D. Market Demand and Elasticity

1. Suppose that Mr. D's demand curve for good X is given in Figure 2. Also suppose that the price of X is less than $1.

   (a) In what numerical range is the price elasticity of demand for X,

   $$e_{X,P_X} = \frac{\%\Delta Q_X^D}{\%\Delta P_X} ?$$

   (b) Suppose the cross-price elasticity

   $$e_{X,P_Y} = \frac{\%\Delta Q_X^D}{\%\Delta P_Y} = +1.3 .$$

   Is the income elasticity of X,

   $$e_{X,I} = \frac{\%\Delta Q_X^D}{\%\Delta I} ,$$

   positive or negative? Remember to use part (a) in answering this.

   (c) If the income-elasticity of a good is positive, is the good normal, inferior, or can you not tell?

2. Suppose a consumer only consumes two goods, X and Y, and he is never satiated in either of the two goods. Suppose the income elasticity of the demand for X is -1 and the cross-price elasticity of the demand for X with respect to the price of Y is -2.

   (a) Is X normal, inferior, or can you not tell? Why?

   (b) Is X Giffen, not Giffen, or can you not tell? Why? (Hint: Find the own-price elasticity of X.)

3. From observations of a consumer who only buys two goods X and Y, a student in an econometrics class calculates the following elasticities:

   $$\frac{\%\Delta Q_x^D}{\%\Delta p_x} = -0.4 \qquad \frac{\%\Delta Q_x^D}{\%\Delta p_y} = +0.7$$
   $$\frac{\%\Delta Q_y^D}{\%\Delta p_x} = -0.1 \qquad \frac{\%\Delta Q_y^D}{\%\Delta p_y} = +0.3.$$

   Assume the consumer always prefers more of both goods.

(a) Calculate the income elasticities for X and Y. Then explain why the econometrics student *must* have calculated the elasticities wrong.

(b) Suppose three of the four elasticities are correct. Which elasticity is probably the one that is wrong? What values for that elasticity might be correct?

4. Suppose a consumer only consumes three goods, X, Y, and Z. Furthermore, suppose that this consumer's income elasticity for X is $-7/10$; his cross-price elasticity of demand for X with respect to the price of Y is $1/5$; and his cross-price elasticity of demand for X with respect to the price of Z is $-2/5$.

   For this consumer, is X a Giffen good?

5. In a two-good economy with goods called X and Y, suppose the income elasticity of demand for X is -0.4 and the own-price elasticity of demand for X is -0.2. What is the cross-price elasticity of demand for X? Explain carefully!

6. Suppose that Person 1's demand for good X is given by

$$Q_{D1} = 100 - 2P$$

   and Person 2's demand for good X is given by

$$Q_{D2} = 160 - 4P\,,$$

   where P is the price of X.

   (a) Putting X on the horizontal axis and P on the vertical axis (as usual), sketch the graph of the two persons' demand curves. You may use one graph or two. Be sure to label any intercepts of the demand curves with the axes with the appropriate numbers.

   (b) Sketch the graph of the market demand curve if Person 1 and Person 2 are the only two people in the market for X. Again, label the points (if any) where the curve hits each axis. Be sure to explain what you are doing.

7. Suppose the quantity of good Y demanded by individual 1 is given by

$$Y_1 = \begin{cases} 24 - 2P_Y & \text{for } P_Y < 12 \\ 0 & \text{for } P_Y \geq 12 \end{cases}$$

and the quantity of Y demanded by individual 2 is

$$Y_2 = \begin{cases} 23 - P_Y & \text{for } P_Y < 23 \\ 0 & \text{for } P_Y \geq 23. \end{cases}$$

(a) Draw one graph showing both the demand curve of individual 1 and the demand curve of individual 2.

(b) On the graph you drew in part (a), draw the market demand curve for individual 1 plus individual 2. Explain briefly.

(c) Find the algebraic equation for the market demand curve you drew in part (b).

