}}_{=1} = \frac{a + (1-a)}{a^a (1-a)^{1-a}} = \frac{1}{a^a (1-a)^{1-a}} \Rightarrow$$

$$e(\underset{\sim}{p}, u_0) = p_1^a p_2^{1-a} \frac{1}{a^a (1-a)^{1-a}} u_0 = \left(\frac{p_1}{a} \right)^a \left(\frac{p_2}{1-a} \right)^{1-a} u_0$$ (compare with p. 4).

d) Instead of doing just this, let's verify that the entire Slutsky substitution matrix on p. 2 is equal to $\begin{bmatrix} \partial h_1/\partial p_1 & \partial h_1/\partial p_2 \\ \partial h_2/\partial p_1 & \partial h_2/\partial p_2 \end{bmatrix}$.

$$h_1 = \left(\frac{a}{1-a}\,\frac{p_2}{p_1}\right)^{1-a} u_0 \quad \text{from p. 5}$$

$$\frac{\partial h_1}{\partial p_1} = \left(\frac{a}{1-a}\,p_2\right)^{1-a} u_0 \frac{\partial}{\partial p_1} p_1^{a-1} = \left(\frac{a}{1-a}\,p_2\right)^{1-a} u_0 \,(a-1)\, p_1^{a-2}.$$

From the indirect utility function (p. 1), $u_0 = \left(\frac{1-a}{p_2}\right)^{1-a}\left(\frac{a}{p_1}\right)^{a} m$. So

$$\frac{\partial h_1}{\partial p_1} = a^{1-a} \cdot a^{a}$$
$$\cdot (1-a)^{a-1} \cdot (1-a)^{1-a} \cdot (1-a)$$
$$\cdot\, p_2^{1-a} \cdot p_2^{a-1}$$
$$\cdot\, p_1^{-a} \cdot p_1^{a-2} \cdot (-m) = (-m)\, a\,(1-a)\, p_1^{-2} = -\frac{a(1-a)m}{p_1^2}.$$

$$\frac{\partial h_1}{\partial p_2} = \left(\frac{a}{1-a}\,\frac{1}{p_1}\right)^{1-a} u_0 \underbrace{\frac{\partial}{\partial p_2} p_2^{1-a}}_{=(1-a)\,p_2^{-a}}$$

$$= a^{1-a} \cdot a^{a}$$
$$\cdot (1-a)^{a-1} \cdot (1-a)^{1-a} \cdot (1-a)$$
$$\cdot\, p_1^{a-1} \cdot p_1^{-a}$$
$$\cdot\, p_2^{a-1} \cdot p_2^{-a} \cdot m = a(1-a)\, p_1^{-1} p_2^{-1} m = \frac{a(1-a)m}{p_1 p_2}.$$

$$h_2 = \left(\frac{1-a}{a} \; \frac{p_1}{p_2}\right)^a u_0 \text{ from p. 5}$$

$$\frac{\partial h_2}{\partial p_2} = \left(\frac{1-a}{a} \; p_1\right)^a u_0 \underbrace{\frac{\partial}{\partial p_2} p_2^{-a}}_{= -a\, p_2^{-a-1}}$$

$$\begin{aligned} = {} & (1-a)^a \cdot (1-a)^{1-a} \\ & \cdot a^{-a} \cdot a^a \cdot a \\ & \cdot p_1^a \cdot p_1^{-a} \\ & \cdot p_2^{a-1} \cdot p_2^{-a-1} \cdot (-m) = (1-a)\, a\, p_2^{-2}(-m) = -\frac{a(1-a)m}{p_2^2}. \end{aligned}$$

$$\frac{\partial h_2}{\partial p_1} = \left(\frac{1-a}{a} \; \frac{1}{p_2}\right)^a u_0 \underbrace{\frac{\partial}{\partial p_1} p_1^{a}}_{= a\, p_1^{a-1}}$$

$$\begin{aligned} = {} & (1-a)^a \cdot (1-a)^{1-a} \\ & \cdot a^{-a} \cdot a^a \cdot a \\ & \cdot p_2^{-a} \cdot p_2^{a-1} \\ & \cdot p_1^{-a} \cdot p_1^{a-1} \cdot m = (1-a)\, a\, p_2^{-1} p_1^{-1} m = \frac{a(1-a)m}{p_1 p_2}. \end{aligned}$$

So $$\nabla_{\underset{\sim}{p}} \underset{\sim}{h} = \begin{bmatrix} \partial h_1/\partial p_1 & \partial h_1/\partial p_2 \\ \partial h_2/\partial p_1 & \partial h_2/\partial p_2 \end{bmatrix} = \begin{bmatrix} -\frac{a(1-a)m}{p_1^2} & \frac{a(1-a)m}{p_1 p_2} \\ \frac{a(1-a)m}{p_1 p_2} & -\frac{a(1-a)m}{p_2^2} \end{bmatrix}$$ exactly as on p. 2.

