## G. Cost Functions

1. Explain everything you know about possible shapes of the expansion path if a firm's isoquants are parallel straight lines.

2. Putting water $W$ on the horizontal axis and fertilizer $F$ on the vertical axis, sketch a situation in which the firm's expansion path is the $W$ axis. Is the Rate of Technical Substitution of Water for Fertilizer less than, greater than, or equal to the price of water divided by the price of fertilizer in this case? Assume the Rate of Technical Substitution is constant in drawing your graph.

3. A firm uses water $W$ and fertilizer $F$ to produce corn $Q$. More fertilizer always results in more corn, but if more than 100 gallons of water is applied, more water starts to cause corn production to fall. Sketch the isoquant map and the expansion path under these conditions. Argue for or against the following statement: "the expansion path may go above the line $W = 100$."

4. Suppose a firm's production function is given by

$$Q = \sqrt{FW}$$

where $F$ is fertilizer, measured in pounds, $W$ is water, measured in gallons, and $Q$ is corn, measured in bushels.

   (a) Find the average product of fertilizer, $AP_F$, and the average product of water, $AP_W$.

   (b) For this particular function $Q = \sqrt{FW}$ it can be shown (using advanced techniques you are not expected to know) that the marginal product of water, $MP_W$, is equal to $(1/2)AP_W$, and that the marginal product of fertilizer, $MP_F$, is equal to $(1/2)AP_F$. Using this information and your answer to part (a), find the rate of technical substitution of water for fertilizer.

   (c) If the price of water is \$10/gallon and the price of fertilizer is \$40/pound, what is the optimum ratio $F/W$?

   (d) Using the answer in (c), if $Q = 6$ how much $F$ and $W$ will the firm buy?

5. Fact: If firms take input prices as given then isocost curves are straight lines.

Fact: If firms do not take input prices as given then isocost curves are not straight lines.

If firms do not take input prices as given then input prices could either fall as quantity purchased rises—a "volume discount"—or input prices could rise as quantity purchased rises—a "volume surcharge."

(a) Does Figure 1 represent a volume discount or a volume surcharge? Does Figure 2 represent a volume discount or a volume surcharge? Hint: starting from a point like $N$ or $M$, is the firm in a better or worse situation than it would be if the isocost curves were straight? If the firm is in a better situation, it is getting a volume discount; if the firm is in a worse situation, it is paying a volume surcharge.

(b) Is $A$ a cost-minimizing point on Figure 1? Why or why not?

(c) Is $B$ a cost-minimizing point on Figure 2? Why or why not?

6. Label all the axes and all the curves in Figure 1. If an arrow goes from one figure to another it means that the second can be derived from the first. If a dotted arrow goes from one figure to another it means that part of the second can be derived from the first. Briefly justify each of your answers. In the case of graphs 1 to 4 and graph 9, a one or two sentence explanation should do. In the case of graphs 5 to 8, you should explain how they are derived from graphs 1 to 4. Drawing a few lines on graphs 1 to 4 is necessary for this, but long explanations are not required—again, two or three sentences will be enough, at least for graphs 5, 6, and 7.

7. Label all the axes and all the curves in Figure 1. If an arrow goes from one figure to another it means that the second can be derived from the first. If a dotted arrow goes from one figure to another it means that part of the second can be derived from the first. Briefly justify each of your answers. In the case of graphs 1 to 4 and graph 9, a one or two sentence explanation should do. In the case of graphs 5 to 8, you should explain how they are derived from graphs 1 to 4. Drawing a few lines on graphs 1 to 4 is necessary for this, but long explanations are not required—again, two or three sentences will be enough, at least for graphs 5, 6, and 7.

8. A farmer faces a price of \$12 for each unit of water he buys and \$6 for each unit of fertilizer he buys. An economist notices that the farmer

chooses to buy 2 units of water and 2 units of fertilizer, and from this the farmer grows 5 bushels of wheat.

(a) What is the value of the farmer's cost function at $Q = 5$ bushels of wheat?

(b) Make a graph showing everything you know about this farmer's $Q = 5$ isoquant.

9. Suppose the production function for pencils is given by $Q = \min(6G, W)$ where $Q$ stands for the quantity of pencils produced, $G$ stands for graphite (or "lead") and $W$ stands for wood. Graphite costs 10 cents per ounce and wood costs 2 cents per ounce. What is the firm's total cost function? What is its average cost function? (Hint: how much does it cost to produce one pencil?)

10. Let $W$ stand for water and $F$ stand for fertilizer. Suppose a firm's production function is $Q = \sqrt{WF}$, and suppose $F$ is fixed in the short run.

(a) Sketch the total product of water curve, putting $Q$ on the vertical axis and $W$ on the horizontal axis, as usual.

(b) Sketch the marginal product of water curve and the average product of water curve.

(c) Sketch the variable cost curve. (Assume the firm takes the price of water as given.)

(d) Sketch the total cost curve.

(e) Sketch the average total cost curve.

11. (a) Let $W$ stand for water and $F$ stand for fertilizer. Suppose a firm uses $W$ and $F$ to produce corn $q$ according to the production function $q = \sqrt{WF}$. What kind of returns to scale does this production function have? What does this imply about the shape of the firm's average cost curve?

(b) For the production function $q = \sqrt{WF}$, it turns out that

$$\text{RTS of } W \text{ for } F = \frac{F}{W}$$

where "RTS" means "rate of technical substitution." If the price of water $p_w = \$1$ and the price of fertilizer $p_f = \$0.25$, in what ratio will the firm use $W$ and $F$? What is the firm's total cost function $C(q)$?

Graph (with numbers) the firm's average and marginal costs. (Remember to label the axes of your graph).

12. Sketch the graph of the long-run average cost curve and the long-run marginal cost curve as a function of output quantity $q$, supposing that the firm produces corn $q$ from water $W$ and fertilizer $F$ according to a production function whose isoquants are shown in Figure 1. You do not have to put any specific numbers on your graph, just show the basic shape. Explain your answer.

13. (a) What things are wrong with Figures 1 and 2?
    (b) Tell me everything you know about inferior inputs.

14. The two parts of this question are not related to each other.
    (a) Explain everything wrong with Figure 1. Justify your answer.
    (b) Explain everything wrong with Figure 2. Also label the axes in Figure 2 and label the unlabeled curves in Figure 2.

15. Suppose a firm produces corn $Q$ from water $W$ and fertilizer $F$. Can the firm have increasing returns to scale but also have diminishing returns to water? Answer this question step-by-step as follows. Do not forget to explain each one of your answers!
    (a) Sketch the long-run total cost curve under increasing returns to scale.
    (b) Sketch the graph relating "quantities of the variable input" to $Q$, supposing that fertilizer $F$ is fixed in the short run. Assume that diminishing returns to water sets in immediately; there is no region of increasing returns to water.
    (c) Use the answer to (b) to derive the graph of short-run total cost.
    (d) Are the answers to (a) and (c) compatible, incompatible, or can you not say? Answer with a graph.

16. Suppose that the LRAC curve in Figure 3 is correct. Find all the mistakes in Figure 3 and correct them on a new figure you draw. Explain!

17. (a) Which of Figures 1–4 are possible cross-sections of a production function? (There might only be one of them which is possible, though this sentence is stated in the plural.) Give your reasons!

    (b) Derive the shape of average variable costs and average total costs for the graphs you named in part (a).

18. Suppose a firm produces corn $Q$ from fertilizer $F$ and water $W$. Let $SRTC_1$ be the short run total cost curve when the fertilizer input is fixed at 10 pounds. Let $SRTC_2$ be the short run total cost curve when the fertilizer input is fixed at 20 pounds. In class we argued that if $SRTC_1$ and $SRTC_2$ are plotted on a graph (with $Q$ on the horizontal axis and dollars on the vertical axis) then $SRTC_1$ and $SRTC_2$ would cross. Let $\hat{Q}$ be the quantity of corn at which $SRTC_1$ and $SRTC_2$ cross.

    In addition, let $Q_a$ be a quantity of corn less than $\hat{Q}$ and let $Q_b$ be a quantity of corn greater than $\hat{Q}$.

    Draw a graph with $W$ on the horizontal axis and $F$ on the vertical axis. Using this graph, explain why $SRTC_1$ and $SRTC_2$ could cross by using the $Q_a$ and $Q_b$ isoquants as examples.

    Hint: to do this you should show that to produce $Q_a$ bushels, it is cheaper to use 10 pounds of fertilizer than 20 pounds of fertilizer, but to produce $Q_b$ bushels, it is cheaper to use 20 pounds of fertilizer than 10 pounds of fertilizer.

19. Refer to Figure 3, which depicts the isoquants of a firm which produces corn $Q$ from water $W$ and fertilizer $F$. The price of water is \$2/gallon and the price of fertilizer is \$4/lb.

    (a) Sketch two points on the short-run cost curve if the firm is restricted to buying exactly 4 lbs. of fertilizer (no more and no less).

    (b) Sketch two points on the long-run cost curve, using the same diagram as the one you drew to answer part (a).

20. A firm produces corn $Q$ from water $W$ and fertilizer $F$. Draw a graph with $W$ on the horizontal axis and $F$ on the vertical axis. Suppose the firm wishes to produce 10 bushels of corn. Mark on your graph the cost-minimizing amount of $W$ and $F$ needed to produce the 10 bushels; label these points $W^*$ and $F^*$. Draw the isocost line which goes through $(W^*, F^*)$.

(a) Suppose the firm is forced to buy *less* than $F^*$ pounds of fertilizer, but it still wants to produce 10 bushels of corn. Will its costs go up or down? Explain this using your graph.

(b) Suppose the firm is forced to buy less than $F^*$ pounds of fertilizer but it still wants to produce 10 bushels of corn, just as in part (a). Is the Rate of Technical Substitution of Water for Fertilizer greater than, equal to, or less than it would be if the firm could buy as much fertilizer as it wanted? Why?

21. Suppose a firm produces corn $Q$ (measured in bushels) from water $W$ (measured in gallons) and fertilizer $F$ (measured in pounds) according to the production function $Q = 2W + 2F$. Suppose the price of water is \$2/gal and the price of fertilizer is \$1/lb.

(a) Find the Long Run Total Cost of producing:

i. 2 bushels;

ii. 4 bushels;

iii. 6 bushels;

iv. 0 bushels.

Your explanation should include a graph showing the 2, 4 and 6 bushel isoquants and showing isocost lines.

(b) Based on your answers to part (a), does the production function have increasing, decreasing, or constant returns to scale, or can you not tell? (If you could not figure out part (a), make up answers for it and then use those answers to work this part.)

(c) Answer parts (a) and (b) again if the price of fertilizer rises to \$4/lb.

22. Suppose a firm uses water $W$ and fertilizer $F$ to produce corn $Q$. Also suppose the firm is in a short-run situation, and is forced to use exactly $F_0$ pounds of fertilizer.

Draw a graph with $W$ on the horizontal axis and $F$ on the vertical axis. Using this graph, show that, depending on how much corn the firm wishes to produce:

(a) the long-run (that is, optimal) amount of fertilizer may be smaller than $F_0$; and

(b) the long-run (that is, optimal) amount of fertilizer may be larger than $F_0$.

Be sure to explain what each line or curve on your graph means. Also:

(c) State whether long-run cost is greater than, equal to, or less than short-run cost in part (a).

State whether this "long-run cost" is long-run total cost, long-run fixed cost, long-run variable cost, long-run average total cost, long-run average variable cost, long-run average fixed cost, long-run marginal total cost, long-run marginal variable cost, or long-run marginal fixed cost.

State whether this "short-run cost" is short-run total cost, short-run fixed cost, short-run variable cost, short-run average total cost, short-run average variable cost, short-run average fixed cost, short-run marginal total cost, short-run marginal variable cost, or short-run marginal fixed cost.

(d) State whether long-run cost is greater than, equal to, or less than short-run cost in part (b).

23. A firm produces corn $Q$ from water $W$ and fertilizer $F$ according to the production function $Q = 2F^{1/2}W^2$.

(a) Does the firm have increasing, constant, or decreasing returns to scale, or can you not tell?

(b) Suppose the same firm (with production function $Q = 2F^{1/2}W^2$) is in a short-run situation: it can only buy exactly 4 pounds of fertilizer. Fertilizer costs \$3/pound and water costs \$5/gallon.

i. Find total cost as a function of $W$. (That is, find an algebraic formula relating total cost and $W$, with no other variables in the formula.)

ii. Find $Q$ as a function of $W$ or $W$ as a function of $Q$. (That is, find an algebraic formula relating $Q$ and $W$, with no other variables in the formula.)

iii. Use your answers for the last two questions to derive a formula for total cost as a function of $Q$. (That is, find an algebraic formula relating total cost and $Q$, with no other variables in the formula.) If you could not solve the last two questions, just make something up for their answers.

iv. Use your answer to the last question to find the short-run marginal cost of the first bushel of corn. If you could not solve the last question, just make something up for its answer.

24. An electric utility has two plants, both producing electricity $Q$ from fuel $F$. Plant 1 has the production function

$$Q_1 = 5\sqrt{F_1}\,,$$

so its marginal product of fuel is

$$MP_{F1} = 5/(2\sqrt{F_1})\,,$$

whereas Plant 2 has the production function

$$Q_2 = 10\sqrt{F_2}\,,$$

so its marginal product of fuel is

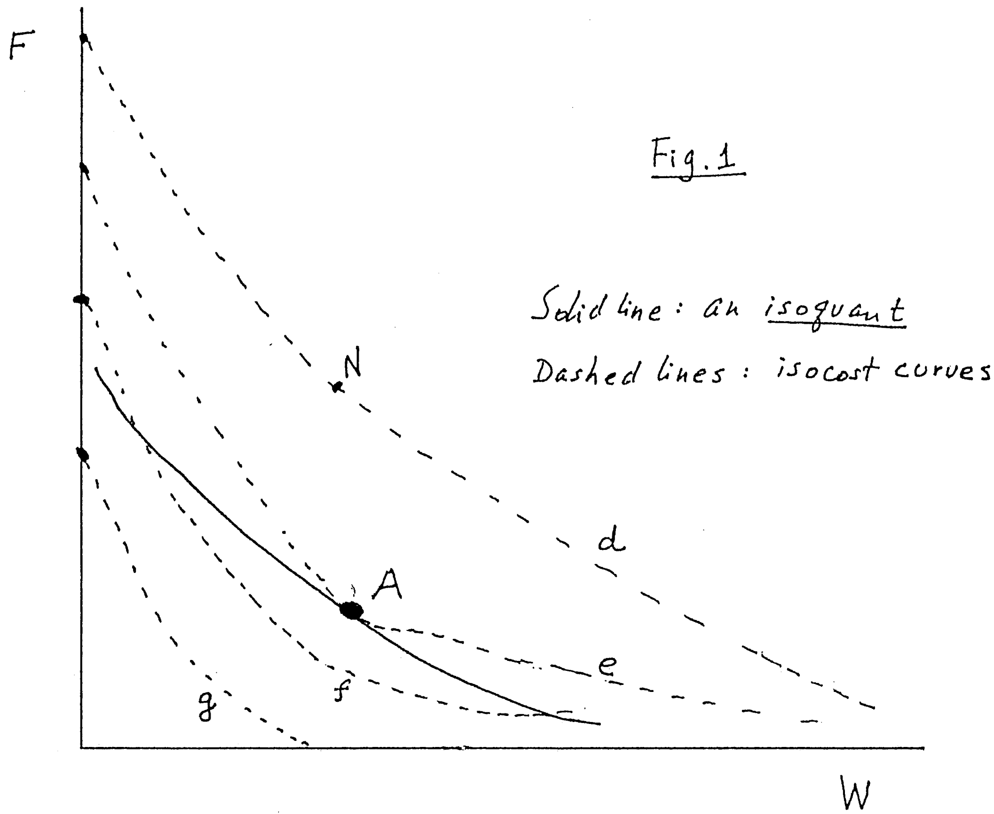
$$MP_{F2} = 5/\sqrt{F_2}\,.$$

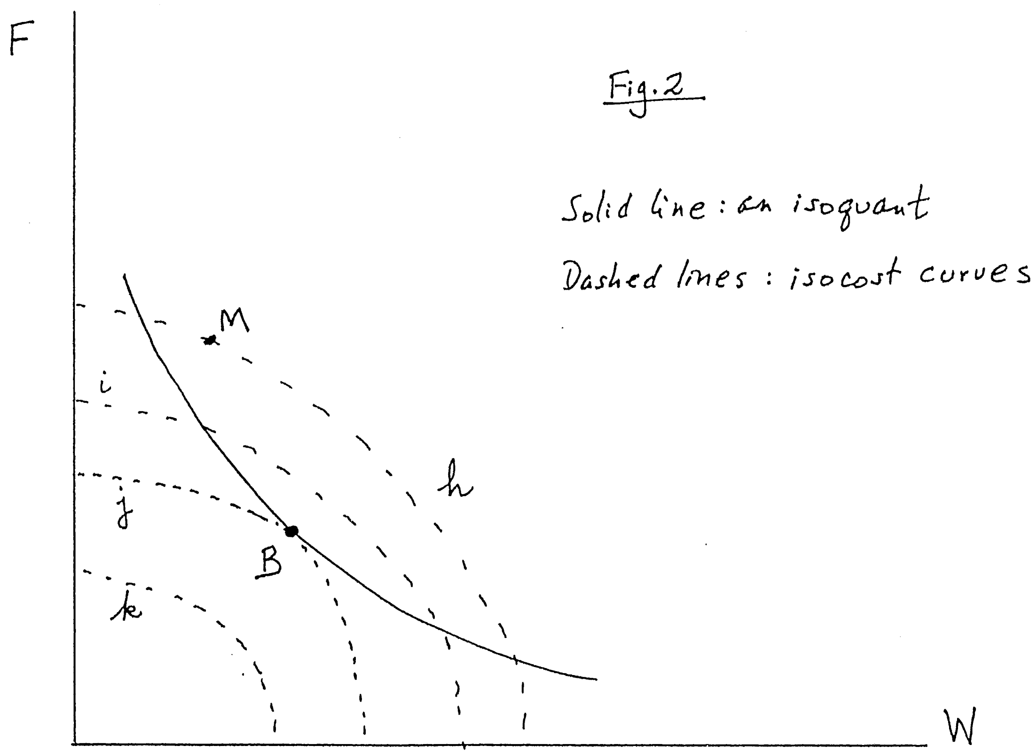
(a) If the utility wishes to minimize the cost of producing electricity, show that it should set $F_2 = 4F_1$.

(b) If the utility sets $F_2 = 4F_1$, show that this implies $Q_2 = 4Q_1$.

(c) If $Q_2 = 4Q_1$, show that total output of electricity $Q$ is equal to $5Q_1$.

(d) From part c, $Q = 5Q_1$. From this fact and the production function of plant 1, show that if the cost of fuel is \$625 per unit, then the utility's cost function is $C(Q) = 5Q^2$.




