The Ellsberg paradox

The Ellsberg paradox concerns subjective probability theory. You are told that an urn contains 300 balls. One hundred of the balls are red and 200 are either blue or green.

Gamble A. You receive \$1,000 if the ball is red.

Gamble B. You receive \$1,000 if the ball is blue.

Write down which of these two gambles you prefer. Now consider the following two gambles:

Gamble C. You receive \$1,000 if the ball is not red.

Gamble D. You receive \$1,000 if the ball is not blue.

It is common for people to strictly prefer A to B and C to D. But these preferences violate standard subjective probability theory. To see why, let R be the event that the ball is red, and $\neg R$ be the event that the ball is not red, and define B and $\neg B$ accordingly. By ordinary rules of probability,

$$\begin{aligned} p(R) &= 1 - p(\neg R) \\ p(B) &= 1 - p(\neg B). \end{aligned} \tag{11.12}$$

Normalize $u(0) = 0$ for convenience. Then if A is preferred to B, we must have $p(R)u(1000) > p(B)u(1000)$, from which it follows that

$$p(R) > p(B). \tag{11.13}$$

If C is preferred to D, we must have $p(\neg R)u(1000) > p(\neg B)u(1000)$, from which it follows that

$$p(\neg R) > p(\neg B). \tag{11.14}$$

However, it is clear that expressions (11.12), (11.13), and (11.14) are inconsistent.

The Ellsberg paradox seems to be due to the fact that people think that betting for or against R is "safer" than betting for or against "blue."

Opinions differ about the importance of the Allais paradox and the Ellsberg paradox. Some economists think that these anomalies require new models to describe people's behavior. Others think that these paradoxes are akin to "optical illusions." Even though people are poor at judging distances under some circumstances doesn't mean that we need to invent a new concept of distance.