K. Monopoly

1. Suppose the market demand curve takes the form

$$P = 2 - Q\,.$$

   (a) If a monopolist faced this demand curve and decided to produce $Q = 1/2$, what would his or her marginal revenue at $Q = 1/2$ be? What would marginal cost at $Q = 1/2$ be?

   (b) If a competitive industry faced this demand curve and produced $Q = 1/2$, what would the marginal revenue curve for an individual firm look like? How much would marginal cost be? (I am not asking for the shape of the whole marginal cost curve.)

   (c) Now, instead of supposing to start with that $Q = 1/2$, suppose that both the monopolist and the competitive firms have a constant-returns-to-scale technology with average costs of \$1. What is the deadweight loss from monopoly?

2. If the demand curve is given by $P = 6 - Q$, and average costs are constant at $AC = 2$, find the deadweight loss, monopoly profit, and consumer surplus if a monopolist supplies the good instead of a competitive industry.

3. (a) Suppose the demand curve in a monopoly market is given by $p = 1/\sqrt{Q}$. Find total revenue as a function of Q. Then sketch the total revenue curve and the marginal revenue curve.

   (b) Suppose the demand curve in a monopoly market is given by $p = 1/Q^2$.

      i. Find total revenue as a function of Q.

      ii. Sketch the total revenue curve and, on another graph, the marginal revenue curve.

      iii. Suppose the monopolist has constant returns to scale. Sketch the total cost curve on the graph you drew the total revenue curve on, and sketch the marginal cost curve on the graph you drew the marginal revenue curve on.

      iv. Find the profit-maximizing Q for the monopolist using the graphs you just drew (the ones from $p = 1/Q^2$), or explain why it is impossible to do.

4. Suppose a monopolist faces a demand curve of the form $p = a - bQ$ where a and b are positive numbers. Suppose the monopolist's average cost curve is constant at a height of d, where $d < a$. Find, as a function of a, b, and d:

   (a) The monopolist's profit-maximizing output.

   (b) The price the monopolist will charge.

   (c) The consumer surplus.

5. A monopolist faces the following demand curve:

$$P = 3 - 0.5Q \,.$$

   The monopolist has two plants, "1" and "2". Let q_1 be the quantity produced in plant 1 and let q_2 be the quantity produced in plant 2 (so $q_1 + q_=Q$). The marginal cost curve for plant 1 is

$$MC_1 = q_1$$

   and the marginal cost curve for plant 2 is

$$MC_2 = 2q_2 \,.$$

   What will the profit-maximizing q_1, q_2, and Q be?

6. A monopolist has two plants, "1" and "2". Plant 1 has marginal costs $MC_1(q_1) = q_1$ and plant 2 has marginal costs $MC_2(q_2) = 2q_2$. The two plants supply a market where the demand curve is $P = 1 - \frac{1}{2}Q$, $Q = q_1 + q_2$. What is the marginal revenue curve? What are the profit-maximizing values of q_1, q_2, and Q? (Assume neither plant shuts down).

7. A monopolist faces two separated markets. The demand curve in Region 1 is given by $P_1 = 4 - 4Q_1$, and the demand curve in Region 2 is given by $P_2 = 3 - 3Q_2$. The monopolist has one plant, producing with constant returns to scale at an average cost of \$1. Find the equilibrium prices and quantities in the two markets.

8. (a) A monopolist can practice price discrimination between two markets, 1 and 2. The demand curves for these two markets are given in Figure 1; they are straight lines. If the monopolist has constant average cost of \$1, graphically illustrate in Fig. 1 how much

profit he makes. Roughly how many dollars in profit does he make? (Just use the graph; working out the algebra is not necessary. Also, if you don't have a calculator with you, you needn't do hard multiplication and addition by hand, just set the equations up.)

(b) Now suppose consumers in markets 1 and 2 can freely trade with each other. What are the monopolist's profits now? Show them on the graph (or on a new graph if you prefer) and give an approximate numerical estimate of them.

9. Using a diagram, prove that a monopolist facing separated markets with different demand curves might choose not to price-discriminate even though it could price-discriminate. (Do not worry if your marginal revenue curves are inconsistent with your sketched demand curves; you get full credit as long as the marginal revenue and demand curves have the correct basic shape).

10. A monopolist can sell in two geographically separated markets called '1' and '2'. The monopolist's customers cannot transport the monopolist's product between '1' and '2'. The demand curve in each of these markets is

$$P_1 = 6 - 3q_1$$
$$P_2 = 2 - q_2.$$

The monopolist also has two plants, called 'A' and 'B'. They have the following average and marginal costs:

$$AC_A = q_A$$
$$MC_A = 2q_A$$
$$AC_B = 3q_B$$
$$MC_B = 6q_B.$$

Let Q be the total quantity that the monopolist produces and sells. Find the optimal q_A, q_B, q_1, q_2, Q, P_1, P_2, TR_1, TR_2, AC_A, AC_B, TC_A, TC_B, and the monopolist's profit. (TR means total revenue and TC means total cost.) [Here is a hint which you should only use to check your answer (don't reason backwards from it): $Q = 1$ and profit $= 2.5$. Some of the other answers are messy fractions.]

11. A monopolist can sell in two geographically separated markets called '1' and '2'. The demand curve in each of these markets is

$$P_1 = 3 - \tfrac{3}{2}q_1$$

$$P_2 = 1 - \tfrac{1}{2}q_2.$$

The monopolist also has two plants, called 'A' and 'B'. They have the following average and marginal costs:

$$\begin{aligned} AC_A &= \tfrac{1}{2}q_A \\ MC_A &= q_A \\ AC_B &= \tfrac{3}{2}q_B \\ MC_B &= 3q_B. \end{aligned}$$

Let Q be the total quantity that the monopolist produces and sells. Find the optimal q_A, q_B, q_1, q_2, Q, P_1, P_2, AC_A, AC_B, and the monopolist's profit. Be sure to check the long-run shut-down rule. [Here is a hint which you should only use to check your answer (don't reason backwards from it): the optimum $Q = 1$ and profit $= 1.25$. Some of the other answers are messy fractions.]

12. Throughout this question, assume that the firm's production function exhibits increasing returns to scale. Also assume that the firm is a monopolist.

    (a) Sketch a situation in which, in the long run, the monopolist would rather shut down than produce. Using your graph, explain why the monopolist is better off not producing.

    (b) Sketch a situation in which, in the long run, the monopolist would rather produce than shut down. Using your graph, explain why the monopolist is better off producing. Show the area of consumer surplus on your graph.

    (c) Just like in part (b), sketch a situation in which, in the long run, the monopolist would rather produce than shut down.

        If the monopolist is producing some quantity "Q_1" (where Q_1 could be any positive number), define "social surplus" as being the total area between the demand curve and the marginal cost curve, from a quantity of zero to a quantity of Q_1. Areas where the demand curve is higher than the marginal cost curve count as positive areas of social surplus; areas where the demand curve is lower than the marginal cost curve count as negative areas of social surplus.

Let $\hat{Q}$ be the quantity which results in the maximum social surplus. Locate $\hat{Q}$ on your graph and explain your answer briefly.

(d) Is the market-clearing price at $\hat{Q}$ greater than, equal to, or less than the marginal cost of producing $\hat{Q}$, or can you not tell? Is average revenue at $\hat{Q}$ greater than, equal to, or less than the average cost of producing $\hat{Q}$, or can you not tell?

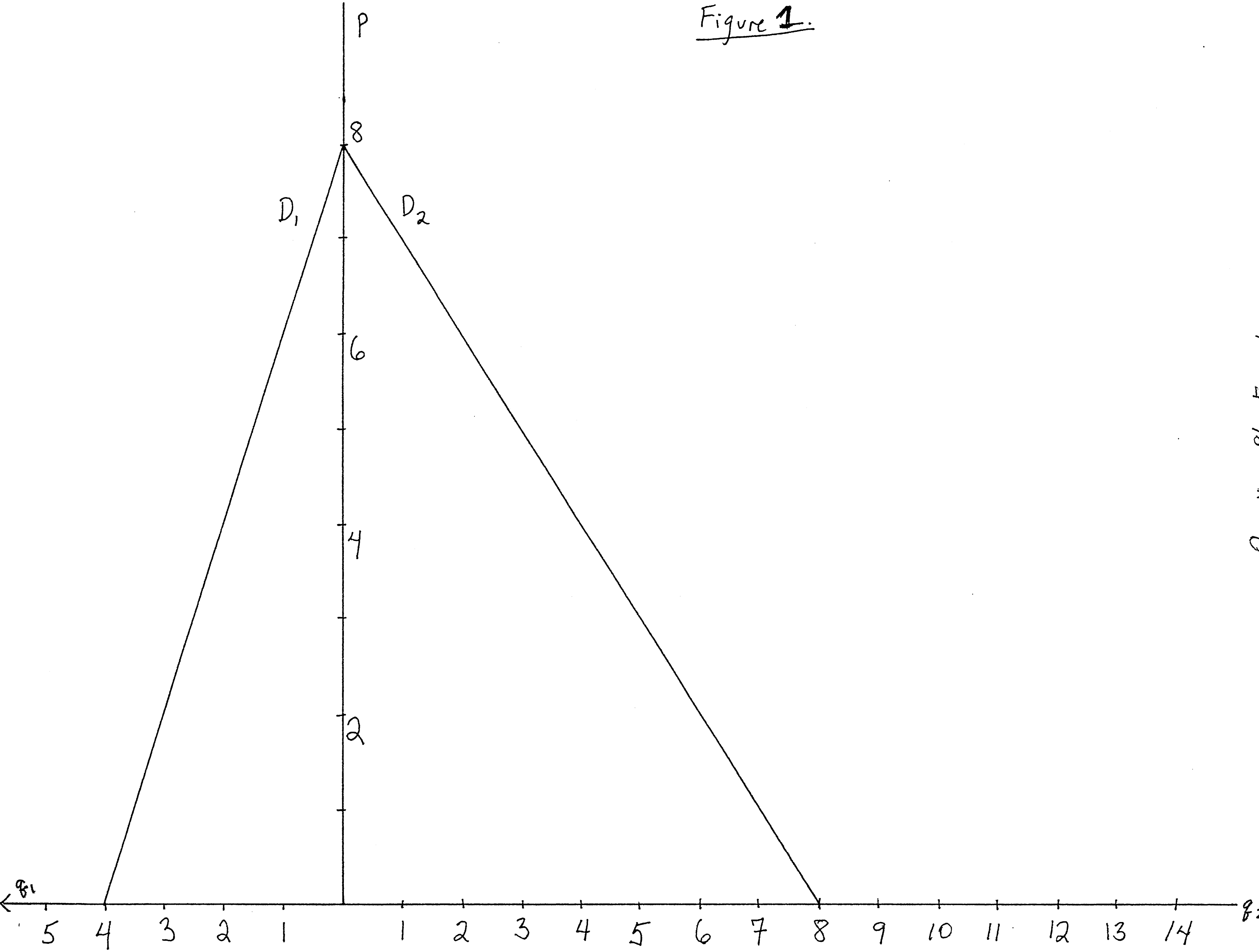
(e) Suppose the government wanted to force the monopolist to produce $\hat{Q}$, so it passed a law saying that the monopolist had to produce $\hat{Q}$ or go out of business. What would happen: would the monopolist produce $\hat{Q}$ or would he go out of business?

