(18.8) Robinson Crusoe Economy. Labor/leisure tradeoff. Criticisms:

↑ assumed good

work hours are always a choice.

leisure is always better than working

labor → consumption w/ no natural resources

Fig. 18.1: the CRS case. $ ←(an abbreviation for "Suppose")

$m(\text{labor}) = 3 \text{ labor}.$

↑ mangoes

Firm: $\pi = p_m\, m(\text{labor}) - w \cdot \text{labor}$

max over labor ⇒

$$0 = p_m\, m' - w$$

⇒ $m' = w/p_m$. Since m' is a constant (3), this determines the real wage, $3 = w/p_m$. So this equilibrium price has been determined w/o knowing demand.

↑ (an abbreviation for "without")

Could choose $p_m \equiv 1 \Rightarrow 3 = w/1 \Rightarrow w = 3$; or

$w \equiv 1 \Rightarrow 3 = 1/p_m \Rightarrow p_m = \frac{1}{3}$; or

$p_m + w \equiv 1 \Rightarrow p_m + 3p_m = 1 \Rightarrow p_m = \frac{1}{4} \Rightarrow w = \frac{3}{4}$.

We'll choose the second. $\pi^* = \frac{1}{3} \cdot 3 \text{ labor} - 1 \cdot \text{labor} \equiv 0$.

Consumer: Budget constraint is $w(\omega - \text{leisure}) = p_m m$ (no profit income)

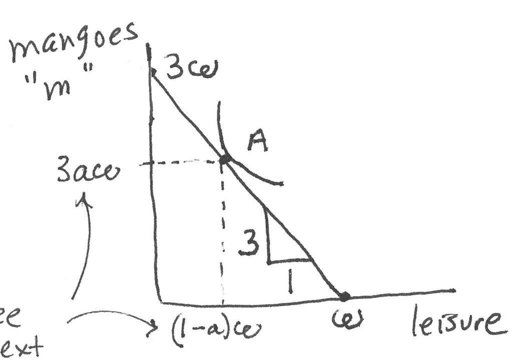
or $w\omega = p_m m + w \text{ leisure}$

or $w\omega = p_m m + w \text{ leisure}$ ⇒

or $m = \frac{w}{p_m}(\omega - \text{leisure})$

$m = \frac{-w}{p_m}\text{leisure} + \frac{w\omega}{p_m}$

$= -\frac{w}{p_m}\text{leisure} + \frac{w}{p_m}\omega$. (It's best not to substitute prices in at this stage so you can get the labor supply & mango demand curves.)



see next page

Note: ω = leisure + labor

Suppose $u = a \ln m + (1-a)\ln(\text{leisure})$. Maximizing u s.t. the B.C. yields $\mathcal{L} = a \ln m + (1-a)\ln(\text{leisure}) + \lambda\left[-\frac{w}{p_m}\text{leisure} + \frac{w}{p_m}\omega - m\right]$.

$0 = \partial\mathcal{L}/\partial m = \frac{a}{m} - \lambda \Rightarrow \lambda = \frac{a}{m}$ and

$$0 = \partial\mathcal{L}/\partial\,\text{leisure} = \frac{1-a}{\text{leisure}} - \frac{\lambda w}{p_m}$$

$$= \frac{1-a}{\text{leisure}} - \frac{a}{m}\frac{w}{p_m} \Rightarrow \frac{aw}{m p_m} = \frac{1-a}{\text{leisure}} \Rightarrow \frac{aw}{1-a}\,\frac{\text{leisure}}{p_m} = m.$$

Substituting into the BC,

$$\frac{aw}{1-a}\,\frac{\text{leisure}}{p_m} = \frac{-w}{p_m}\,\text{leisure} + \frac{w}{p_m}\,\omega$$

$$\left(\frac{aw}{1-a}\,\frac{1}{p_m} + \frac{w}{p_m}\right)\text{leisure} = \frac{w\,\omega}{p_m}$$

$$\left(\frac{aw}{(1-a)p_m} + \frac{(1-a)w}{(1-a)p_m}\right)\text{leisure} = \frac{w\,\omega}{p_m}$$

$$\left[\frac{a}{(1-a)p_m} + \frac{1-a}{(1-a)p_m}\right]\text{leisure} = \frac{\omega}{p_m}$$

$$\frac{1}{(1-a)p_m}\,\text{leisure} = \frac{\omega}{p_m} \Rightarrow \frac{1}{1-a}\,\text{leisure} = \omega \Rightarrow$$

$\text{leisure}^* = (1-a)\,\omega$, the demand curve for leisure.

The corresponding supply curve for labor is

$\omega - \text{leisure} = \omega - (1-a)\,\omega = a\,\omega.$

Also, the demand for mangoes is

$$m = \frac{aw}{1-a}\,\frac{(1-a)\,\omega}{p_m} = a\omega\,\frac{\overset{=1}{w}}{p_m} = a\omega\,\frac{1}{1/3}$$

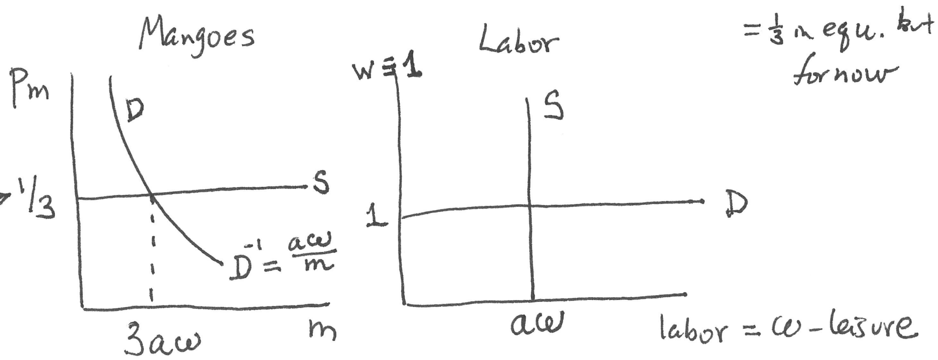
$= \frac{1}{3}$ in equ. but leave unspecified for now

From firm's problem → 1/3

$AC = \frac{w \cdot \text{labor}}{m}$

$= \frac{w(m/3)}{m}$

$= \frac{w}{3} = \frac{1}{3}.$



↳ either from the prod^n fnct^n or from $D_{\text{mangoes}} = a\omega\,\frac{1}{1/3} = 3a\omega.$

↑ (an abbreviation for "production function")

Fig. 18.2 $\downarrow$RS case.

