C. Changes in Income and Prices

1. Assume that X is a normal good and Y is an inferior good.
   (a) Graph the income expansion path for X and Y. (Illustrate at least three different points on the income expansion path.) Show indifference curves on your graph and explain your graph.
   (b) Sketch the Engel Curves for X and Y. Explain!

2. Referring to Figure 1, fill in a chart like the following one. Under "X" write 'normal,' 'inferior,' and/or 'Giffen.' (Be sure to write all that apply.) Under "Y" do the same thing. Under "X and Y" write either 'complements' or 'substitutes.'

	X	Y	X and Y
1			
2			
3			
4			
5			

3. Referring to Figure 3, fill in a chart like the following one. Under X write "normal," "inferior," and/or "Giffen." Under Y do the same. Under "X and Y" write either "complements" or substitutes."

	X	Y	X and Y
1			
2			
3			
4			
5			

   (Write your answers on your answer sheet, not on this piece of paper.)

4. Referring to Figure 1, fill in a chart like the following one on your answer sheet. Under "X" write 'normal,' 'inferior,' and/or 'Giffen.' (Write all that apply.) Under "Y" do the same. Under "X and Y" write either 'complements' or 'substitutes.' Also, very briefly explain each of your answers.

	X	Y	X and Y
1			
2			
3			
4			
5			

5. Assume that X is a normal good and Y is a normal good. Also, assume that X and Y are complements. Graphically illustrate the income and substitution effects of a rise in the price of Y. Put Y on the vertical axis and X on the horizontal axis, as we usually do. Explain your answer step by step.

6. Assume that X is a normal good and Y is an inferior good. Also, assume that X and Y are complements. Graphically illustrate the income and substitution effect of a rise in the price of Y. Put Y on the vertical axis and X on the horizontal axis, as we usually do.

7. A consumer buys two goods X and Y. Graph X on the horizontal axis and Y on the vertical axis. Suppose the price of Y falls. Suppose X and Y are just on the border between being complements and being substitutes.

   (a) Which is stronger: the income effect for X or the substitution effect for X?

   (b) Which is stronger: the income effect for Y or the substitution effect for Y?

   There are two correct answers to (a); give both of them. There are also two correct answers to (b); give both of them. Explain.

8. In a two-good economy, suppose the price of good Y rises while the price of X is unchanged. Illustrate the income and substitution effects in a graph in which you plot X on the horizontal axis and Y on the vertical axis. Are X and Y complements or substitutes in your graph? Is X a normal good or an inferior good? Is Y a normal good or an inferior good? Thoroughly explain.

9. Suppose there are only two goods in the economy, X and Y. By drawing a graph with X on the horizontal axis and Y on the vertical axis, prove that the following statement is FALSE: "If X and Y are normal goods, then they must be complements."

10. Suppose a consumer's budget constraint passes through the bliss point of his preference map (the map is the X-Y plane, where X and Y are the two goods).

    (a) Does he consume where his marginal rate of substitution equals the price ratio? Explain, making a sketch in the X-Y plane. Also: explain what is meant by the "price ratio," and explain that the "marginal rate of substitution" is really the "marginal rate of substitution of [fill in the blank] for [fill in the blank]."

    (b) Suppose the price of X drops a little. How does the consumer's demand for X change? How does the consumer's demand for X change if the price of X increased a little? Sketch these changes in the X-Y plane. Then, sketch the consumer's demand curve for X.

11. Farmer Sam consumes only two goods, x and y. The price of x is $p_x = \$1/\text{unit}$ and the price of y is $p_y = \$1/\text{unit}$.

    Farmer Sam grows 4 units of x and he grows 3 units of y.

    (a) Sketch Farmer Sam's budget constraint.

    (b) Suppose Farmer Sam decides to consume more than 4 units of x. Sketch the indifference curve passing through Farmer Sam's utility-maximizing bundle of x and y.

    (c) Show on your graph that a *small* rise in the price of x would make Farmer Sam worse off.

    (d) Re-draw the graph you drew in part (b) and show on this graph that a *large* rise in the price of x could make Farmer Sam better off.

12. Suppose: there are two goods X and Y; their prices are $p_X = 1$ and $p_Y = 1$ respectively; and the consumer's income is \$2. Then the budget constraint looks as in Figure 1. (The consumer is not forced to buy an integer number of units of X or Y such as 0, 1, 2, etc. units; he could buy a fractional number of units of X or Y, such as 1.523 units of X, if he wished.)

    (a) Now suppose the seller of good X offers buyers the following "volume discount": if the buyer buys more than 1 unit of X, the additional units of X beyond 1 unit will have their price cut from \$1.00 per unit to \$0.50 per unit.

        On Figure 1, draw the new budget constraint.

(b) Turn to Figure 2. Figure 2 is identical to Figure 1, and it is based on the same assumptions: $p_X = 1$, $p_Y = 1$, and the consumer's income is \$2.

