

Ch 4

"lightning calculations": Friedman behave as if. Rats, wolves.

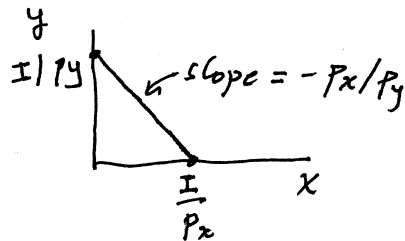
Altruism & selfishness: echoes of E3.4 p. 112, second-party preferences

§ 4.1

Fig. 4.1 Budget constraint $p_x x + p_y y \leq I$.

§ 4.2

With unsaturation: $= I$.



Replace with § 4.3

$$\begin{aligned} \textcircled{2} \quad & \mathcal{L} = u(x, y) + \lambda [I - p_x x - p_y y] \\ & 0 = \mathcal{L}'_{\lambda} = I - p_x x - p_y y \\ & 0 = \mathcal{L}'_x = u'_x - \lambda p_x \\ & 0 = \mathcal{L}'_y = u'_y - \lambda p_y \end{aligned} \quad \left. \vphantom{\begin{aligned} \textcircled{2} \quad & \mathcal{L} = u(x, y) + \lambda [I - p_x x - p_y y] \\ & 0 = \mathcal{L}'_{\lambda} = I - p_x x - p_y y \\ & 0 = \mathcal{L}'_x = u'_x - \lambda p_x \\ & 0 = \mathcal{L}'_y = u'_y - \lambda p_y \end{aligned}} \right\} \lambda = \frac{u'_x}{p_x} = \frac{u'_y}{p_y} \Rightarrow \frac{u'_x}{u'_y} = \frac{p_x}{p_y}$$

MRS of x for y $\nwarrow$ -slope

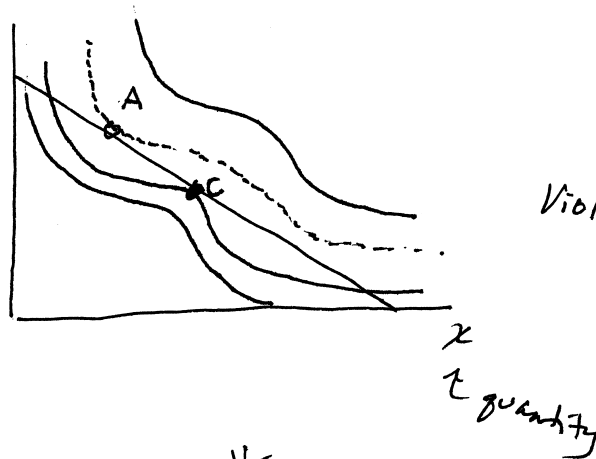


Find the utility-maximizing point.

§ 4.2, cont.

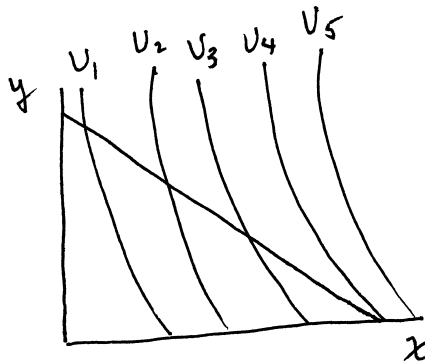
quantity y

Fig 4.3



Violates second-order conditions.

Fig. 4.4



Indifference curves are the contour lines of the utility function.

Quasiconcave functions have convex upper level sets.

Fig. 4.3 is not a depiction of the contour lines of a quasiconcave function.

But Fig 4.2 is.

Hence: S.O.C. are satisfied by quasiconcave functions.