Question 5's Fig. 1 and Fig. 2

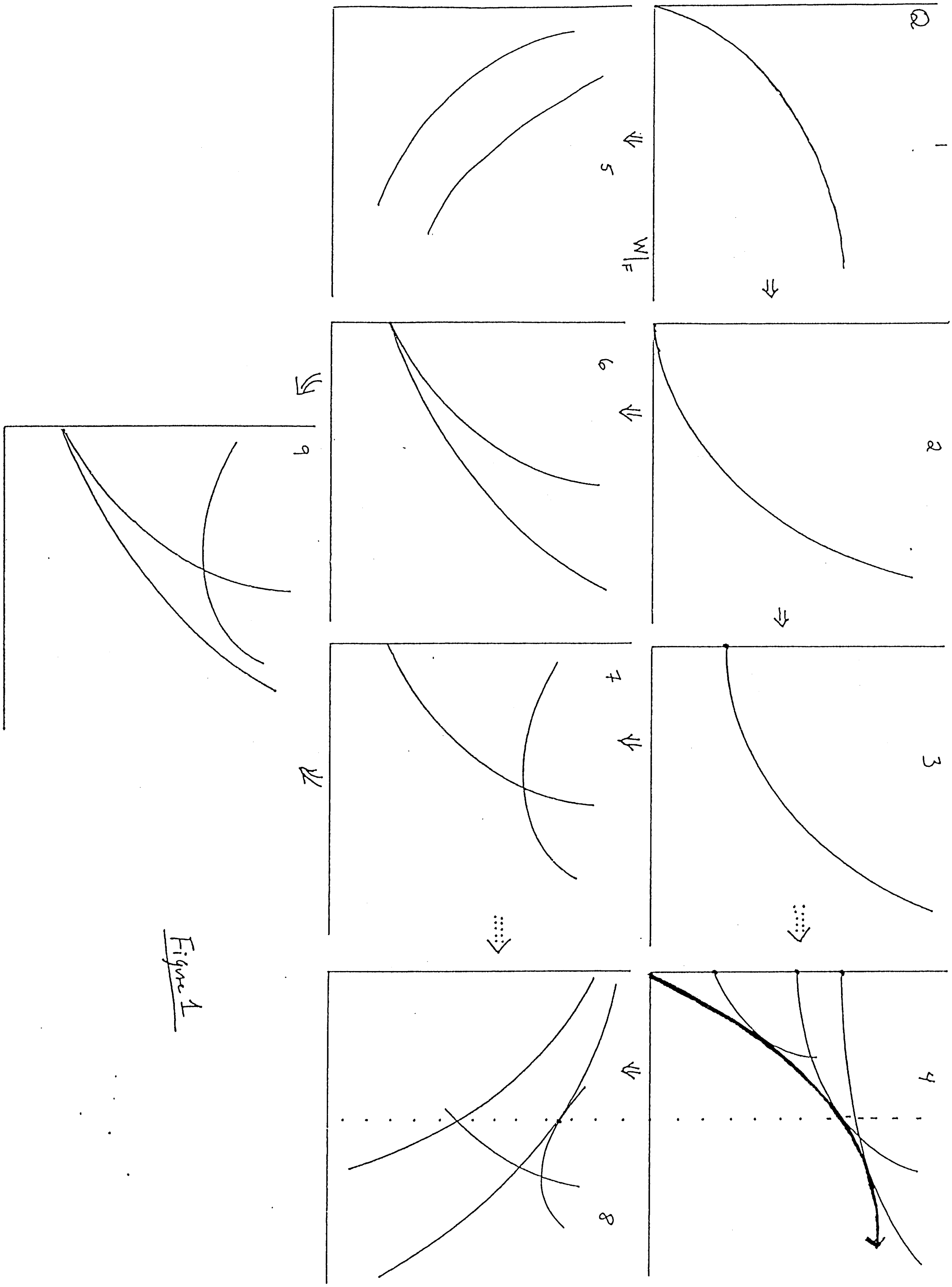


Figure 1

Question 6's Fig. 1

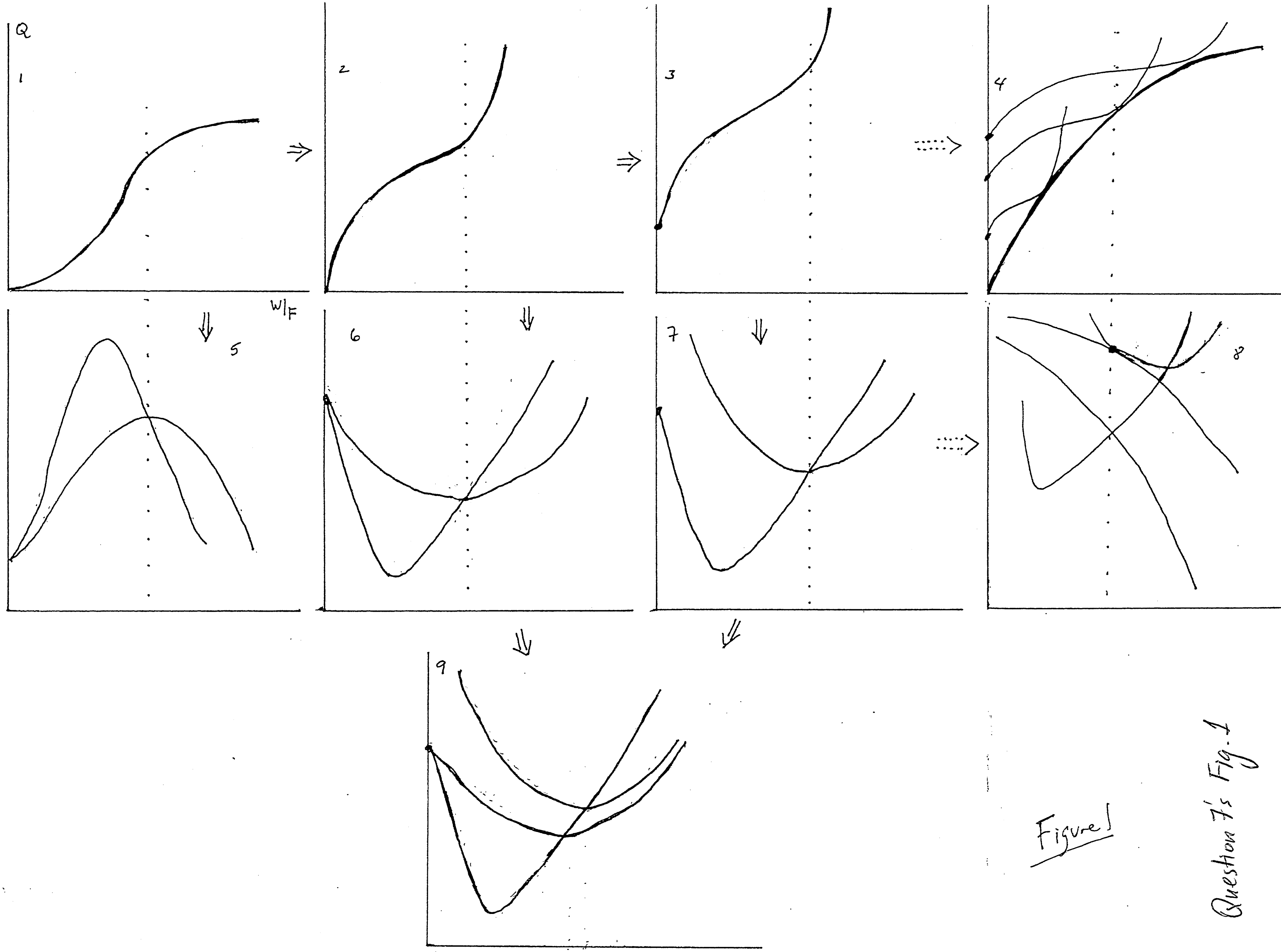



Figure 1

Question 7's Fig. 1

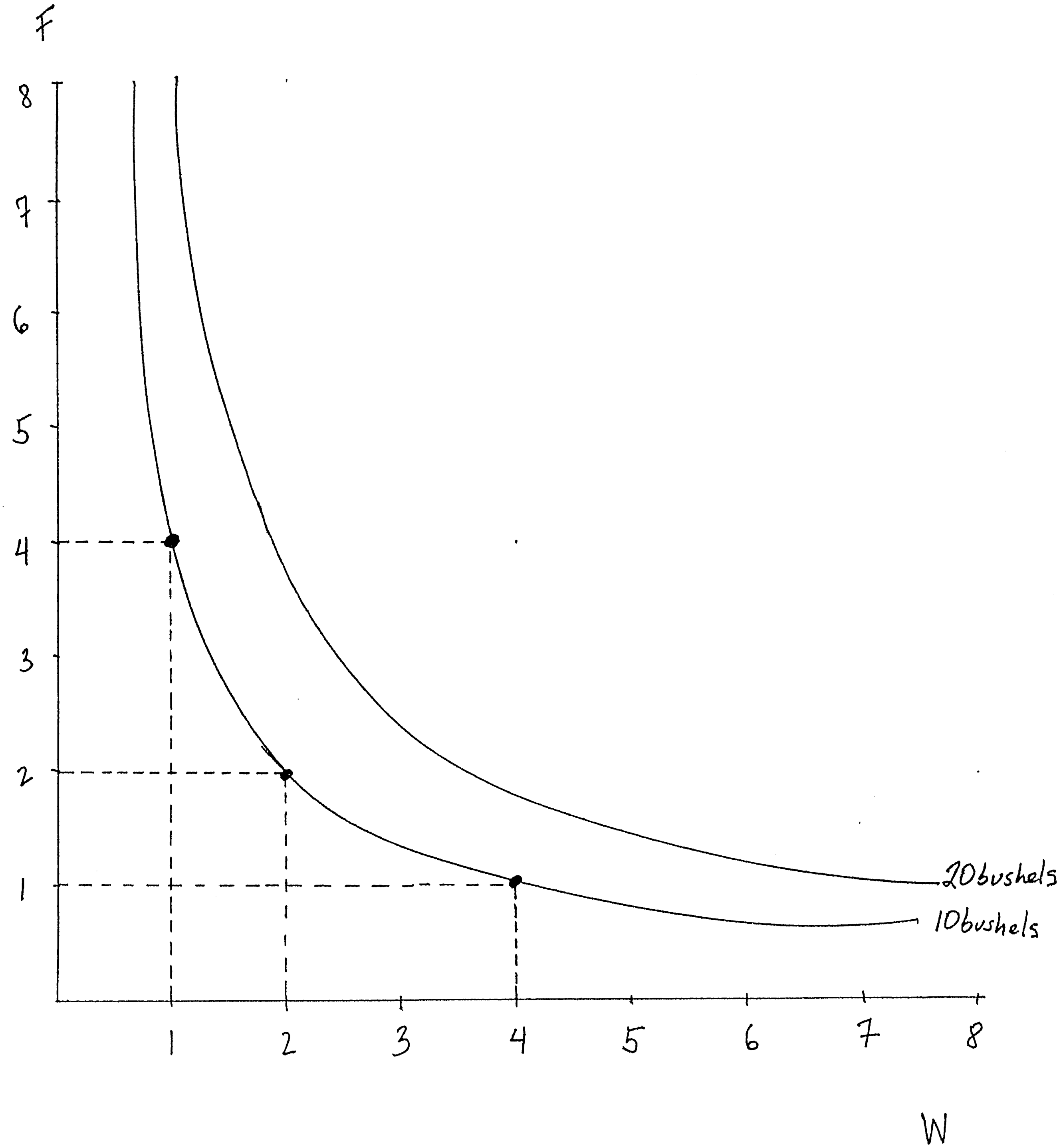



Figure 1

Question 12's Fig. 1

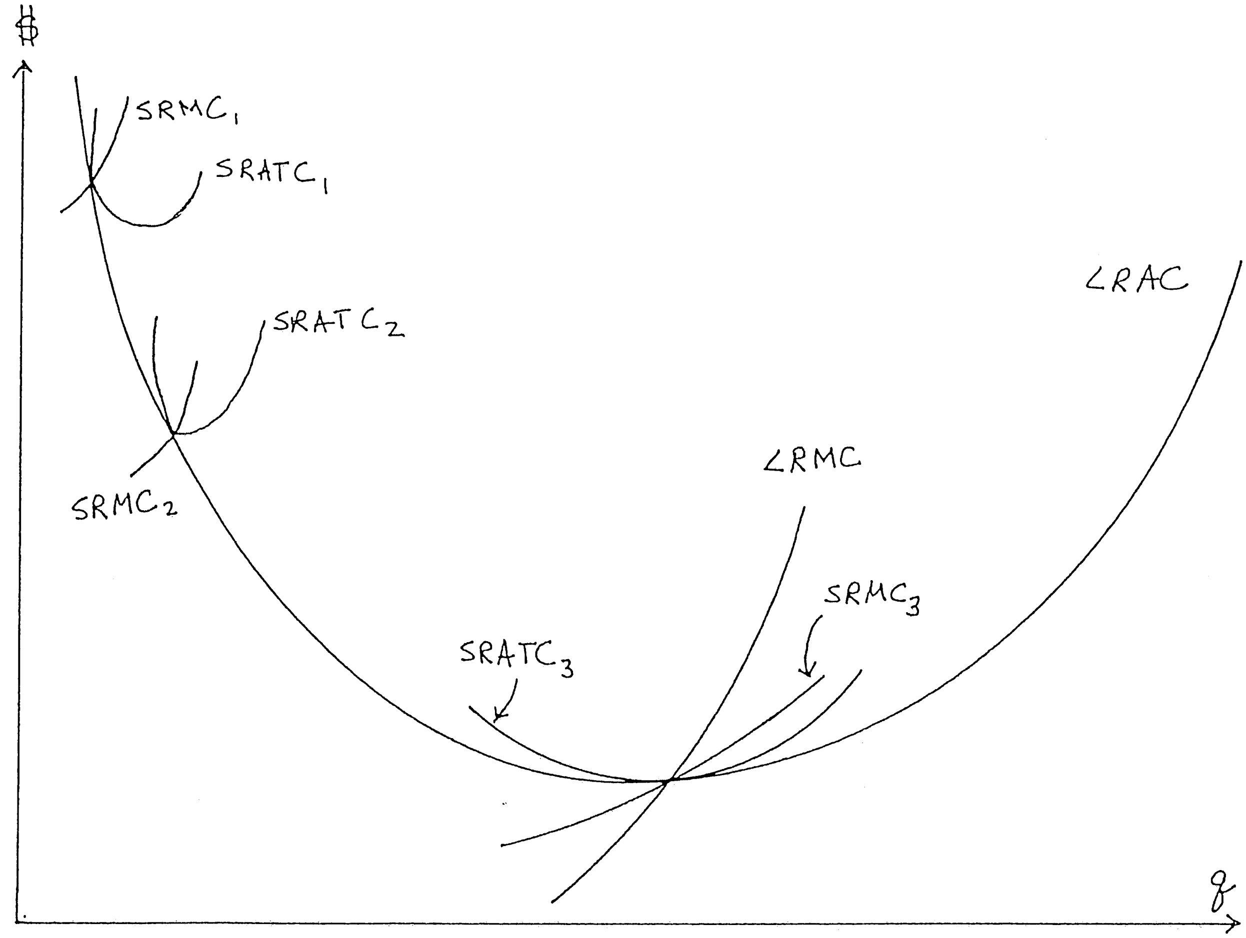



Fig. 1

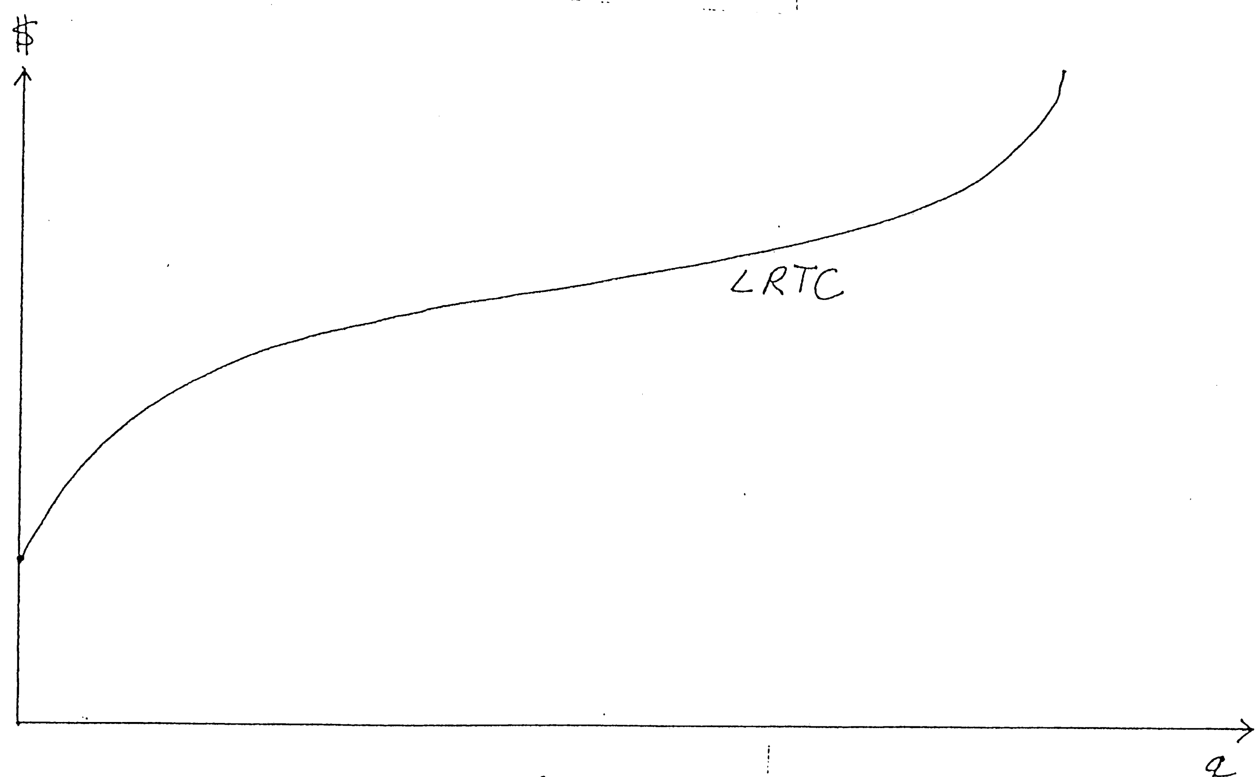



Fig. 2

Question 13's Fig. 1 and Fig. 2

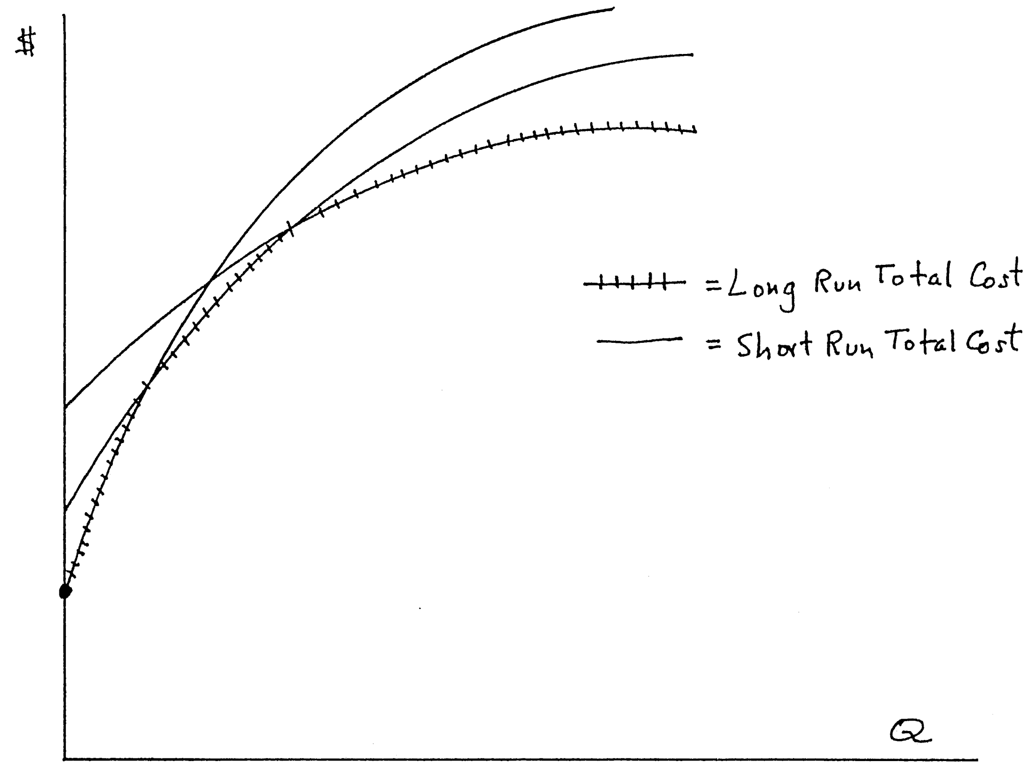



Figure 1

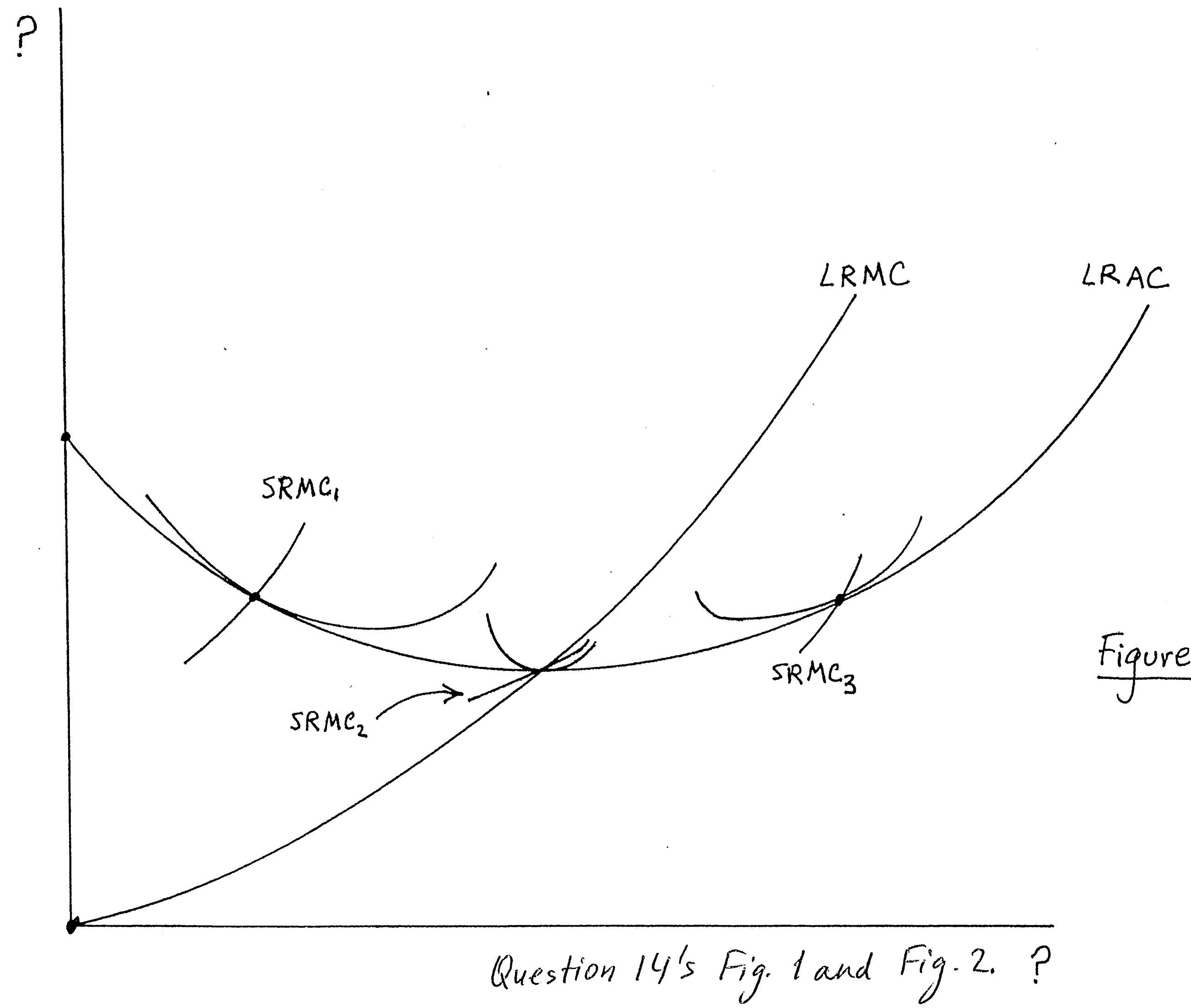



Figure 2

Question 14's Fig. 1 and Fig. 2. ?

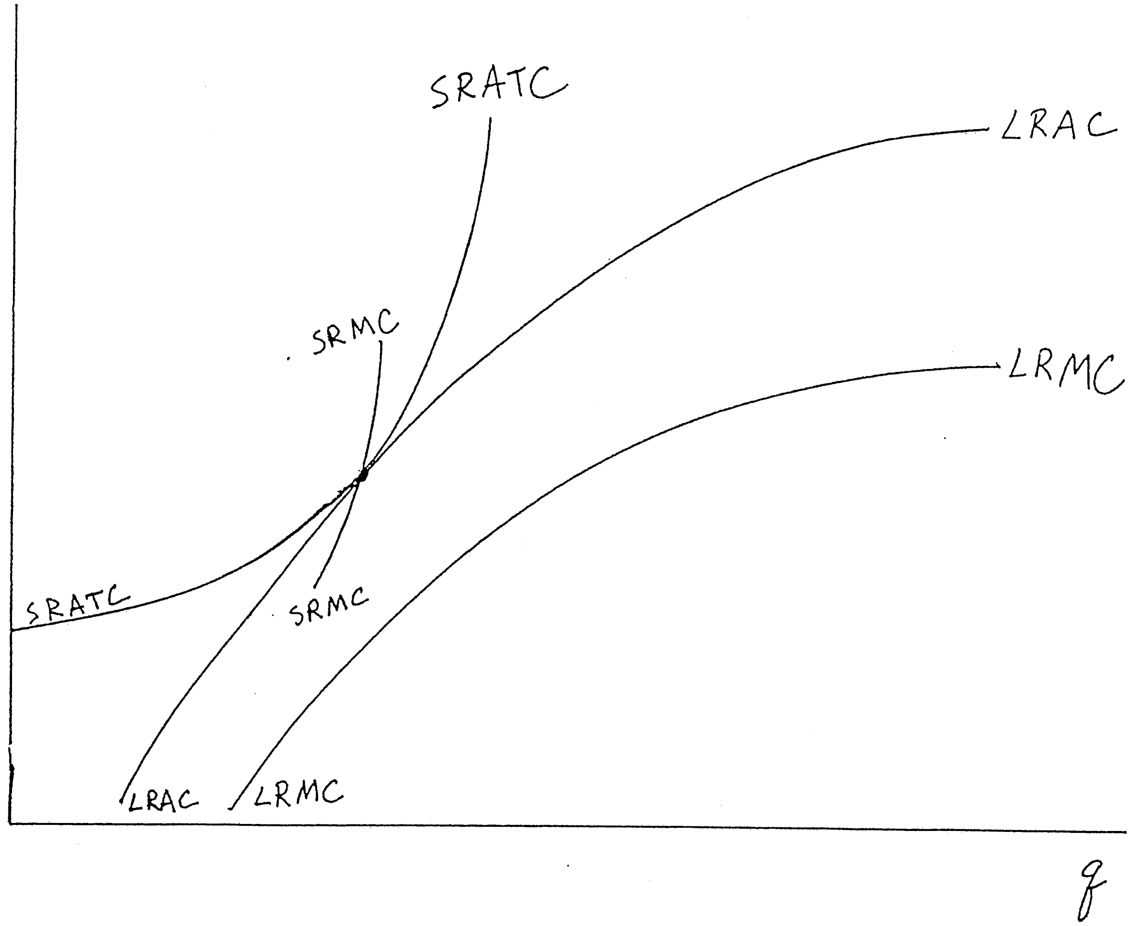
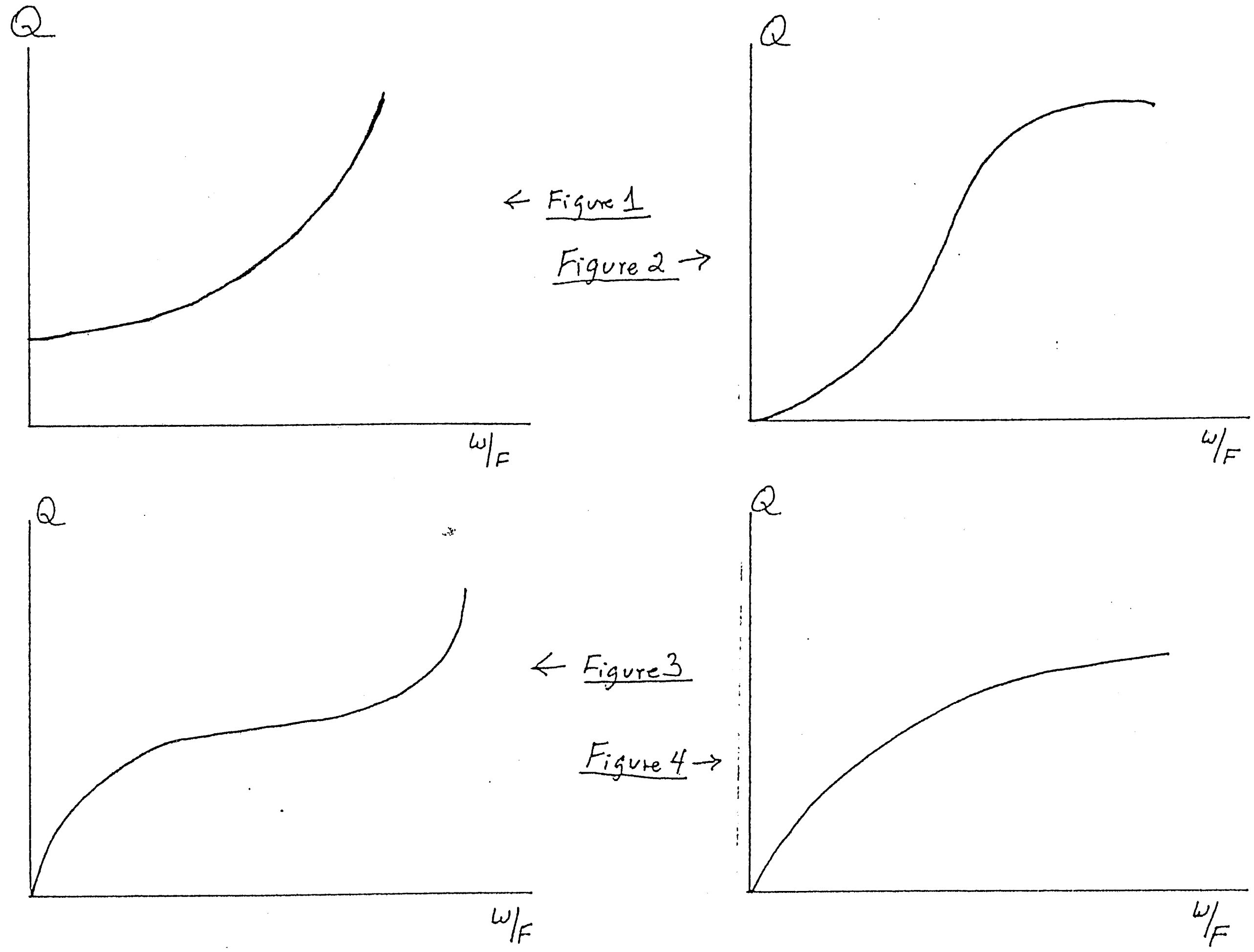
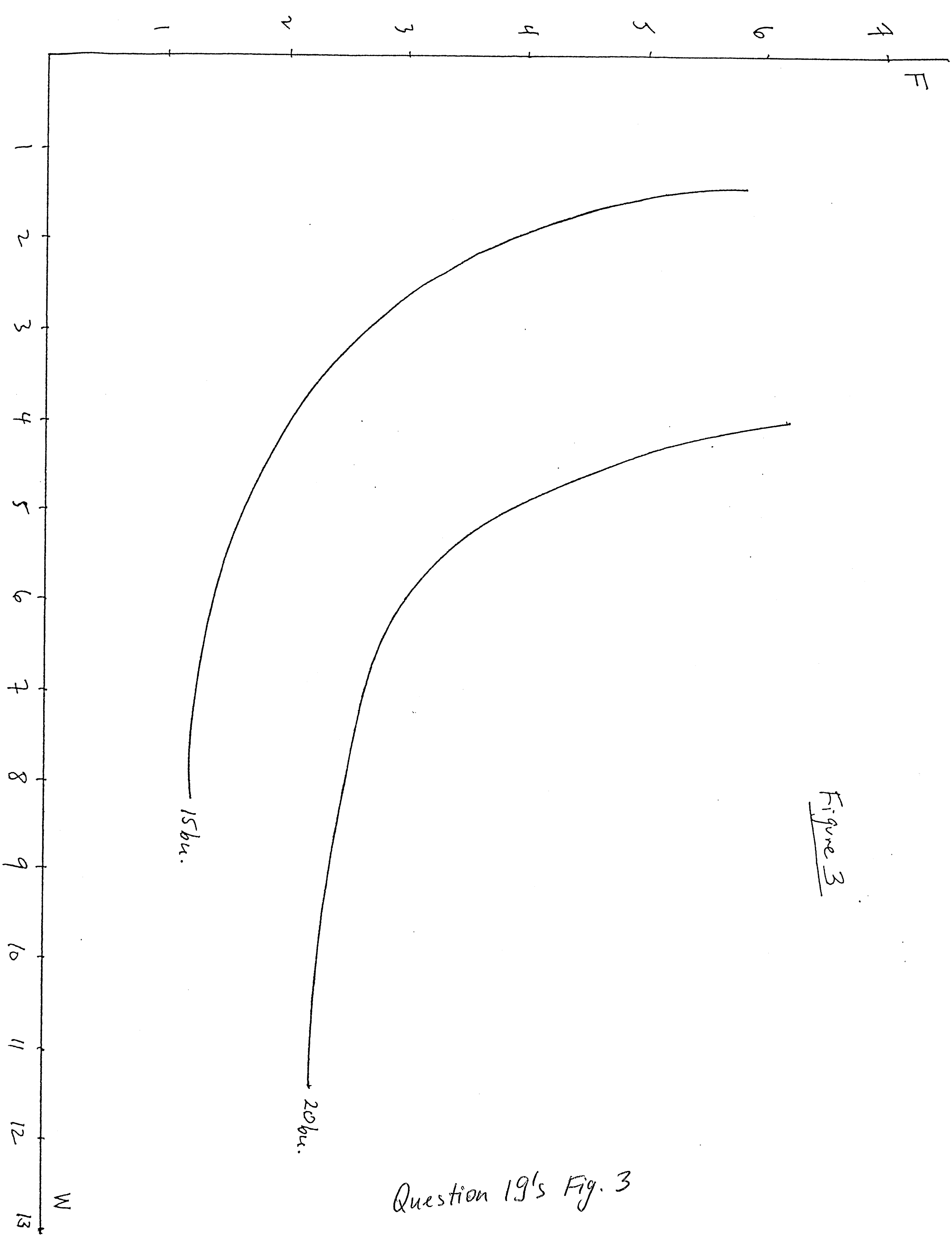



