B. The Theory of Choice

1. Figure 4 shows a recent ranking of some large US cities by their Crime Rate and the number of Artistic Events held there.

   (a) If a person likes artistic events—in other words, the more artistic events there are in a city the more he likes the city—but he does not care about different crime rates, what would his indifference curves look like on the graph? Which indifference curves represent higher and which represent lower utility? (Do not put your answer on the graph itself because I would like you to put the answer to part (c) on the graph.)

   (b) If a person dislikes crime—in other words, the higher the crime rate is in a city the more he dislikes the city—but does not care about artistic events, what would his indifference curves look like on the graph? Which indifference curves represent higher and which represent lower utility? (Do not put your answer on the graph itself because I would like you to put the answer to part (c) on the graph.)

   (c) Suppose a person likes artistic events but dislikes high crime rates. What would his indifference curves look like? Which indifference curves represent higher and which represent lower utility? You may draw them on Figure 4. Particularly explain the meaning of the concavity, convexity, or lack thereof of the curves which you draw.

2. Suppose there are only two goods in an economy, and call the two goods "lima beans" and "pinto beans." Increases in the consumption of lima beans lead to increases in utility until some point, but after that, increases in the consumption of lima beans lead to decreases in utility. The same is true of pinto beans.

   (a) Sketch a few of the consumer's indifference curves, and indicate the bliss point. Explain why the indifference curves are shaped as they are (but to save time, you do not have to thoroughly explain each unusual part of an indifference curve—just explain one of the unusual parts).

   (b) Graphically show that if the consumer's income is small, he will spend all his income. Also show that if the consumer's income is large, he will not spend all his income.

3. Mr. A, rather than having diminishing Marginal Rate of Substitution of X for Y, has constant Marginal Rate of Substitution of X for Y. X and Y are the only two goods.

   (a) Sketch three or four of Mr. A's indifference curves, and indicate, for your sketch, what the numerical value of the Marginal Rate of Substitution of X for Y is.

   (b) Show, on the graph you made in part (a), a situation (that is, a budget constraint) leading Mr. A to buy only good Y. Is p_X/p_Y less than, equal to, or greater than Mr. A's MRS of X for Y in the situation you have illustrated?

4. Mrs. C gets utility from consuming good X and good Y. The price of good X is \$2 and the price of good Y is \$6. Mrs. C's income is \$12.

   (a) Graph Mrs. C's budget constraint.

   (b) If Mrs. C's utility function is $U(X,Y) = X + 4Y$, how much X and Y will she buy?

   (c) How will your answer change if $U(X,Y) = X + 3Y$?

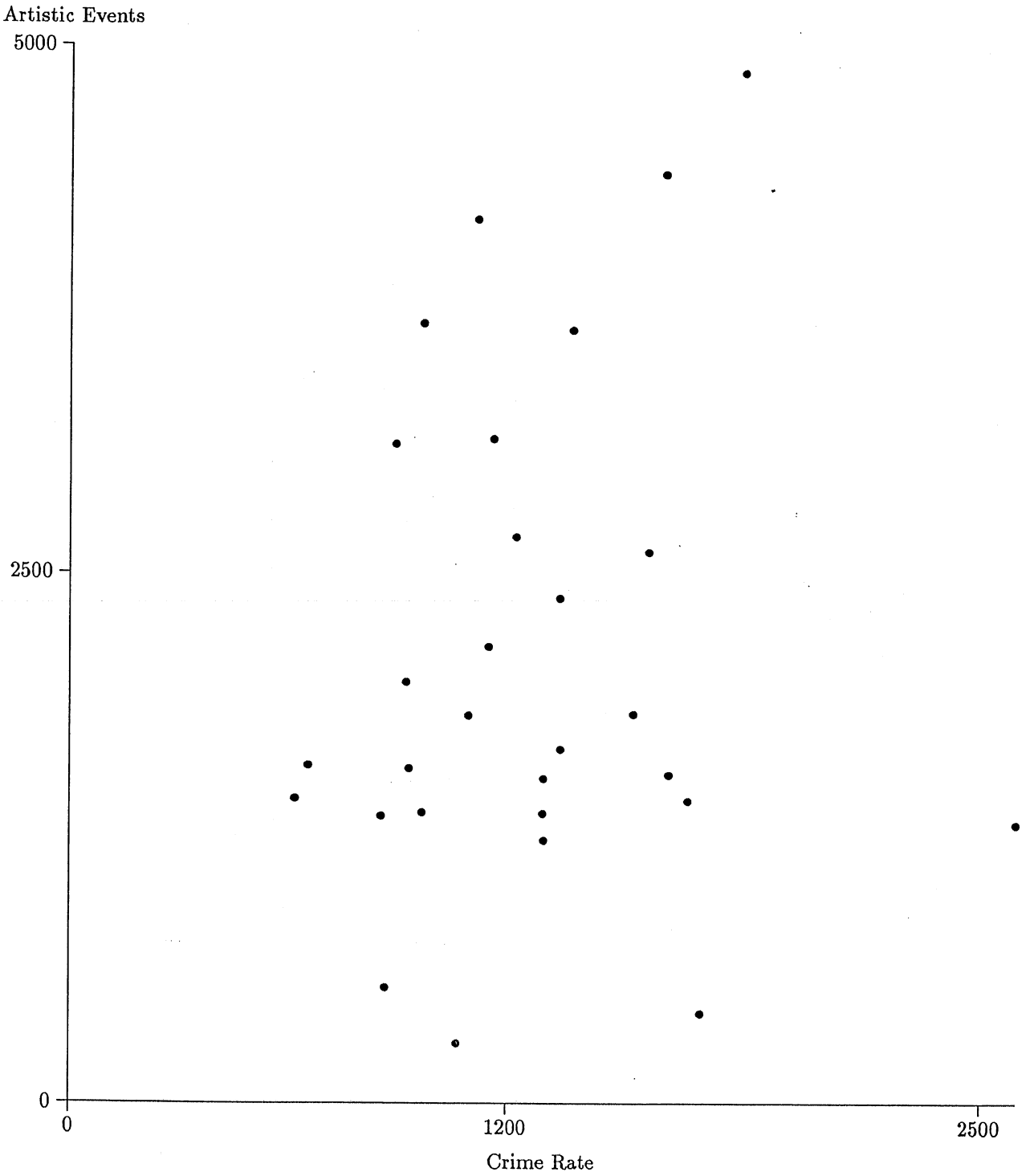
5. Mr. B enjoys fish (F) and chips (C) according to the function: $U(F,C) = F + C$.