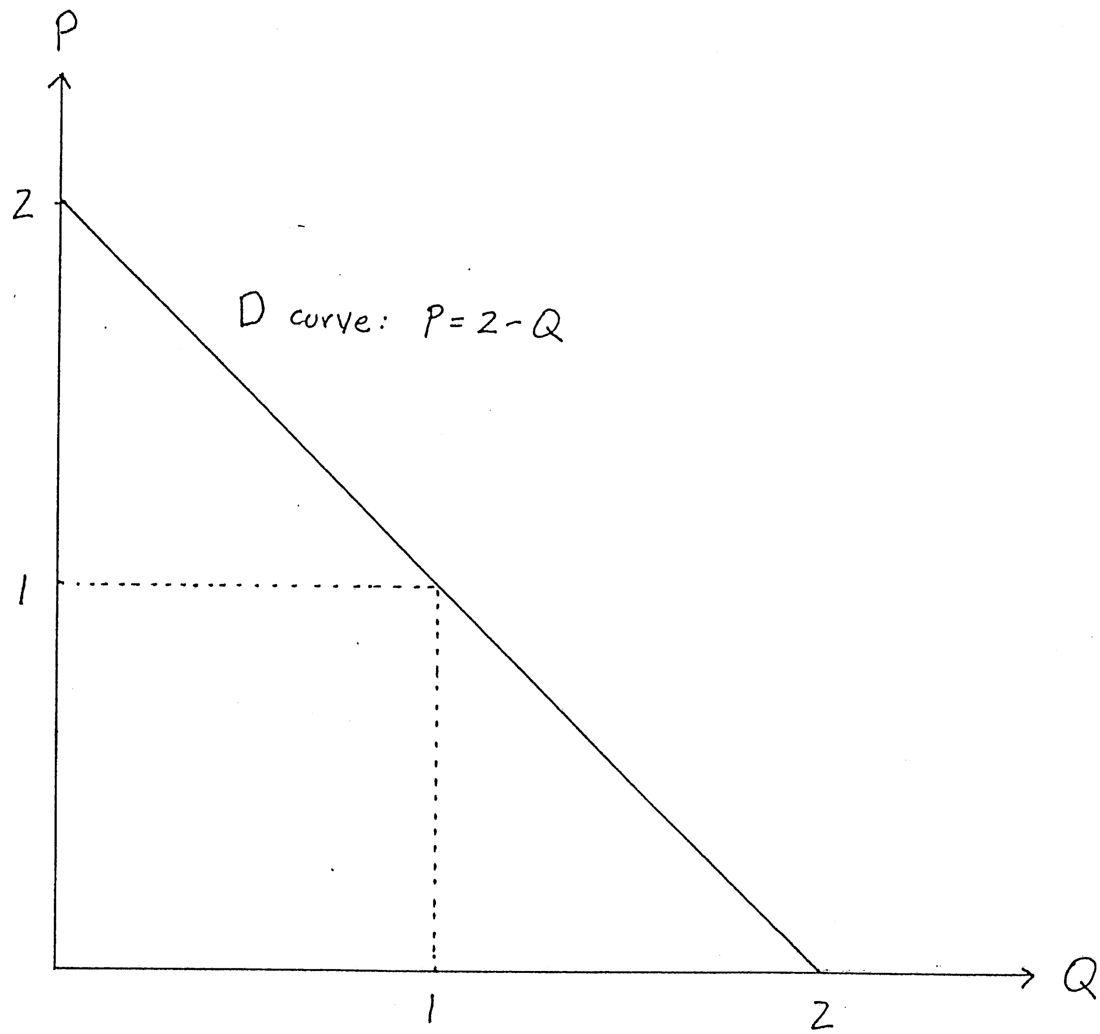
8. Suppose person A's demand curve for good x is $P = 2 - Q$ and person B's demand curve for good x is $P = 4 - 2Q$, where "P" stands for the price of x and "Q" stands for the quantity of x. What is the market demand curve for x if persons A and B are the only consumers in the market? (You should give me the answer both as an algebraic equation and as a graph).

9. Consumer A has a demand curve for good Q of the form $p = 2 - 2Q_A$ where p is the price of the good. Consumer B has a demand curve for good Q of the form $p = 3 - Q_B$.

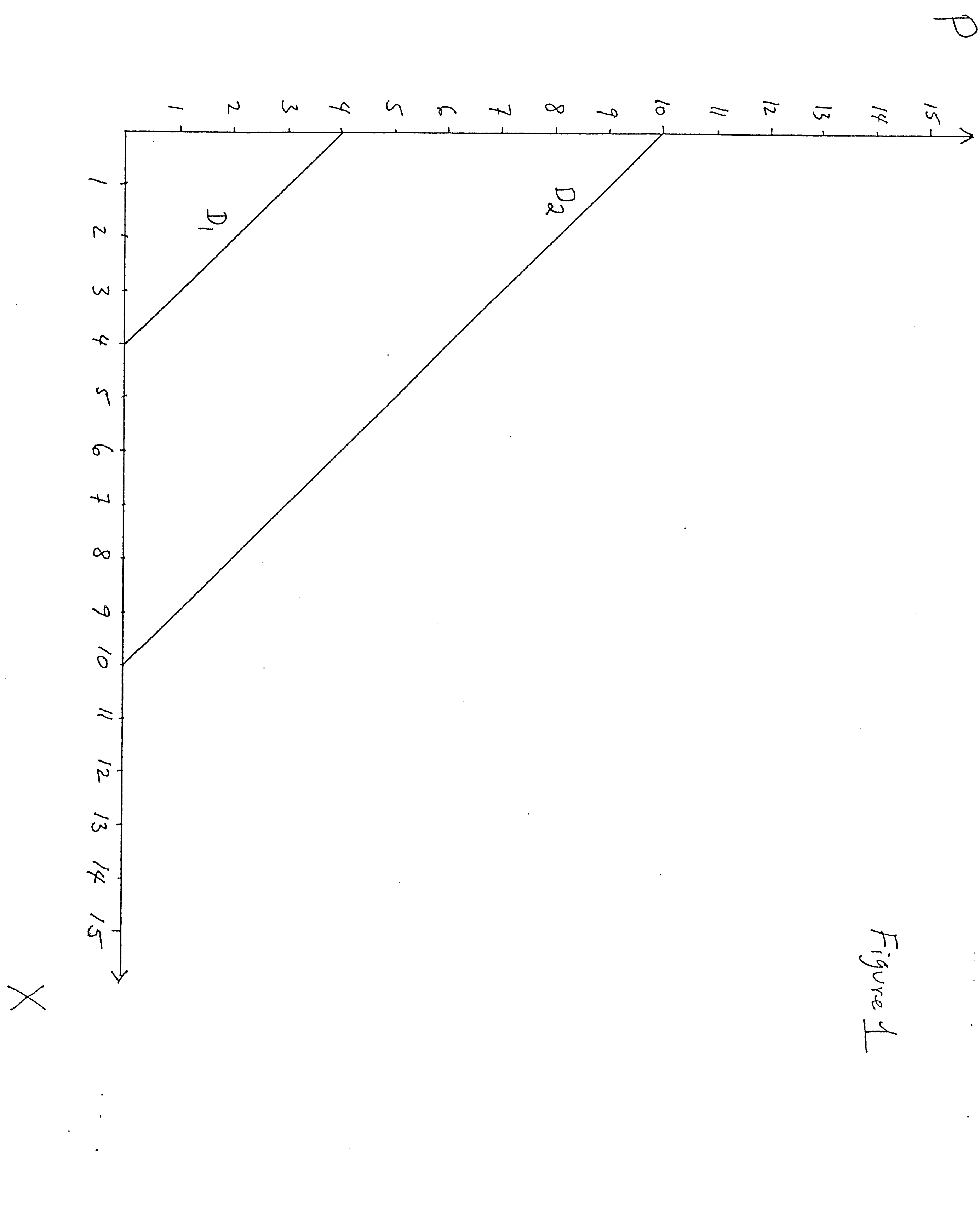
   (a) Sketch the demand curves of A, B, and the sum of A and B.