2. Varian, problem 7.4. $v(p_1, p_2, m) = \dfrac{m}{p_1+p_2}$.

a) Roy's Identity $x_i = -\dfrac{\partial v/\partial p_i}{\partial v/\partial m}$. Here, $\dfrac{\partial v}{\partial p_i} = \dfrac{-m}{(p_1+p_2)^2}$ (1) and

$\dfrac{\partial v}{\partial m} = \dfrac{1}{p_1+p_2}$, so $x_i = +\dfrac{m}{(p_1+p_2)^2}\,\dfrac{p_1+p_2}{1} = \boxed{\dfrac{m}{p_1+p_2} \text{ for } i = 1, 2.}$

b) $v(\underset{\sim}{p}, e(\underset{\sim}{p}, u)) \equiv u$ (Section 7.4). Since $v(\underset{\sim}{p}, m) = \dfrac{m}{p_1+p_2}$ is given,

$v(\underset{\sim}{p}, e) = \dfrac{e}{p_1+p_2}$ and $v(\underset{\sim}{p}, e) \equiv u \Rightarrow \dfrac{e}{p_1+p_2} = u \Rightarrow \boxed{e(\underset{\sim}{p}, u) = (p_1+p_2)u.}$

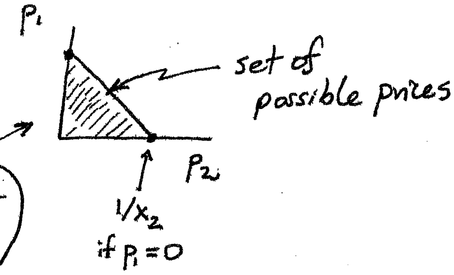
c) From Section 8.6, bottom of p. 129:

$$u(\underset{\sim}{x}) = \min_{\underset{\sim}{p}} v(\underset{\sim}{p}) \text{ s.t. } \underset{\sim}{p}\cdot\underset{\sim}{x} = 1 \ (= m)$$

$$= \min_{p_1, p_2} \frac{1}{p_1+p_2} \text{ s.t. } \underbrace{p_1 x_1 + p_2 x_2 = 1}.$$

$p_1 x_1 = 1 - p_2 x_2$

$p_1 = \frac{1}{x_1} - \frac{x_2}{x_1} p_2$, a straight line



$$= \min_{p_2} \frac{1}{\frac{1}{x_1} - \frac{x_2}{x_1} p_2 + p_2} = \min_{p_2} \frac{x_1}{1 - p_2 x_2 + p_2 x_1} = \min_{p_2} \frac{x_1}{1 + p_2(x_1 - x_2)}.$$

If $x_1 = x_2$ this is $\min_{p_2} \frac{x_1}{1}$ so $p_2^* =$ anything and $u(\underset{\sim}{x}) = x_1$.

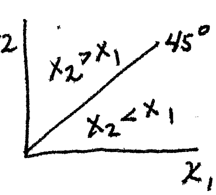
If $x_1 < x_2$ this is $\min_{p_2} \dfrac{x_1}{1 + p_2 (\text{negative } \#)}$ so $p_2^* = 0$ and $u(\underset{\sim}{x}) = x_1$.

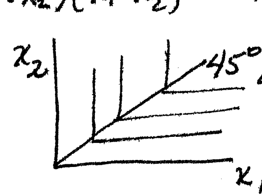
If $x_2 > x_1$, $u = x_1$, so indifference curves are vertical.

If $x_1 > x_2$ this is $\min_{p_2} \dfrac{x_1}{1 + p_2 (\text{positive } \#)}$ so p_2^* should be as large as possible. From the diagram above, $p_2^* = \frac{1}{x_2}$. Then $u(\underset{\sim}{x}) = \dfrac{x_1}{1 + (\frac{1}{x_2})(x_1 - x_2)} = \dfrac{x_1 x_2}{x_2 + x_1 - x_2} = x_2$.

So $u(\underset{\sim}{x}) = \begin{cases} x_1 & \text{if } x_1 \le x_2 \\ x_2 & \text{if } x_1 > x_2. \end{cases}$

$= \min\{x_1, x_2\}$.





Indiff curves.
Leontief utility fnctn.
If $x_2 < x_1$, $u = x_2$, so indiff. curves are flat.

3. 8.11 No, because his demand behavior violates GARP. When prices are $(2,4)$ he spends 10. At these prices he could afford the bundle $(2,1)$, but rejects it; therefore, $(1,2) \succ (2,1)$. When prices are $(6,3)$ he spends 15. At these prices he could afford the bundle $(1,2)$ but rejects it; therefore, $(2,1) \succ (1,2)$.

4. 8.14.a This is an ordinary Cobb-Douglas demand: $S_1 = \frac{\alpha}{\alpha+\beta+\gamma}Y$ and $S_2 = \frac{\beta}{\alpha+\beta+\gamma}Y$.

8.14.b In this case the utility function becomes $U(C, S_1, L) = \cancel{\bar{S}_1^\alpha L^\beta C^\gamma}$. $L^\alpha S_2^\beta C^\gamma$
The L term is just a constant, so applying the standard Cobb-Douglas formula $\cancel{S_1 = \frac{\alpha}{\alpha+\gamma}Y}$.