   (a) The price of fish is \$2 and the price of chips is \$1. If Mr. B's income is \$2, how much fish and how many chips will he buy? Draw a graph of Mr. B's indifference curves and budget constraint in this situation and show how to use the graph to answer the question.

   (b) Is Mr. B's marginal rate of substitution of fish for chips decreasing? Why do we ordinarily have diminishing marginal rate of substitution?

   (c) Now suppose the price of fish drops to \$1. Will Mr. B be better off? How much F and C will he buy? What does the diagram now look like?

6. Mr. C has the following utility function: $U(X,Y) = 2X + Y$. Some of his indifference curves are drawn in Figure 1. Suppose the price of good Y is \$1. For what values of the price of good X will Mr. C decide to spend all of his income buying X and, therefore, buy no Y at all? (Of course, as always it is very important that you explain your answer thoroughly.)



Question 1's Figure 4

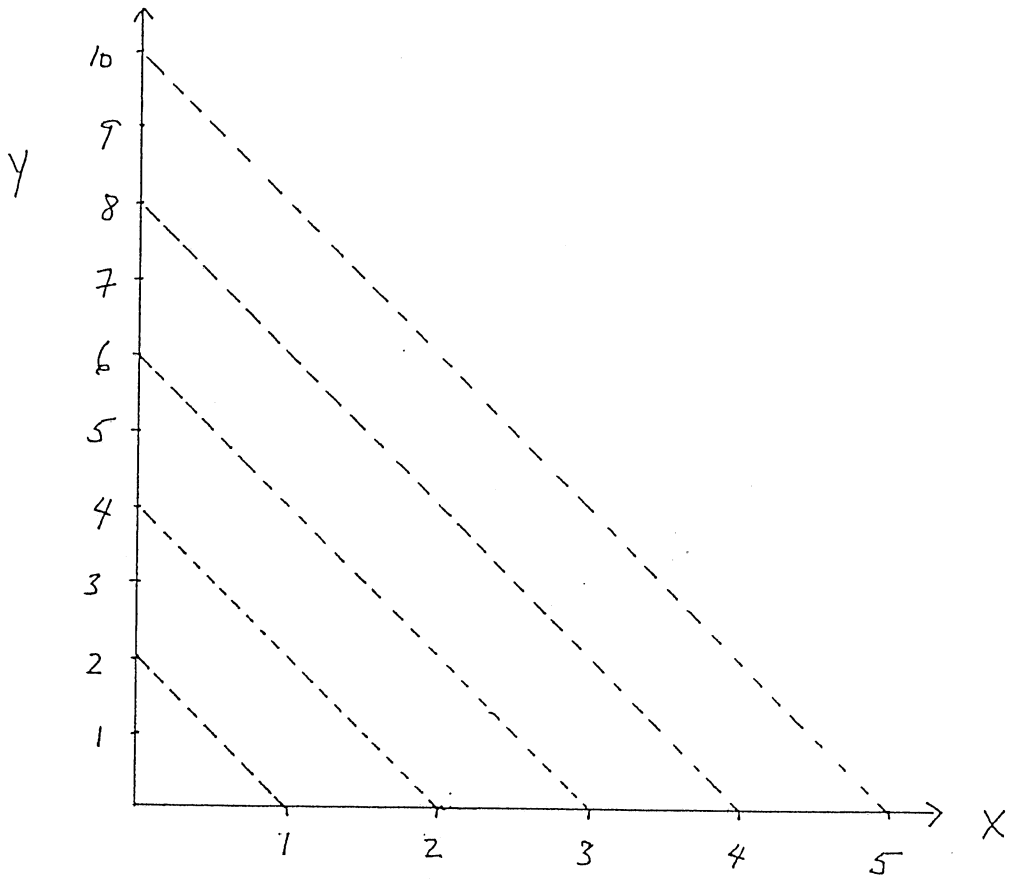
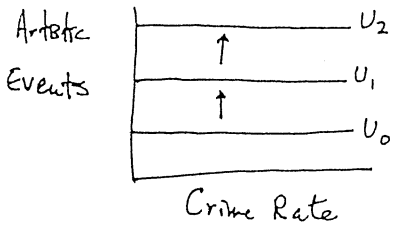


Figure 1

Question 6's Fig. 1

Answers

(1) a)

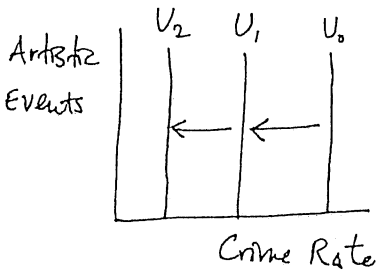


7 pts →

The indifference curves are horizontal because motion in the horizontal direction does not change utility, and if utility doesn't change then you are on the same indifference curve.