   (b) Give the algebraic formula for the sum of A and B's demand curves. (A few of you may want to use the fact that if two points (x_1, y_1) and (x_2, y_2) are on a straight line, then the equation for the line is $\frac{y - y_2}{x - x_2} = \frac{y_2 - y_1}{x_2 - x_1}$.)

10. Person 1's demand curve for good X is shown in Figure 1. Person 2's demand for good X is also shown in Figure 1. If Person 1 and Person 2 are the only consumers in the market, graph the market demand curve for X. (It should be convenient to do this on Figure 1 itself.)



Question 1's Figure 2.



Question 10's Fig. 1

<u>Answers</u>

① a. This is the inelastic portion of the demand curve, so

$$-1 < e_{X, P_X} < 0. \qquad (8 \text{ points})$$

b. An $x\%$ fall in all prices is equivalent to an $x\%$ rise in income. For concreteness here, suppose $x=3$. Then a 3% fall in P_x together with a 3% fall in P_y is equivalent to a 3% rise in income.

• 3% ↓ in P_x: $e_{X, P_X} = \dfrac{\% \Delta Q_X^D}{\% \Delta P_X} \Rightarrow$

$$\Delta Q_X^D = e_{X, P_X} \cdot \% \Delta P_X = (\text{between } \underset{\text{from part (a)}}{-1 \text{ and } 0}) \cdot (-3)$$

$$= \underline{\text{between } 3 \text{ and } 0.}$$

• 3% ↓ in P_y: $e_{X, P_y} = \dfrac{\% \Delta Q_X^D}{\% \Delta P_y} \Rightarrow$

$$\underline{\Delta Q_X^D = e_{X, P_y} \cdot \% \Delta P_y = +1.3 \cdot (-3) = -3.9.}$$

• 3% ↑ in income: $e_{X, I} = \dfrac{\% \Delta Q_X^D}{\% \Delta I}$

$$= \frac{(\% \Delta Q_X^D \text{ due to } 3\% \downarrow P_X) + (\% \Delta Q_X^D \text{ due to } 3\% \downarrow P_y)}{3\% \uparrow \text{ in income}}$$

$$= \frac{(\text{between } 3 \text{ and } 0) + (-3.9)}{+3} = \frac{\text{between } -0.9 \text{ and } -3.9}{3}$$

$$= \text{between } -0.3 \text{ and } -1.3 \; \underline{\underline{< 0}}. \qquad (8 \text{ points})$$

c) A good is normal if $\frac{\Delta QD}{\Delta I} > 0$.

" " " inferior " $\frac{\Delta QD}{\Delta I} < 0$.

Income elasticity is $\frac{\% \Delta QD}{\% \Delta I}$, which has the same sign as $\frac{\Delta QD}{\Delta I}$. Therefore, a positive income elasticity implies a normal good. (9 points)

② $e_{x,I} = -1$

$e_{x,P_y} = -2$

a) X is inferior because as income I increases, purchases of X drop. This is because $-1 = e_{x,I} = \dfrac{\%\Delta Q^D_x}{\%\Delta I}$, so if $\%\Delta I > 0$ (that is, $I\uparrow$) then $\%\Delta Q^D_x < 0$ (that is, $Q^D_x \downarrow$).

points
3 definition of inferior
3 definition of inc. elasticity
3 reasoning for answer

b) If X is Giffen then when $P_x\uparrow$, $Q^D_x\uparrow$. So Giffen goods have an own-price elasticity $e_{x,P_x} = \dfrac{\%\Delta Q^D_x}{\%\Delta P_x}$ which is positive.

3pts

3pts

To find the own-price elasticity of X, use the fact that a 5% ↑ in I is equivalent to a 5% ↓ in both prices.