Starting from Figure 2, now suppose the seller of good X offers buyers the following "two for one" offer: if the buyer buys 1 or more units of X (but less than 2 units of X), the seller will give the buyer an additional unit of X free; if the buyer buys 2 or more units of X (but less than 3 units of X), the seller will give the buyer an additional 2 units of X free; and so forth.

i. On Figure 2, draw the new budget constraint.

ii. On Figure 2, draw old and new optimal indifference curves.

iii. Based on the indifference curves you drew in (ii), was the effect of the "two for one" offer to increase purchases of Y or to decrease purchases of Y?

iv. Based on the indifference curves you drew in (ii), was the effect of the "two for one" offer to increase expenditures on Y or to decrease expenditures on Y?

v. Based on the indifference curves you drew in (ii), was the effect of the "two for one" offer to increase total consumption of X or to decrease total consumption of X?

vi. Based on the indifference curves you drew in (ii), was the effect of the "two for one" offer to increase expenditures on X or to decrease expenditures on X? (Hint: remember your answer to part (iv).)

13. The government wishes to give a fixed amount of money to a consumer who only buys two goods. Should the government give the money to the consumer as a lump sum, or as a subsidy on the price on one of the two goods? Illustrate your answer with a carefully drawn graph. An algebraic proof is not necessary; you can reason by analogy with a lecture I gave on a similar topic.

14. Show that a lump-sum subsidy is superior to a subsidy on the price of one good. You can do this in one of two ways:

i. Show that if the government spends the same amount on a lump-sum subsidy or on a subsidy on one good, consumers will reach a higher level of utility under the first plan than under the second plan.

or

ii. Show that the government can save money by switching from a subsidy on one good to a lump-sum subsidy, and keep consumers just as well off as before.

Method (i) is probably easier since it is closer to something that was done in class.

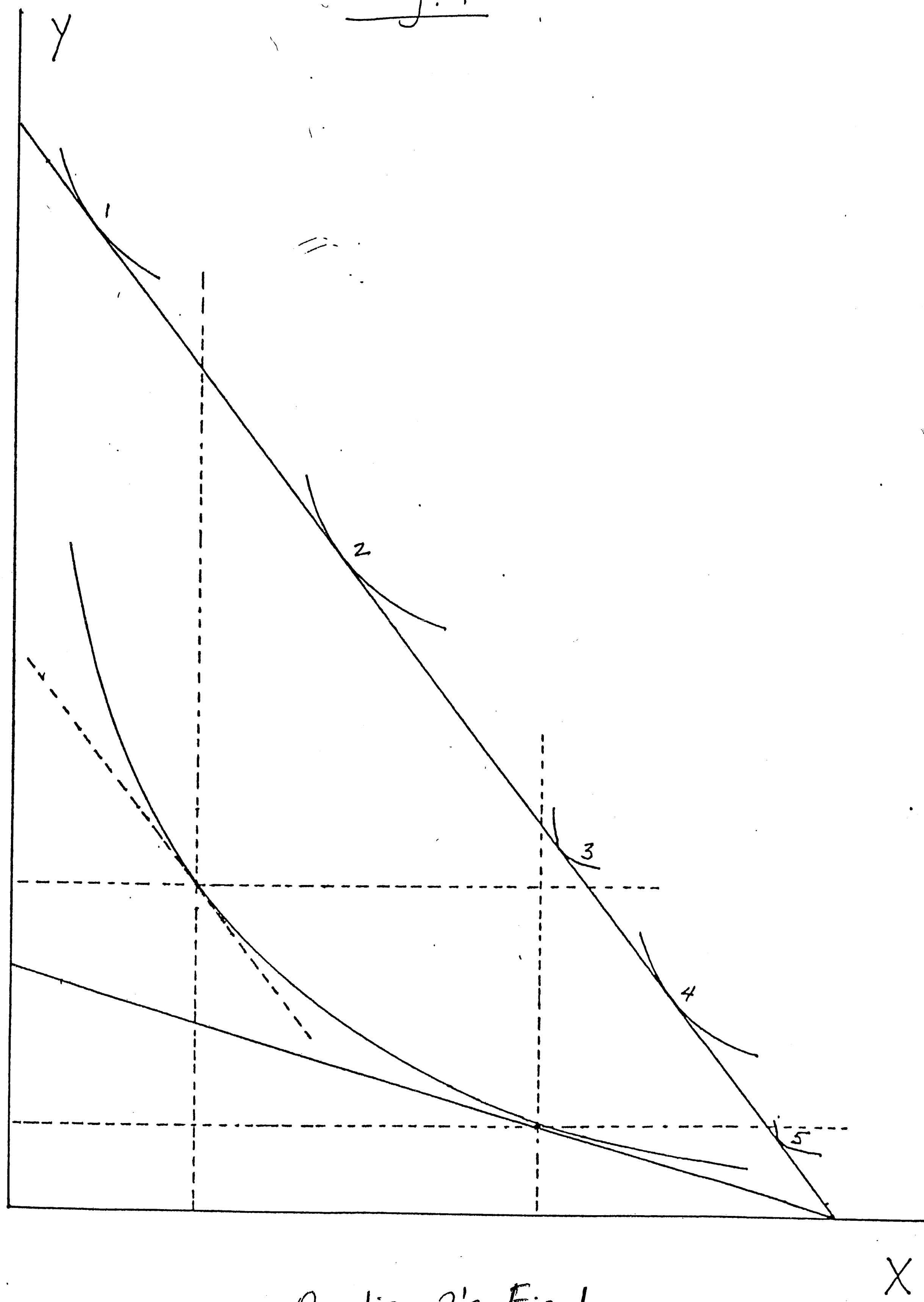
15. Prove, using a diagram, that a consumer is better off under a lump-sum tax than under a specific tax on one good that produces equal revenue. Label your graph carefully.

