E. Price Indexes and the Edgeworth Box

1. Consider the following chart:

	1972	1986
P_x	1	8
Q_x	2	7
P_y	3	6
Q_y	4	5

   (a) Given this chart, has real income risen between 1972 and 1986? Use the Laspeyres Index.

   (b) Given this chart, has the price level risen between 1972 and 1986? Use base year quantities.

   Show all your work.

2. Based on our study of index numbers, give two estimates of the rate of inflation between 1972 and 1987 if the prices and quantities of goods X and Y were as follows.

	1972	1987
P_x	\$1	\$5
Q_x	3	6
P_y	\$3	\$7
Q_y	4	8

3. Consider the following chart.

	1972	1987
Q_x	5	4
P_x	2	9
Q_y	1	10
P_y	10	3

   (a) Given this chart, show that the Laspeyres Price Index is exactly 2.4.

   (b) Given this chart, show that the Paasche Price Index is $33/54 \approx 0.61$.

   (c) Has the price level risen or fallen between 1972 and 1987? (As always, explain your answer thoroughly.)

4. (a) Draw an Edgeworth Box showing everything you know about the indifference curves of two agents Smith and Jones if the contract curve is a straight line.

   (b) Re-draw the graph from part (a). Mark on the graph a point A which is *not* on the contract curve. What price ratio will lead Smith and Jones, starting at A, to demand commodity bundles which are on the contract curve even if they cannot talk to each other? Your answer should be: "the correct price ratio is the slope of (such-and-such a) line which I drew on the graph." Explain briefly.

   (c) Definition: An "allocation" is a list of: Smith gets a certain amount of X and a certain amount of Y, and at the same time Jones gets a certain amount of X and a certain amount of Y. Graphically, an allocation is just a single point in an Edgeworth Box.

   Definition: A price ratio is called "efficient" if it results in a Pareto Optimal allocation.

   Pick two or three other initial allocations besides A and find what the efficient price ratios are for those points. Are the efficient price ratios flatter, steeper, or the same as the efficient price ratio for point A?

   (d) Now suppose the contract curve is *not* a straight line. Pick two initial allocations A_1 and A_2 not on the contract curve; pick A_2 to be above and to the right of A_1. Draw in the efficient price ratios for A_1 and for A_2. Are the efficient price ratios the same?

   (e) Using the graph you just drew, suppose the initial allocation gives more of both goods to Jones. Which initial allocation is that, A_1 or A_2?

   Starting from this situation, would it still be possible to reach an efficient allocation and have an efficient price ratio if, before anything else happened, the government confiscated some X and some Y from Jones and gave it to Smith?

5. Draw a contract curve and explain precisely what it is. Be sure to fully explain the sort of graph you are using. Show what indifference curves look like where they touch the contract curve.

Answers

① a) Laspeyres = base year (here, base year prices)

$$\text{Las. Index} = \frac{P_X^{72} Q_X^{86} + P_Y^{72} Q_Y^{86}}{P_X^{72} Q_X^{72} + P_Y^{72} Q_Y^{72}} = \frac{1\,(7) + 3\,(5)}{1\,(2) + 3\,(4)} > 1$$

④ ③

Yes - real income ↑ ③

b)

$$\text{Index} = \frac{P_X^{86} Q_X^{72} + P_Y^{86} Q_Y^{72}}{P_X^{72} Q_X^{72} + P_Y^{72} Q_Y^{72}} = \frac{8\,(2) + 6\,(4)}{1\,(2) + 3\,(4)} > 1$$

④ ③

Yes - price level ↑ ③

(2) "The rate of inflation" means "the rate of increase in prices." So we want price indexes. ← 10 points

Base Year Quantities: $$\frac{P_X^{87} Q_X^{72} + P_Y^{87} Q_Y^{72}}{P_X^{72} Q_X^{72} + P_Y^{72} Q_Y^{72}} = \frac{5(3) + 7(4)}{1(3) + 3(4)} = \frac{15+28}{3+12}$$ ← 10 points

$$= \frac{43}{15}.$$

Current Year Quantities: $$\frac{P_X^{87} Q_X^{87} + P_Y^{87} Q_Y^{87}}{P_X^{72} Q_X^{87} + P_Y^{72} Q_Y^{87}} = \frac{5(6) + 7(8)}{1(6) + 3(8)} = \frac{30+56}{6+24}$$ ← 10 points

$$= \frac{86}{30} = \frac{43}{15}.$$

0 points (I didn't ask about this)

The index using base year quantities overestimates the rate of inflation.*
The index using current year quantities underestimates the rate of inflation.**
Since these two indexes agree for this problem, we know the true rate of inflation. It is 187%, since

$$\frac{43}{15} - 1 = 1.87.$$

* This is because its denominator represents an optimal choice of Q's for the given P's, while its numerator has Q's belonging to different P's. Whenever Q's and P's are mismatched, expenditure is measured too high. So the numerator is too big, while the denominator is OK, giving a result that's too big.