$S_2 = \frac{\beta}{\beta+\gamma}Y.$

$$S_2 = \frac{\beta}{\beta+\gamma}(Y - L)$$

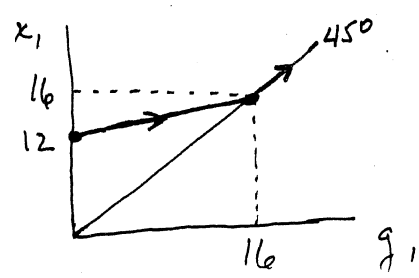
5. 8.16.a True. With the grant, the consumer will maximize $u(x_1, x_2)$ subject to $x_1 + x_2 \le m + g_1$ and $x_1 \ge g_1$. We know that when he maximizes his utility subject to $x_1 + x_2 \le m$, he chooses $x_1^* \ge g_1$. Since x_1 is a normal good, the amount of good 1 that he will choose if given an unconstrained grant of g_1 is some number $x_1' > x_1^* \ge g_1$. Since this choice satisfies the constraint $x_1' \ge g_1$, it is also the choice he would make when forced to spend g_1 on good 1.

8.16.b False. Suppose for example that $g_1 = x_1^*$. Then if he gets an unconstrained grant of g_1, since good 1 is inferior, he will choose to reduce his consumption to less than $x_1^* = g_1$. But with the constrained grant, he must consume at least g_1 units of good 1. Incidentally, he will accept the grant, since with the grant he can always consume at least as much of both goods as without the grant.

8.16.c If he got an unconstrained grant of g_1, he would spend $(48 + g_1)/4$ on good 1. This is exactly what he will spend if $g_1 \le (48 + g_1)4$. But if $g_1 > (48+g_1)/4$, he will spend g_1 on good 1. The curve therefore has slope 1/4 if $g_1 < 16$ and slope 1 if $g_1 > 16$. Kink is at $g_1 = 16$.

→ Unconstrained: $x_1 + x_2 = m + g$

$\frac{x_2^*}{x_1^*} = 3 = \frac{x_2}{x_1}$ always since prices don't change & preferences are homothetic. So $x_2 = 3x_1$ and $x_1 + 3x_1 = m + g$. With $m = 48$, $x_1 = 12 + g/4$. But the constraint is $x_1 \ge g_1$, so $12 + g/4 \ge g_1 \Rightarrow 16 \ge g_1$.



6. $U(q_1, q_2) = \ln q_1 + 2\sqrt{q_2}$

$$\mathcal{L} = \ln q_1 + 2\sqrt{q_2} + \lambda(m - p_1 q_1 - p_2 q_2)$$

$$\left.\begin{array}{l}\dfrac{\partial \mathcal{L}}{\partial q_1} = 0 = \dfrac{1}{q_1} - \lambda p_1 \Rightarrow \dfrac{1}{q_1} = \lambda p_1 \\[2ex] \dfrac{\partial \mathcal{L}}{\partial q_2} = 0 = \dfrac{1}{\sqrt{q_2}} - \lambda p_2 \Rightarrow \dfrac{1}{\sqrt{q_2}} = \lambda p_2\end{array}\right\} \dfrac{\sqrt{q_2}}{q_1} = \dfrac{p_1}{p_2}.$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 = m - p_1 q_1 - p_2 q_2$$

So $q_2 = \left(\frac{p_1}{p_2} q_1\right)^2$ and

$q_1 = \frac{p_2}{p_1}\sqrt{q_2}$.

$$m = p_1 q_1 + p_2 \left(\frac{p_1}{p_2} q_1\right)^2$$

$$m = p_1 q_1 + p_2 \frac{p_1^2}{p_2^2} q_1^2$$

$$m = p_1 q_1 + \frac{p_1^2}{p_2} q_1^2$$

implicit form for the demand for q_1

$$m = p_1\left(\frac{p_2}{p_1}\sqrt{q_2}\right) + p_2 q_2$$

$$m = p_2\sqrt{q_2} + p_2 q_2$$

implicit form for the demand for q_2

Evidently purchases of q_2 are affected only by m and p_2, not p_1. Therefore $\partial q_2 / \partial p_1 = 0$.

To see how purchases of q_1 are affected by changes in p_2, totally differentiate with respect to all the endogenous variables (in this case: only q_1) and with respect to the exogenous variable of interest (in this case: p_2) (so $dm = dp_1 = 0$).

$$0 = dm = p_1\, dq_1 + \frac{p_1^2}{p_2}(2q_1\, dq_1) - \frac{p_1^2}{p_2^2} q_1^2\, dp_2$$

$$\frac{p_1^2}{p_2^2} q_1^2\, dp_2 = \left[p_1 + \frac{2p_1^2}{p_2} q_1\right] dq_1 \Rightarrow \frac{dq_1}{dp_2} = \frac{(p_1 q_1 / p_2)^2}{p_1 + 2p_1^2 q_1 / p_2} > 0.$$

Symmetry only tells us that $\frac{\partial h_i}{\partial p_j} = \frac{\partial h_j}{\partial p_i}$; it says nothing about Marshallian demand curves, which are what were in this problem.