§4.3

$$\max u(\tilde{x}) \text{ s.t. } \underset{\sim}{p} \cdot \underset{\sim}{x} \leq I$$

$\downarrow$

$$\sum_{i=1}^n p_i x_i$$

$$\mathcal{L} = u(x) + \lambda [I - p_1 x_1 - p_2 x_2 - \dots - p_n x_n]$$

$$0 = \mathcal{L}'_{\lambda} = I - p_1 x_1 - \dots - p_n x_n$$

$$0 = \mathcal{L}'_1 = u'_1 - \lambda p_1$$

$$0 = \mathcal{L}'_i = u'_i - \lambda p_i$$

$$0 = \mathcal{L}'_j = u'_j - \lambda p_j \quad \left. \vphantom{\begin{matrix} 0 = \mathcal{L}'_i \\ 0 = \mathcal{L}'_j \end{matrix}} \right\} \lambda = \frac{u'_i}{p_i} = \frac{u'_j}{p_j} \Rightarrow \frac{u'_i}{u'_j} = \frac{p_i}{p_j}$$

$\nearrow$
MRS of x_i for x_j

Also,

$$\frac{\partial u^*}{\partial I} = \frac{\partial \mathcal{L}^*}{\partial I} \text{ Envelope Thm.}$$
$$= \lambda.$$

Cornersolutions.

→ Ex. 4.1

→ Ex. 4.2

§ 4.4

$$v(p_1, \dots, p_n, I) = u(x_1^*(p_1, \dots, p_n, I), \\ x_2^*(p_1, \dots, p_n, I), \dots, \\ x_n^*(p_1, \dots, p_n, I))$$

the indirect utility function, which is a value function

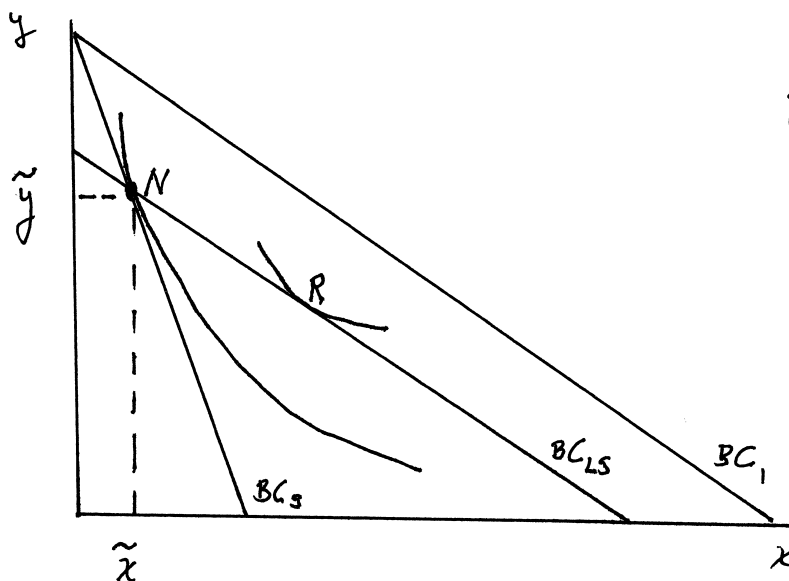
$$\text{Envelope Thm.: } \frac{\partial v}{\partial p_i} = \frac{\partial \mathcal{L}^*}{\partial p_i} = \frac{\partial}{\partial p_i} \left[u(x_1, \dots, x_n) + \lambda [I - \sum p_j x_j] \right] \\ = -\lambda x_i$$

$$\frac{\partial v}{\partial I} = \frac{\partial \mathcal{L}^*}{\partial I} = \frac{\partial}{\partial I} \left[u(x_1, \dots, x_n) + \lambda [I - \sum p_j x_j] \right] \\ = \lambda$$

$$\text{So } \lambda = \frac{-1}{x_i} \frac{\partial v}{\partial p_i} = \frac{\partial v}{\partial I} \text{ and}$$

$$x_i = - \frac{\partial v / \partial p_i}{\partial v / \partial I}, \text{ Roy's Identity.}$$

§4.5 The Lump-Sum Principle : specific taxes vs. lump-sum taxes



Draw: BC_1 ; BC_s ; $\tilde{x}$, $\tilde{y}$,
 N , and indiff. curve.
 Then do proof below.
 Then draw BC_{LS} and R .

BC_s is $p_x x + t x + p_y y = I$.

By assumption, $(\tilde{x}, \tilde{y})$ is on BC_s : $p_x \tilde{x} + t \tilde{x} + p_y \tilde{y} = I$. (1)

The Lump Sum Tax revenue is by assumption equal to the specific tax revenue, which is $t \tilde{x}$.

So $BC_{LS} = p_x x + p_y y = I - t \tilde{x}$.