13. Thoroughly explain the two-part pricing system depicted in Figure 1. What kinds of returns to scale does this industry have? What is the long run competitive output if a two-part pricing system is not used? What is the significance of point C? What is the firm's profit? Why?

14. (a) Explain by drawing a graph how a two-part pricing scheme could enable a publicly regulated monopolist to make positive profit using an increasing returns to scale technology.

(b) Graph profit π versus quantity Q for the two-part-pricing-scheme regulated monopolist described in part (a). Remember that π will be increasing as Q increases if and only if the marginal unit makes a positive contribution to the firm's profit.

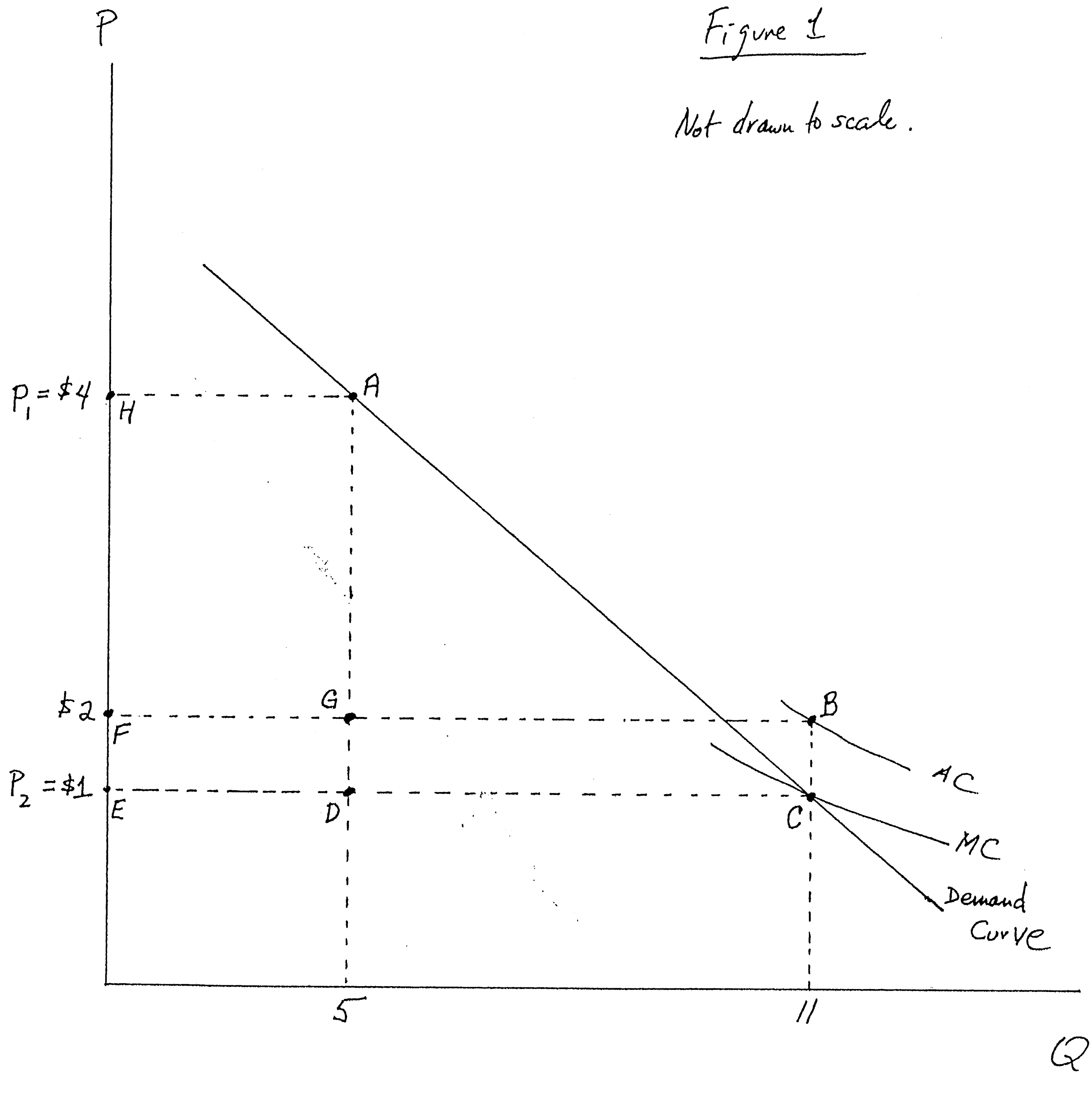
Figure 1.



Question 8's Fig. 1.

Figure 1

Not drawn to scale.

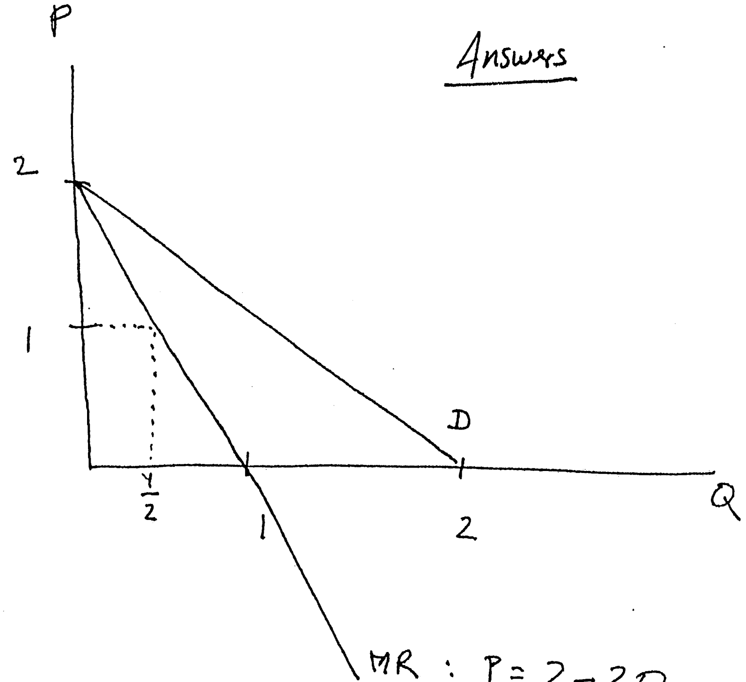


(AC downward sloping)

Question 13's Fig. 1

① a.

Answers

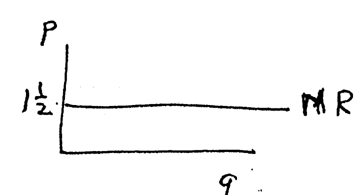


At $Q = \frac{1}{2}$, $MR = 2 - 2(\frac{1}{2}) = 1$.

If the monopolist decided to produce there then that had to be the point at which $MR = MC$. So $MC = 1$.