** Here, the numerator's OK and the denominator's too big.

(3.) a. "Laspeyres" implies base year quantities:

(9 points)

$$\frac{P_x^{87} Q_x^{72} + P_y^{87} Q_y^{72}}{P_x^{72} Q_x^{72} + P_y^{72} Q_y^{72}} = \frac{(9\cdot 5) + (3\cdot 1)}{(2\cdot 5) + (10\cdot 1)} = \frac{48}{20} = 2.4$$

b. "Paasche" implies current year quantities:

(9 points)

$$\frac{P_x^{87} Q_x^{87} + P_y^{87} Q_y^{87}}{P_x^{72} Q_x^{87} + P_y^{72} Q_y^{87}} = \frac{(9\cdot 4) + (3\cdot 10)}{(2\cdot 4) + (10\cdot 10)} = \frac{66}{108} = \frac{33}{54}$$

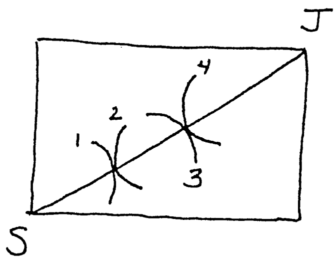
c. Since $2.4 > 1$, the price level has risen according to the Laspeyres Index.
Since $\frac{33}{54} < 1$, the price level has fallen according to the Paasche Index.

(7 points)

The Laspeyres Price Index always overstates the amount of inflation: its numerator is "too large", since the prices are assumed to go along with quantities which are not optimal at these prices. Similarly, the Paasche Price Index always understates the amount of inflation.

(4) a)

pts
2 Box
3 Contract curve
2 Smith's indiff curves
2 Jones's " "
3 tangent on contract curve



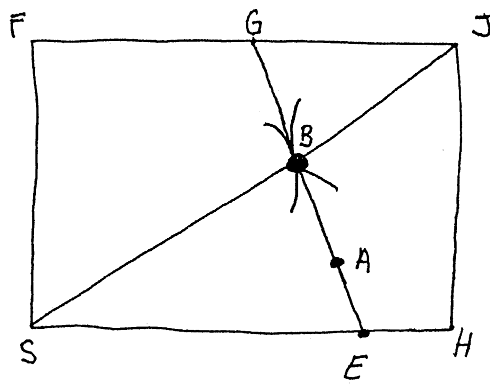
2 and 4 are indifference curves for Smith

1 and 3 are indifference curves for Jones

The indifference curves are tangent along the contract curve, which is the straight diagonal line joining the corners of the Edgeworth Box (the S and J corners, that is).

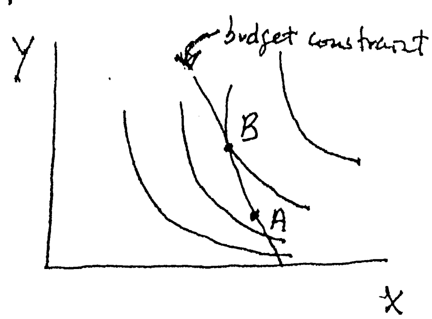
b)

pts
2 price ratio
2 tangency on contract curve

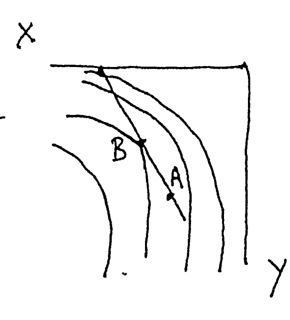


The price ratio is the slope of the line AB. With this ratio, both Smith and Jones will want to move from A to B, because B their utility is highest within their affordable sets.

Note (not required for full credit on part (b)) : Smith's affordable set with this price ratio is GESF and Jones's affordable set is GEHJ. So the problem for Smith alone looks like



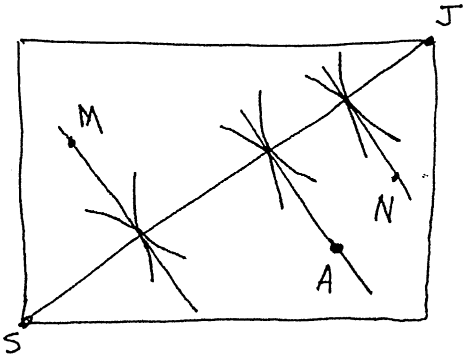
whereas for Jones the problem looks like



. (End of optional note.)