CLAIM $\rightarrow$ Is $(\tilde{x}, \tilde{y})$ on BC_{LS} ? $\Leftrightarrow$ Is $p_x \tilde{x} + p_y \tilde{y} = I - t \tilde{x}$?

Yes because of (1). ■

Given this, the government is indifferent between N and R , while consumers prefer R .

- Are income taxes lump-sum? (Taxes on soft drinks; "sin taxes" on alcohol, gambling,)
- Is this generalizable?
 Th. Second Best (Taxes on pollution & other externalities.)

§4.6

min expenditures s.t. a utility target is reached

$$\min_{\underline{x}} p_1 x_1 + \dots + p_n x_n \text{ s.t. } u(x_1, \dots, x_n) = \bar{u}$$

$$\Rightarrow x_i(p_1, \dots, p_n, \bar{u}) \leftarrow \text{Hicksian, or compensated, demand curve}; \text{ and the value function } \left\{ \begin{array}{l} h(p, \bar{u}) \\ \tilde{h}(p, \bar{u}) \end{array} \right.$$

$$\Rightarrow p_1 x_1^* + \dots + p_n x_n^* = e(p_1, \dots, p_n, \bar{u})$$

minimized expenditures expenditure function.

Example 4.4: students should work. Note:

$$v(\underline{p}, I)$$

$$v(\underline{p}, e(\underline{p}, \bar{u})) = \bar{u}$$

$$\uparrow$$

$$v(\underline{p}, I)$$

$$\mathcal{L} = p_1 x_1 + \dots + p_n x_n$$

$$+ \lambda [\bar{u} - u(\underline{x})]$$

$$\frac{\partial e}{\partial p_i} = \frac{\partial \mathcal{L}^*}{\partial p_i} = x_i^* \text{ or } h_i(p, \bar{u})$$

§5.5.1 Shepherd's Lemma

End of section: compensating for a price change

$$u_0 = v(p_0, m_0)$$

$$E(\underline{p}', u') - E(\underline{p}', u_0) \text{ compensating variation, } \text{Kaldor}$$

$$u' = v(p', m')$$

$$E(\underline{p}', u') - E(p_0, u_0) \text{ equivalent variation, } \text{Hicks}$$

Contrary to §5.8.1.

Optional: These definitions are from Hal Varian's Ph.D. - level textbook.

If $m_0 = m'$ then $m_0 = E(p_0, u_0) = m' = E(p', u')$ and the above expression for CV is equal to $E(p_0, u_0) - E(p', u_0)$, which is -1 times the Econ. 6610 textbook's equation 5.57 in §5.8.1.

The -1 is not a mistake, just a different convention. In general, $m_0 \neq m'$, so Varian's definitions are better.

Confirms my definitions of CV and EV.

think of $(\mathbf{p}^0, m^0)$ as being the status quo and $(\mathbf{p}', m')$ as being a proposed change, although this is not the only interpretation.

Then the obvious measure of the welfare change involved in moving from $(\mathbf{p}^0, m^0)$ to $(\mathbf{p}', m')$ is just the difference in indirect utility:

$$v(\mathbf{p}', m') - v(\mathbf{p}^0, m^0).$$

If this utility difference is positive, then the policy change is worth doing, at least as far as this consumer is concerned; and if it is negative, the policy change is not worth doing.

This is about the best we can do in general; utility theory is purely ordinal in nature and there is no unambiguously right way to quantify utility changes. However, for some purposes it is convenient to have monetary measure of changes in consumer welfare. Perhaps the policy analyst wants to have some rough idea of the magnitude of the welfare change for purposes of establishing priorities. Or perhaps the policy analyst wants to compare the benefits and costs accruing to different consumers. In circumstances such as these, it is convenient to choose a "standard" measure of utility differences. A reasonable measure to adopt is the (indirect) money metric utility function described in Chapter 7, page 109.