    Also, explain how the following algebraic steps help in your argument. Here I is the consumer's income, p_x is the price of good X, p_y is the price of good Y, t is the amount of the specific tax, and (X_s, Y_s) is the optimum consumption bundle of a consumer facing a specific tax.

    (a) $I = p_x X + tX + p_y Y$.
    (b) $I = p_x X_s + tX_s + p_y Y_s$.
    (c) $I -$ lump sum tax $= p_x X + p_y Y$.
    (d) lump sum tax = specific tax.
    (e) $I - tX_s = p_x X + p_y Y$.
    (f) $I - tX_s = p_x X_s + p_y Y_s$?

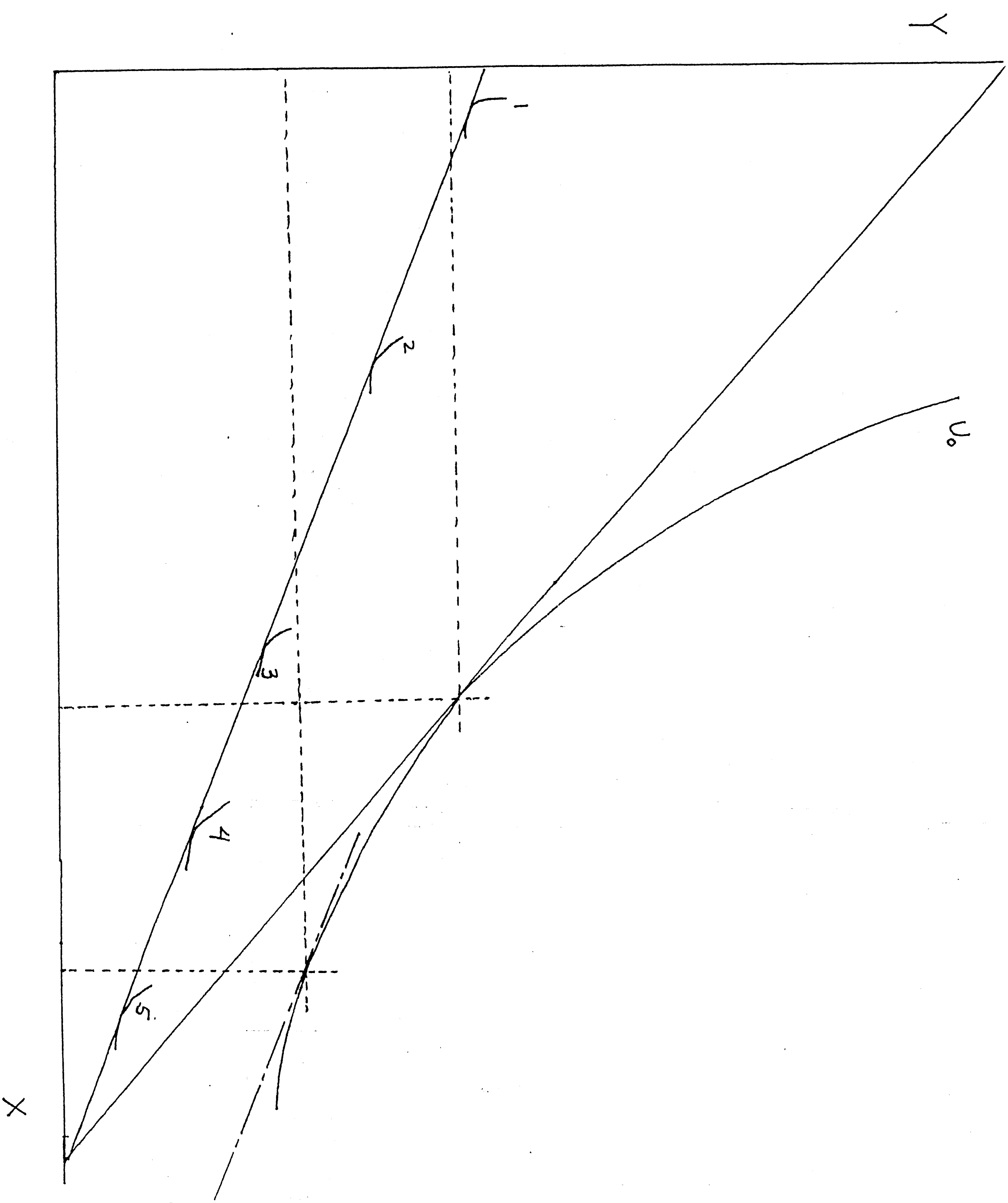
16. Mr. B has indifference curves as in Figure 2. His income is \$10 and the price of Y is \$2. Sketch three points on his demand curve for X.