3pts for approach

a: If I ↑ by 5% then $e_{x,I} = -1$

$$\frac{\%\Delta Q^D_x}{\%\Delta I} = -1 \Rightarrow \frac{\%\Delta Q^D_x}{5} = -1 \Rightarrow \%\Delta Q^D_x = -5.$$

4pts

b: If P_y ↓ by 5% then $e_{x,P_y} = -2$

$$\frac{\%\Delta Q^D_x}{\%\Delta P_y} = -2 \Rightarrow \frac{\%\Delta Q^D_x}{-5} = -2 \Rightarrow \%\Delta Q^D_x = +10$$

4pts

If P_x ↓ by 5% then

$$\frac{\%\Delta Q^D_x}{\%\Delta P_x} = e_{x,P_x} \Rightarrow \frac{\%\Delta Q^D_x}{-5} = e_{x,P_x} \Rightarrow \%\Delta Q^D_x = (-5)(e_{x,P_x})$$

ⓐ and ⓑ are equivalent, so

$$-5 = +10 + (-5)(e_{x,P_x})$$
$$-15 = (-5)\, e_{x,P_x}$$
$$3 = e_{x,P_x}. \leftarrow \text{6pts}$$

So $e_{x,P_x} > 0$ and X is Giffen. ← 1 point

You did not have to do this by finding e_{x,P_x}.
(Following "hints" on exams is optional.)
But if you did it another way, assign 21 possible points to that approach.

③ a) A 1% rise in income is equivalent to a 1% fall in both prices. A 1% fall in P_x causes Q_x^D to rise by 0.4%. A 1% fall in P_y causes Q_x^D to fall by 0.7%. So a 1% fall in both prices together would result in a fall in Q_x^D by $+0.4\% - 0.7\% = -0.3\%$. Therefore $\frac{\%\Delta Q_x^D}{\%\Delta I} = -0.3$, the income elasticity of X.

The income elasticity of Y can be calculated similarly. A 1% fall in both prices causes a $\%\Delta Q_y^D = -0.3 + 0.1 = -0.2$, where Q_y^D falls by 0.3% because of the change in P_y, and it rises by 0.1% because of the change in P_x.
Therefore $\frac{\%\Delta Q_y^D}{\%\Delta I} = -0.2$, the income elasticity of Y.

pts
8 basic reasoning here ↑
4 answer for X
4 answer for Y

Both X and Y thus have negative income elasticities (they are inferior goods).
6 pts → This must be wrong because it is impossible for all goods to be inferior: if they were, with more income consumers would choose to buy less of everything. Such behavior is inconsistent (they could have chosen to buy less of everything before their income went up, so why didn't they buy less of everything then?), and it violates the assumption that the consumer always prefers more of both goods.

b) The elasticity $\frac{\% \Delta Q_y^D}{\% \Delta P_y} = +0.3$ is probably wrong because it implies that

6pts

Y is a Giffen good (as its price goes up, you buy more of it). From part (a), the income elasticity of Y is $\left[-\frac{\% \Delta Q_y^D}{\% \Delta P_y} + 0.1\right]$. To make this positive, one needs $\frac{\% \Delta Q_y^D}{\% \Delta P_y}$ to be less than 0.1 (instead of 0.3, as it is given in the problem).

5pts

(4) A fall in all prices is equivalent to a rise in income of the same amount.

For example, a 10% $\downarrow$ in P_x and a 10% $\downarrow$ in P_y and a 10% $\downarrow$ in P_z, all together, is equivalent to a 10% $\uparrow$ in I.

quantity demanded of X

10% $\downarrow$ in P_z with $e_{x,P_z} = \frac{-2}{5} = \frac{\%\Delta Q_x^D}{\%\Delta P_z} = \frac{\%\Delta Q_x^D}{-10}$

$$\Rightarrow \%\Delta Q_x^D = \frac{-2}{5}(-10) = +4$$

10% $\downarrow$ in P_y with $e_{x,P_y} = \frac{+1}{5} = \frac{\%\Delta Q_x^D}{\%\Delta P_y} = \frac{\%\Delta Q_x^D}{-10}$

$$\Rightarrow \%\Delta Q_x^D = \frac{1}{5}(-10) = -2$$

10% $\downarrow$ in P_x with $e_{x,P_x} = \frac{\%\Delta Q_x^D}{\%\Delta P_x} = \frac{\%\Delta Q_x^D}{-10}$

$$\Rightarrow \%\Delta Q_x^D = -10 \cdot e_{x,P_x}$$

Is equivalent to

10% $\uparrow$ in I with $e_{x,I} = \frac{-7}{10} = \frac{\%\Delta Q_x^D}{\%\Delta I} = \frac{\%\Delta Q_x^D}{+10}$

$$\Rightarrow \%\Delta Q_x^D = \frac{-7}{10}(10) = -7.$$

(Note: e_{X,P_Z} is the cross-price elasticity of demand for X with respect to the price of Z; similarly with e_{X,P_Y}, e_{X,P_X}, and $e_{X,I}$.)

So: $\begin{pmatrix} \%\Delta Q^D_X = +4 \\ \%\Delta Q^D_X = -2 \\ \%\Delta Q^D_X = -10\, e_{X,P_X} \end{pmatrix}$ is equivalent to $\left(\%\Delta Q^D_X = -7\right)$.