Recall that $\mu(\mathbf{q}; \mathbf{p}, m)$ measures how much income the consumer would need at prices $\mathbf{q}$ to be as well off as he or she would be facing prices $\mathbf{p}$ and having income m . That is, $\mu(\mathbf{q}; \mathbf{p}, m)$ is defined to be $e(\mathbf{q}, v(\mathbf{p}, m))$. If we adopt this measure of utility, we find that the above utility difference becomes

$$\mu(\mathbf{q}; \mathbf{p}', m') - \mu(\mathbf{q}; \mathbf{p}^0, m^0).$$

It remains to choose the base prices $\mathbf{q}$. There are two obvious choices: we may set $\mathbf{q}$ equal to $\mathbf{p}^0$ or to $\mathbf{p}'$. This leads to the following two measures for the utility difference:

$$\begin{aligned} EV &= \mu(\mathbf{p}^0; \mathbf{p}', m') - \mu(\mathbf{p}^0; \mathbf{p}^0, m^0) = \mu(\mathbf{p}^0; \mathbf{p}', m') - m^0 \\ CV &= \mu(\mathbf{p}'; \mathbf{p}', m') - \mu(\mathbf{p}'; \mathbf{p}^0, m^0) = m' - \mu(\mathbf{p}'; \mathbf{p}^0, m^0). \end{aligned} \quad (10.1)$$

The first measure is known as the **equivalent variation**. It uses the current prices as the base and asks what income change at the current prices would be equivalent to the proposed change in terms of its impact on utility. The second measure is known as the **compensating variation**. It uses the new prices as the base and asks what income change would be necessary to compensate the consumer for the price change. (Compensation takes place after some change, so the compensating variation uses the after-change prices.)

Both of these numbers are reasonable measures of the welfare effect of a price change. Their magnitudes will generally differ because the value of a dollar will depend on what the relevant prices are. However, their sign will

A money metric indirect utility function can be constructed in this manner for any price vector $\bar{p} \gg 0$. Two particularly natural choices for the price vector $\bar{p}$ are the initial price vector p^0 and the new price vector p^1 . These choices lead to two well-known measures of welfare change originating in Hicks (1939), the *equivalent variation* (EV) and the *compensating variation* (CV). Formally, letting $u^0 = v(p^0, w)$ and $u^1 = v(p^1, w)$, and noting that $e(p^0, u^0) = e(p^1, u^1) = w$, we define

$$EV(p^0, p^1, w) = e(p^0, u^1) - e(p^0, u^0) = e(p^0, u^1) - w \quad (3.I.1)$$

and

$$CV(p^0, p^1, w) = e(p^1, u^1) - e(p^1, u^0) = w - e(p^1, u^0). \quad (3.I.2)$$

The equivalent variation can be thought of as the dollar amount that the consumer would be indifferent about accepting in lieu of the price change; that is, it is the change in her wealth that would be *equivalent* to the price change in terms of its welfare impact (so it is negative if the price change would make the consumer worse off). In particular, note that $e(p^0, u^1)$ is the wealth level at which the consumer achieves exactly utility level u^1 , the level generated by the price change, at prices p^0 . Hence, $e(p^0, u^1) - w$ is the net change in wealth that causes the consumer to get utility level u^1 at prices p^0 . We can also express the equivalent variation using the indirect utility function $v(\cdot, \cdot)$ in the following way: $v(p^0, w + EV) = u^1$.²²

The compensating variation, on the other hand, measures the net revenue of a planner who must *compensate* the consumer for the price change after it occurs, bringing her back to her original utility level u^0 . (Hence, the compensating variation is negative if the planner would have to pay the consumer a positive level of compensation because the price change makes her worse off.) It can be thought of as the negative of the amount that the consumer would be just willing to accept from the planner to allow the price change to happen. The compensating variation can also be expressed in the following way: $v(p^1, w - CV) = u^0$.

Figure 3.I.2 depicts the equivalent and compensating variation measures of welfare change. Because both the EV and the CV correspond to measurements of the changes in a money metric indirect utility function, both provide a correct welfare ranking of the alternatives p^0 and p^1 ; that is, the consumer is better off under p^1 if and only if these measures are positive. In general, however, the specific dollar