Firm $\pi = p_m \, m(\text{labor}) - w \cdot \text{labor}$

max over labor $\Rightarrow$

$0 = p_m \, m' - w$

$\Rightarrow m' = w/p_m.$

$$\left.\begin{matrix} \pi = p_m \, m(\text{labor}) - w \cdot \text{labor} \\ \text{max over labor} \Rightarrow \\ 0 = p_m m' - w \\ \Rightarrow m' = w/p_m \end{matrix}\right\} \text{As in CRS case.}$$

Since $m'(\text{labor}) = w/p_m$, the demand for labor is $\text{labor} = (m')^{-1}(w/p_m)$, and the supply of mangoes is $m\left[(m')^{-1}(w/p_m)\right]$. Also,

$$\pi = p_m \, m\left[(m')^{-1}(w/p_m)\right] - w \cdot (m')^{-1}(w/p_m).$$

Consumer: Budget Constraint $\quad w(\omega - \text{leisure}) + \pi = p_m m$

$$\Leftrightarrow \frac{w}{p_m}\omega - \frac{w}{p_m}\text{leisure} + \frac{\pi}{p_m} = m \quad \rightarrow m = \frac{-w}{p_m}\text{leisure} + \frac{w\omega + \pi}{p_m}$$

$$\Leftrightarrow \frac{-w}{p_m}\text{leisure} + \frac{w}{p_m}\omega - \left(m - \frac{\pi}{p_m}\right) = 0$$

which is the same as the budget constraint in the CRS case except that "$m - \frac{\pi}{p_m}$" here replaces "m" there. One can check that the $0 = \partial\mathcal{L}/\partial m$ and $0 = \partial\mathcal{L}/\partial\,\text{leisure}$ F.O.C.'s are the same as in the CRS case. Substituting into the B.C.,

$$\frac{-w}{p_m}\text{leisure} + \frac{w}{p_m}\omega - \underbrace{\frac{a\,w}{1-a}\,\frac{\text{leisure}}{p_m}}_{\text{same as the CRS case's "m"}} + \frac{\pi}{p_m} = 0$$

$$\Rightarrow \frac{w\,\omega}{p_m} + \frac{\pi}{p_m} = \text{leisure}\left[\frac{w}{p_m} + \frac{1}{p_m}\frac{a w}{1-a}\right]$$

$$= \text{leisure}\left[\frac{w(1-a) + a w}{p_m(1-a)}\right] = \text{leisure}\ \frac{w}{p_m(1-a)}$$

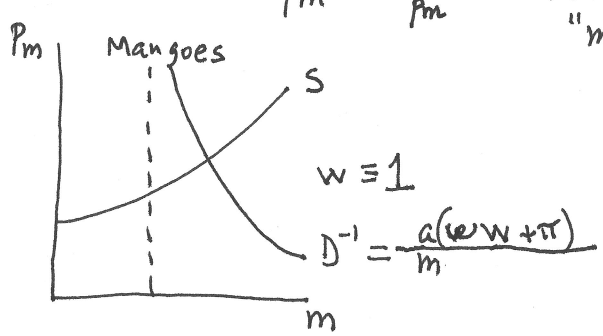
$$\frac{\omega}{p_m} + \frac{\pi}{w\,p_m} = \frac{\text{leisure}}{p_m(1-a)} \Rightarrow \text{leisure}^* = (1-a)\,\omega + \frac{(1-a)\cdot\pi}{w},$$

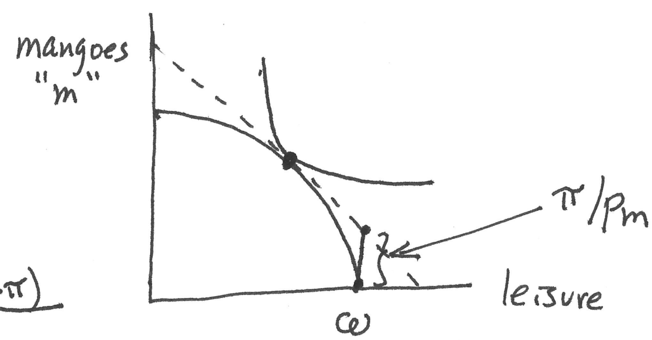
the <u>demand for leisure</u>. The corresponding <u>supply of labor</u> is

$\omega - \text{leisure} = a\omega - \frac{(1-a)\cdot\pi}{w}$. The <u>demand for mangoes</u> is

$$m = \frac{a w}{1-a}\frac{1}{p_m}\left[(1-a)\,\omega + \frac{(1-a)\cdot\pi}{w}\right]$$

$$= a w \frac{\omega}{p_m} + \frac{a\pi}{p_m}.$$





Note: If we take labor as the numeraire*, then equilibrium can be obtained by clearing either the mango market or the labor market.

* or mangoes as the numeraire,
or use a simplex,