pts	
3	Basic equivalence of ↓ p's with ↑ I
1	deduction from e_{X,P_Z}
1	" " e_{X,P_Y}
1	" " $e_{X,I}$
2	answer

$$+4 - 2 - 10\, e_{X,P_X} = -7$$

$$9 = 10\, e_{X,P_X}$$

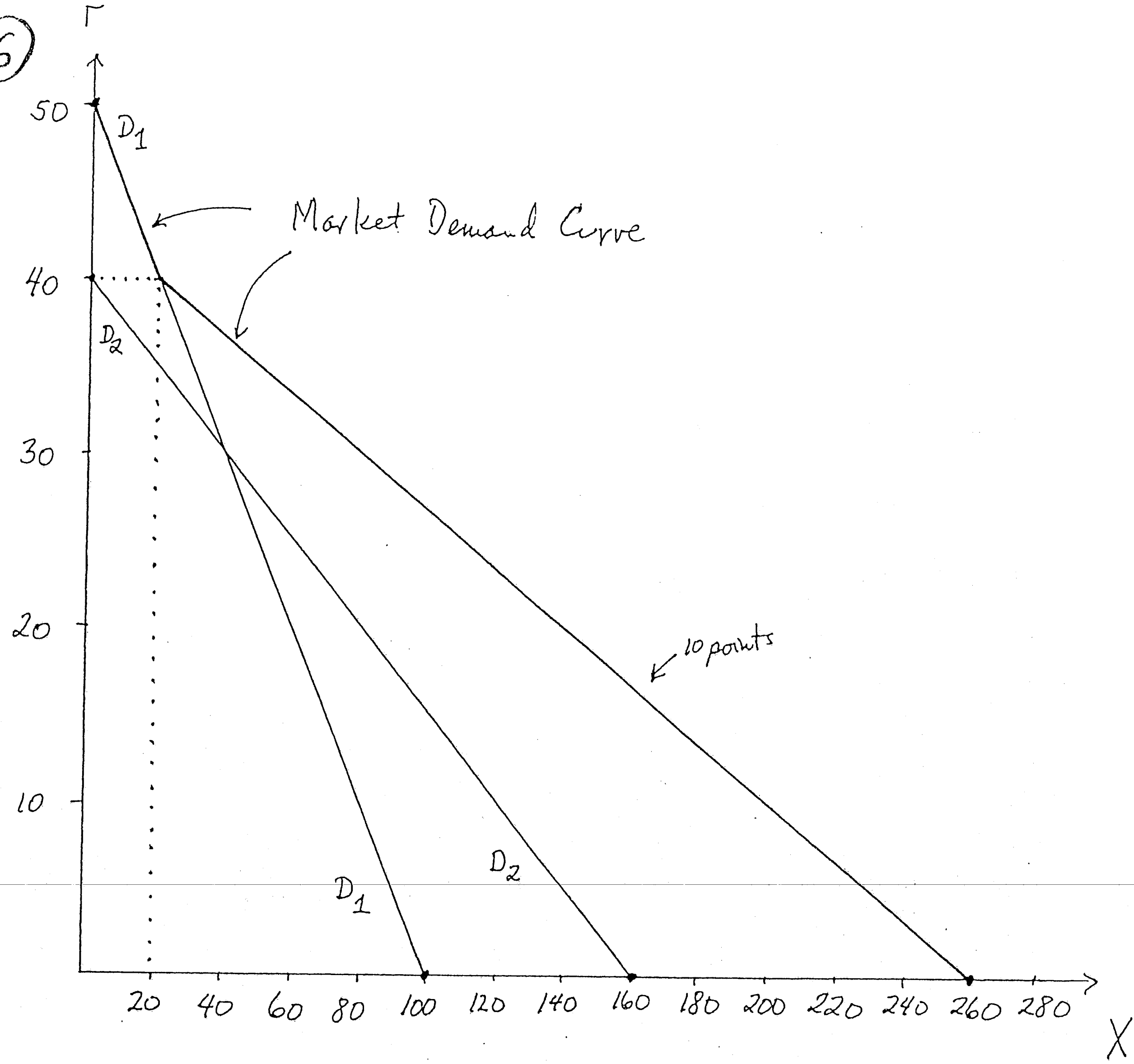
$$\frac{9}{10} = e_{X,P_X}$$

So $e_{X,P_X} = \frac{9}{10}$ and X is therefore Giffen (since $e_{X,P_X} > 0$).

(5) As explained in class, a 10% rise in income is (almost) equivalent to a 10% fall in all prices. ←9pts Since $e_{X,I} = \frac{\%\Delta Q^D_X}{\%\Delta I} = -0.4$, ←3pts if $\%\Delta I = +10$ then $\%\Delta Q^D_X = -4$. 4pts ↓ So the effect of a 10% drop in P_X plus the effect of a 10% drop in P_Y should be $\%\Delta Q^D_X = -4$. Now the effect of a 10% drop in P_X is: $e_{X,P_X} = \frac{\%\Delta Q^D_X}{\%\Delta P_X} = \frac{\%\Delta Q^D_X}{-10} = -0.2$, ←3pts so that $\%\Delta Q^D_X = +2$. ←4pts Therefore the effect of a 10% drop in P_Y must be $\%\Delta Q^D_X = -6$ (since $+2-6=-4$). ←5pts Hence

$$e_{X,P_Y} = \frac{\%\Delta Q^D_X}{\%\Delta P_Y} = \frac{-6}{-10} = \boxed{0.6}.$$ ←5pts

(6)



a) $Q_{D1} = 100 - 2P$ is linear with intercepts $P = 0 \Rightarrow Q = 100$, 6 points

$Q = 0 \Rightarrow P = 50$.