U_0 represents the lowest utility and U_2 is the highest because more artistic events are better than fewer.

b)

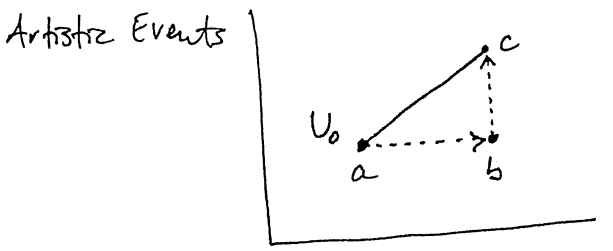


7 pts →

The indifference curves are vertical because motion in the vertical direction does not affect utility.

Since lower crime rates are better than higher crime rates, U_2 is the highest level of utility and U_0 is the lowest among the three indifference curves drawn.

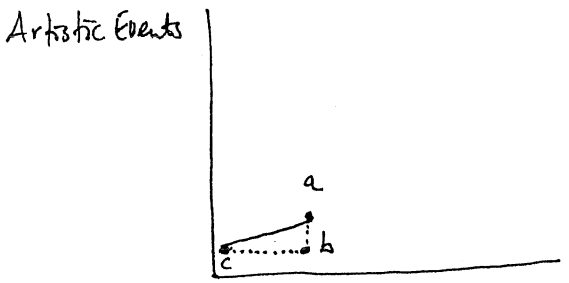
c)



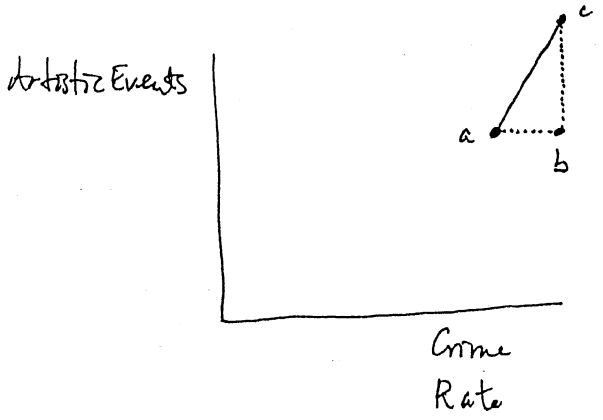
Start out at point a with level of utility U_0. Try to draw the U_0 indifference curve. If you went from a to b,

13 pts

utility would fall below U_0 because there's more crime. To get back up to the U_0 utility level, you have to add artistic events (from b to c). U_0 may be reachieved at c. So a and c lie on the same, U_0 indifference curve. Hence indifference curves are upward sloping in this example.



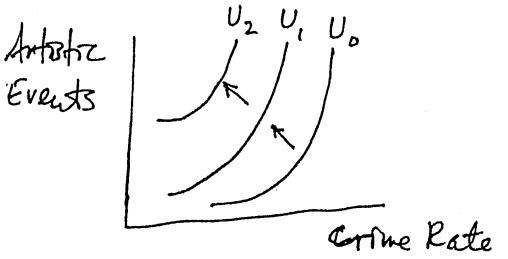
Start out at a, with very few artistic events but a very safe city. Now remove some of the few artistic events which there were. If a mix of artistic events and safety is preferred to having much of one and little of the other, then the compensation for the a → b move would have to be a large increase in safety (b → c). So in the lower left-hand corner of the graph, the indifference curves are rather flat.



Now begin in the upper right-hand corner of the graph, with many artistic events but little safety (point a). Move to point b, which is even more dangerous. The compensation (if "mixing" is preferred) would have to be a very large

increase in artistic events (to c). So indifference curves in the upper right-hand corner are rather steep.

Hence the indifference curves are upward sloping and convex:



6 pts for convexity or concavity plus discussion

As you go from U_0 to U_2, crime falls and artistic events rise, so utility goes up.

OPTIONAL: For your information, Figures 5 and 6 give the city names, as reported in the Places Rated Almanac (Prentice-Hall 1989) by Richard Boyer and David Savageau. The "Artistic Events" measure is a composite including measures of museum size and quality, public radio and TV stations, and opera, dance, and classical music performances.* The "Crime Rate" is the rate of violent crime plus $\frac{1}{10}$ the rate of property crimes. By "rate" is meant $\frac{\text{number of crimes}}{\text{size of permanent population}}$. Cities in Florida may have misleadingly high crime rates because the size of their permanent population is much less than the size of their total population.

The cities I chose to graph are the biggest US cities plus some others I was interested in. The book ranks 333 cities in all.

What would your indifference curves look like?

* But excluding "CMSA access."