Figure 3

Question 16's Fig. 3



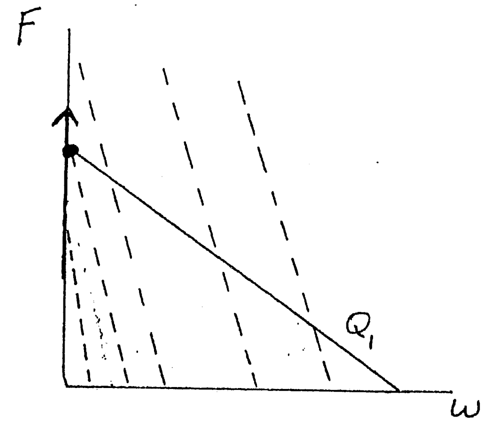

Question 17's Figs. 1, 2, 3, and 4




Question 19's Fig. 3

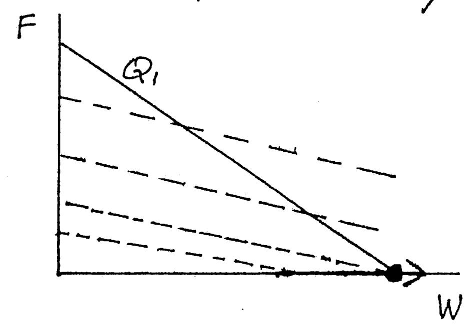
# Answers

① First, suppose the isoquants are flatter than the isocost lines. An example would be:

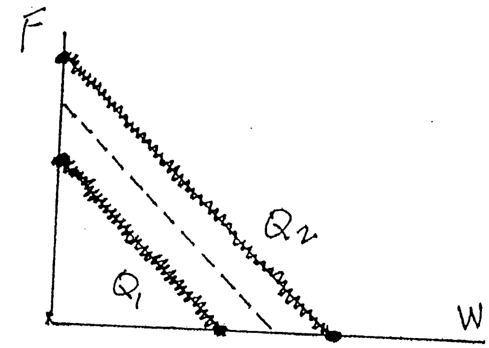



Here, clearly the smallest isocost line still touching $Q_1$ would be the one intersecting $Q_1$ on the F axis. Since all the isoquants are parallel, and all the isocost lines are parallel, one has $W=0$ for all Q. So as $Q\uparrow$, the optimal input bundle is just the F axis, so the F axis is the expansion path.