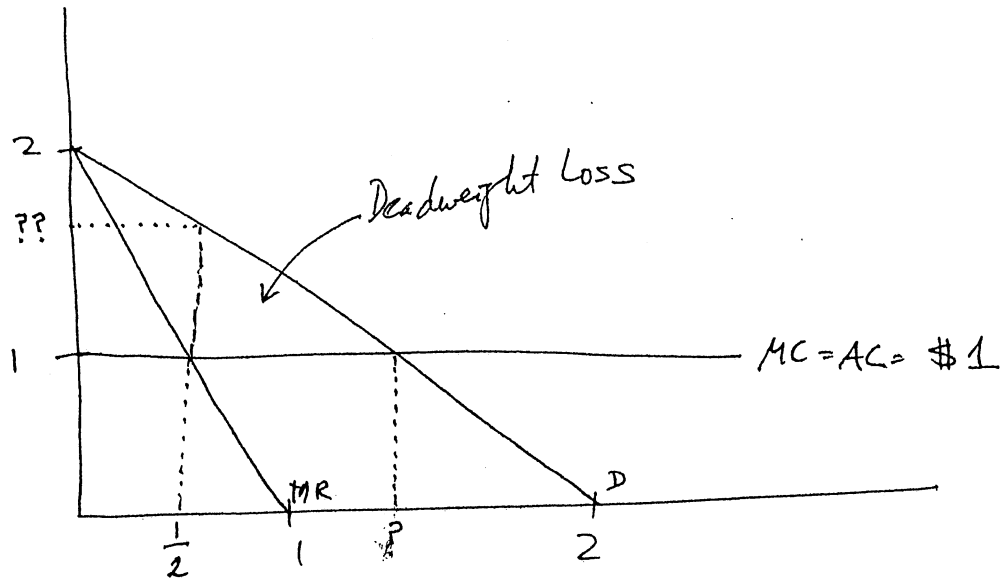
b. D: $P = 2 - Q$

At $Q = \frac{1}{2}$, $P = 1\frac{1}{2}$. For 1 firm, $MR = AR = P$, so $MR(q) = 1\frac{1}{2}$ for all q:



$MC = 1\frac{1}{2}$ for the same reason as in part a: $MC = MR$ when a firm produces.

c.

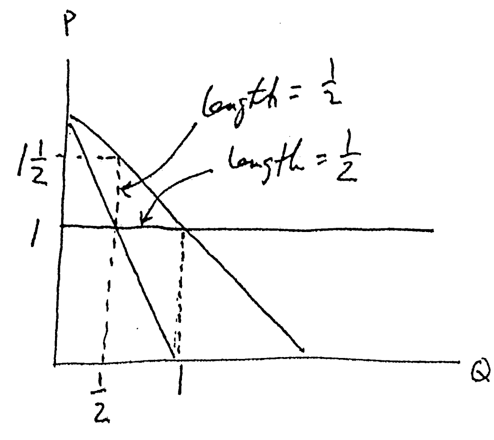


D: $P = 2 - Q$

for ??: $Q = \frac{1}{2}$ so $P = 1\frac{1}{2}$: $?? = 1\frac{1}{2}$

for ?: $P = 1$ so $Q = 2 - P = 1$: $? = 1$. (So the graph

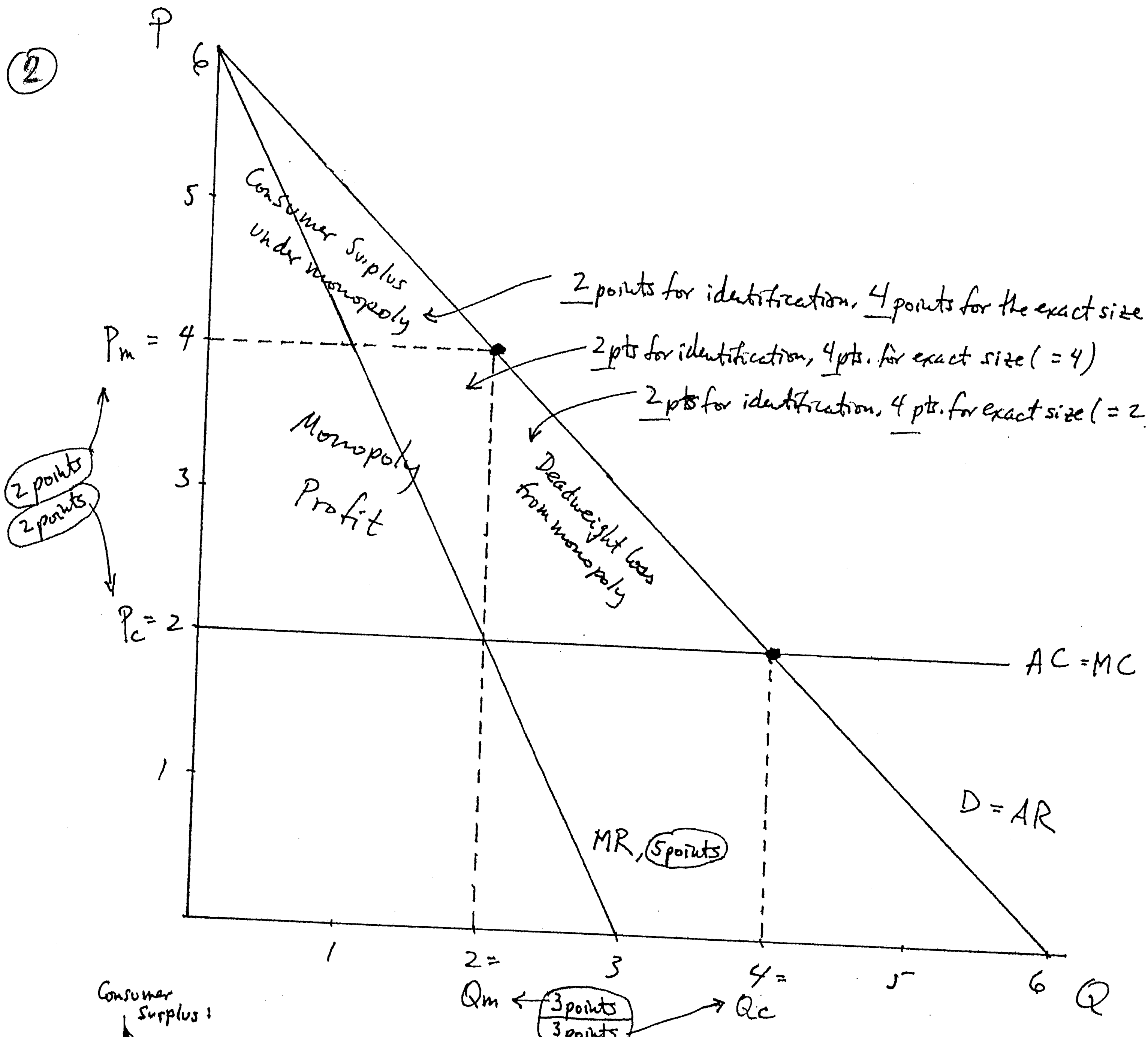
ought to look more like

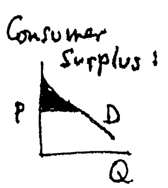


.)

Deadweight loss is $\frac{1}{2}bh = \frac{1}{2}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \$\frac{1}{8}$.

2

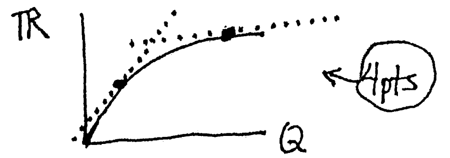


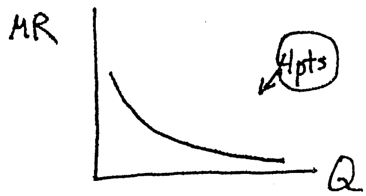


Under competition, the supply curve is flat at $P = 2$, so $P_c = \$2$ and $Q_c = 4$. Consumer surplus is the area under D and above the price line, so it is $\frac{1}{2}bh = \frac{1}{2}(4)(4) = 8$, using the formula for the area of a triangle. The monopolist sets $MR = MC$ at $Q_m = 2$, for which the market-clearing price is \$4. So under monopoly, consumer surplus is $\frac{1}{2}bh = \frac{1}{2}(2)(2) = \underline{2}$. Monopoly profit is AR − AC times Q_m, or $(4-2)(2) = \underline{4}$. Deadweight loss is the triangle shown, with area $\frac{1}{2}bh = \frac{1}{2}(2)(2) = \underline{2}$.

3) a) $p = \frac{1}{\sqrt{Q}}$

$TR = pQ = \left(\frac{1}{\sqrt{Q}}\right) Q = \sqrt{Q}$, total revenue as a function of Q. 4pts



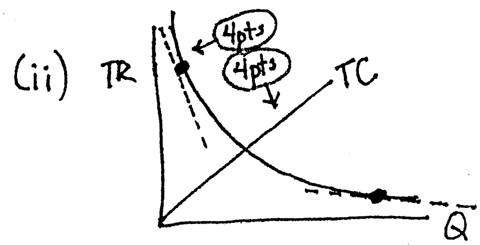


You can tell MR is positive and falling by looking at tangent lines to TR, like the dotted lines I have drawn.

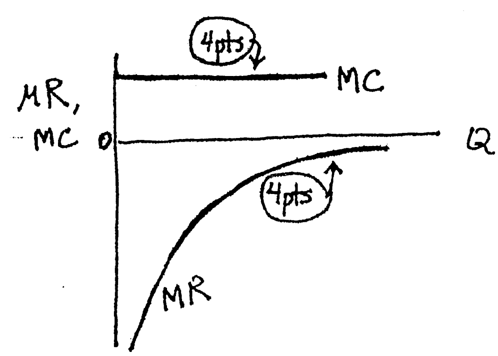
b) (i) $p = \frac{1}{Q^2}$

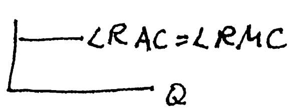
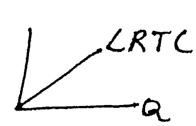
$TR = pQ = \left(\frac{1}{Q^2}\right) Q = \frac{1}{Q}$, total revenue as a function of Q. 3pts

(ii)



The dashed tangent lines are downward-sloping (so $MR < 0$) but their slopes are getting close to zero (so MR is getting close to zero).