Figure 1. Notes: (1) Oakland lies between Houston and Detroit. (2) Newark lies between Phoenix and Detroit. (3) Salt Lake City has almost the same position as Anaheim, Louisville, Raleigh-Durham, and San Jose. (4) Albany has almost the same position as Nassau-Suffolk Counties (NY).

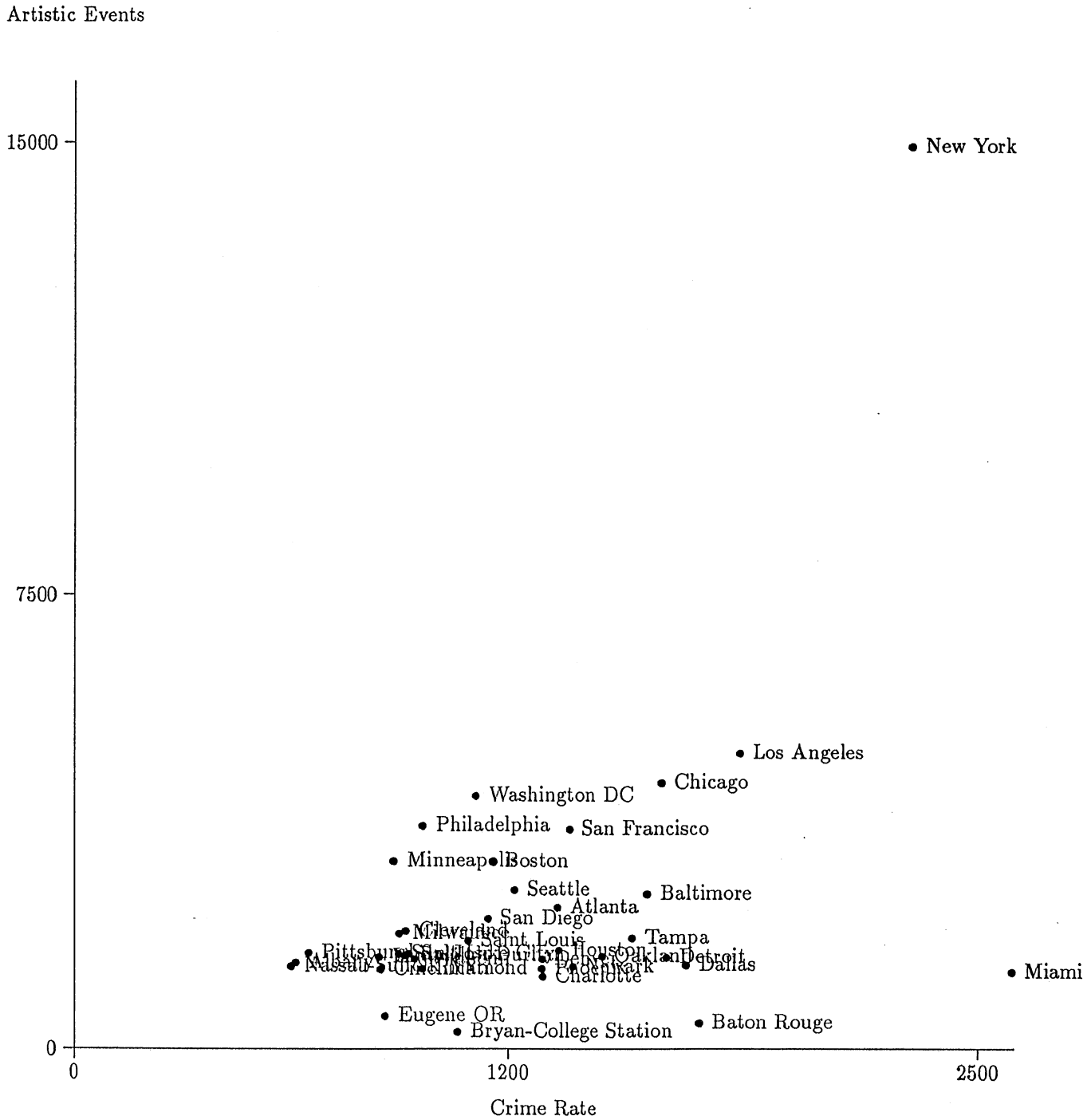
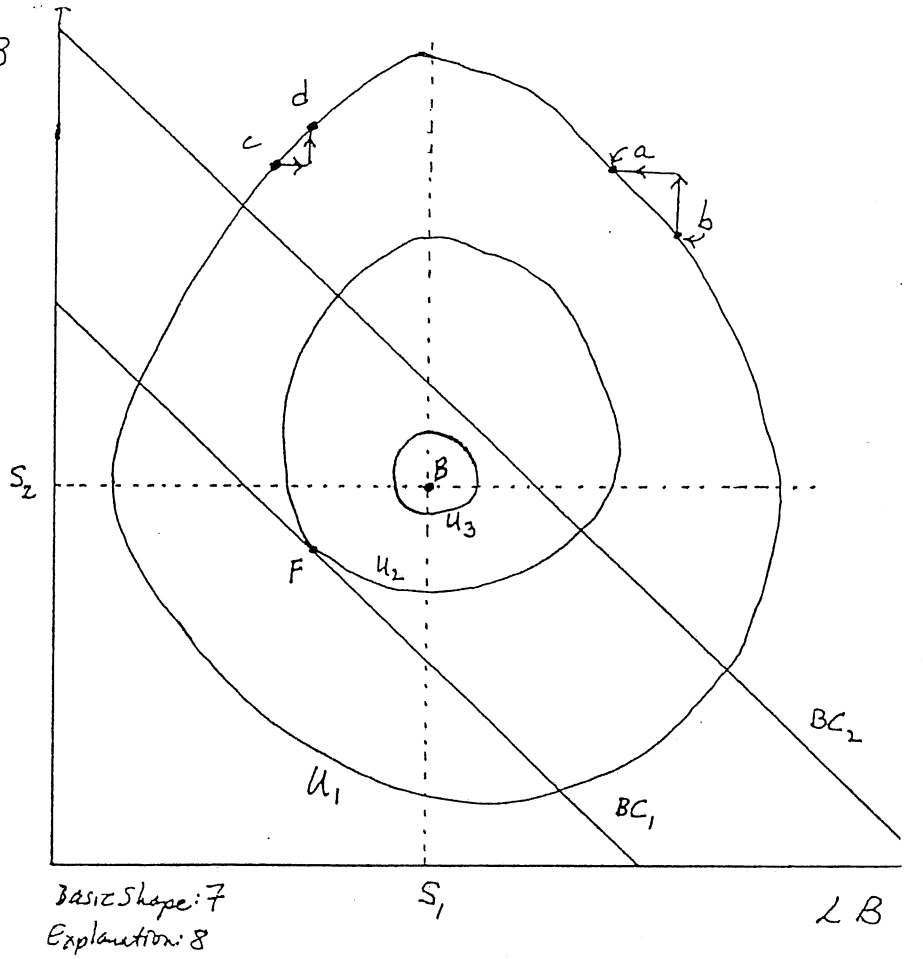