$Q_{D2} = 160 - 4P$ is linear with intercepts $P = 0 \Rightarrow Q = 160$, 6 points

$Q = 0 \Rightarrow P = 40$.

b)

At a price greater than 40, $Q_{D2} = 0$. Hence for $P > 40$,

market quantity $D_{1+2} = D_1$. At a price below 40, however, the two

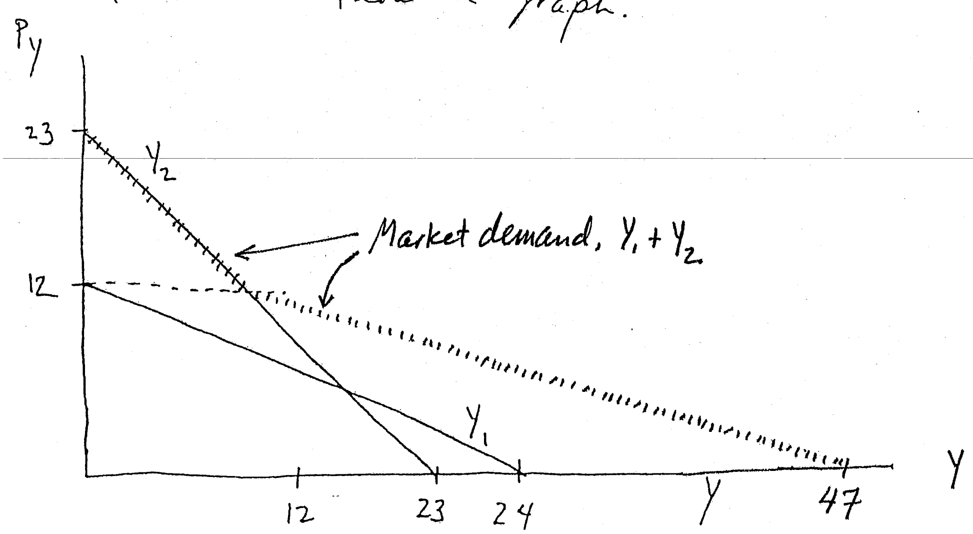
11 points

demand curves add. For example, if $P = 0$ then $Q_{D1} = 100$ and $Q_{D2} = 160$, so the total Q is 260. Algebraically, the market demand curve is:

$$Q_{D(1+2)} = \begin{cases} 100 - 2P & \text{for } P > 40 \\ 260 - 6P & \text{for } 0 < P \le 40. \end{cases}$$

(For $P > 50$, $Q_{D(1+2)} = 0$, of course.)

(7) This is hard to do without a graph.



(12) → For $12 \le P_Y \le 23$, market D for $Y = Y_2 = 23 - P_Y$

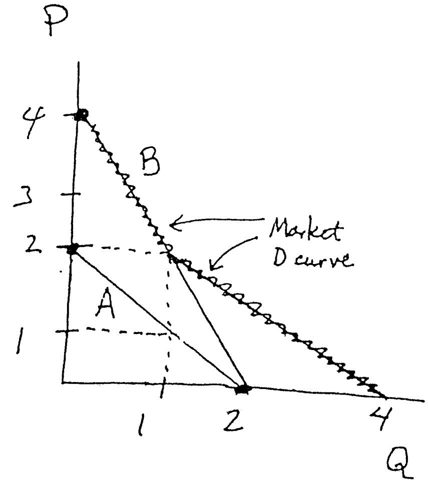
(8) → For $0 \le P_Y < 12$, — " — $= Y_1 + Y_2 = 47 - 3P_Y$.

If $P_Y > 23$, neither individual buys Y, so demand is zero.

For P_Y between 12 and 23, only individual 2 buys Y, so his demand is the market demand.

For $P_Y < 12$, the market demand consists of positive (i.e., nonzero) demand by both individuals.

(8)



$A: P = 2 - Q$

$B: P = 4 - 2Q$

Person A's demand curve goes between $Q=0$, $P=2$ and $P=0$, $Q=2$. Person B's demand curve has $P=4$ when $Q=0$, and $P=0$ when $Q=2$. So A and B's curves are as sketched on the left.

Above \$2, B is the only one in the market, so above \$2, the market demand curve is $P = 4 - 2Q$. This is the same as

$$2Q = 4 - P$$

$$Q = 2 - \tfrac{1}{2}P.$$

Below $p = \$2$, Person A comes into the market. For instance, at $p = 1$, $Q^D_A = 1$, $Q^D_B = 2 - \tfrac{1}{2} = 1\tfrac{1}{2}$, and market demand is $2\tfrac{1}{2}$. At $p = 0$, $Q^D_A = 2$, $Q^D_B = 2$, so market demand is 4. This is indicated by the line marked . Algebraically, for person A, $Q = 2 - P$, whereas for person B,

$Q = 2 - \tfrac{1}{2}P$. The sum is

Market $Q = 4 - \tfrac{3}{2}P$. Check: at $P = 2$, is Market $Q = 1$?

Check: at $P = 0$, is Market $Q = 4$?

Check: at $P = 1$, is Market $Q = 2\tfrac{1}{2}$?

over→

The answer, then, is

$$Q = 2 - \tfrac{1}{2}P \quad \text{if } P > 2$$

$$Q = 4 - \tfrac{3}{2}P \quad \text{if } P < 2$$

(and either one if $P = 2$).