Fig. 1



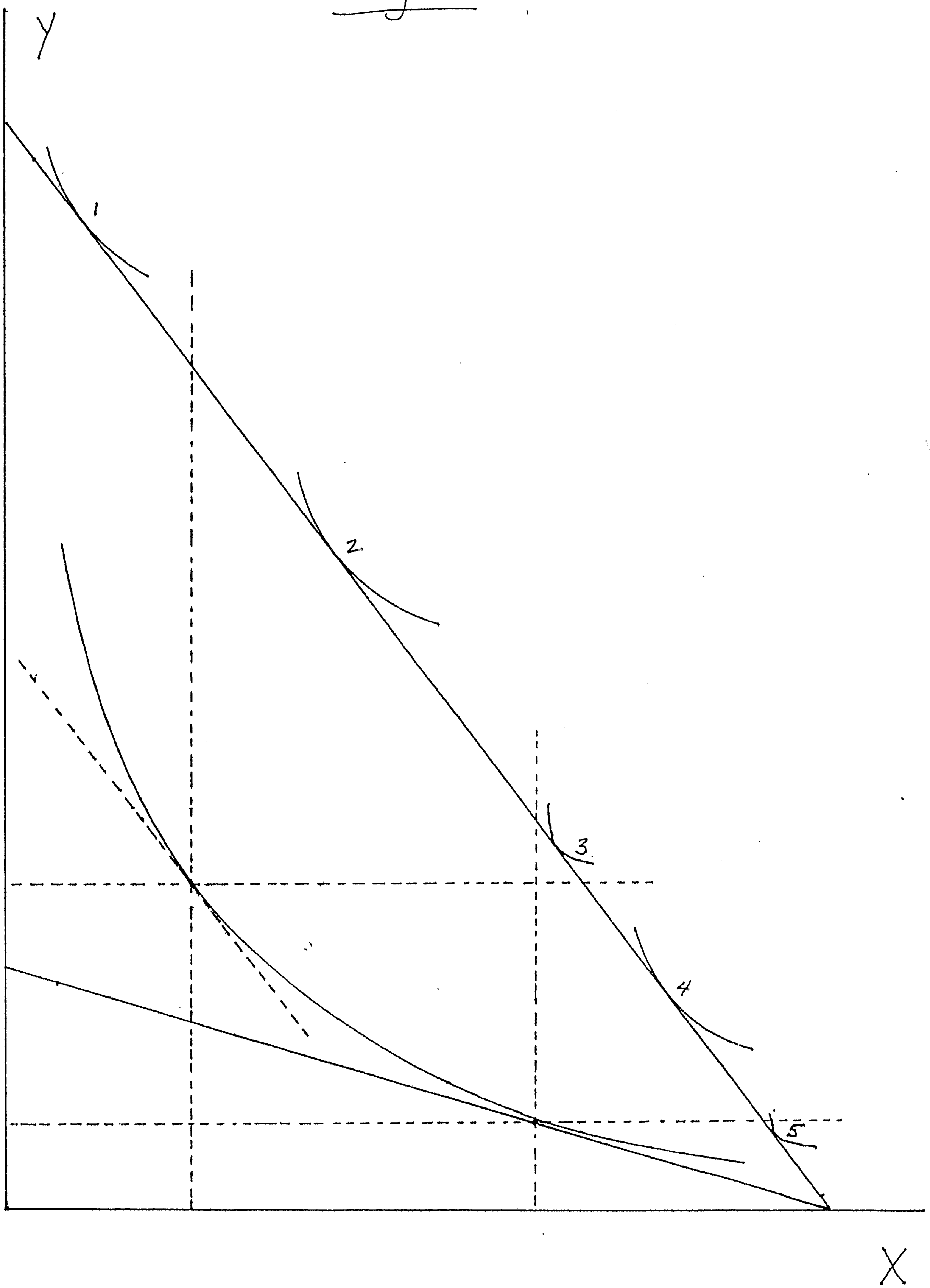
Question 2's Fig. 1

Figure 3



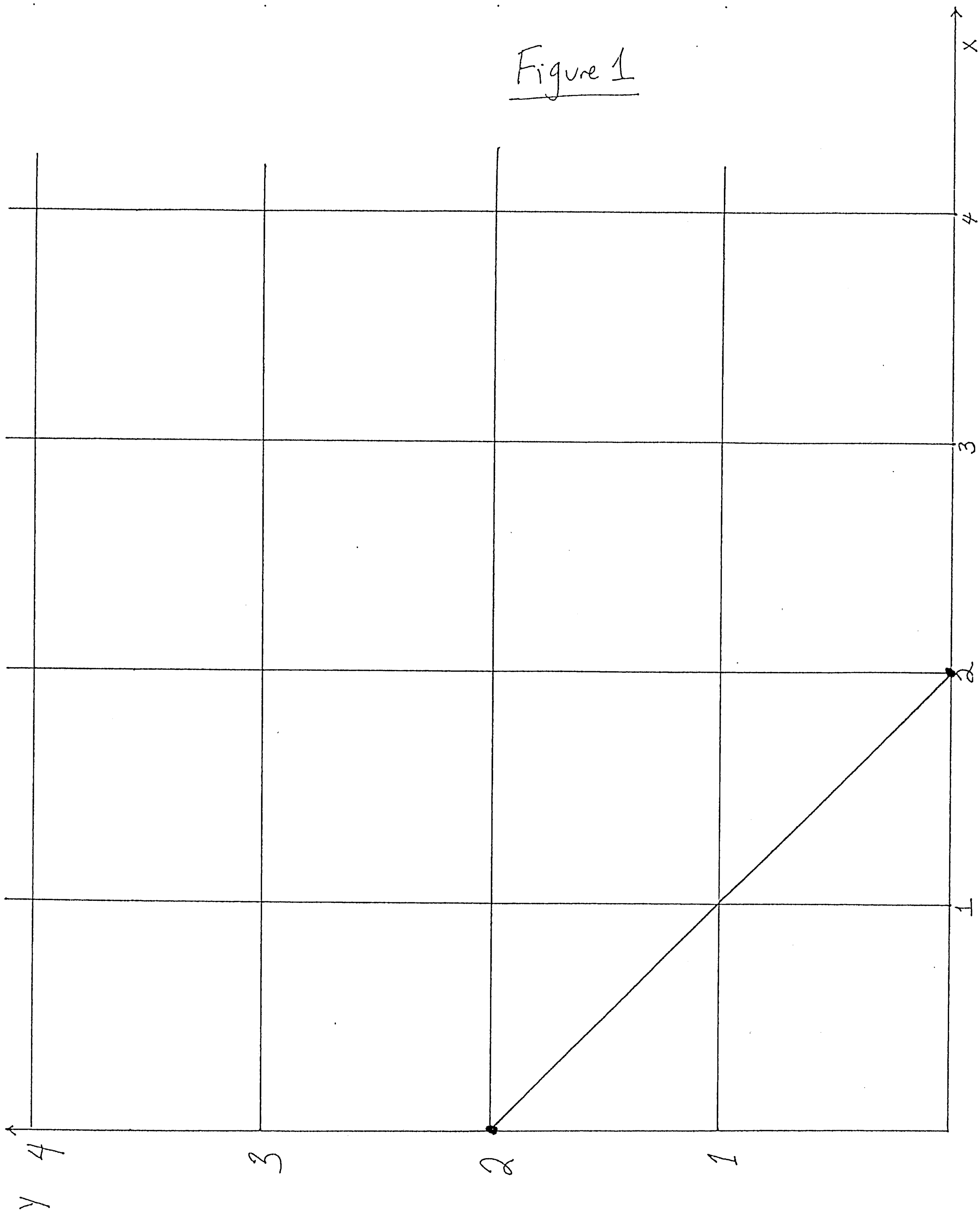
Question 3's Fig. 3

Fig. 1



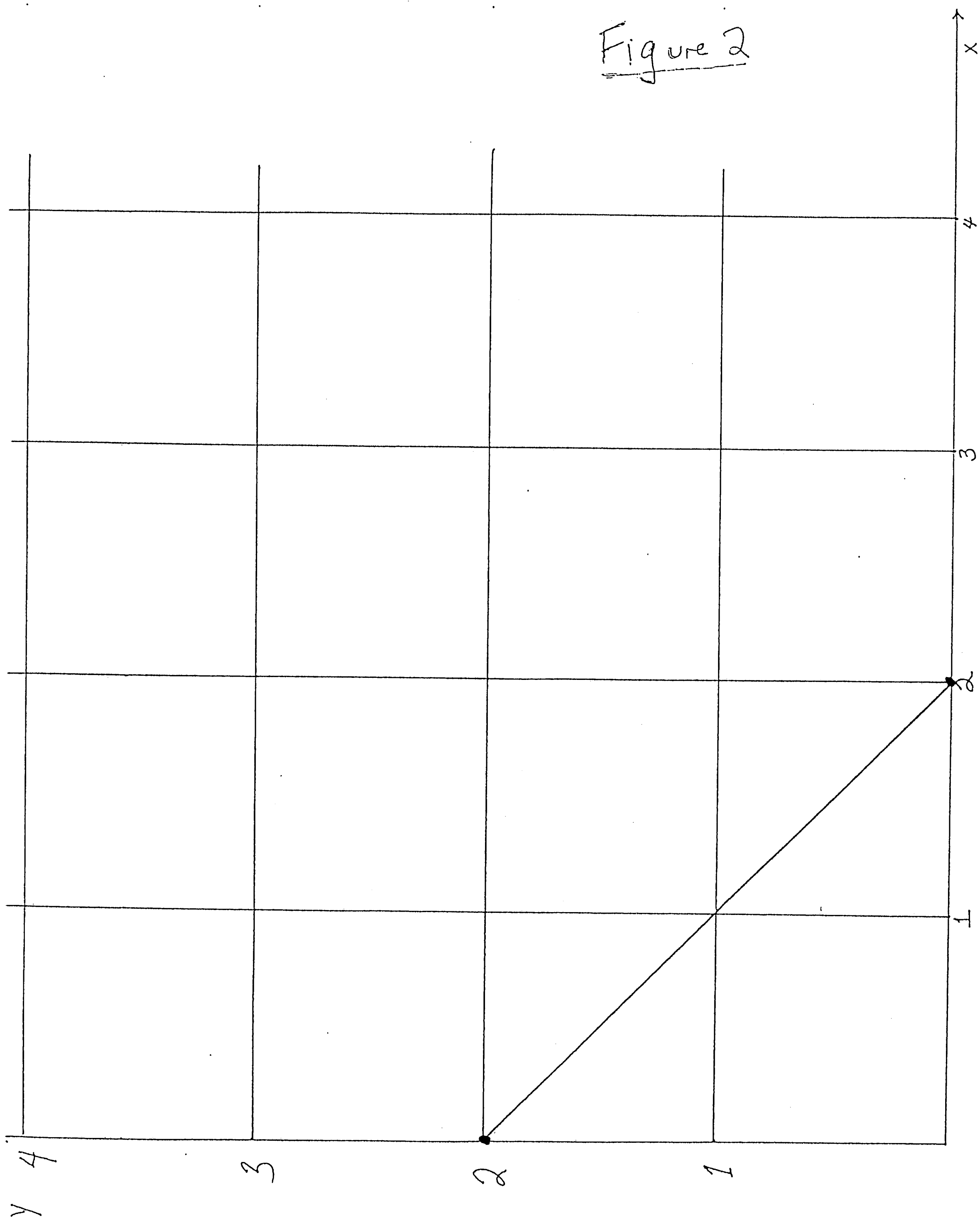
Question 4's Fig. 1

Figure 1

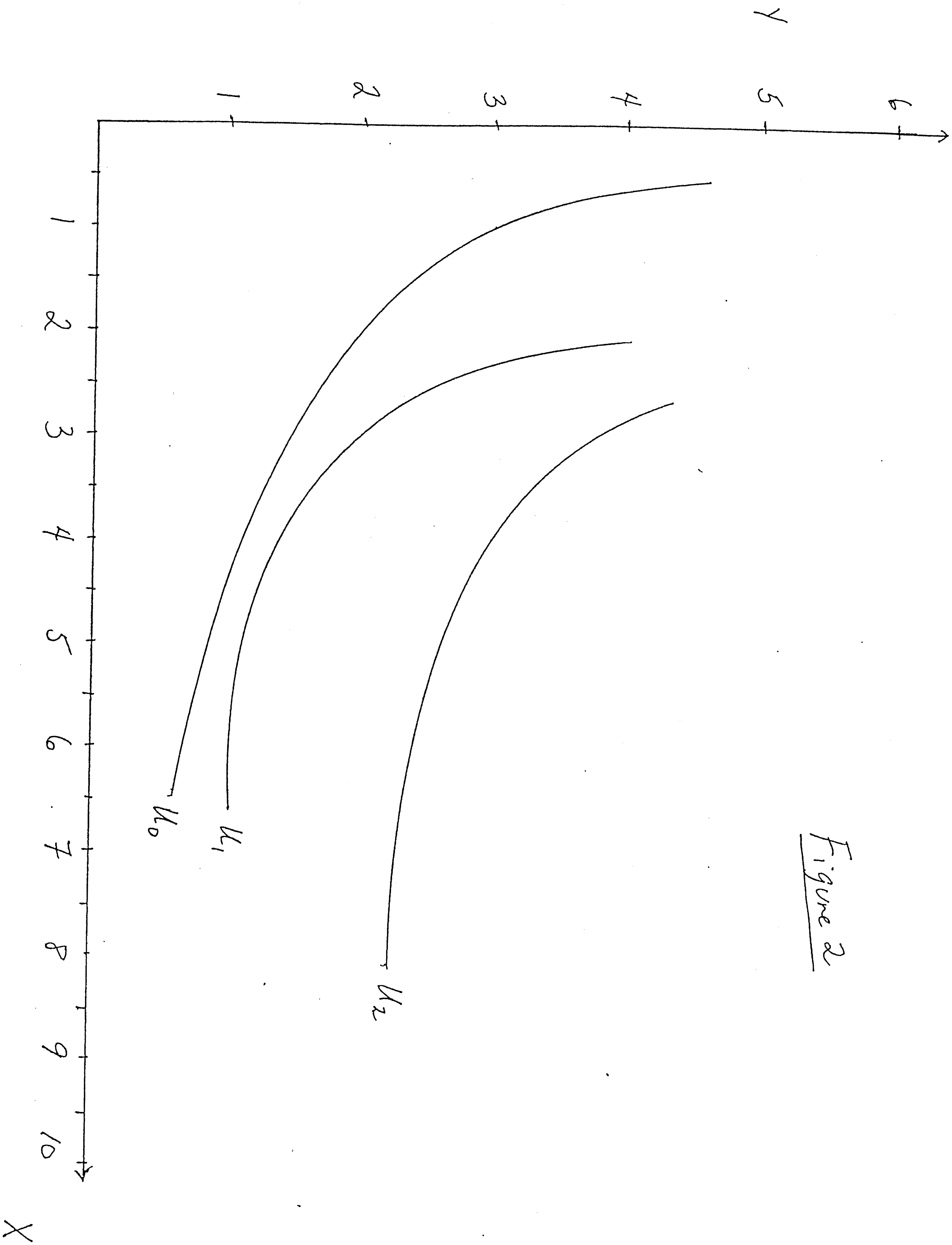


Question 12's Fig. 1

Figure 2



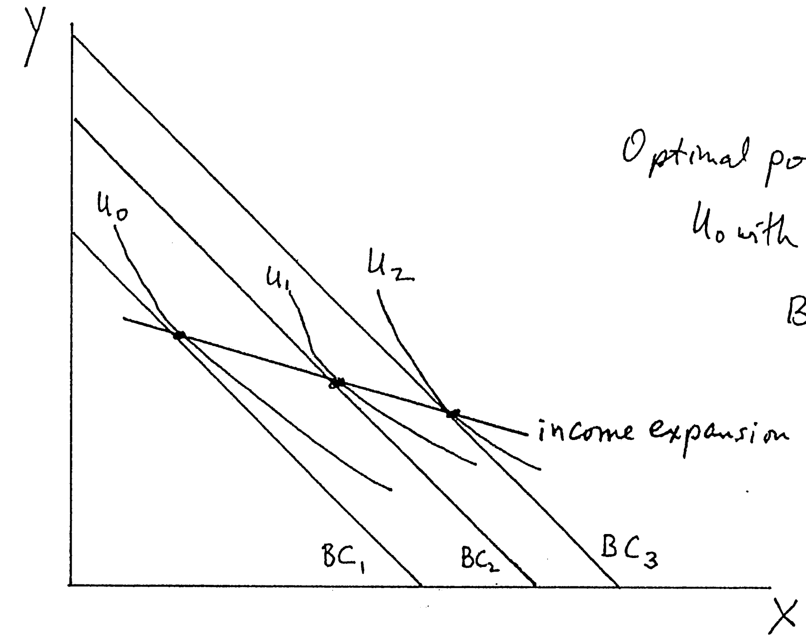
Question 12's Fig. 2



Question 16's Fig. 2

① a.