Artistic Events
15000
7500
0
0
1200
2500
Crime Rate
New York
Los Angeles
Chicago
Washington DC
Philadelphia
San Francisco
Minneapolis
Boston
Seattle
Baltimore
Atlanta
San Diego
Tampa
Dallas
Miami
Eugene OR
Bryan-College Station
Baton Rouge

Figure 6



Basic Shape: 7
Explanation: 8

② a) The consumer's indifference curves are like circles. (To be more precise, they are closed curves with positive slope if pinto beans $PB > S_2$ and lima beans $LB < S_1$, or if $PB < S_2$ and $LB > S_1$; otherwise, the slope is negative.) S_1 is the amount of LB beyond which more LB lowers utility; S_2 is the corresponding amount of PB. For $LB < S_1$ and $PB < S_2$, everything is "normal": downward sloping convex indifference curves. To explain the slope between points a and b, consider starting at b and giving the consumer more PB. This makes him worse off. In order to compensate him, LB must be taken away (since $LB > S_1$)

(since he's got too many PB: b lies above S_2)

This results in a movement above and to the left (more PB, less LB). Or consider starting at point c and increasing LB. Since $LB < S_1$, the consumer is not satiated in LB, so this move to the right makes him better off. In order to bring down his utility to the original level U_1, more PB must be given to him (since more make him worse off: c is above S_2). This results in a movement to the right and up.