Although I would not recommend it, this can be rewritten. $Q = 2 - \frac{1}{2}P$ is equivalent to $P = 4 - 2Q$, and $Q = 4 - \frac{3}{2}P$ is equivalent to $2Q = 8 - 3P \Rightarrow 3P = 8 - 2Q \Rightarrow P = \frac{8}{3} - \frac{2}{3}Q$. Also, at $P = 2$, Q is equal to 1. So you could write the market demand curve as

$$P = 4 - 2Q \quad \text{if } Q < 1$$

$$P = \tfrac{8}{3} - \tfrac{2}{3}Q \quad \text{if } Q > 1$$

(and either one if $Q = 1$).

Points

a) 4 sketch of A's demand curve

b) 5 " " B's " "

c) 5 " " mkt. " " above \$2

d) 5 algebraic description of mkt. D curve above \$2

e) 7 sketch of mkt. D curve below \$2

f) 7 algebraic description of mkt. D curve below \$2

⑨ a) The graphs are shown on the next page. One has the following demands:

P	D by A	D by B	total D
≥ 3	0	0	0
2	0	1	1
1	$\frac{1}{2}$	2	$2\frac{1}{2}$
0	1	3	4

This results in the horizontal addition of demand curves shown. Note the kink point at $P=2, Q=1$; to the left of it, the slope of the market D curve is -1, but to the right it is $\frac{-2}{3}$.

3pts 5pts 1pt

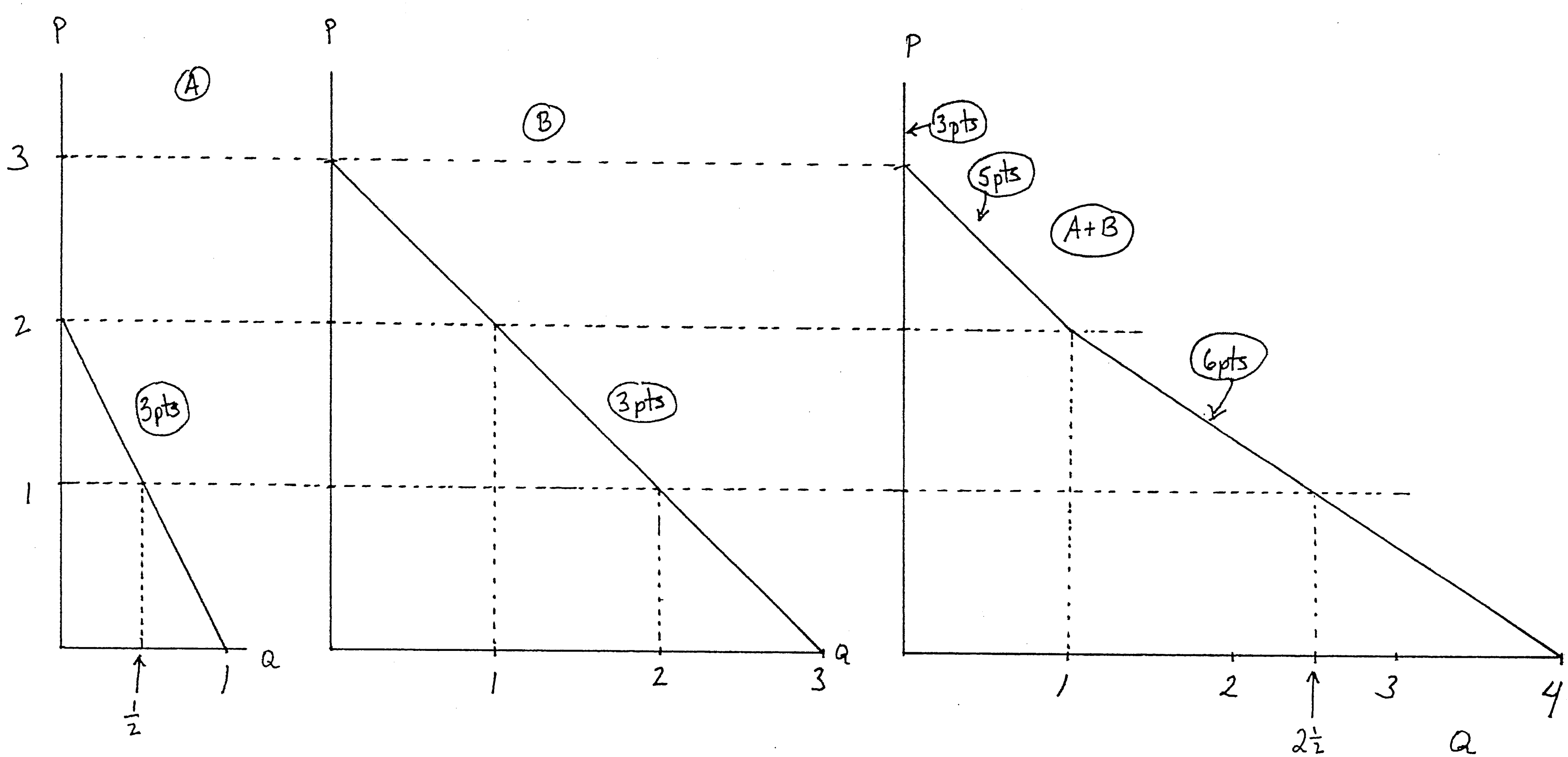
b) For $P \geq 3$, $Q=0$. For $2 \leq P < 3$, $Q = Q_B = 3-p$. For $0 \leq P < 2$, two points on the demand curve are $(1,2)$ and $(4,0)$ (in (q,p) form), so using the formula $\frac{X-X_2}{Y-Y_2} = \frac{X_1-X_2}{Y_1-Y_2}$ yields $\frac{Q-4}{p-0} = \frac{1-4}{2-0}$, replacing x with Q and Y with → over two pages