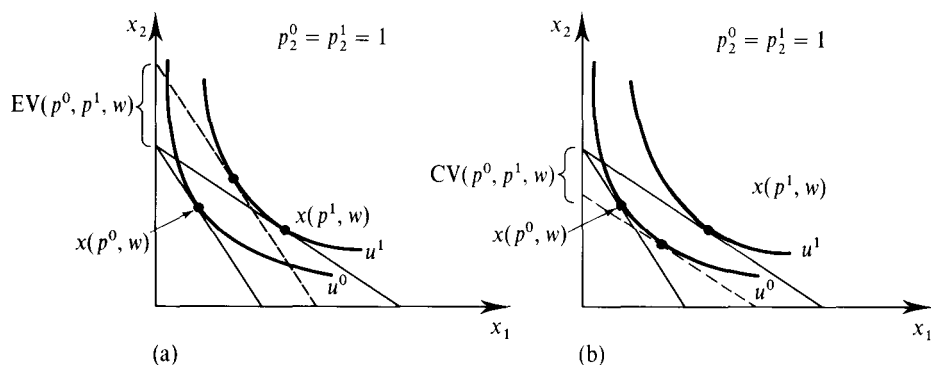


Figure 3.I.2

The equivalent (a) and compensating (b) variation measures of welfare change.

22. Note that if $u^1 = v(p^0, w + EV)$, then $e(p^0, u^1) = e(p^0, v(p^0, w + EV)) = w + EV$. This leads to (3.I.1).

§4.7

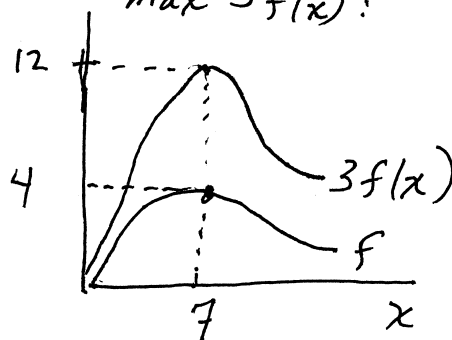
$$e(\underset{\sim}{p}, \bar{u}) = \min_{\underset{\sim}{x}} \underset{\sim}{p} \cdot \underset{\sim}{x} \text{ s.t. } u(\underset{\sim}{x}) = \bar{u}$$

$$e(\lambda \underset{\sim}{p}, \bar{u}) = \min_{\underset{\sim}{x}} \lambda \underset{\sim}{p} \cdot \underset{\sim}{x} \text{ s.t. } u(\underset{\sim}{x}) = \bar{u} \text{ . Claim:}$$

$$= \lambda \min_{\underset{\sim}{x}} \underset{\sim}{p} \cdot \underset{\sim}{x} \text{ s.t. } u(\underset{\sim}{x}) = \bar{u}$$

$$= \lambda' e(\underset{\sim}{p}, \bar{u}) \text{ homogeneity of degree one.}$$

Reason: compare $\max f(x)$ with $\max 3f(x)$:

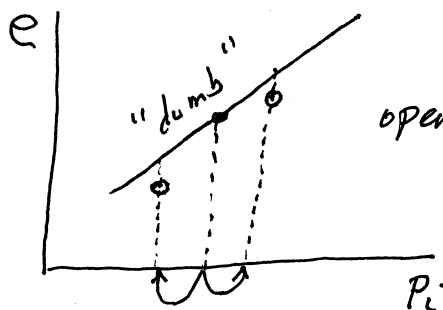


$$\frac{\partial e}{\partial p_i} = \frac{\partial x^*}{\partial p_i} = \frac{\partial}{\partial p_i} \left\{ \underset{\sim}{p} \cdot \underset{\sim}{x} + \lambda [\bar{u} - u(\underset{\sim}{x})] \right\}$$

$= x_i(\underset{\sim}{p}, \bar{u})$, the Hicksian demand curve for x_i .

≥ 0 (6 intuition)

$e(\underset{\sim}{p}, \bar{u})$ is concave in $\underset{\sim}{p}$. Argument in one dimension:



open circles: "smart"

$$\partial^2 e / \partial p_i^2 < 0 \therefore$$

$$0 > \frac{\partial}{\partial p_i} \frac{\partial e}{\partial p_i} = \frac{\partial}{\partial p_i} h_i$$

Hicksian D curves are downward sloping. §5.5.1

Ch. 4 problems

4.1

4.2 a, b; do c after Ch. 5

4.3

4.4 maximize U^2

4.5 a

4.6 read only

4.10 a, b

4.11 }
4.12 } read only
4.13 }

4.14 will be discussed in class (Ch. 3 extensions E3.4)

Ch 4 extensions

E 4.1 variability of budget shares

E 4.2 Stone-Geary utility function

E 4.3 CES utility & NAFTA, computable general equilibrium
(markets always clear)

E 4.4 AIDS. read after (ix)