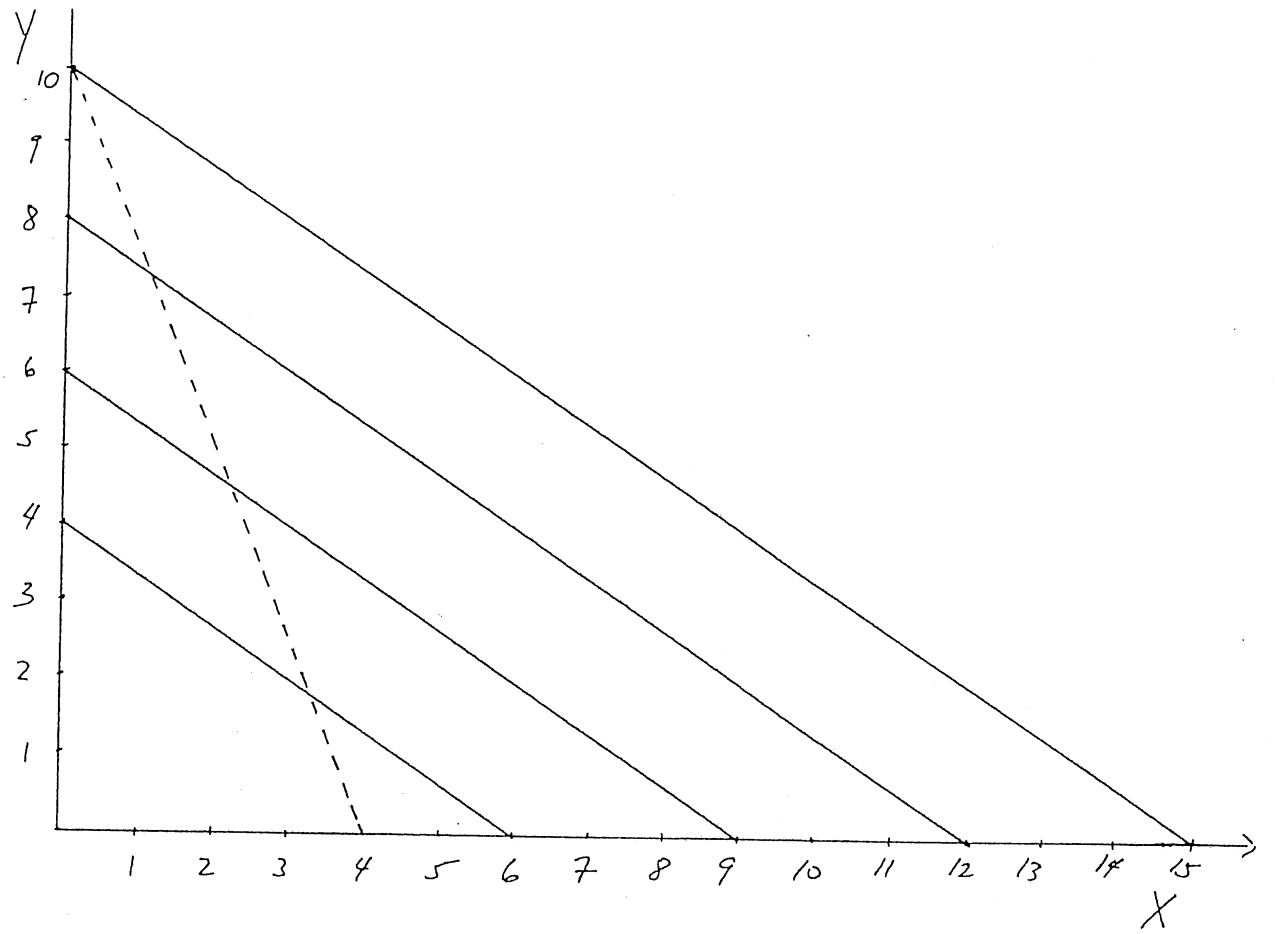
over →

Utility is increasing from U_1 to U_2 to U_3. Maximum utility is at $\angle B = S_1$, $PB = S_2$; this is the bliss point B. ← 5 pts

b) If the consumer's income is small, his budget constraint would look like BC_1. In this case the optimal consumption bundle is at point F, which is on the budget constraint. Hence all the income is used. 5 pts ↑ 8 pts ↓

If the consumer's income is large, his budget constraint would look like BC_2. In this case, the bliss point B is affordable, so it is optimal ("B" makes the consumer the happiest he can be). Yet the consumer could afford to buy more $\angle B$ and PB than he is buying at B (since B is not on the budget constraint). So the consumer is not spending all his income.

(3)



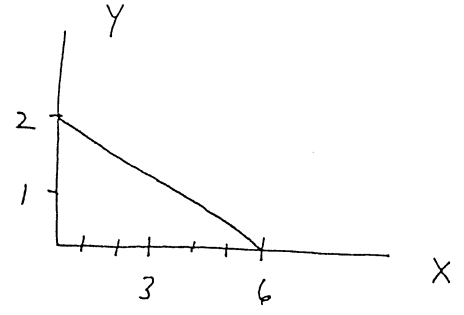
a) A constant MRS ⇒ linear indifference curves. I chose some with a slope of $-2/3$. So the MRS of X for Y in this graph is $+\frac{2}{3}$. (10 pts)

b) If the budget constraint is like the dotted line, then the consumer will buy all Y and no X. In this case the budget constraint is steeper than the indifference curves: $-\frac{P_X}{P_Y}$ = slope of B.C. < slope of indifference curves = $-\frac{2}{3}$

$\Rightarrow \frac{P_X}{P_Y} > \frac{2}{3}$. This makes sense: if the price of X relative to Y is too great, this consumer will only buy Y. Since here MRS of X for Y = $\frac{2}{3}$, we have $\frac{P_X}{P_Y}$ > MRS of X for Y. (9 points)

14 pts

(4) a.



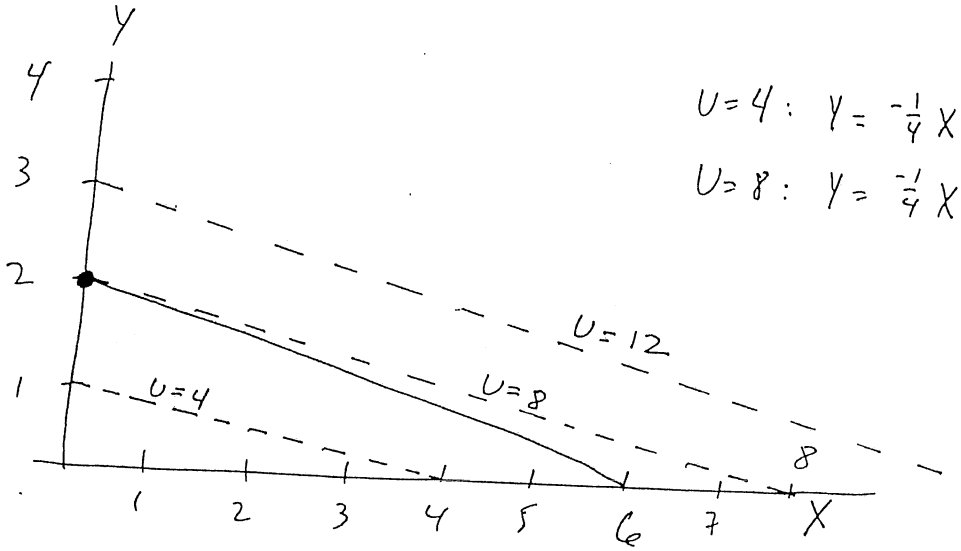
(10 points)

maximum X purchasable is $\frac{\$12}{\$2} = 6$

" Y " " $\frac{\$12}{\$6} = 2$

hence the line from (6,0) to (0,2).