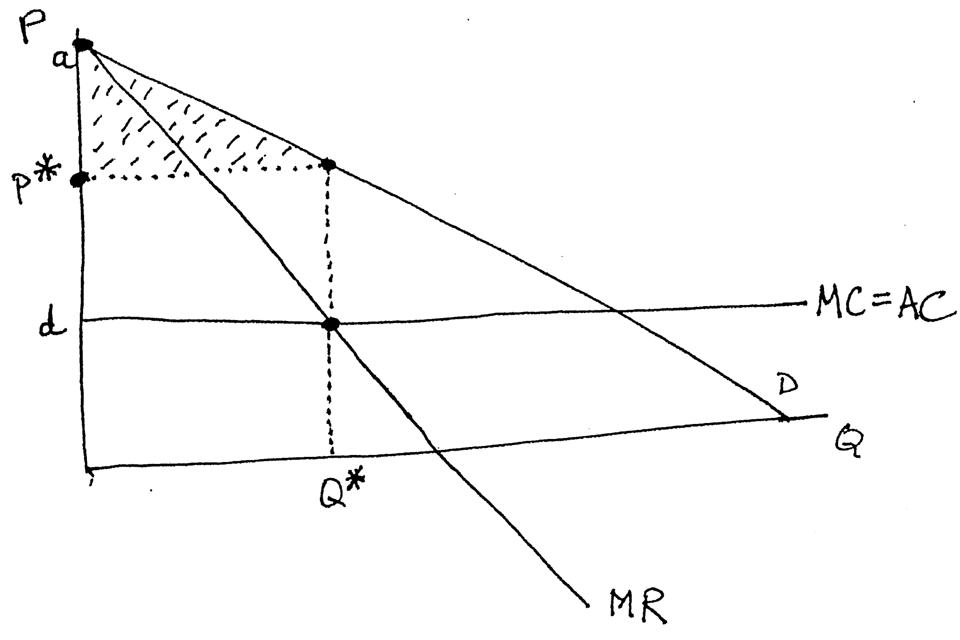
(iii) Constant Returns to scale implies LRAC is constant and equal to LRMC :  . This means that LRTC is a straight line from the origin :  . These are sketched above.

4pts for explanation

(iv) There is no way to make $MR = MC$, so one suspects there is no way to maximize profit. Profit is the gap between the TR and TC curves. That gap gets bigger and bigger as $Q \to 0$, so the monopolist wants to produce Q as close to zero as he can get. However, if he actually produces $Q = 0$, profit is zero because revenues $= 0$ and long-run costs $= 0$. So there is no profit-maximizing Q.

By the way, the demand curve in (a) is elastic because TR↑ when Q↑. The demand curve in (b) is inelastic because TR↓ as Q↑. A monopolist will never produce on the inelastic portion of a demand curve, because there $MR < 0$. If $MR < 0$ then the firm can ↑TR by ↓Q; since ↓Q also results in ↓ total cost TC usually, by ↓Q the monopolist could ↑TR and ↓TC. Certainly he would do that. So he will never be happy if $MR < 0$ (if the demand curve is inelastic).

4



Demand: $p = a - bQ$

Marginal Revenue: $MR = a - 2bQ$ (If the demand curve is a straight line, the marginal revenue curve is also a straight line, with the same p intercept and exactly twice the slope.)

5pts

5pts

a) $MR = MC \Rightarrow a - 2bQ = d$, find Q: $a - d = 2bQ$

$$\frac{a-d}{2b} = Q = Q^*.$$

5pts

b) Using $Q = Q^*$, find p^* from the demand curve: 2pts

$$p^* = a - bQ^* \quad \text{1pt}$$

$$= a - b\left(\frac{a-d}{2b}\right) = a - \frac{a-d}{2} = a - \left(\frac{a}{2} - \frac{d}{2}\right)$$

$$= a - \frac{a}{2} + \frac{d}{2} = \frac{a}{2} + \frac{d}{2}$$

$$= \frac{a+d}{2}.$$ 3pts, this also OK; unsimplified −1.

c) The area of a triangle is $\frac{1}{2}$(base)(height). 1pt The base of the consumer surplus triangle is the distance from 0 to Q^*, which is just Q^*, which is $\frac{a-d}{2b}$ from part (a). 3pts The height of the consumer surplus triangle is the distance from "a" to p^*, 2pts which is $a - p^* = a - \frac{a+d}{2} = a - \frac{a}{2} - \frac{d}{2} = \frac{a}{2} - \frac{d}{2} = \frac{a-d}{2}$, using part (b). So consumer surplus is

$$\frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}\,\frac{a-d}{2b}\,\frac{a-d}{2} = \frac{(a-d)^2}{8b}.$$

3pts

3pts for Correct Consumer Surplus region on graph or in words

2pts (−1 if unsimplified)

5 $P = 3 - \frac{1}{2}Q$

$MC_1 = q_1$

$MC_2 = 2q_2$

$Q = q_1 + q_2$.

First of all, if $P = 3 - \frac{1}{2}Q$ then $MR = 3 - Q$. ← 6 points

Second, MC_1 should equal MC_2, or else production should be shifted to where MC is the smallest. Hence $q_1 = 2q_2$. ← 4pts

6 points

Finally, $MR = MC$. (MC is either MC_1 or MC_2 — it does not matter, since they're equal.)

2pts.

So:

$$3 - (q_1 + q_2) = 3 - Q = MR = MC = q_1 \quad \leftarrow \text{7pts}$$

$$3 - q_2 = 2q_1$$

$$3 - \left(\tfrac{1}{2}q_1\right) = 2q_1$$

$$3 = \tfrac{5}{2}q_1 \Rightarrow q_1 = \tfrac{6}{5}; \quad q_2 = \tfrac{1}{2}q_1 \Rightarrow$$

$$q_2 = \tfrac{3}{5}; \quad Q = q_1 + q_2 \Rightarrow$$

$$Q = \tfrac{9}{5}.$$

4 points for the algebra.

You can also check that in this situation, $P = 3 - \frac{1}{2}Q = 3 - \frac{9}{10} = 2.1$. ← 0 points (not asked for)

(6) $MC_1 = MC_2 \Rightarrow \boxed{q_1 = 2q_2}$ (1) 8 pts.

$P = 1 - \frac{1}{2}Q \Rightarrow \boxed{MR = 1 - Q}$ (2) 8 pts.

$= 1 - (q_1 + q_2)$

$= 1 - (2q_2 + q_2)$ from (1)

$= 1 - 3q_2.$

$MR = MC_1 = MC_2$ so $MR = MC_2$ 8 pts.

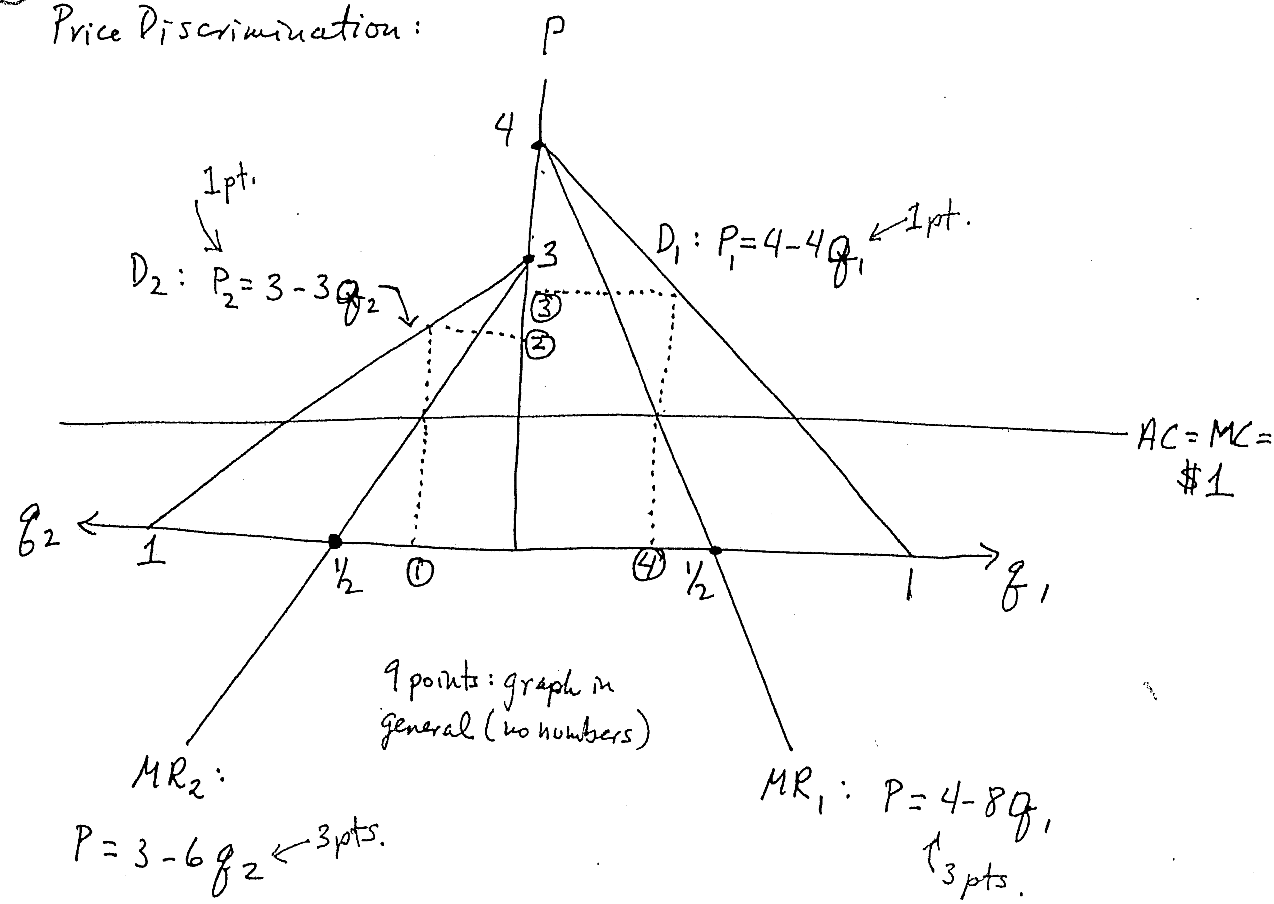
$1 - 3q_2 = 2q_2$

$1 = 5q_2$

$\boxed{\frac{1}{5} = q_2}$ so $\boxed{q_1 = \frac{2}{5}}$ and $\boxed{Q = \frac{3}{5}}$.