Similarly, if the isoquants are steeper than the isocost lines then the expansion path is just the W axis:




Finally, if the isoquants are just as steep as the isocost lines, then any point on (say) the $Q_1$ isoquant is cost-minimizing. Increasing Q to another level, $Q_2$, simply makes all points on $Q_2$ cost-minimizing. So the "expansion path" is the entire F-vs.-W graph.




Defn. of expansion path as (W, F) when $Q\uparrow$ (5pts)

Isocost line . . . . . . . . . . . (5pts)

Cost-Minimization: Attempted (5pts)

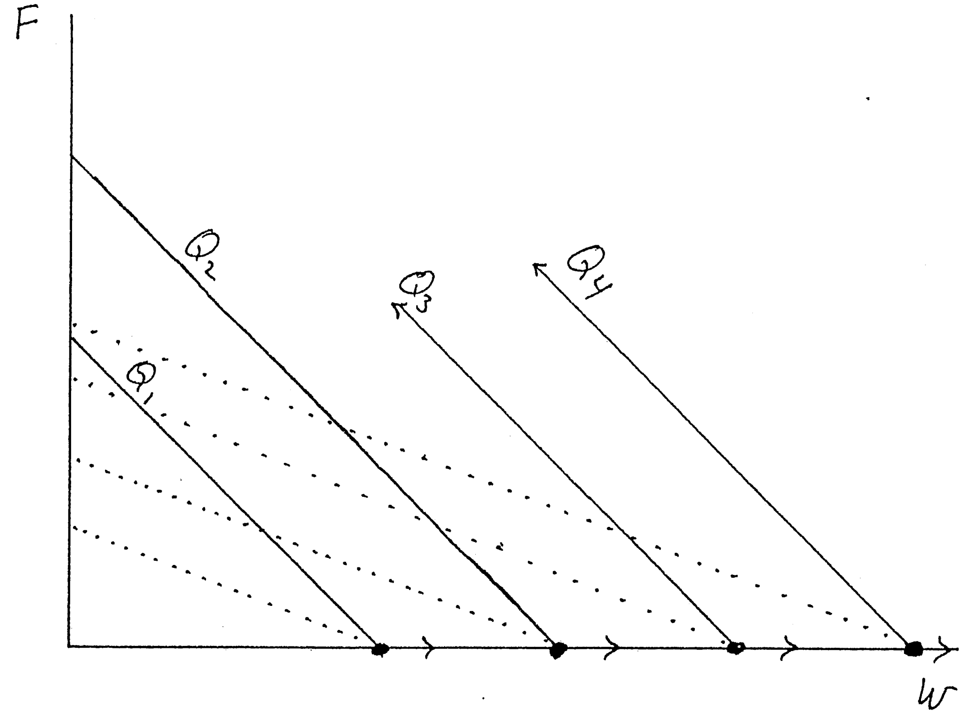
Correct (6pts)

Expansion Path along an axis (6pts)

" " " the other axis (3pts)

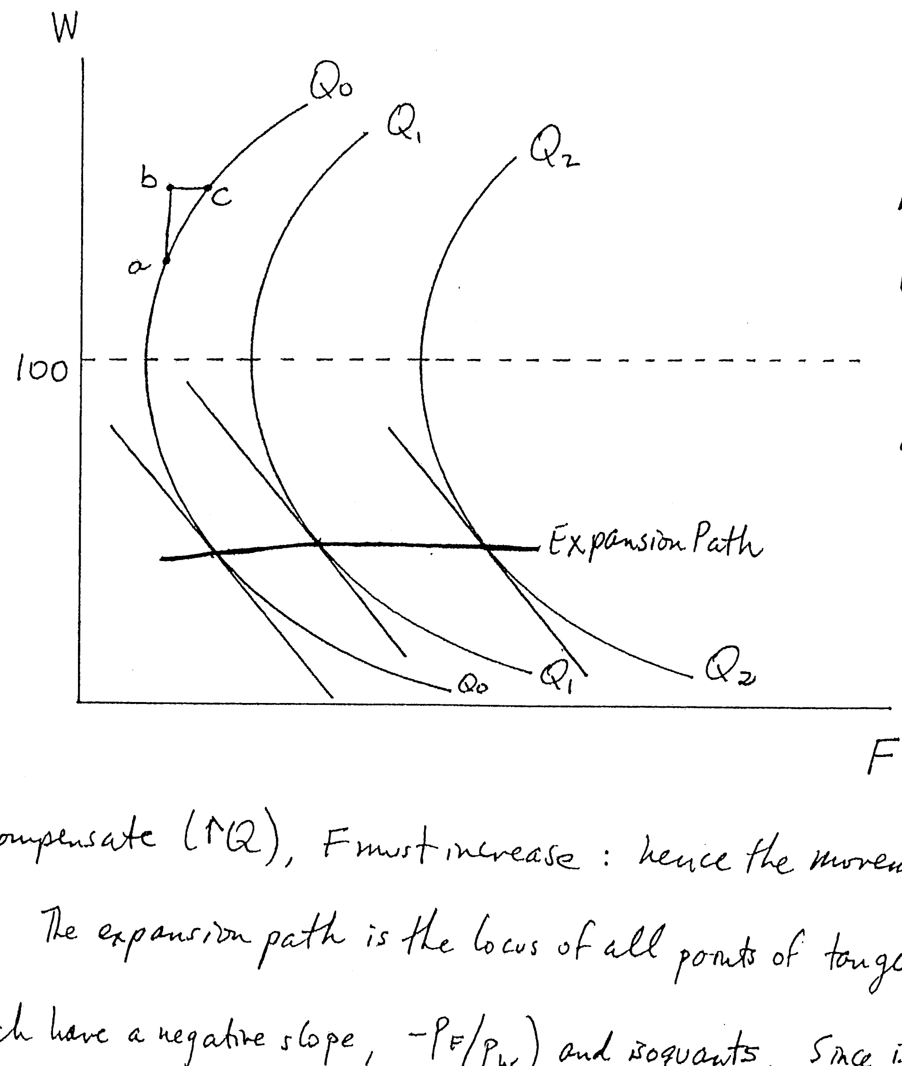
" " as entire graph (3pts)

②




If the RTS is constant then isoquants are linear, and all have the ↙5pts same slope. If the isocost lines look like the dotted lines in the above figure, then the cheapest way to produce any Q is to use only W (and no F) ↙10pts; by looking at $Q_1$ you can see that this is true. Hence as output increases from $Q_1$ to $Q_4$ via $Q_2$ and $Q_3$, the optimal choice of inputs traces the 'W' axis; the four black dots on the 'W' axis show the optimal inputs. Since the definition of the expansion path is the line connecting optimal input choices as Q changes, the expansion path for this example is clearly ↙10pts the 'W' axis. We have $-\frac{P_W}{P_F}$ = slope of isocost line > slope of isoquant = − RTS of W for F, or $\boxed{\frac{P_W}{P_F} < \text{RTS of } W \text{ for } F,}$ ← 10pts which makes sense: water is relatively cheap. (The slope of the isocost lines is greater than the slope of the isoquants because both slopes are negative and the isoquants are steeper, so their slopes are more negative. We got from $-\frac{P_W}{P_F} > -\text{RTS of } W \text{ for } F$ to $\frac{P_W}{P_F} < \text{RTS of } W \text{ for } F$ because when you multiply an inequality by a negative number (such as $-1$), you change the order of the inequality.)

(3)




Below $W = 100$, the isoquants have their usual shape. Above $W=100$, the isoquants are positively sloped. To see this, consider trying to maintain output level $Q_0$ while moving from point "a" to point "b." This $\uparrow W$ causes a $\downarrow Q$ because $W > 100$. In order to compensate ($\uparrow Q$), F must increase: hence the movement from b to c to stay on $Q_0$

The expansion path is the locus of all points of tangency between isocost lines (which have a negative slope, $-P_F/P_W$) and isoquants. Since isocost lines' slopes are negative, at tangency points the isoquants' slope must be negative too. Hence W must be less than 100.

→ So the expansion path can never go above the line $W = 100$.

[Explanation of this: 8 points]

$W<100$ isoquants: 4 pts
$W>100$ ": 6 pts
Explanation of $W>100$ ": 6 pts

(4)

a) $AP_F = \frac{Q}{F} = \frac{\sqrt{FW}}{F} = \sqrt{\frac{W}{F}}$ (Algebra for the last step:

$\frac{\sqrt{FW}}{F} = \frac{\sqrt{F}\sqrt{W}}{\sqrt{F}\sqrt{F}} = \frac{\sqrt{W}}{\sqrt{F}} = \sqrt{\frac{W}{F}}$ or $\frac{\sqrt{FW}}{F} = \frac{\sqrt{F}\sqrt{W}}{F} = \frac{\sqrt{W}}{\sqrt{F}} = \sqrt{\frac{W}{F}}$

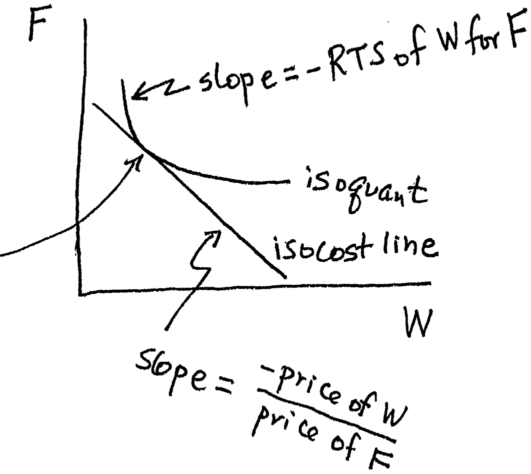
or $\frac{\sqrt{FW}}{F} = \frac{\sqrt{FW}}{\sqrt{F^2}} = \sqrt{\frac{FW}{F^2}} = \sqrt{\frac{W}{F}}$ or $\frac{\sqrt{FW}}{F} = F^{1/2}W^{1/2}F^{-1} = W^{1/2}F^{-1/2}$.)

$AP_W = \frac{Q}{W} = \frac{\sqrt{FW}}{W} = \sqrt{F/W}$.

b) RTS of W for F $= \frac{MP_W}{MP_F} = \frac{\frac{1}{2}\sqrt{\frac{F}{W}}}{\frac{1}{2}\sqrt{\frac{W}{F}}} = \sqrt{\frac{F}{W}} \div \sqrt{\frac{W}{F}} = \sqrt{\frac{F}{W}} \cdot \sqrt{\frac{F}{W}} = \frac{F}{W}$.

c) RTS of W for F $= \frac{\text{price of W}}{\text{price of F}} \Rightarrow \frac{F}{W} = \frac{10}{40} \Rightarrow \underline{\frac{F}{W} = \frac{1}{4}}$ or

$\underline{W = 4F}$. Explanation for first step:




At the optimum, these slopes are equal, so

$-\text{RTS of W for F} = \frac{-\text{price of W}}{\text{price of F}} \Leftrightarrow$

$\text{RTS of W for F} = \frac{\text{price of W}}{\text{price of F}}$.

this is F/W from part (b)

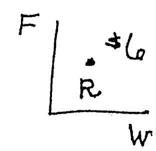
d) $6 = Q = \sqrt{FW}$

$= \sqrt{F(4F)} = \sqrt{4F^2} = 2F \Rightarrow F = \frac{6}{2} = 3$ and

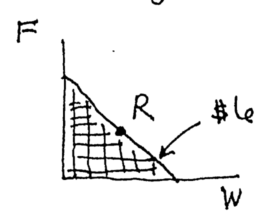
$W = 4F = 12$.

(5)

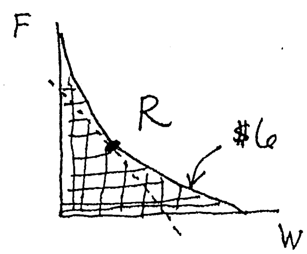
a) Start from a point in the (W, F) plane called "R." Suppose it costs \$6 to buy the W and F represented by R.




If input prices are given then the isocost curves are straight lines. The set of all (W, F) costing \$6 or less is hatched in the following graph:

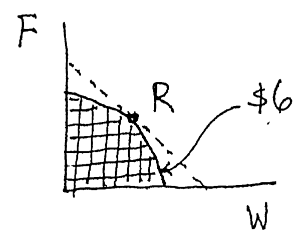



If the firm gets volume discounts then it can purchase more inputs for \$6 than it could before; for instance, \$6 might be able to buy this area:



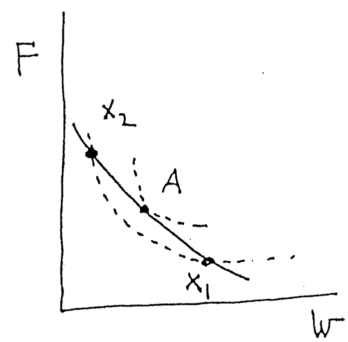

This is like Fig. 1.

On the other hand, if the firm pays volume surcharges then it can purchase fewer inputs for \$6 than it could before; for instance, \$6 might only be able to buy this area:




This is like Fig. 2.

b) 'A' is not a cost-minimizing point because it is on isocost curve 'e', whereas isocost curve 'f' also intersects the isoquant and 'f' represents less cost than 'e'. In other words, by moving from point A to point '$X_1$' or '$X_2$', output stays the same while costs fall.




c) 'B' is a cost-minimizing point because any other point on the same isoquant (solid line) intersects a higher (more costly) dotted line (isocost curve).

| part: | a, Fig. 1 | a, Fig. 2 | b | c |
|---|---|---|---|---|
| points: | both correct: 5<br>only one correct: 3 | | 2 | 2 |

⑥

pts → #1) This is a cross-section of the production function, holding F fixed.

1pts → #2) The graph of Q versus VC looks just like (#1), except that the horizontal axis is shrunk or expanded. Rotating the axes gives #2.

4pts → #3) TC = FC + VC, so (#3) is just like (#2) moved up by FC.

pts → #4) LRTC is the lower envelope of various SRTC curves, each corresponding to a different amount of the fixed input. This is because in the long run you can choose the level of all inputs, so you choose the levels which

(continues →)

.jc

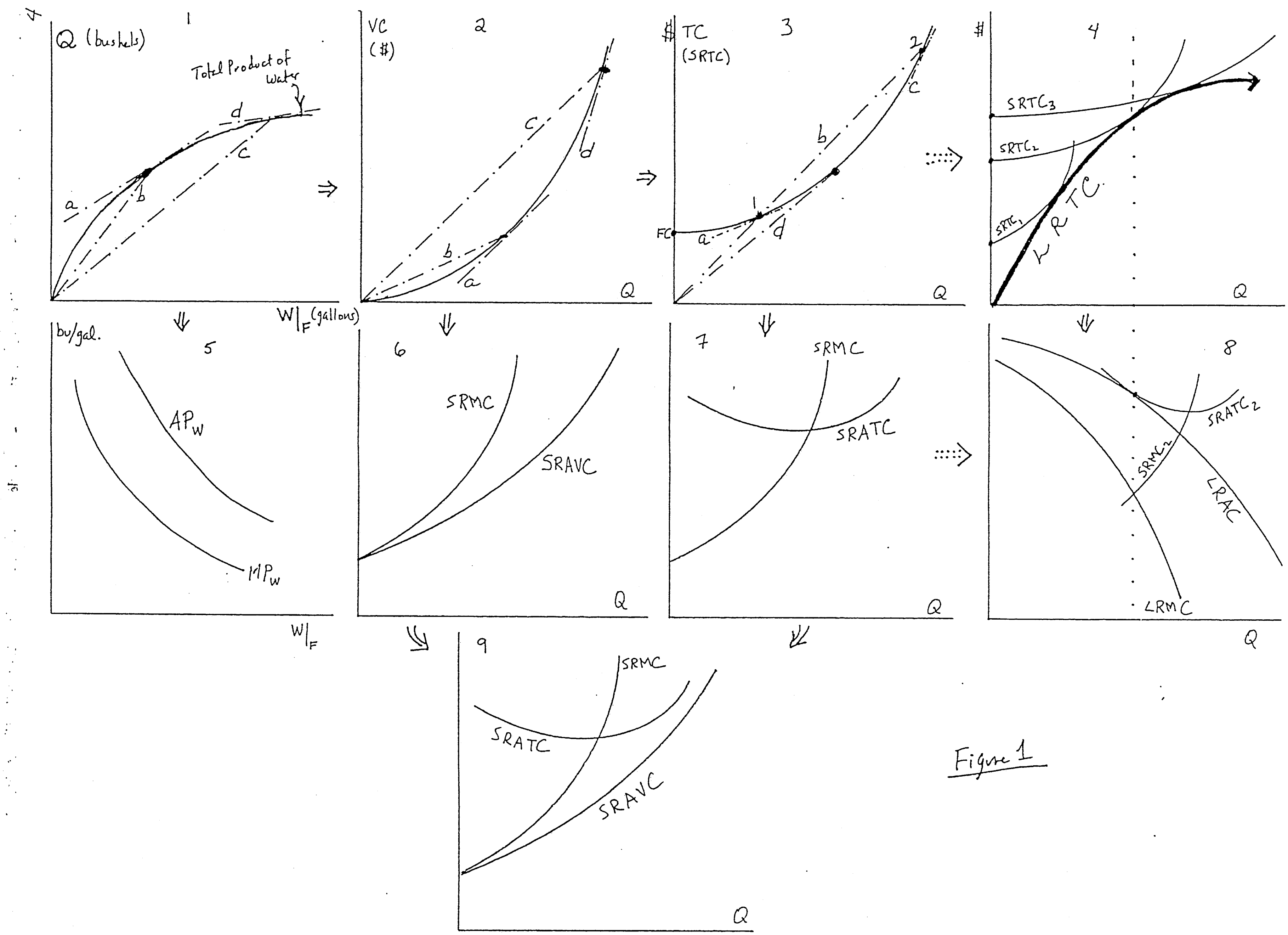

1
Q (bushels)
Total Product of water
a
b
c
d
W/F (gallons)
2
VC ($)
a
b
c
d
Q
3
$ TC (SRTC)
FC
a
b
c
d
1
2
Q
4
$
SRTC3
SRTC2
SRTC1
LRTC
Q
5
bu/gal.
APW
MPW
W/F
6
SRMC
SRAVC
Q
7
SRMC
SRATC
Q
8
SRATC2
SRMC2
LRAC
LRMC
Q
9
SRMC
SRATC
SRAVC
Q

Figure 1

minimize costs for each $Q$.

#5) a & d show that MP is falling. b & c show that AP is falling. These are
4pts ↗
because the slopes of these lines are falling. $MP < AP$ because b is steeper than a and c is steeper than d.

#6) a & d show that MC is rising. b & c show that AVC is rising.
4pts ↗
$AVC < MC$ because b and c are flatter than a and d, respectively.

#7) a, d, & c show that MC is rising. AC is falling from (b1) to d, and
4pts. ↗
is rising from d to (b2); AC at 1 clearly equals AC at 2. at d, $AC = MC$. So AC is U-shaped, and equals MC at AC's minimum.