(8 points) b. If $U = X + 4Y$ then $4Y = U - X \Rightarrow Y = \frac{-1}{4}X + \frac{1}{4}U$. So, indifference curves look like these dotted lines:



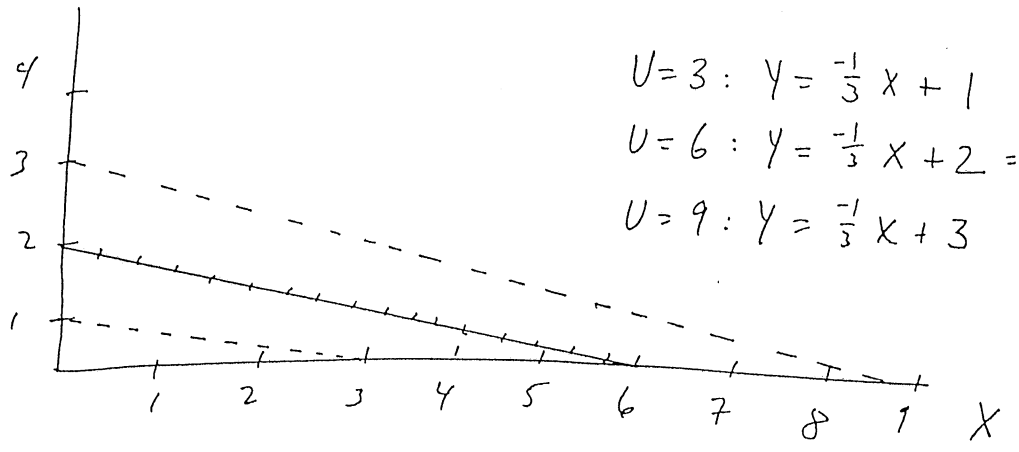
$U=4$: $Y = \frac{-1}{4}X + 1$

$U=8$: $Y = \frac{-1}{4}X + 2$

Mrs. C buys $(X=0, Y=2)$. That gives her more utility ($U=8$) than any other point, since $U=8$ is above any affordable point except for one.

c. Now $U = X + 3Y \Rightarrow Y = \frac{-1}{3}X + \frac{1}{3}U$.

(7 points)



$U=3$: $Y = \frac{-1}{3}X + 1$

$U=6$: $Y = \frac{-1}{3}X + 2$ = budget constraint

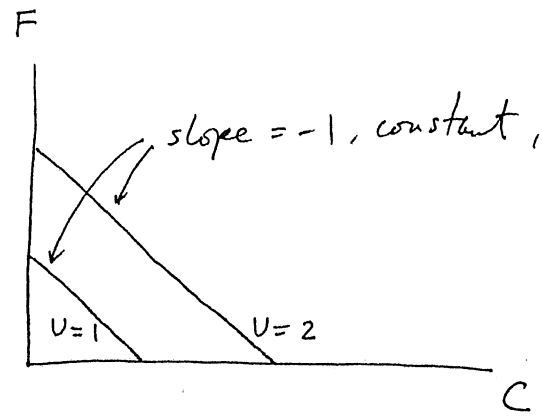
$U=9$: $Y = \frac{-1}{3}X + 3$

Mrs. C. is indifferent about where on her budget constraint $Y = \frac{-1}{3}X + 2$ to be.

(5) $U(F, C) = F + C$

a) Indifference Curves.

(4)



$U=1$: $1 = F + C \Rightarrow F = 1 - C$

$U=2$: $2 = F + C \Rightarrow F = 2 - C$

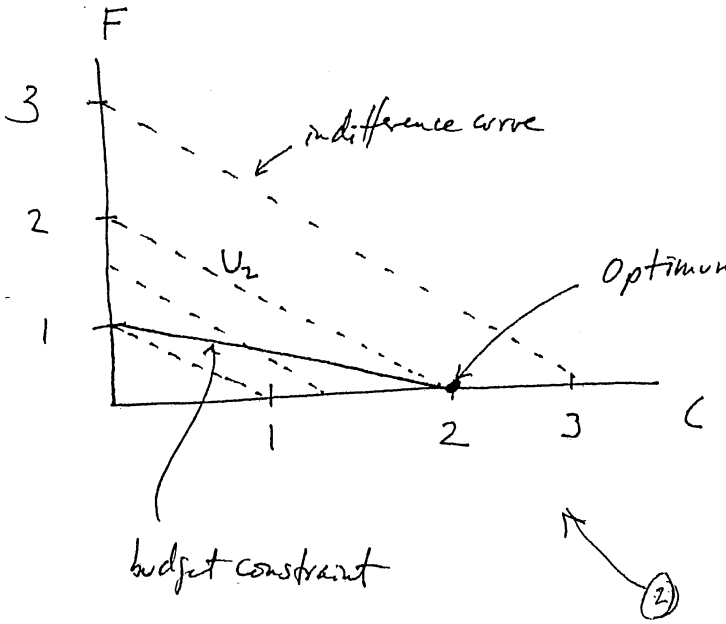
Usually, ↓ MRS because as you get less and less of a good you become less willing to substitute other goods for it.

$$MRS \text{ of } X \text{ for } Y = -\left.\frac{\Delta Y}{\Delta X}\right|_U$$

As $x \uparrow$, need less and less ΔY for any ΔX : ↓ MRS.