6 pts. for algebra

(7) Price Discrimination:



(1) $1 = 3 - 6q_2 \Rightarrow 6q_2 = 2 \Rightarrow q_2 = \frac{1}{3}$

(2) $P_2 = 3 - 3\left(\frac{1}{3}\right) = 2$

(4) $1 = 4 - 8q_1 \Rightarrow 8q_1 = 3 \Rightarrow q_1 = \frac{3}{8}$

(3) $P_1 = 4 - 4\left(\frac{3}{8}\right) = 2\frac{1}{2}$

2 points each

Because of CRS, this is just like solving two unrelated monopoly problems, one for each market.

(8) a) See Figure 1. The marginal revenue curves have the same P-intercept as the demand curves, and twice the slope. Hence their q-intercepts are at 2 and 4 for MR_1 and MR_2, respectively. q_1^* and q_2^* are found where MR = MC (at points "a" and "g"); $q_1^* = 1.75$ and $q_2^* = 3.5$ (any answer close to this will do). To find the prices, go up from q_1^* and q_2^* to their respective demand curves (points "b" and "d"). By coincidence, these points correspond to the same price "c" = \$4.50, but in general the prices will differ and if in your diagram the prices do differ a bit then that is OK.

Profits are the area "abch" for market 1 (1 point) and "cdgh" for market 2 (1 point).

Algebraically, $\pi = (P_1 Q_1 - AC_1 Q_1) + (P_2 Q_2 - AC_2 Q_2)$

$$= \left[4\tfrac{1}{2}\left(1\tfrac{3}{4}\right) - 1\left(1\tfrac{3}{4}\right)\right] + \left[4\tfrac{1}{2}\left(3\tfrac{1}{2}\right) - 1\,(3.5)\right]$$

$$= \underset{\text{1 pt.}}{6\tfrac{1}{8}} + \underset{\text{1 pt.}}{12\tfrac{1}{4}} = \boxed{18\tfrac{3}{8}.}$$

b) If consumers can freely trade with each other then price discrimination is not possible. Hence there is no economic separation between markets 1 and 2, and to solve the problem one gets the relevant "market" demand curve by summing D_1 and D_2 to get "$D_1 + D_2$" in Fig. 1. (Note that $D_1 + D_2$

hits the Q axis at $12 = 4 + 8$, the sum of the q intercepts of the individual markets.) The MR of $D_1 + D_2$ hits the Q axis at $\frac{12}{2} = 6$, and hits the MC curve at 5.25 (point "f"). Going up to the demand curve (to "e") gives a price of $4\frac{1}{2}$, and profit of area "cefh," or $\pi = (P - AC)Q = (4\frac{1}{2} - 1)(5\frac{1}{4}) = \boxed{18\frac{3}{8}.}$ as before. However, this is a coincidence; usually profit will be lower if price discrimination is not possible any more, and it is OK if you said this. 2 points

OPTIONAL: The algebra is as follows.

Market 1: $P = 8 - 2Q_1$

$MR_1 = 8 - 4Q_1$

$MR_1 = MC(=1)$ at $Q_1 = 1\frac{3}{4}$ since $1 = 8 - 4Q_1 \Rightarrow 4Q_1 = 7 \Rightarrow Q_1 = \frac{7}{4} = 1\frac{3}{4}$,

and hence $P = 8 - 2(1\frac{3}{4}) = 4\frac{1}{2}$.

Market 2: $P = 8 - Q_2$

$MR_2 = 8 - 2Q_2$

$MR_2 = MC(=1)$ at $1 = 8 - 2Q_2$

$2Q_2 = 7$

$Q_2 = \frac{7}{2} = 3\frac{1}{2}$,

and then

$P = 8 - 3\frac{1}{2} = 4\frac{1}{2}$.

Both Markets: $Q_1 = 4 - \frac{1}{2}P$

$Q_2 = 8 - P$

$Q_1 + Q_2 = 12 - \frac{3}{2}P$.

If $Q = Q_1 + Q_2$ then

$Q = 12 - \frac{3}{2}P$

$\frac{3}{2}P = 12 - Q$

$P = 8 - \frac{2}{3}Q$ and

You could also get this from the graph.

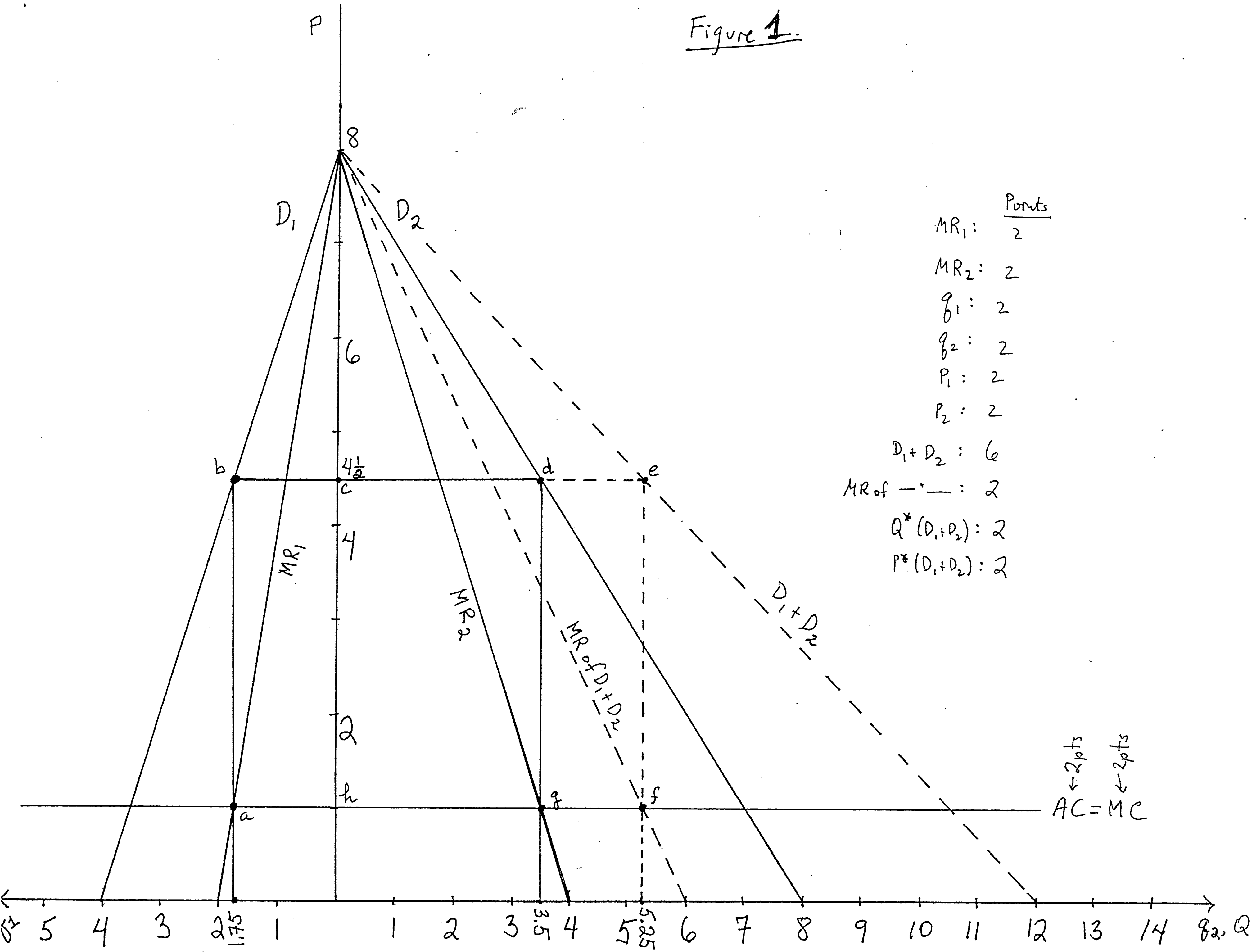
$MR = 8 - \frac{4}{3}Q$.

$MR = MC(=1)$ at $1 = 8 - \frac{4}{3}Q \Rightarrow$

$\frac{4}{3}Q = 7 \Rightarrow Q = \frac{21}{4} = 5\frac{1}{4}$,

and $P = 8 - \frac{2}{3}\left(\frac{21}{4}\right) = 8 - \frac{7}{2} = 4\frac{1}{2}$.

Figure 1.