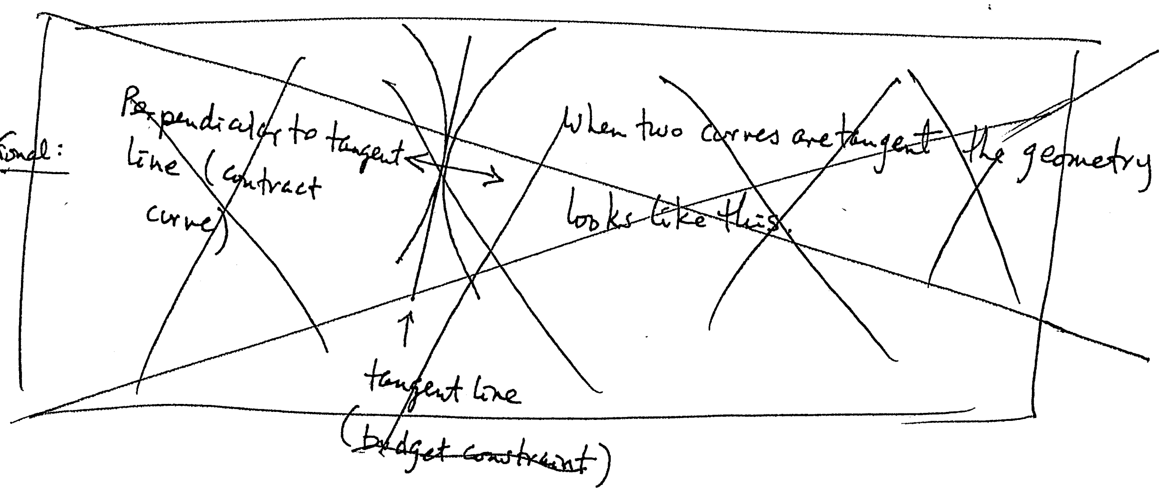
c)

pts
2 graph
3 = price ratios



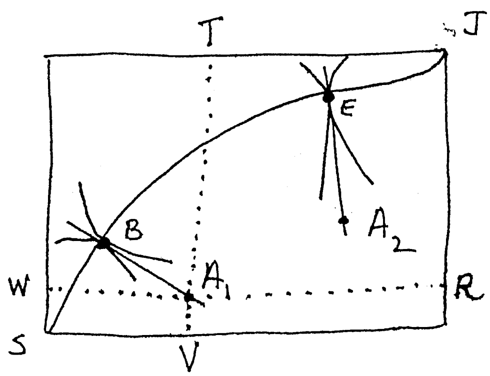
The efficient price ratios for M and N are the same as for A.

~~optional~~:



d)

pts
2 graph
3 ≠ price ratios



The efficient price ratios, A_1B for A_1 and A_2E for A_2, are not the same.

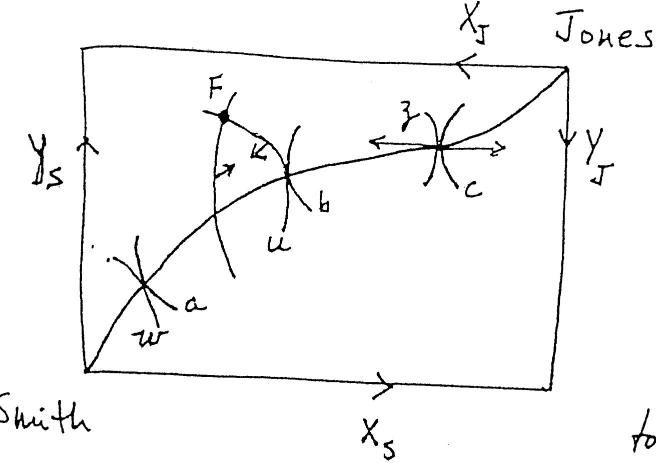
e) A_1 gives more to Jones. At A_1, Jones gets JT units of X while Smith only gets SV units of X, and Jones gets RJ units of Y while Smith only gets SW units of Y.

3 pts

3 pts

Yes, government confiscation and transfer could lead to moving from A_1 to A_2, from which E is an efficient allocation and A_2E is an efficient price ratio.

5



Suppose there are two people in an economy, Smith and Jones, and two goods in the economy, X and Y. Let the X axis' length be the total amount of X which Smith and Jones together have, and construct the Y axis similarly. Measuring Smith's X and Y in the usual way, Jones's X and Y is represented by the difference between the length of the axis and Smith's X and Y, respectively. This can be measured from the top right-hand corner downwards and to the left, as shown.

The contract curve is the locus of points where Smith's indifference curves, such as a, b, and c, are tangent to Jones's indifference curves, such as w, u, and z. The curve is so named because free contracts (unrestricted contracts) between Smith and Jones will result in an allocation of X and Y being on this curve. If they are not on this curve (for example, if they are at point F), then Smith and Jones will want to trade, because both can be made better off (note the arrows showing each person's preferred direction of motion do not contradict each other). If they are on this curve, mutual gains from trading are lost (see the arrows from z and c ∴ they point in opposite directions, so mutual gains do not exist). So a trading process which continues until all mutually beneficial trades have been undertaken will end up on the contract curve.

basic idea of "Edgeworth-Bowley Box" (axes) : 7pts
explanation of box : 7pts
basic shape of indiff. curves : 7pts
" " " contract curve : 7pts
connection betw. indiff. curves and " " : 7pts