#8) LRAC and LRMC come from LRTC just as $AP_w$ and $MP_w$ come from
4pts ↗
Total Product of Water in #5 and #1 : the geometry is the same.
The shape of $SRATC_2$ and $SRMC_2$ are from #7. At the dotted line, $SRTC_2$ is tangent to LRTC, which means that $SRMC_2 = LRMC$ and $SRATC_2 = LRAC$ on the dotted line.

#9) This just combines #6 and #7, showing that they have the same marginal
4pts ↗
cost curve ($SRMVC = SRMTC = SRMC$) and that $SRATC > SRAVC$ since $TC = VC + FC$ with $FC > 0$.

(7) #1) This is a cross-section of the production function, holding F fixed.

#2) Rotating the axes would give graph #2, except that the vertical axis is shrunk or expanded depending on whether the price of water is less than or greater than one.

#3) TC = FC + VC, so #3 is just like #2 moved up by FC.

#4) LRTC is the lower envelope of various SRTC curves, each corresponding to a different amount of the fixed input. This is because in the long run you can choose the level of all inputs, so you choose the levels which minimize costs for each Q.

#5) MP first rises ($e \rightarrow a$) and then falls ($a \rightarrow b \rightarrow c$ on Fig. 1 on answers).

AP first rises (d's first intersection with the curve $\rightarrow b$) then falls ($b \rightarrow d$).

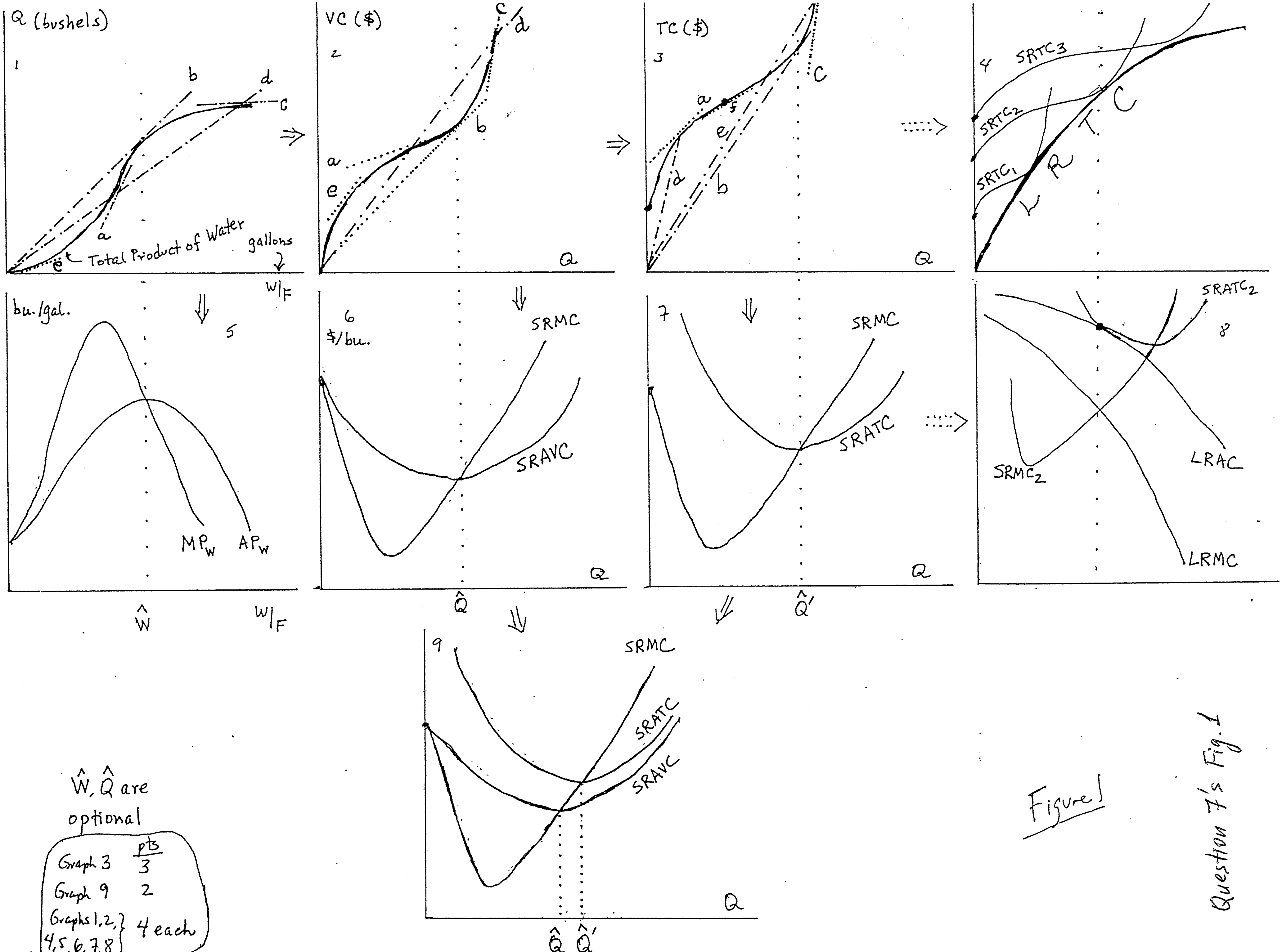
At $\hat{W}$, AP = MP (line b).

At $\hat{w}$, AP reaches its maximum. Before that, AP is rising, so MP > AP; afterwards, it's the reverse.

#6) MC first falls ($e \rightarrow a$) then rises ($a \rightarrow b \rightarrow c$).

AVC first falls (d's first intersection with the curve $\rightarrow b$) then rises ($b \rightarrow d$).

At $\hat{Q}$, MC = AVC (line b).




$\hat{W}$, $\hat{Q}$ are optional

| | pts |
|---|---|
| Graph 3 | 3 |
| Graph 9 | 2 |
| Graphs 1, 2, 4, 5, 6, 7, 8 | 4 each |

Figure

Question 7's Fig. 1

At $\hat{Q}$, AVC reaches its minimum. Before that, AVC is falling, so MC < AVC; afterwards, it's the reverse.

#7) MC first falls ($a \to f$) then rises ($f \to b \to c$).

ATC first falls ($d \to b$) then rises ($b \to e$).

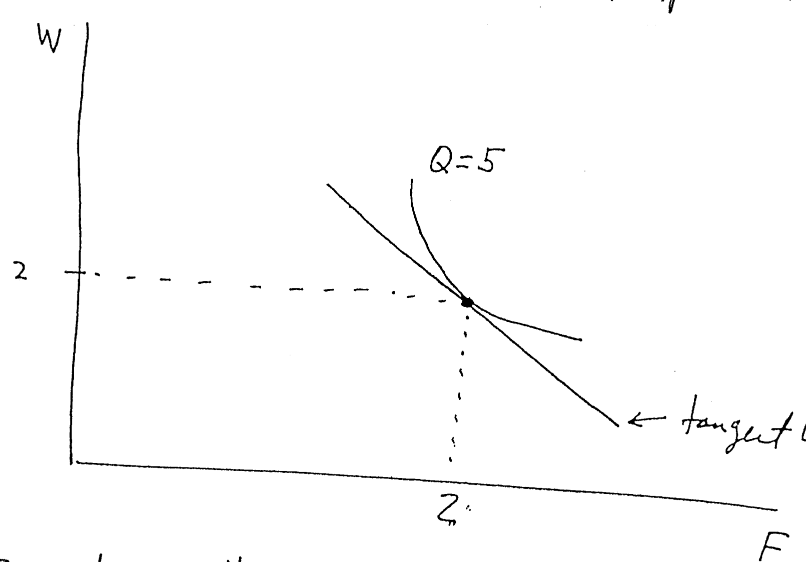
At $\hat{Q}'$, MC = ATC (line b).

#8)
#9) } See answer to #8 and #9 of Question 6b above

(8) a. $(2 \times 12) + (2 \times 6) = \$36$

(units of water * price of water) + (units of fertilizer) * (price of fertilizer) for 5 bu. of wheat (5 points)

b.




The Q=5 isoquant passes through (2F, 2W). The firm produces there when $P_W = \$12$/unit and $P_F = \$6$/unit, so the (F, W) = (2, 2) point must be cost-minimizing for Q = 5, so the Q = 5 isoquant must be tangent to the isocost curve at (2, 2). The slope of the isocost curve is $-P_F/P_W = -6/12 = -1/2$.

coordinates: 5 points
isoquant: 5 points
tangent line: 10 points

15pts.

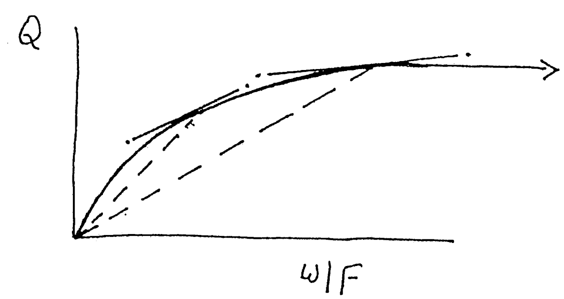
(9) Cost minimization implies that $6G = W$, because if $6G > W$, $Q = W$ (so the extra G did no good), and if $W > 6G$, $Q = 6G$ (so the extra W did no good). To make 1 pencil in a cost-minimizing

5pts → way, then, use $G = \frac{1}{6}$ oz. and $W = 1$ oz. The cost is $\frac{1}{6}(10¢) + 1(2¢)$. $3\frac{2}{3}¢ = \frac{11}{3}¢$. (Just to check, if $G = \frac{1}{6}$ and $W = 1$ then $Q = \min(1,1) = 1$ penc

TC: 5pts
AC: 5pts

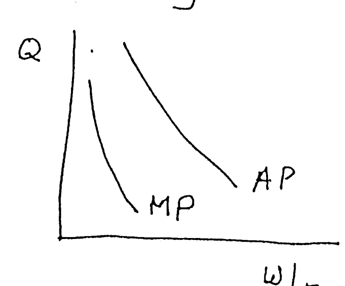
Total Cost then is $C = \frac{11¢}{3} q$, and $AC = \frac{11}{3}¢$.

(10) a. $Q = \sqrt{WF}$ with F fixed implies that $Q(W) = (\sqrt{F})\sqrt{W}$. In other words, Q is proportional to $\sqrt{W}$.



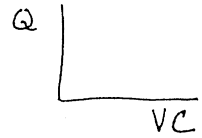

8 points

b. From the dashed lines in (a), AP must be falling. From the lines between two dots in (a), MP must be falling. In addition, since these lines are flatter than the corresponding dashed lines, MP must be less than AP.

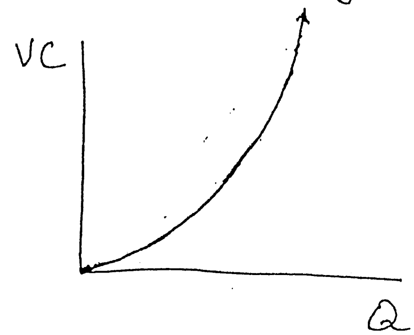



4 points each for MP and AP

c. Since the price of water is given, $VC = P_{water} \cdot Water$ is proportional to Water. So if we were to graph

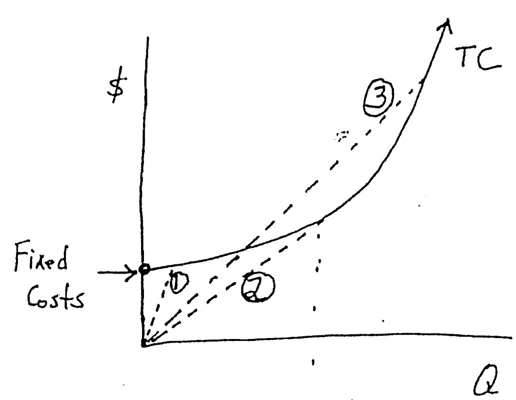



, it would look just like the graph in (a), only expanded or shrunk horizontally. To get the cost curve, we just flip the axes:




8 points

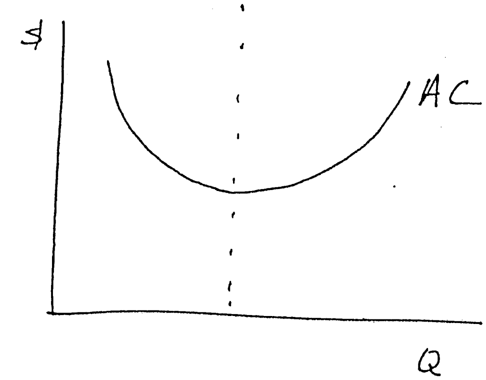
d.




To get total costs, just add Fixed Costs to Variable Costs.

8 points

e.




From the line marked 1 to that marked 2, AC falls; from 2 to 3, AC rises.

8 points

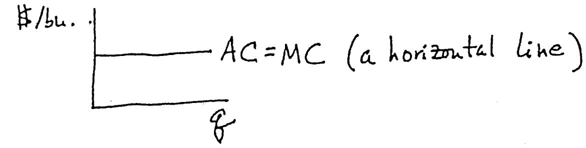
(11)

a) doubling inputs $\Rightarrow$ ? output change:

$$\text{old } q = \sqrt{WF}$$

$$\text{new } q = \sqrt{(2W)(2F)} = \sqrt{4WF} = 2\sqrt{WF} = 2\,(\text{old } q).$$

So there are constant returns to scale. This leads to AC = MC:




b) The "RTS of W for F" is defined to be minus the slope of an isoquant when the axes are $F$ (vertical), $W$ (horizontal). The slope of an isocost line with these axes is $-p_W/p_F$. At a cost-minimizing point, usually:

slope of isoquant = slope of isocost line $\Rightarrow$

$$-\text{RTS of } W \text{ for } F = \frac{-p_W}{p_F} \Rightarrow$$

$$-\frac{F}{W} = -\frac{1}{0.25} = \frac{-1}{1/4} = -4$$

$$\Rightarrow \frac{F}{W} = 4 \quad \text{or} \quad F = 4W \quad \text{or} \quad W = \tfrac{1}{4}F.$$

This answers the ratio in which the firm will use W and F.

As for the total cost function, we have

$$\text{cost} = p_W W + p_F F = p_W W + p_F(4W) = p_W W + 4p_F W$$

$$= (p_W + 4p_F)\,W.$$

Also,

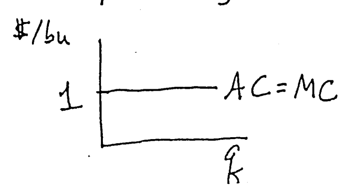
$$q = \sqrt{WF} = \sqrt{W(4W)} = \sqrt{4W^2} = 2W \Rightarrow W = \tfrac{1}{2}q.$$

Substituting this into the previous equation for cost yields

$$\text{cost} = (p_W + 4p_F)\,\tfrac{1}{2}q$$

$$= (1 + 4\cdot 0.25)\,\frac{q}{2} = 2\cdot\frac{q}{2} = q.$$

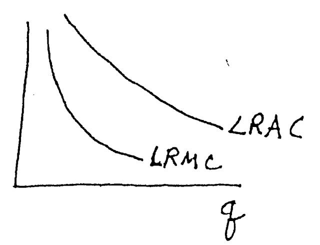
So $C(q) = q$. Average cost is $C(q)/q = q/q = \$1/\text{bu}$, and since it is constant (which you already knew it would be from part (b)), AC = MC:




a) CRS - 1 pt.
graph - 1 pt.

(12) Refer to Figure 1. Doubling all inputs from A leads to point A′; since A′ is beyond the Q = 20 isoquant, and since A was on the Q = 10 isoquant, doubling all inputs from A more than doubled output. At A, this production function therefore has increasing returns to scale. Similarly, doubling output from B yields B′, which is beyond the Q = 10 isoquant, and doubling output from C yields C′, also more than doubling output. So this production function has increasing returns to scale.

Accordingly, one has




. Increasing returns to scale implies

3 pts each $\left\{ \begin{array}{l} \text{LRAC falling} \\ \text{LRMC "} \\ \text{LRMC} < \text{LRAC} \end{array} \right\}$

that LRAC is falling, because $AC = \dfrac{\text{Total Cost}}{Q}$; when inputs are all doubled, this numerator doubles (assuming competition in the input market), but Q more than doubles, so AC↓. When AC is falling, MC is below it.

10 pts: the issue is returns to scale
9 pts: A→A′, B→B′, or C→C′ correctly interpret
[illegible]

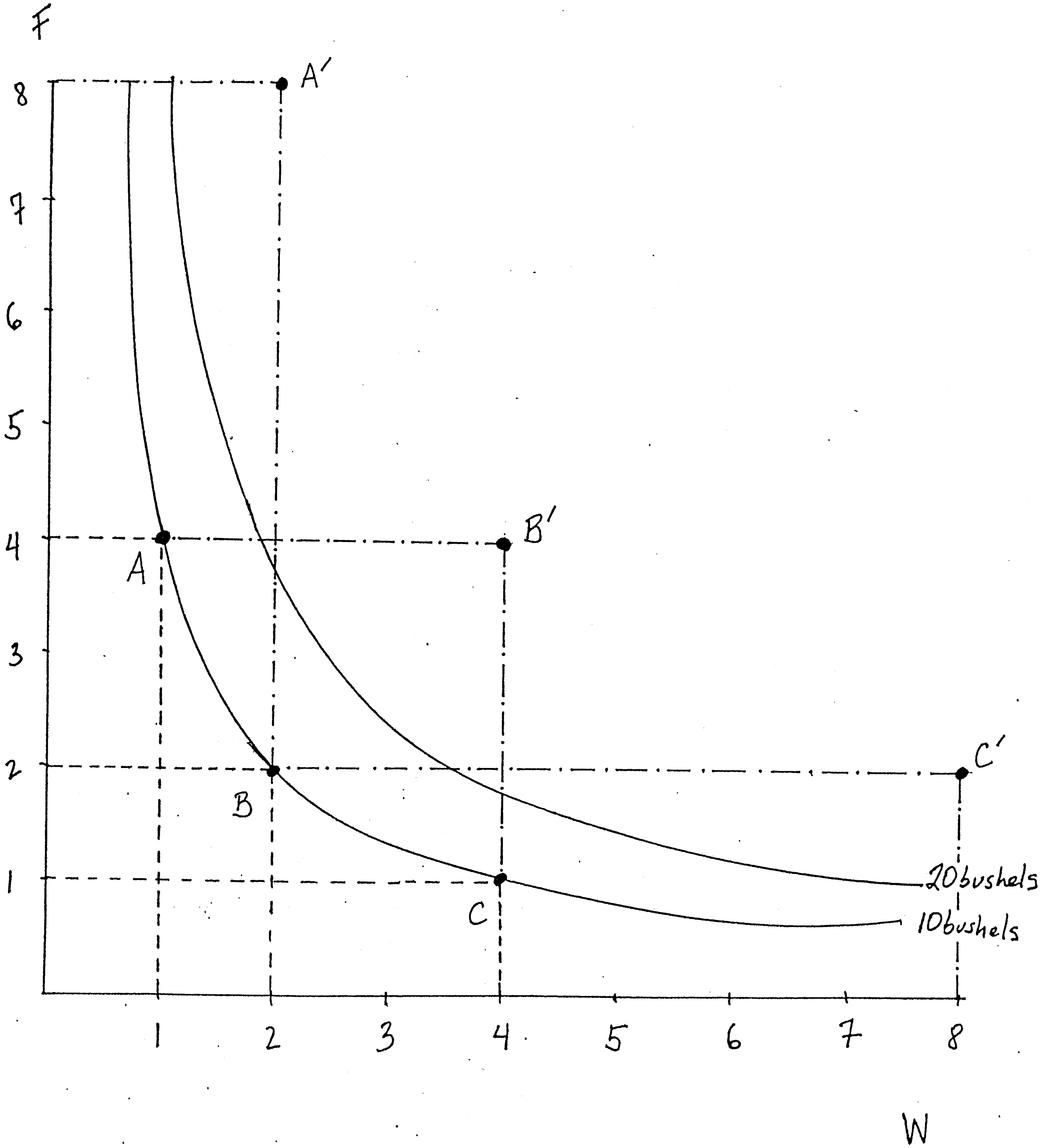



Figure 1

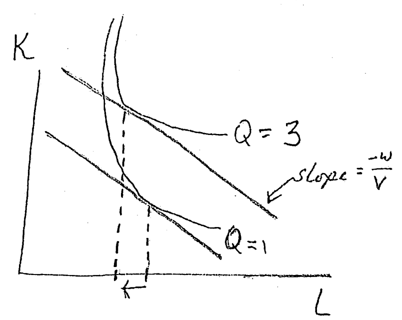
(13) (a) Fig. 1: $SRMC_1$ should pass through the bottom of $SRATC_1$; this bottom point is to the right of LRAC (same for $SRATC_2$ and $SRMC_2$).