Answers



Optimal points are at the tangent points of U_0 with BC_1, U_1 with BC_2, and U_2 with BC_3.

↑ income ⇒ BC shifts out. So BC_1, BC_2, and BC_3, which are parallel, denote ever-increasing income.

As you go from BC_1 to BC_3, the optimal amount of X ↑, so X is normal.

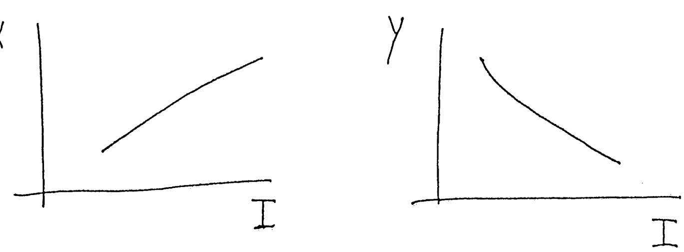
As you go from BC_1 to BC_3, income ↑ but the optimal amount of Y ↓, so Y is inferior.

The income expansion path is therefore downward sloping.

⑤ budget constraints

⑤ locating proper points on budget constraints (with indifference curves (2 points)) for X normal, Y inferior

⑤ income expansion path drawn correctly

b. 

I: income

normal: X↑ when I↑ ⇒ Engel Curve (+) sloped (5)

inferior: Y↓ when I↑ ⇒ Engel Curve (−) sloped. (5)

(2) The changes in X and Y tabulated in rows 1 to 5 are measured from the point I've labeled "B" on the next page. The "Giffen" result just below row 5 is measured from point "A," as are the changes in the "X and Y" column.

	X	Y	X and Y
1	X↓ inferior	Y↑ normal	X↓ Y↑ substitutes
2	X↑ normal	Y↑ normal	X↓ Y↑ substitutes
3	X↑ normal	Y↑ normal	X↑ Y↑ complements
4	X↑ normal	Y↓ inferior	X↑ Y↑ complements
5	X↑ normal	Y↓ Giffen and inferior	X↑ Y↓ substitutes