(3)

b)

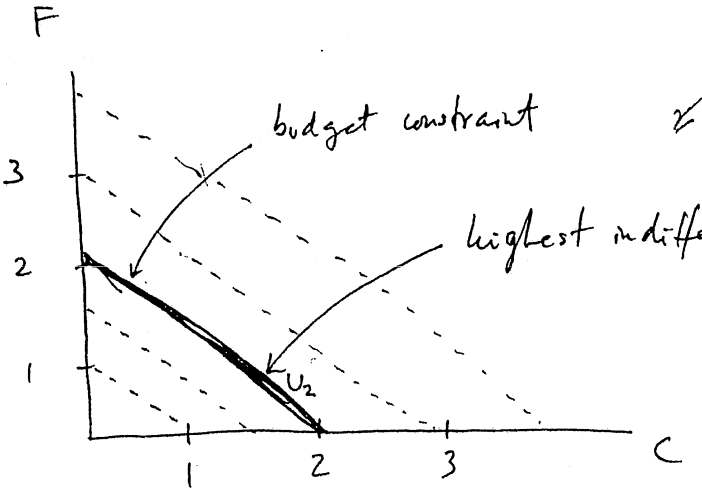


Optimum: 2 chips, no fish (2)

[Explain why this is optimal.] (2)

(2)

c)



(3)

You don't know how much F and C he'll buy; it'll be somewhere on the budget line. He is not better off than before since he's still on U_2. (4)

⑥ The indifference curves have slope -2. If the budget constraint had slope < -2, the budget constraint would be steeper than the indifference curves (e.g. line ①). Mr. C would buy all Y. If the budget constraint had slope > -2, it would be flatter than the indifference curves (e.g. line ②). Mr. C would buy all X. This is what we want. So the slope of the budget constraint, which is $-\frac{P_X}{P_Y}$, should be > -2. Hence

$$\frac{P_X}{P_Y} < 2$$

$$P_X < 2P_Y = 2(1) = 2. \quad \text{So } \boxed{P_X < 2}.$$

⑤ budget constraint
⑤ slope of budget constraint $= -P_X/P_Y$
⑩ explain optimal choice
⑤ get right answer

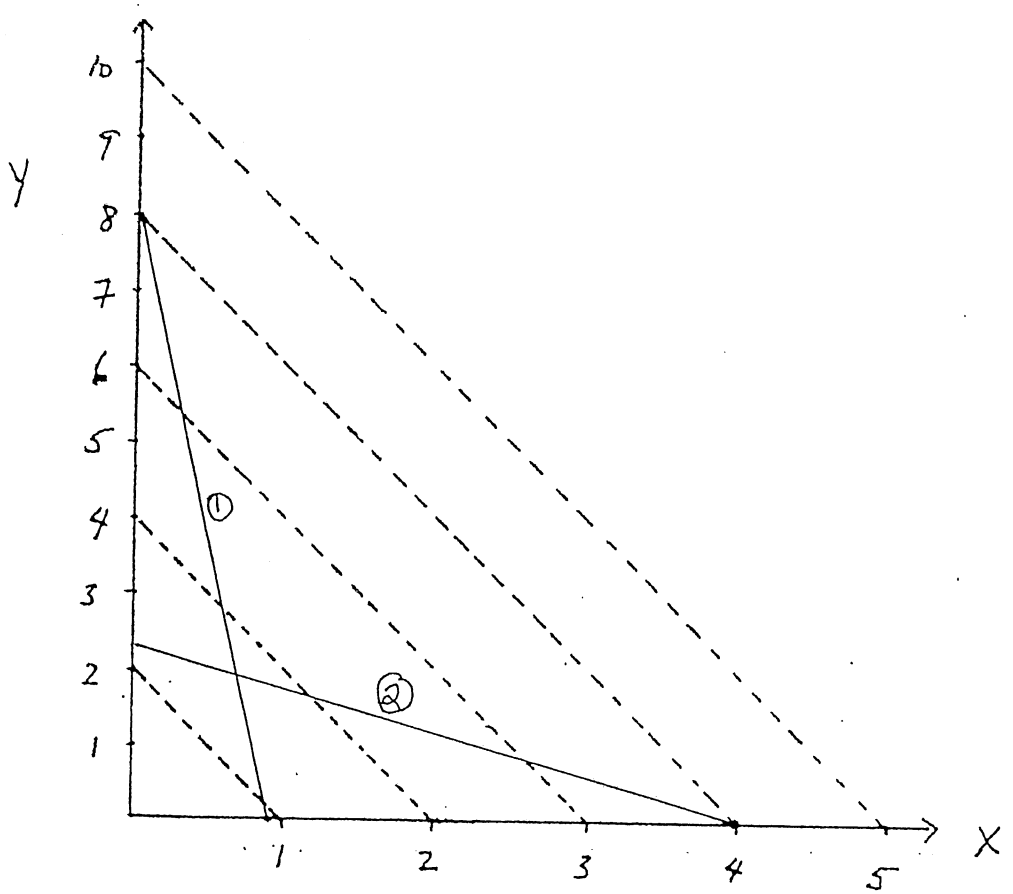


Figure 1