LRMC must be flatter than $SRMC_3$.

[Optional: the vertical axis should be \$/unit of output, not just \#. Also, at all "q" where SRATC = LRAC, SRMC should equal LRMC.]

Fig. 2: LRTC at $q = 0$ should be 0.

(b) Inferior inputs are those for which an increase in quantity produced coincides with a drop in the use of the input. A graphical example would be:

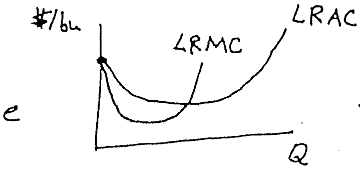



L is inferior ∴ as Q↑ from 1 to 2, L↓.

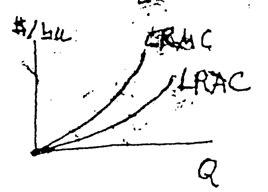
(14) a) LRTC at $Q=0$ should be zero, not positive as on the graph, because in the long run if you produce nothing then you spend nothing on inputs. (1pt)

(1pt) The SRTC curves should be convex instead of concave in order to satisfy the Law of Diminishing Returns.

b) LRMC is incorrectly drawn; it should be like




. LRMC should equal LRAC at $Q=0$. [Another correct answer is to say that LRMC is correct but LRAC is wrong:




.] (1pt)

Line ⓐ on the graph is the correct $SRMC_1$: it hits $SRATC_1$ at $SRATC$'s minimum, (1pt) and $SRMC_1 = LRMC$ where $SRATC_1 = LRAC$ (see the dotted line). (1pt) Similar reasoning works for line ⓒ, which is the correct $SRMC_3$. (2pts)

Line ⓑ is the correct $SRMC_2$ because it is steeper than LRMC near the place where $SRMC_2 = LRMC$. (1pt)

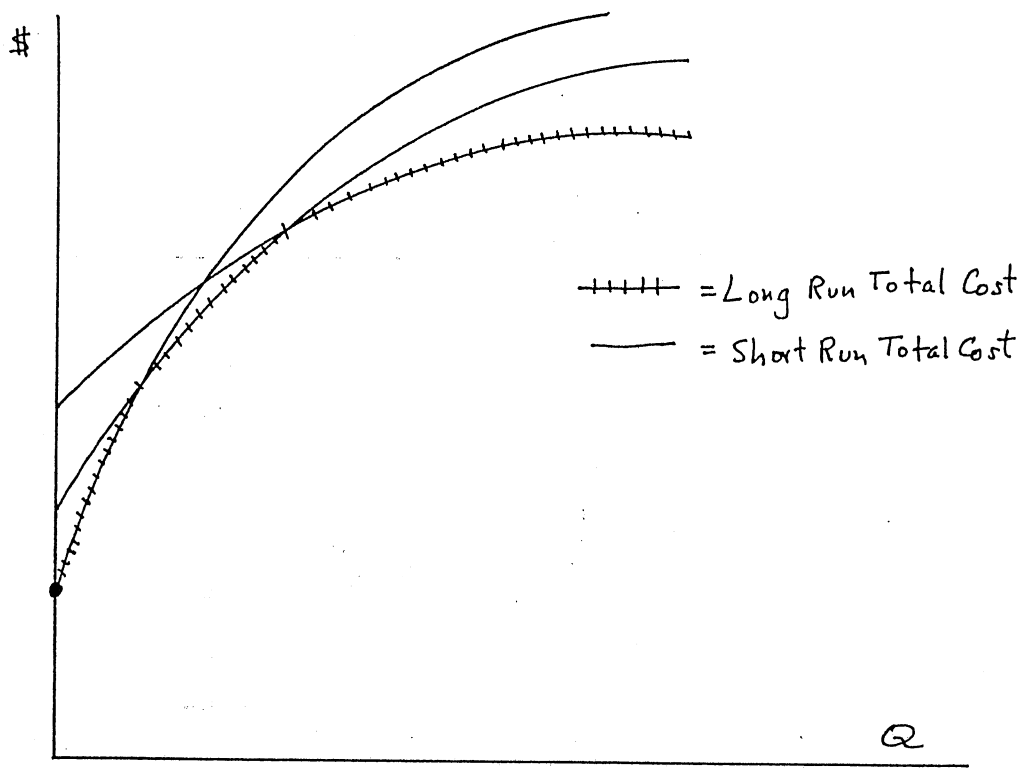



Figure 1

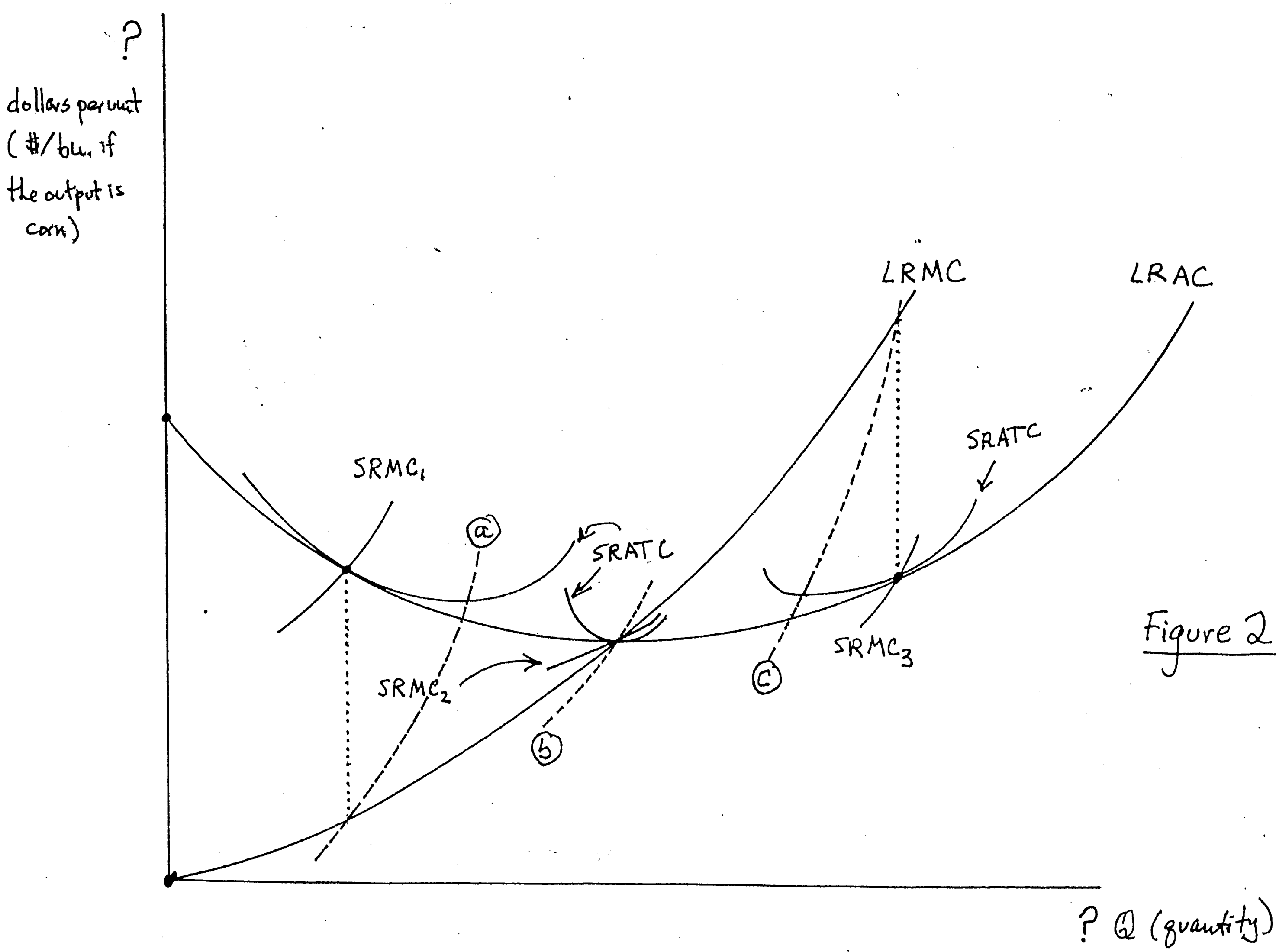
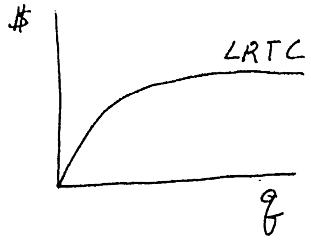



Figure 2

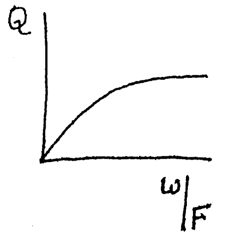
(15) a)




6pts

The LRTC under ↑RS is concave. This corresponds to a falling LRAC curve, which is correct: if inputs double then costs double, while $q$ more than doubles, so $AC = \dfrac{TC \leftarrow \times 2}{q \leftarrow \text{more than } \times 2} \Big\} \downarrow$.

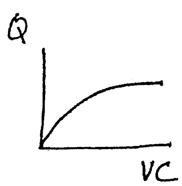
b)




6pts.

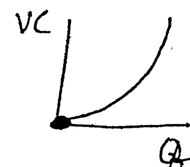
W is the variable input. The graph of Q vs. W holding F constant is concave. This is correct because it results in the marginal product of water, $MP_w$, always falling, which is what diminishing returns to water means.

c) (b) leads to



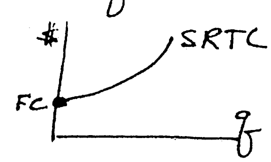

5pts. →

, since $VC = p_w W$ (just a shrinking or expanding of the horizontal scale in graph (b)). Rotating about a 45° line gives




← 3pts

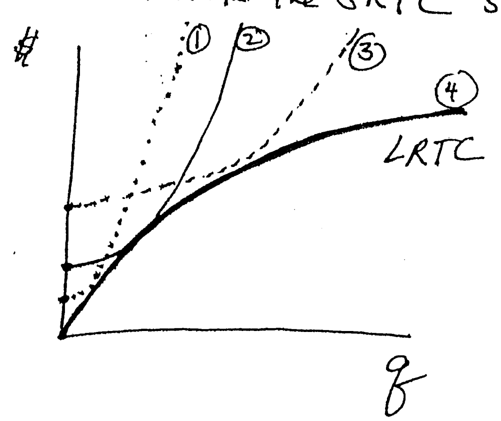
. Since SRTC = SRVC + SRFC, adding fixed cost to this gives total cost:




5pts ↙

d) LRTC is the lower envelope of a series of SRTC's. To show that the shape of the LRTC can be derived from the SRTC shape, see this graph:

8pts →




①, ②, and ③ are SRTC curves shaped just like in part (c). ④ is a LRTC curve shaped just like in part (a). So increasing returns to scale (line ④) is compatible with diminishing returns to water (lines ① - ③).

(16)

The mistakes are :

Because if average is rising, marginal is above it.

(5,3) a) Since LRAC is rising, LRMC should be <u>above</u> it, not below it.

(5,3) b) SRATC should be U-shaped, not always rising.

(5,3) c) SRMC should cut SRATC at the latter's minimum (see reason for part (a)).

(6, 3) d) At the q for which SRATC = LRATC, one should have SRMC = LRMC.

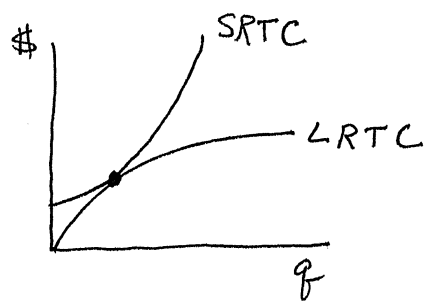
graph correct?

fact correct?

One thus arrives at a graph like that at the bottom of the next page.

- Reason for (b) : $SRATC(Q=0) = \infty$ since $SRATC = \frac{VC+FC}{Q}$, which equals $\frac{0+FC}{0} = \frac{FC}{0} = \infty$ at $Q = 0$.

- Reason for (d) : Where SRTC = LRTC, SRTC and LRTC are also tangent :



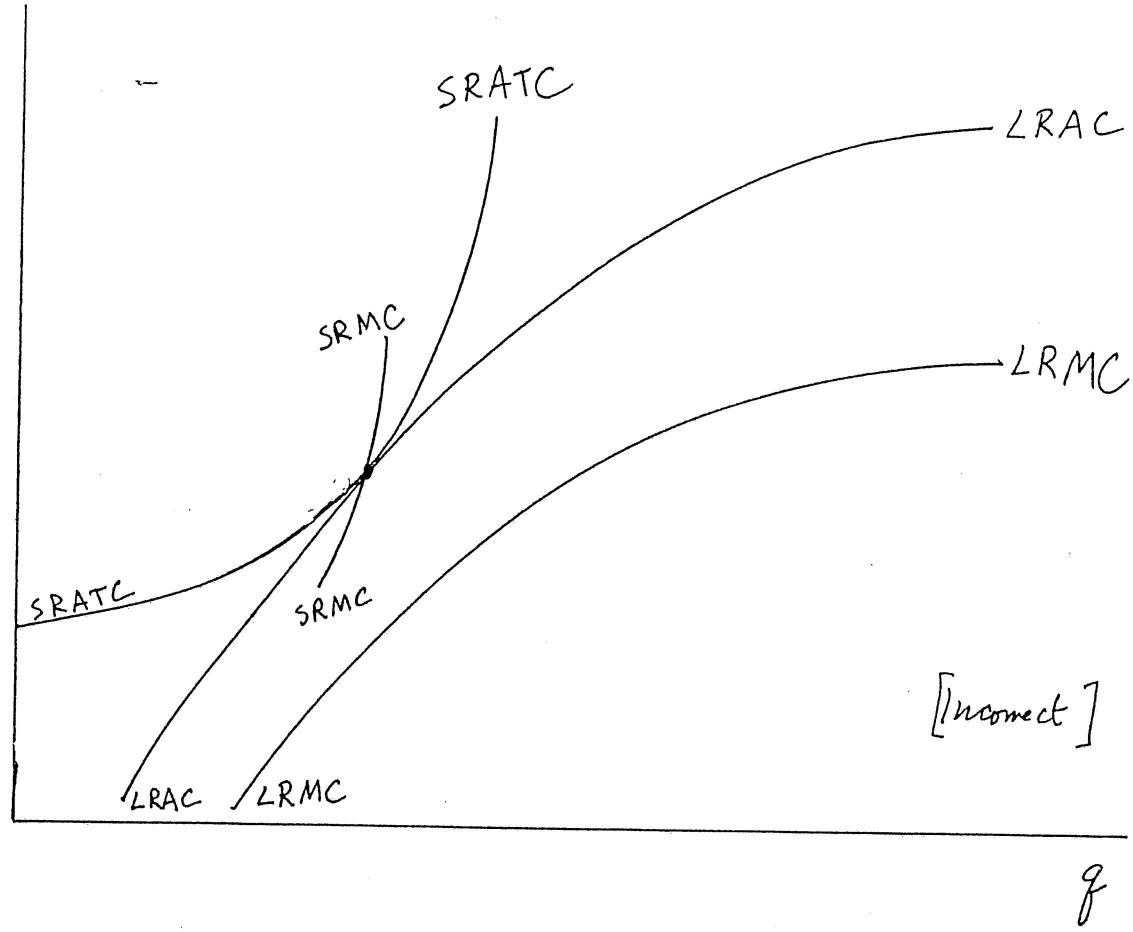
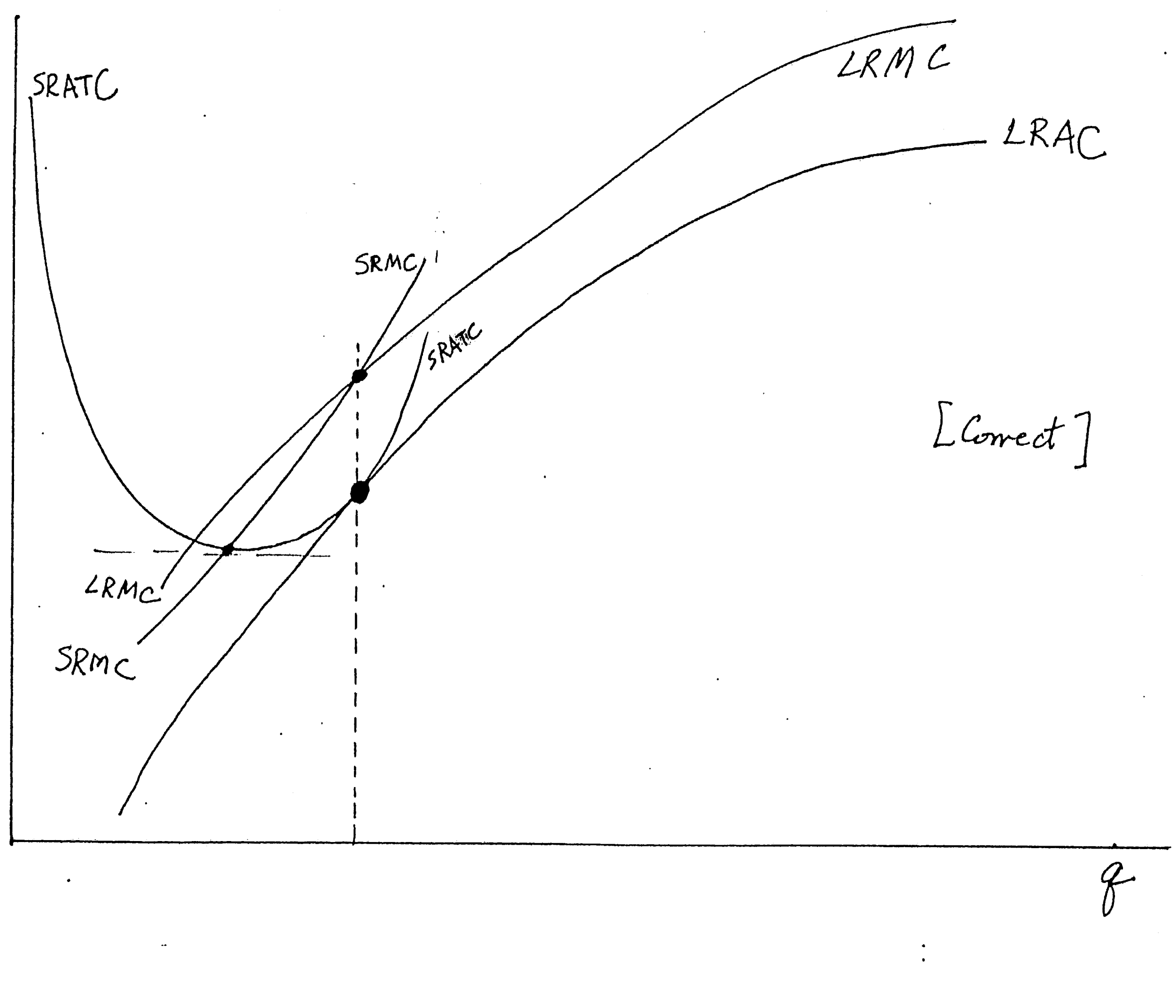
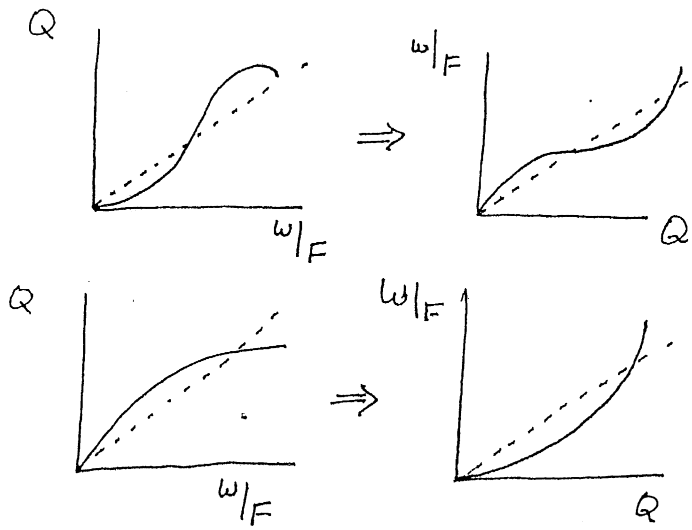