4 pts for this or any other correct, unsimplified form

2pts (or solve for p)

p. So $\frac{Q-4}{P} = \frac{-3}{2}$, $Q-4 = \frac{-3}{2}P$, and $Q = \frac{-3}{2}P+4$. Putting all this together,

$$Q = \begin{cases} 0 & \text{for } P \geq 3 \\ 3-p & \text{for } 2 \leq P < 3 \\ 4-\frac{3}{2}p & \text{for } 0 \leq P < 2. \end{cases}$$



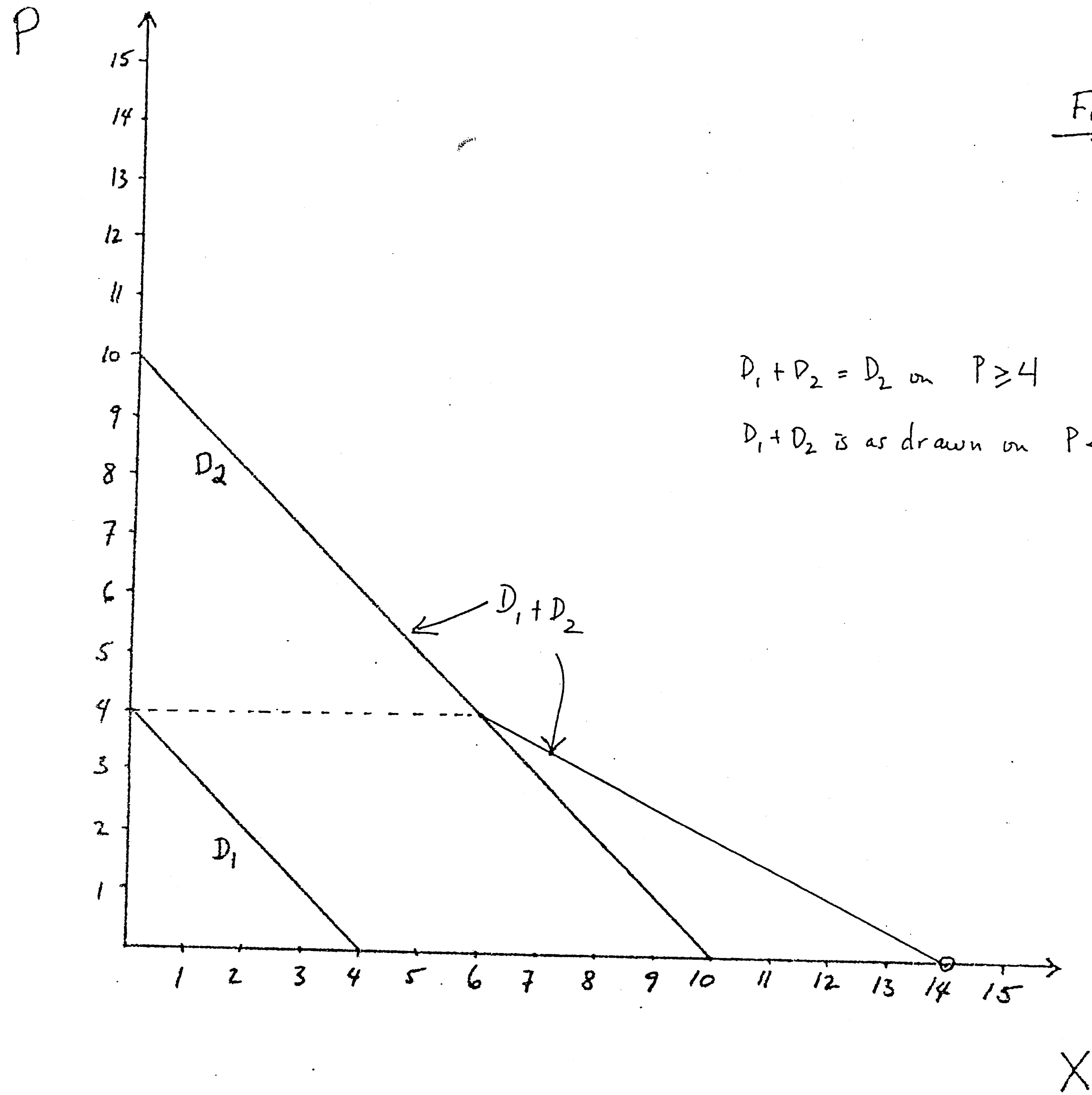
P
A
3
2
1
3pts
Q
1
1/2
P
B
3pts
Q
1
2
3
P
3pts
5pts
A+B
6pts
1
2
3
4
2 1/2
Q

(10) For $P \geq 4$, $D_1 = 0$. So $D_1 + D_2 = D_2$. (13)

For $P < 4$, $D_1 > 0$. At $P = 0$, $D_1 = 4$ and $D_2 = 10$, so $D_1 + D_2 = 14$. (6)

Between $P = 4$ and $P = 0$, $D_1 + D_2$ is a straight line. (6) (OK if just drew correct general shape - flatter ~~line~~ below $P = 4$)

Figure 1



$D_1 + D_2 = D_2$ on $P \geq 4$

$D_1 + D_2$ is as drawn on $P < 4$.