	Points
MR_1:	2
MR_2:	2
q_1:	2
q_2:	2
P_1:	2
P_2:	2
D_1+D_2:	6
MR of — " —:	2
$Q^*(D_1+D_2)$:	2
$P^*(D_1+D_2)$:	2

9

pts
3 2-quadrant graph
3 constant AC
3 correct Q, graph 1
3 ——— 2
2 correct p, graph 1
2 ——— 2
9 same p in both graphs

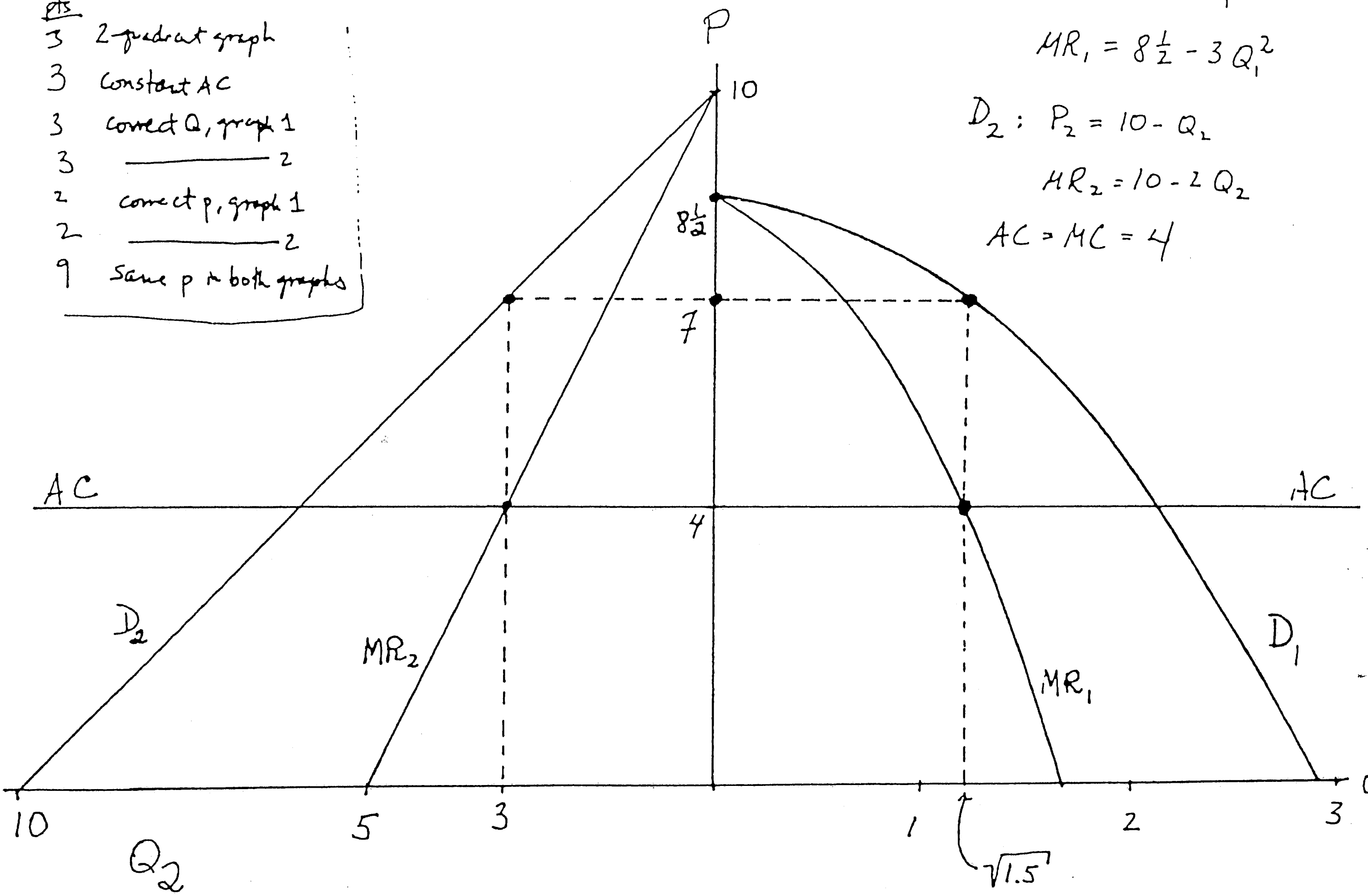
$D_1: \; P_1 = 8\tfrac{1}{2} - Q_1^2$

$MR_1 = 8\tfrac{1}{2} - 3Q_1^2$

$D_2: \; P_2 = 10 - Q_2$

$MR_2 = 10 - 2Q_2$

$AC = MC = 4$



You are not supposed to actually be able to calculate an example such as in the demand curve D_1. However, the basic idea is clear: even with demand curves of different shapes, a monopolist setting $MR = MC$ in both markets may arrive at the same profit-maximizing price in both places. Since the definition of price-discrimination is that different prices are charged for the same good, such a monopolist is not price-discriminating even though he could.

Definition of price discrimination: 8pts

(10) $P_1 = 6 - 3q_1 \Rightarrow MR_1 = 6 - 6q_1$

$P_2 = 2 - q_2 \Rightarrow MR_2 = 2 - 2q_2$

$MC_A = 2q_A$

$MC_B = 6q_B$ Optimum: $MR_1 = MR_2 = MC_A = MC_B$ – 1 pt

$q_1 + q_2 = Q$

$q_A + q_B = Q.$

$MR_1 = MR_2 \Rightarrow 6 - 6q_1 = 2 - 2q_2$ 1 pt

$6 + 2q_2 = 2 + 6q_1$

$4 + 2q_2 = 6q_1$

$2q_2 = 6q_1 - 4$

$q_2 = 3q_1 - 2.$

$MC_A = MC_B \Rightarrow 2q_A = 6q_B$ 1 pt

$q_A = 3q_B$

$MR_1 = MC_B \Rightarrow 6 - 6q_1 = 6q_B$

$1 - q_1 = q_B$ therefore $q_A = 3q_B$

$= 3(1 - q_1) = 3 - 3q_1.$

$q_1 + q_2 = q_A + q_B \Rightarrow q_1 + (3q_1 - 2) = (3 - 3q_1) + (1 - q_1)$

$4q_1 - 2 = 4 - 4q_1$

$8q_1 = 6 \Rightarrow q_1 = \frac{6}{8} = \frac{3}{4}$ – 1 pt

$q_2 = 3(\frac{3}{4}) - 2 = \frac{9}{4} - \frac{8}{4} = \frac{1}{4}$ – 1 pt

$q_A = 3 - 3(\frac{3}{4}) = \frac{12}{4} - \frac{9}{4} = \frac{3}{4}$ – 1 pt

$q_B = 1 - \frac{3}{4} = \frac{1}{4}$ – 1 pt

(There are other ways to arrive at these answers.)

[Optional Check: $MC_A = 2\left(\frac{3}{4}\right) = \frac{6}{4} = \frac{2}{3}$

$MC_B = 6\left(\frac{1}{4}\right) = \frac{6}{4} = \frac{2}{3}$

$MR_1 = 6 - 6\left(\frac{3}{4}\right) = 6 - 3\left(\frac{3}{2}\right) = \frac{12}{2} - \frac{9}{2} = \frac{3}{2}$

$MR_2 = 2 - 2\left(\frac{1}{4}\right) = 2 - \frac{1}{2} = \frac{3}{2}$.]

$Q = q_1 + q_2 = 1$ — 1 pt

$P_1 = 6 - 3\left(\frac{3}{4}\right) = \frac{24}{4} - \frac{9}{4} = \frac{15}{4} = 3\frac{3}{4}$ — 1 pt

$P_2 = 2 - \frac{1}{4} = 1\frac{3}{4} = \frac{7}{4}$ — 1 pt

$TR_1 = P_1 q_1 = \frac{15}{4} \cdot \frac{3}{4} = \frac{45}{16}$ — 1 pt

$TR_2 = P_2 q_2 = \frac{7}{4}\,\frac{1}{4} = \frac{7}{16}$ — 1 pt

$AC_A = \frac{3}{4}$ — 1 pt $\quad TC_A = AC_A\, q_A = \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$ — 1 pt

$AC_B = 3\left(\frac{1}{4}\right) = \frac{3}{4}$ $\quad TC_B = AC_B\, q_B = \frac{3}{4} \cdot \frac{1}{4} = \frac{3}{16}$ — 1 pt

— 1 pt

$\pi = (TR_1 + TR_2) - (TC_A + TC_B)$

$= \frac{52}{16} - \frac{12}{16} = \frac{40}{16} = 2\frac{8}{16} = 2\frac{1}{2}$ — 1 pt

(11) $P_1 = 3 - \frac{3}{2} q_1$ (1)

Remark. This problem puts together Chapter 8's problem 7 (for which one has to equate MC) and Chapter 9's problem 8 (for which one has to equate MR).

(1) implies $\underline{MR_1 = 3 - 3q_1}$ (2). (2 points)

$P_2 = 1 - \frac{1}{2} q_2$ (3). This implies

$\underline{MR_2 = 1 - q_2}$ (4). (2 points)

$AC_A = \frac{1}{2} q_A$ (5)

$MC_A = q_A$ (6)

$AC_B = \frac{3}{2} q_B$ (7)

$MC_B = 3 q_B$ (8).

$\boxed{MR_1 = MR_2} \Rightarrow 3 - 3q_1 = 1 - q_2$ (9) ← 3 pts

$\boxed{MC_A = MC_B} \Rightarrow q_A = 3 q_B$ (10). ← 3pts

11 points ↓

$\boxed{MR_1 = MR_2 = MC_A = MC_B} \Rightarrow MR_2 = MC_A \Rightarrow 1 - q_2 = q_A$ (11).