SRATC
LRAC
SRMC
LRMC
SRATC
SRMC
LRAC
LRMC
[Incorrect]
q

Figure 3


SRATC
LRMC
LRAC
SRMC
SRATC
[Correct]
LRMC
SRMC
q

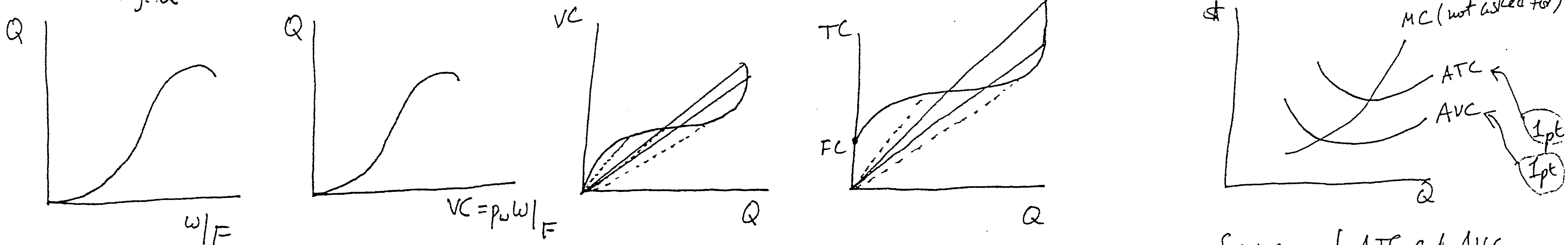
(17) a) The Law of Diminishing Returns states that the marginal product of any input eventually falls. Applied to these graphs, this means that the slope of the curves (which is the marginal product of water) must eventually fall. Figures 1 and 3 have $MP_W$ rising for large W, so they violate the Law of Diminishing Returns. Figures 2 and 4 have $MP_W$ falling eventually, so they are possible cross-sections of the production function.

(Refer to the graphs on the next page.)




For part (b), the third graph in each series can also be derived by first rotating the left-hand graph around a 45° line and then stretching or shrinking the vertical axis as it changes from W/F to VC. Only the second graph in each explanation would change.

Fig. 2




A stretching or shrinking of the horizontal axis.

2pts

Rotation about a 45° line.

AVC first falls (dotted lines flatten out) then rises (solid lines get steeper)

1pt

2pts

TC = VC + FC, so add FC to the previous graph.

ATC first falls, then rises (same reasons as before).

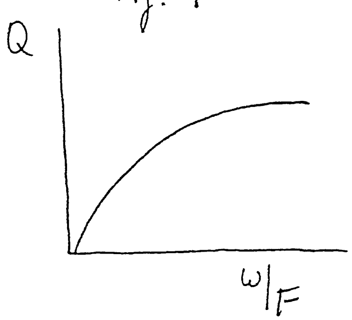
2pts

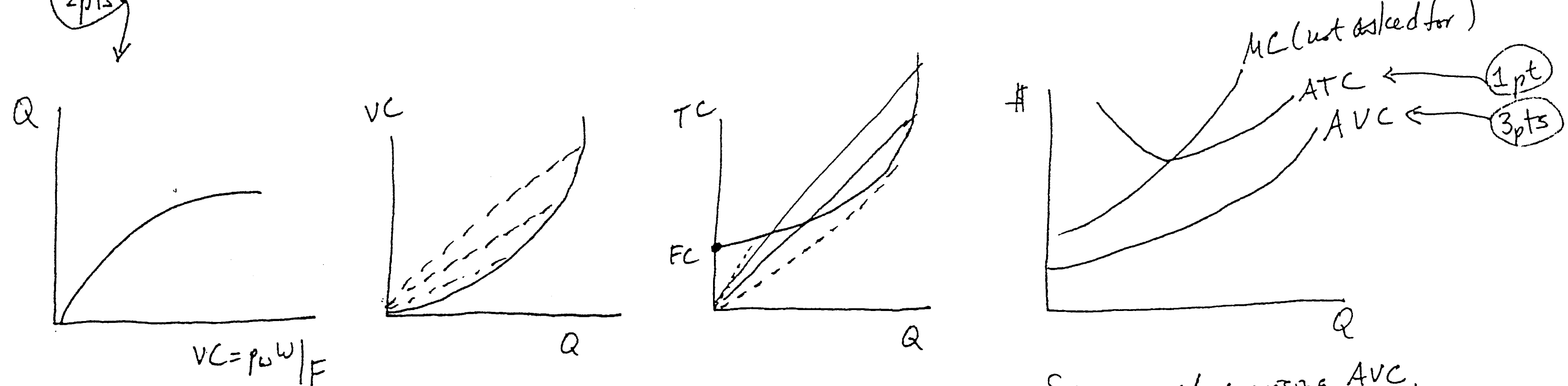
1pt

Summary of ATC and AVC from last two graphs. ATC > AVC since ATC = AVC + AFC, and AFC > 0. U-shaped ATC and AVC.

2pts

Fig. 4




(See reason above.)

(Rotation as above.)

AVC always rising.

1pt

2pts

(Add FC as above.)

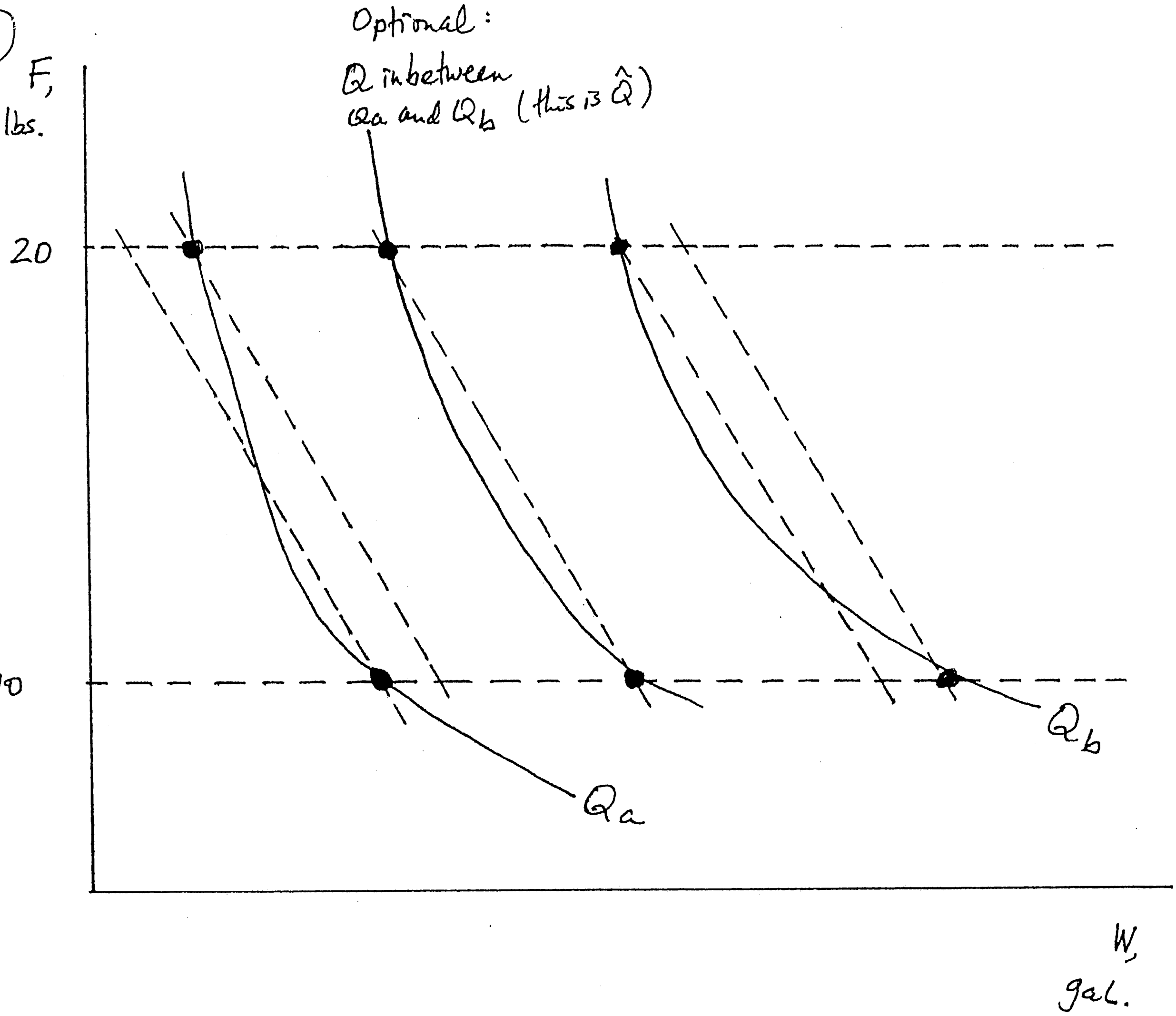
ATC first falling, then rising.

2pts

1pt

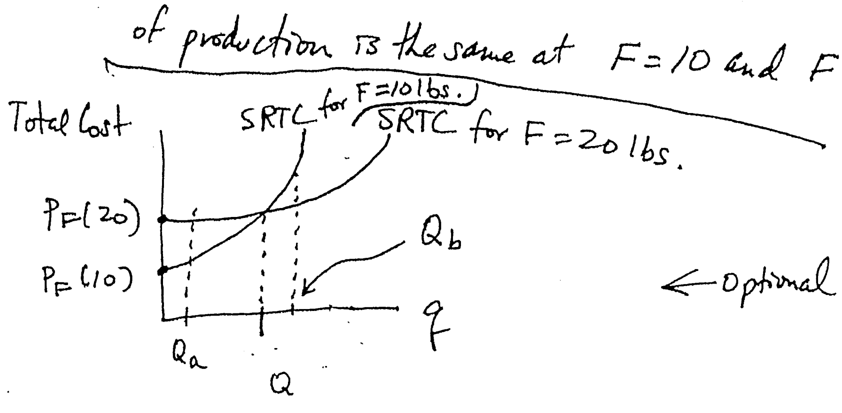
Summary: always rising AVC, U-shaped ATC.

(18)




The slanted lines are isocost curves; those closest to the origin represent smaller cost.

From the graph, the statements given in the problem's "hint" are obvious. For the optional "in-between" isoquant, total cost of production is the same at $F = 10$ and $F = 20$.




Isoquants: 7 pts
Isocost lines: 10 pts
Correct conclusion about the best way to produce one of the quantities: 10 pts
Correct other quantity: 7 pts

(19)

a) With $F=4$, one needs $W=2$ (4pts) to produce 15 bushels and $W=5$ (4pts) to produce 20 bu. $F=4$ lbs. costs \$16, $W=2$ costs \$4, and $W=5$ costs \$10. So the total cost of producing $Q=15$ is \$16 + \$4 = \$20 (1pt), and the total cost of producing $Q=20$ is \$16 + \$10 = \$26 (1pt). This gives the short-run total cost curve sketched below (see two pages from now).

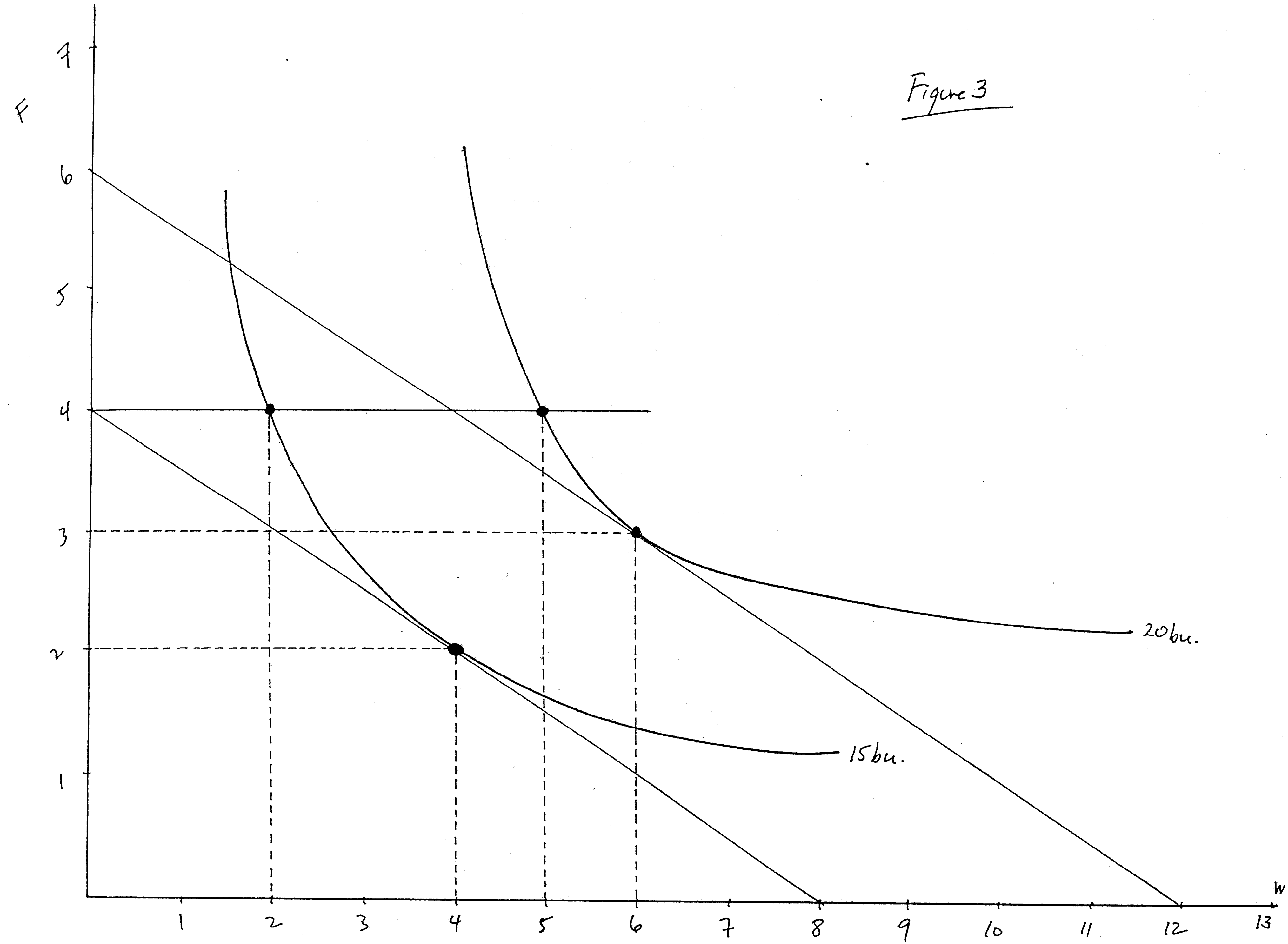
b) The isocost lines have a slope of $\frac{-P_W}{P_F} = \frac{-1}{2}$. The closest-in isocost lines touching the $Q=15$ bu. and $Q=20$ bu. isoquants are sketched in the figure on the next page. The least-cost way of producing $Q=15$ is to use $W=4$ (3pts) and $F=2$; together this costs \$8 + \$8 = \$16 (1pt). The least-cost way of producing $Q=20$ is to use $W=6$ (3pts) and $F=3$; together this costs \$12 + \$12 = \$24 (1pt). This gives the long-run total cost curve, sketched two pages from now.

+ 6pts

There is a short-cut to figuring out total costs. Take $Q=15$ bu. as an example. The

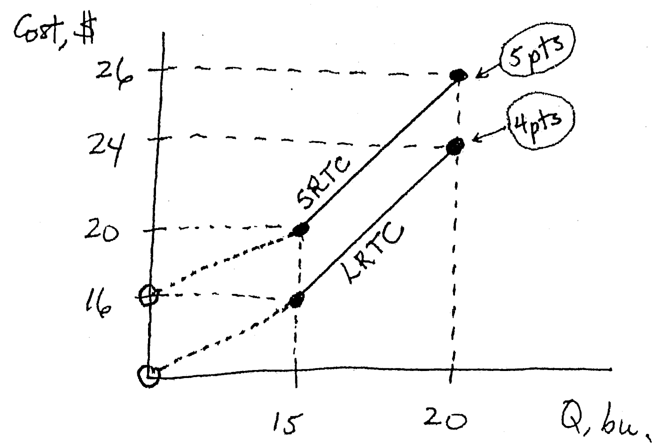
→

Figure 3



least-cost point ($W = 4$, $F = 2$) has to have the same cost as the point ($W = 0$, $F = 4$) and also the same cost as ($W = 8$, $F = 0$), since all three of these points lie on the same isocost line. Calculating costs at either one of these two points is very easy: costs at ($W = 0$, $F = 4$) are just 4 lbs. of F × \$4/lb. = \$16, and costs at ($W = 8$, $F = 0$) are 8 gal. of W × \$2/gal. = \$16. Of course, this is the same answer we got for the cost of ($W = 4$, $F = 2$).

A sketch of the cost curves is:

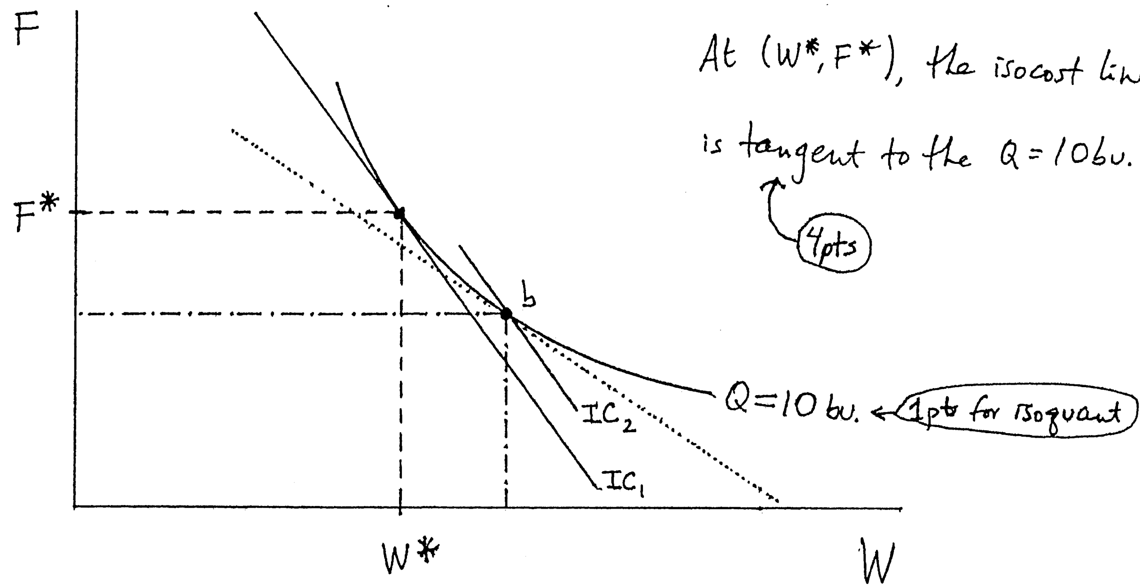



As one expects, SRTC > LRTC. This will be true except at one particular Q, where SRTC will equal LRTC. At that point, the cost-minimizing long-run amount of fertilizer to buy will be 4 lbs., which is exactly what was contracted for in the short run.

Notice also that at $Q = 0$:

- LRTC must be zero
- SRTC = fixed cost = $p_F F_0$ = (\$4/lb.)·(4 lbs.) = \$16.

(20)




At $(W^*, F^*)$, the isocost line ($IC_1$) is tangent to the Q = 10bu. isoquant.

4pts

3pts

a) If the firm is forced to buy less F, it moves to a point like "b" on the Q = 10bu. isoquant. The isocost line going through this point is $IC_2$, which represents a higher cost than $IC_1$, so costs have gone up. 5pts, including "costs ↑"

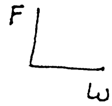
b) The dotted line in the graph is tangent to the Q = 10bu isoquant at point b, while $IC_1$ is tangent to the Q = 10bu isoquant at $(W^*, F^*)$. $IC_1$ has a slope which is more negative than the dotted line. So the slope of the isoquant

2pts increases (gets closer to zero) from $(W^*, F^*)$ to b. RTS of W for F = −slope of the isoquant, so this decreases going from $(W^*, F^*)$ to b.

4pts

1pt

(21)

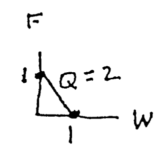
a. If the axes are  then the slope of the isocost lines is $-\frac{P_W}{P_F} = \frac{-2}{1} = -2.$

2 bushel isoquant: production function $Q = 2W + 2F$

$Q = 2$

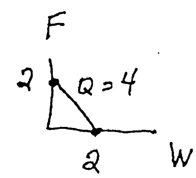
So $2 = 2W + 2F$

$1 = W + F$

$1 - W = F$ which looks like 


4 bushel isoquant: $4 = 2W + 2F$

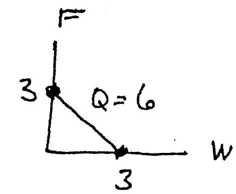
$2 = W + F$

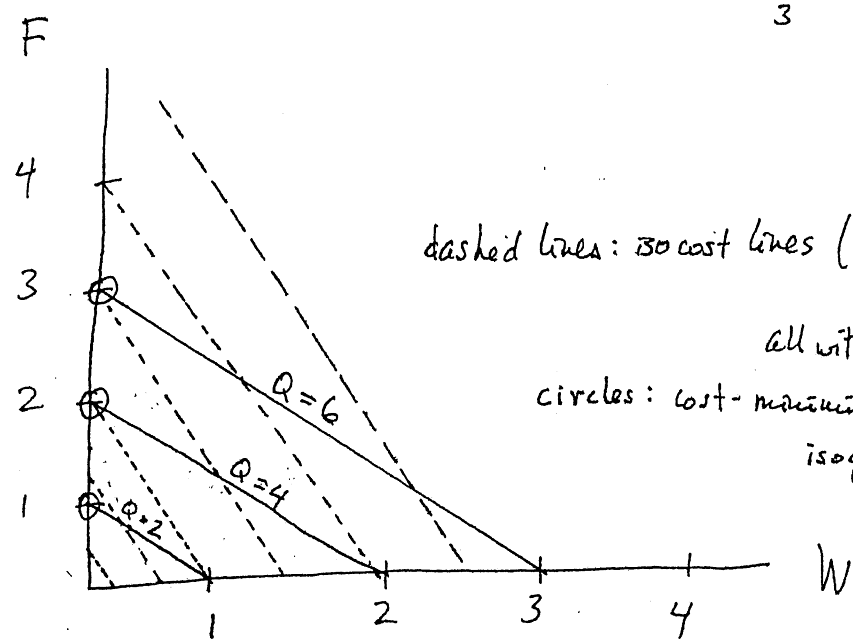
$F = 2 - W$ 

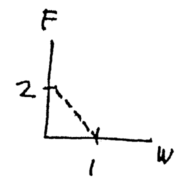
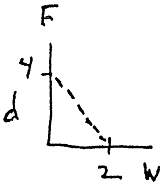
6 bushel isoquant: $6 = 2W + 2F$

$3 = W + F$

$F = 3 - W$




dashed lines: iso cost lines (such as  and 

all with slope -2

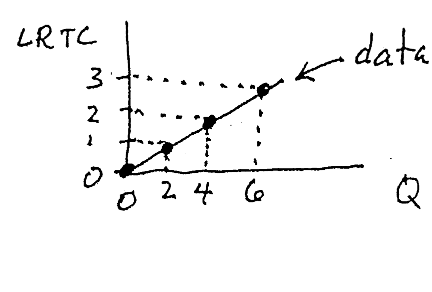
circles: cost-minimizing points on each isoquant (Q=2, Q=4, Q=6)

| Q | least-cost W | least-cost F | cost of least-cost W | cost of least-cost F | total Cost | average cost = total cost / Q |
|---|---|---|---|---|---|---|
| 2 | 0 | 1 | 0 | \$1 | \$1 | .5 |
| 4 | 0 | 2 | 0 | \$2 | \$2 | .5 |
| 6 | 0 | 3 | 0 | \$3 | \$3 | .5 |
| 0 | 0 | 0 | 0 | \$0 | \$0 | |

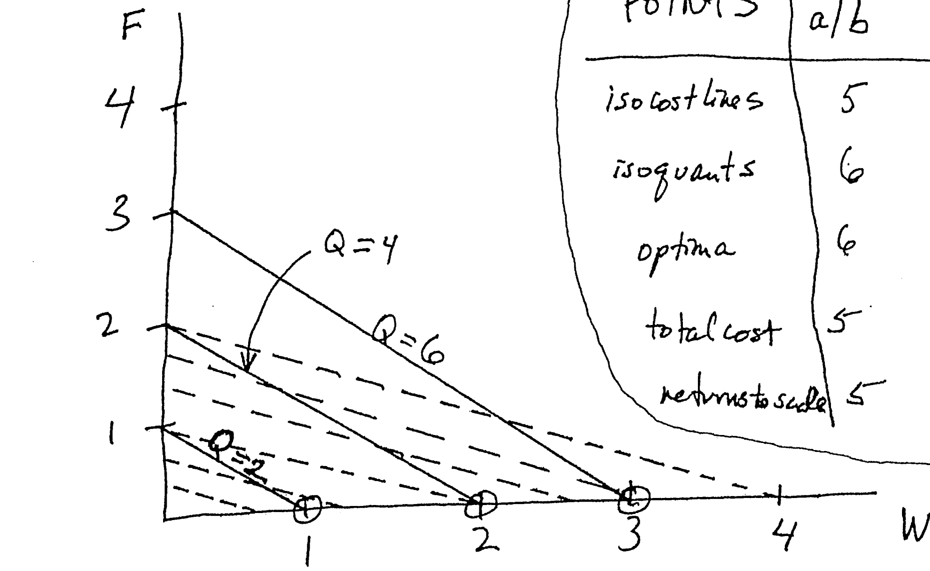
b)

LRAC constant ⇒ constant returns to scale

Or, alternatively:



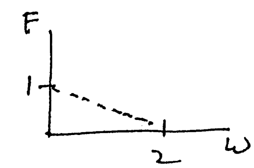

data points from the table in part (a); since LRTC is linear, it's constant returns to sca c)



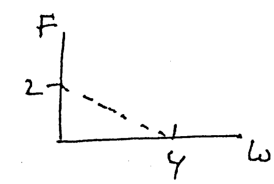

| POINTS | part a/b | part c |
|---|---|---|
| isocost lines | 5 | 1 |
| isoquants | 6 | 1 |
| optima | 6 | 3 |
| total cost | 5 | 1 |
| returns to scale | 5 | 1 |

dashed lines are isocost lines; their slope is $\frac{-P_W}{P_F} = \frac{-2}{4} = \frac{-1}{2}$, so

for example

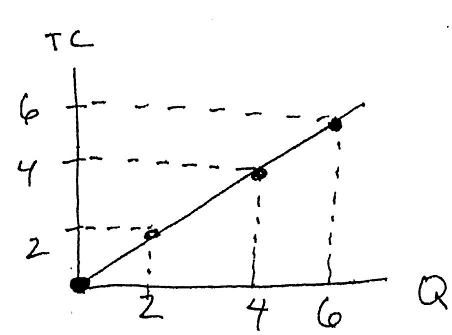



and




circles: cost-minimizing points on each isoquant

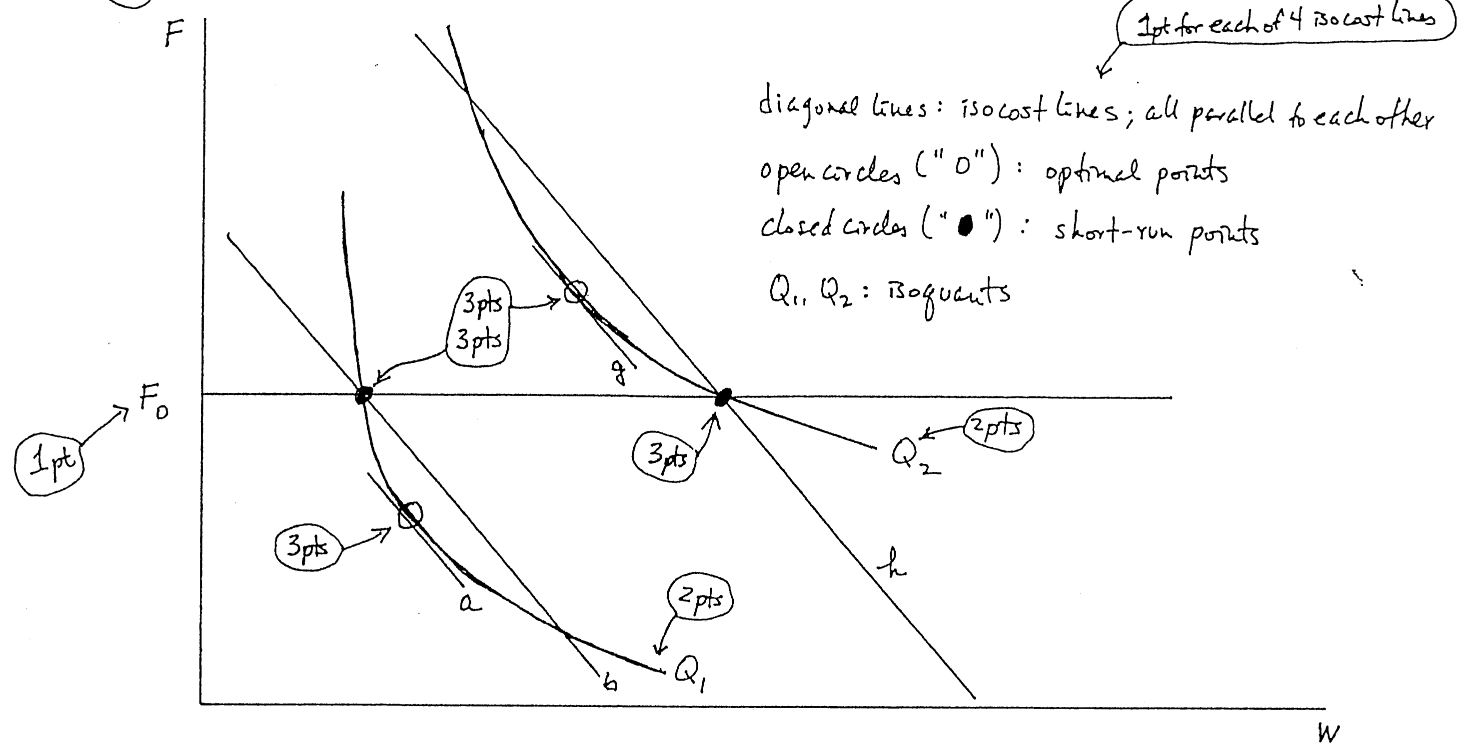
| Q | least-cost W | least-cost F | cost of least-cost W | cost of least-cost F | total cost | AC |
|---|---|---|---|---|---|---|
| 2 | 1 | 0 | \$2 | \$0 | \$2 | 1 |
| 4 | 2 | 0 | \$4 | \$0 | \$4 | 1 |
| 6 | 3 | 0 | \$6 | \$0 | \$6 | 1 |
| 0 | 0 | 0 | \$0 | \$0 | \$0 | |




From either source:

constant returns to scale

(22)




a) To produce $Q_1$, the optimal amount of F is below $F_0$.

b) To produce $Q_2$, " " " " " above $F_0$.

c) Isocost lines represent the total cost of production. From least costly to most costly, they are ranked a, b, g, and h. For this question, long-run cost is "a", short-run cost is "b," so long-run cost is less. "a" is long-run total cost. (4pts) (4pts)

d) "g" is the long-run cost and "h" is the short-run cost, so the long-run cost is less than the short-run cost. In fact, long-run cost will always be less than short-run cost except for one single Q, where long-run and short-run cost will equal. (4pts)

(23) a) $Q = 2F^{1/2}W^2$ Now double all inputs:

new $Q = 2(2F)^{1/2}(2W)^2$ ← 5pts for doubling

$= 2 \cdot 2^{1/2} \cdot F^{1/2} \cdot 4 \cdot W^2$ 6pts for algebra

$= (2F^{1/2}W^2)(2^{1/2}2^2) = 2^{5/2}(2F^{1/2}W^2)$

$= 2^{5/2}(\text{old } Q) > 2(\text{old } Q)$ so increasing returns to scale.

5pts for inequality and answer

b) $F = 4$ lb. $p_F = \$3/\text{lb}$ $p_W = \$5/\text{gal}$.

(i) $TC = p_F F + p_W W = (3)(4) + 5W = 12 + 5W$ ← 5pts

(ii) $Q = 2F^{1/2}W^2 = 2(4)^{1/2}W^2 = 2(2)W^2 = 4W^2$, and

$\frac{Q}{4} = W^2 \Rightarrow \frac{\sqrt{Q}}{2} = W.$ $\boxed{\text{5pts for } Q = 4W^2 \text{ or } W = \sqrt{Q}/2}$

(iii) $TC = 12 + 5W$ from (i), but $W = \sqrt{Q}/2$ from (ii), so

$TC = 12 + 5\sqrt{Q}/2.$ ← 7pts

(iv) $TC(Q=0) = 12$

$TC(Q=1) = 12 + \frac{5}{2} = 14\frac{1}{2}$ 2pts

$MC = \frac{TC(Q=1) - TC(Q=0)}{1-0} = \frac{14\frac{1}{2} - 12}{1} = 2\frac{1}{2}$ since $MC = \frac{\Delta TC}{\Delta Q}$. Other answers

3pts

2pts ...... are possible; calculus shows the exact answer is 1.25.

(24)

a) $MP_{F1} = MP_{F2}$ for cost minimization

(7 points)

$$\frac{5}{2\sqrt{F_1}} = \frac{5}{\sqrt{F_2}} \Rightarrow \frac{\sqrt{F_1}}{\sqrt{F_2}} = \frac{1}{2} \Rightarrow \frac{F_1}{F_2} = \frac{1}{4}, \quad F_2 = 4F_1.$$

(6 points)

b) $\frac{Q_1}{Q_2} = \frac{5\sqrt{F_1}}{10\sqrt{F_2}} = \frac{1}{2}\sqrt{\frac{F_1}{F_2}} = \frac{1}{2}\left(\frac{1}{2}\right)$ from part (a) (that is, from $F_2 = 4F_1$), so $\frac{Q_1}{Q_2} = \frac{1}{4}$ or $Q_2 = 4Q_1$.

(6 points)

c) $Q = Q_1 + Q_2$

$= Q_1 + 4Q_1 = 5Q_1$

(6 points)

d) $Q = 5Q_1$

$= 5 \cdot 5\sqrt{F_1} = 25\sqrt{F_1}$.

So $\sqrt{F_1} = \frac{Q}{25}$ and $F_1 = \frac{Q^2}{625}$. Similarly, $F_2 = 4F_1 = \frac{4Q^2}{625}$.

Hence total cost $= p_F(F_1 + F_2) = 625\left(\frac{Q^2}{625} + \frac{4Q^2}{625}\right) \Rightarrow$

$C(Q) = 5Q^2$.