→ Or you can use $MR_1 = MC_A$ or $MR_2 = MC_B$ or $MR_1 = MC_B$.

In addition,

$q_1 + q_2 = Q$ (12) ← 2pts.

$q_A + q_B = Q$ (13). ← 2pts.

(9)–(13) is a system of 5 equations in 5 unknowns q_1, q_2, q_A, q_B, and Q.

First, let's get rid of q_1:

$(9) \Rightarrow \quad -3q_1 = -2 - q_2$

$$\underline{q_1 = \frac{2}{3} + \frac{1}{3} q_2} \text{ . Then (12)} \Rightarrow$$

$$\left(\frac{2}{3} + \frac{1}{3} q_2\right) + q_2 = Q$$

$$\frac{4}{3} q_2 = Q - \frac{2}{3} \Rightarrow \quad q_2 = \frac{3}{4} Q - \frac{1}{2}. \qquad (14)$$

Also, let's get rid of q_B:

$$(10) \Rightarrow \underline{q_B = \frac{1}{3} q_A}, \text{ and substituting into (13),}$$

$$q_A + \frac{1}{3} q_A = Q \Rightarrow \frac{4}{3} q_A = Q, \quad q_A = \frac{3}{4} Q. \qquad (15)$$

Plug (14) and (15) into (11):

$$1 - \left(\frac{3}{4} Q - \frac{1}{2}\right) = \frac{3}{4} Q$$

$$\frac{3}{2} = \frac{6}{4} Q \Rightarrow \boxed{Q = 1.}$$

8 points for all the algebra

Then

$$q_2 = \frac{3}{4} Q - \frac{1}{2} = \underline{\frac{1}{4}}$$

$$q_A = \frac{3}{4} Q = \underline{\frac{3}{4}}$$

and from (12) and (13) [or from $q_1 = \frac{2}{3} + \frac{1}{3} q_2$ and $q_B = \frac{1}{3} q_A$],

$$q_1 = \underline{\frac{3}{4}}$$

$$q_B = \underline{\frac{1}{4}}.$$

So

$$TR_1 = P_1 q_1 = \left(3 - \frac{3}{2} q_1\right) q_1 = \left(3 - \frac{3}{2} \cdot \frac{3}{4}\right) \frac{3}{4} = \frac{24-9}{8} \cdot \frac{3}{4} = \frac{45}{32}$$

$$TR_2 = P_2 q_2 = \left(1 - \frac{1}{2} q_2\right) q_2 = \left(1 - \frac{1}{2} \cdot \frac{1}{4}\right) \frac{1}{4} = \frac{8-1}{8} \cdot \frac{1}{4} = \frac{7}{32}.$$

} 2 points

On the other hand,

$$TC_A = AC_A \cdot q_A = \left(\tfrac{1}{2} q_A\right) q_A = \left(\tfrac{1}{2} \cdot \tfrac{3}{4}\right) \tfrac{3}{4} = \tfrac{9}{32}$$

$$TC_B = AC_B \cdot q_B = \left(\tfrac{3}{2} q_B\right) q_B = \left(\tfrac{3}{2} \cdot \tfrac{1}{4}\right) \tfrac{1}{4} = \tfrac{3}{32}.$$

} 2 points

Hence

$$\pi = TR - TC = (TR_1 + TR_2) - (TC_A + TC_B)$$

$$= \frac{45 + 7 - 9 - 3}{32} = \frac{40}{32} = \frac{5}{4} = \underline{1.25}.$$

} 1 point

Since $\pi > 0$, producing is clearly better than shutting down. The long-run shut-down rule is simply "produce if $\pi \geq 0$," or "produce if $P > AC$." (2 points) Now actually, we should check all combinations: $P_1 > AC_A$, $P_1 > AC_B$, $P_2 > AC_A$, $P_2 > AC_B$. But since

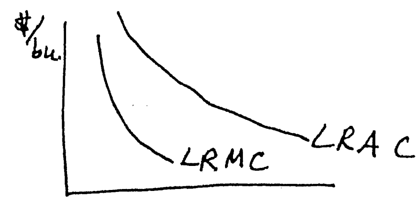
$$P_1 = \tfrac{15}{8} \qquad AC_A = \tfrac{3}{8}$$

$$P_2 = \tfrac{7}{8} \qquad AC_B = \tfrac{3}{8},$$

all of these inequalities will hold. If one of them didn't, we'd have to consider the effect of shutting down one or both of the plants or not selling in one or both of the markets, which gets much too complicated.

(12)

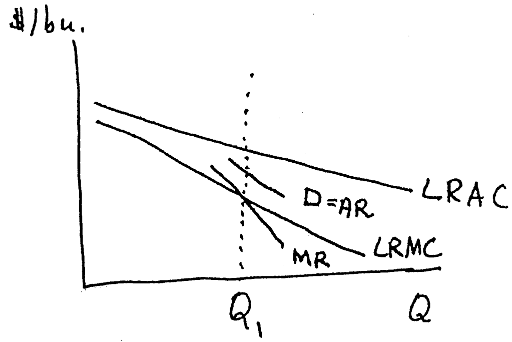
Increasing Returns to Scale $\Rightarrow$



AC: 3pts.
MC: 3pts.

(Should be like this in all the graphs below.)

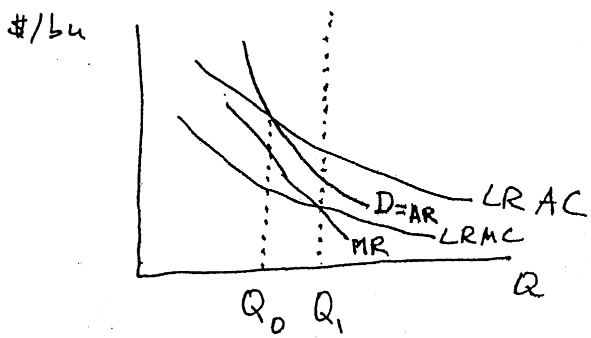
a)



MR 1pt
D 1pt
$\pi<0$ 4pts.

Setting MR = MC gives Q_1, but at Q_1, AR < AC so $\pi < 0$. When $Q = 0$, $\pi = 0$; so, it's better to shut down. (P < AC is another way to say this.)

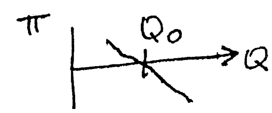
An aside: Can you figure out what is wrong with this graph?



Below Q_0, AR > AC, so $\pi > 0$.

Above Q_0, AR < AC, so $\pi < 0$.

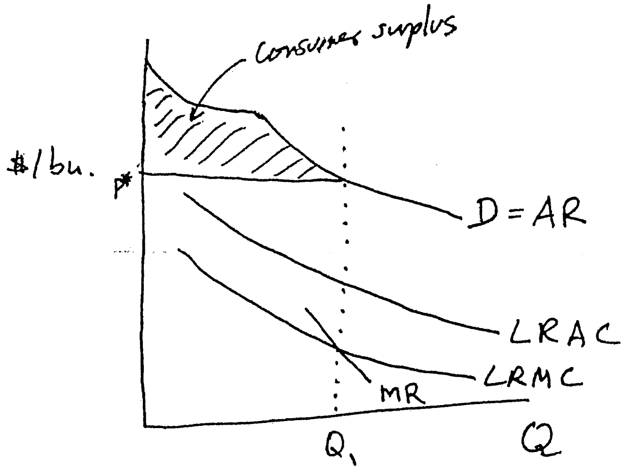
At Q_0, AR = AC, so $\pi = 0$.



Therefore at Q_0, $M\pi < 0 \Rightarrow MR < MC$. This contradicts the graph, because it shows $MR > MC$ at Q_0.

(No points were taken off if you drew an incorrect graph like this one.)

b)



MR 1pt
D 1pt
$\pi > 0$ 4pts

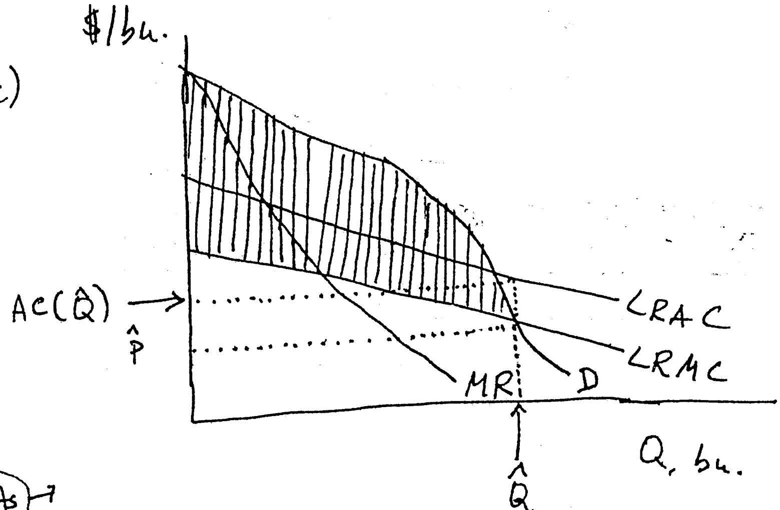
-2 for wrong Consumer surplus

At Q_1, $MR = MC$.

At Q_1, $AR > AC$ so $\pi > 0$ and the firm would want to produce.

($P > AC$ is another way to say this.)

c)



4pts

Below $\hat{Q}$, $D > MC$, so social surplus is positive.

For $Q > \hat{Q}$, $D < MC$, so social surplus is negative.

$\hat{Q}$ maximizes social surplus because it includes all of the positive area and none of the negative area.

d) $\hat{P}$ is the market-clearing price.

4pts $\hat{P} = MC$.

4pts $AR(\hat{Q}) < AC(\hat{Q})$

$= D(\hat{Q}) = \hat{P}$

e) At $\hat{Q}$, $\pi < 0$ since $AR < AC$.

If $Q = 0$ then $\pi = 0$.

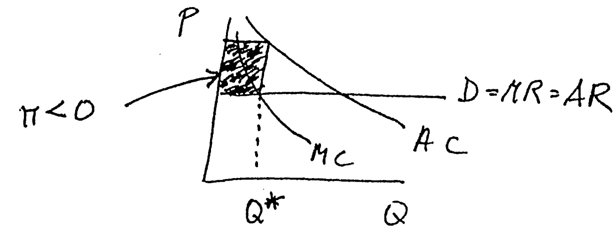
So the firm would want to shut down.

4pts

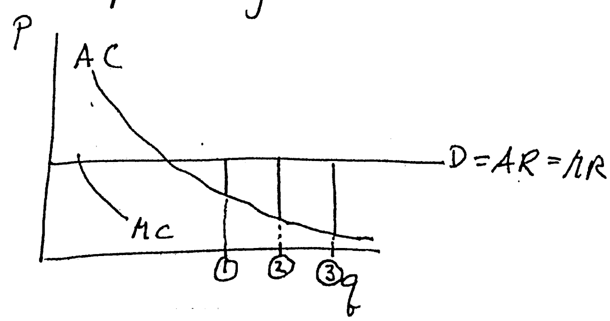
⑬ Since AC is downward sloping, this technology has increasing returns to scale. This implies that the long-run competitive output is zero:

↑ 4pts

↑ 4pts

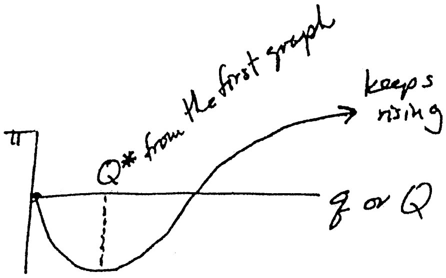


(It is also OK if you said that the LR competitive output of each firm is infinity, since as q increases, π keeps rising:



① small π
② bigger π
③ even bigger π ... etc. ...

I think the graph of π versus q (or Q) looks like this



.)

In any case, the technology cannot be given to competitive firms, because then $Q = 0$ (or $Q = \infty$, which would be hard to match with supply = demand).

9 pts..

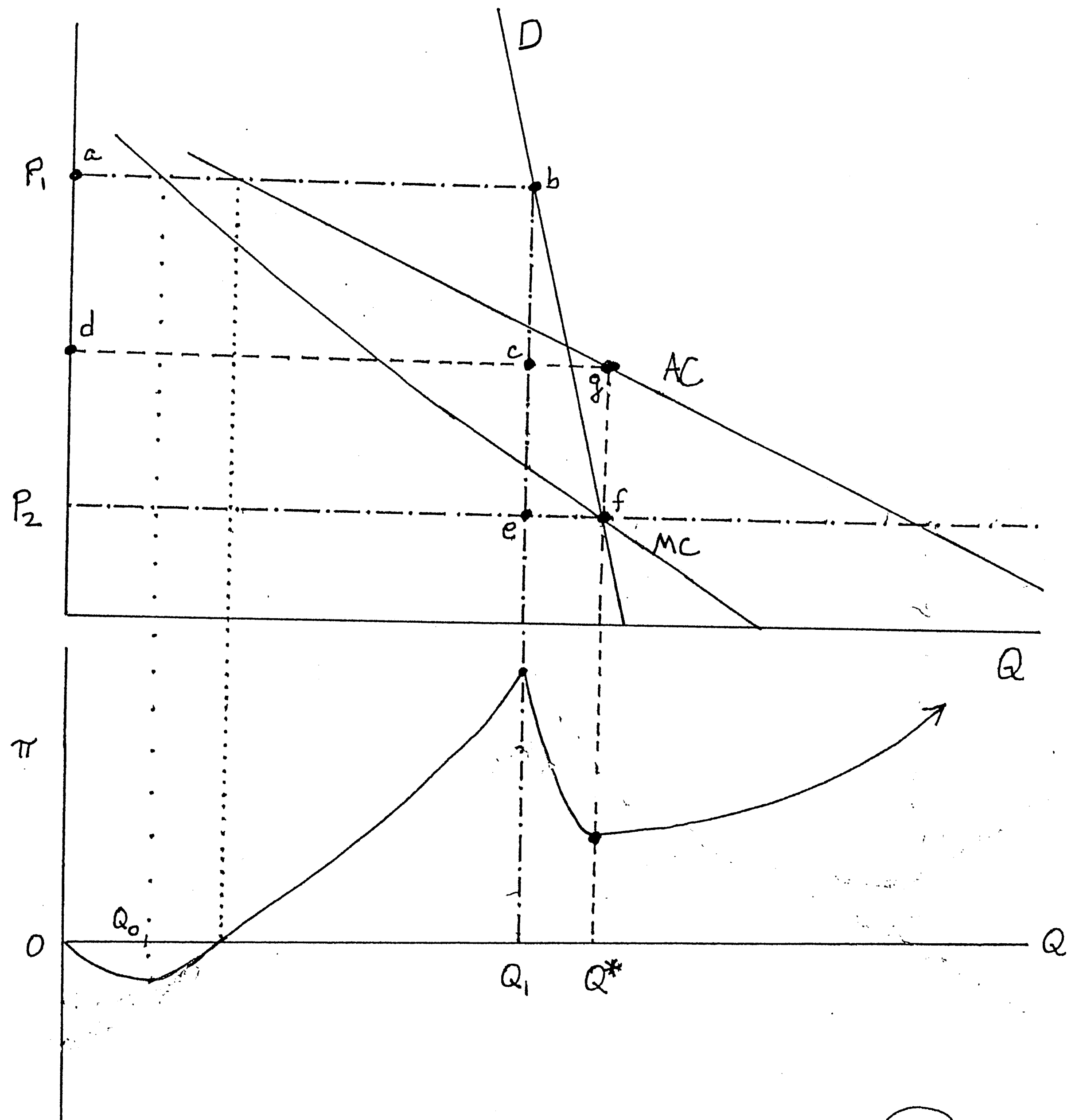
The significance of point C (and hence of $Q = 11$) is that if the technology could be given to competitive firms, then MC would be the industry S curve, and point C would be the point where supply equalled demand. This makes it desireable to find a way to convince the firm to produce Q = 11, if the firm is a publically regulated monopoly.

In the two-part pricing scheme shown, the firm charges \$4 (4pts.) to some consumers, who buy Q = 5. The firm charges \$1 (4pts.) to the remaining consumers, who buy 6 (= 11 - 5). Since we suppose that the government only allows the firm to charge \$4 or \$1, and it already has charged \$4 to everyone who'd buy at that price, all that remains is for the firm to charge \$1. Hence beyond Q = 5, the relevant MR = AR curve for the firm is a flat line at \$1; setting MR = MC makes producing Q = 11 be in the monopolist's best interest.

Q = 11 and so AC = \$2. Call the origin point "O." Then total revenue is:
O - H - A - the point marked "5" (from the consumers who pay \$4) plus
D - C - the point marked "11" - the point marked "5" (from the consumers who pay \$1).
Total cost is: F - B - the point marked "11" - O. Profit, then, is AGFH plus negative GBCD, or $(\$4 - \$2)(5) - (\$2 - \$1)(11 - 5) = 10 - 6 = \$4$.

4pts ↑ 4pts ↑

14



4pts

a) The monopolist is permitted to price-discriminate, charging p_1 to one group of consumers, who buy Q_1, and 4pts p_2 to the remainder, who buy $Q^* - Q_1$. Total production is at Q^*, 4pts since firms wish to produce Q^* (there, MC is equal to p_2, and p_2 is marginal revenue because the firm cannot change p_2 due to government regulation) and since consumers wish to buy Q^* at price p_2. At Q^*, average costs are given by the height of point g, while average revenue is p_2 for Q between Q_1 and Q^* (and beyond Q^*) and average revenue is p_1 for Q less than Q_1. The firm's positive profits are therefore abcd, (3pts) while its losses are cgfe. (3pts) If abcd is larger than cgfe, the monopolist will make positive profit. ← (2pts)

The increasing returns to scale assumption implied that AC was falling and that MC was below AC. ← (3pts)

b) Below Q_0, price P_1 is less than MC. Because the price is regulated at P_1, price is marginal revenue. So below Q_0, MR < MC and π is falling.

Between Q_0 and Q_1, $MR = P_1 > MC$, so π is rising. (Where $P_1 = AR$ is equal to AC, $\pi = 0$.)

Between Q_1 and Q^*, $MR = P_2 < MC$, so π is falling.

For $Q > Q^*$, $MR = P_2 > MC$, so π is rising. (Alternatively, if the monopolist doesn't believe the demand curve is flat at $p = P_2$ for $Q > Q^*$ but instead believes the market demand curve is the true market demand curve "D", then MR is negative for $Q > Q^*$, and the profit function jumps to below $\pi = 0$ at Q^* and then is downward-sloping.)

	pts
π increasing just to the left of Q_1	4
" decreasing " " " right " Q_1	4
correct graph near $Q = 0$	1
correct graph for $Q > Q^*$	1