P_y↓, Y↓

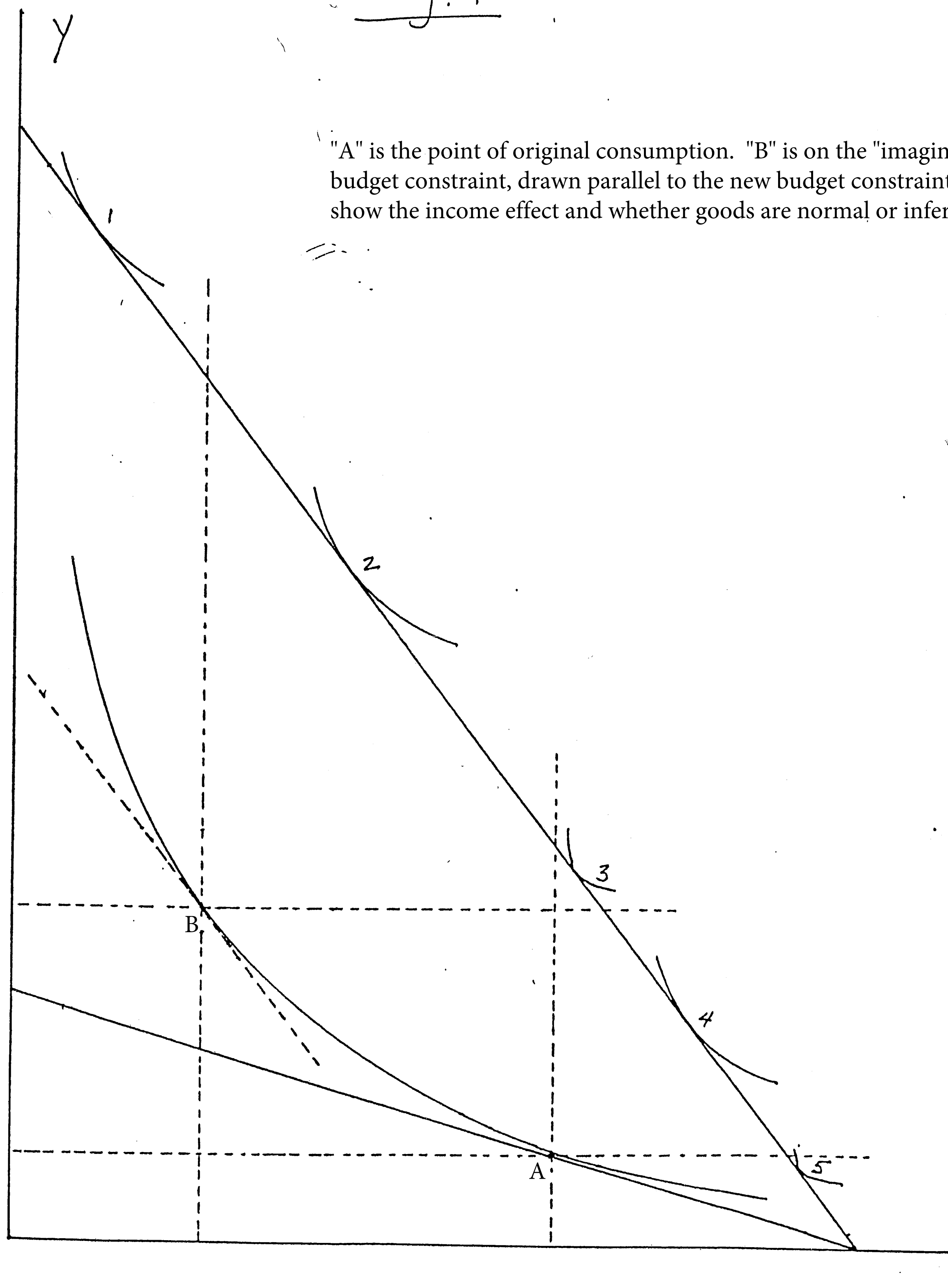
(Here, the price of Y has fallen) so it's like income has risen, even though it hasn't

[5/3 point each, rounded off to the nearest point]

Complements: X & Y move in the same direction. Substitutes: they move in opposite directions. Measure from the initial consumption bundle (rightmost vertical dashed line, lower horizontal dashed line). Inferior: I↑, consumption ↓. Normal: I↑, consumption ↑. Measure using 2 parallel lines, new B.C. and "imaginary" B.C. Measure from leftmost vertical dashed line's intersection with upper horizontal dashed line.

Fig. 1

"A" is the point of original consumption. "B" is on the "imaginary" budget constraint, drawn parallel to the new budget constraint to show the income effect and whether goods are normal or inferior.



Question 2's Fig. 1

(3)

	X	Y	X and Y
1)	normal X↓	inferior, Giffen Y↑	substitutes X↓ Y↑
2)	" X↓	inferior Y↑	complements X↓ Y↓
3)	" X↓	normal Y↓	" X↓ Y↓
4)	" X↓	" Y↓	substitutes X↑ Y↓
5)	inferior X↑	normal Y↓	" X↑ Y↓

Since the affordable set has shrunk in the Y direction, P_y has ↑ in this graph. This resembles a ↓ in income even though income is unchanged.

$1\frac{1}{4}$ pts each entry

Original point: intersection of leftmost vertical dashed line and upper horizontal dashed line

Imaginary intermediate point: " " rightmost " " " " lower " " "

Complements: X & Y move in the same direction (substitutes is the opposite). Measure from original point.

Normal: ↑ income ⇒ ↑ consumption (inferior is the opposite). Measure from "imaginary intermediate point" to get a parallel shift (like a change in income).

Giffen: P_y ↑ ⇒ demand for Y ↑ (or vice versa, P_y ↓ ⇒ demand for Y ↓). Giffen goods have upward-sloping demand curves.

④

	X	Y	X & Y
1	inferior	normal	subs.
2	normal	normal	subs.
3	normal	normal	comp.
4	normal	inferior	comp.
5	normal	inferior + Giffen	subs.

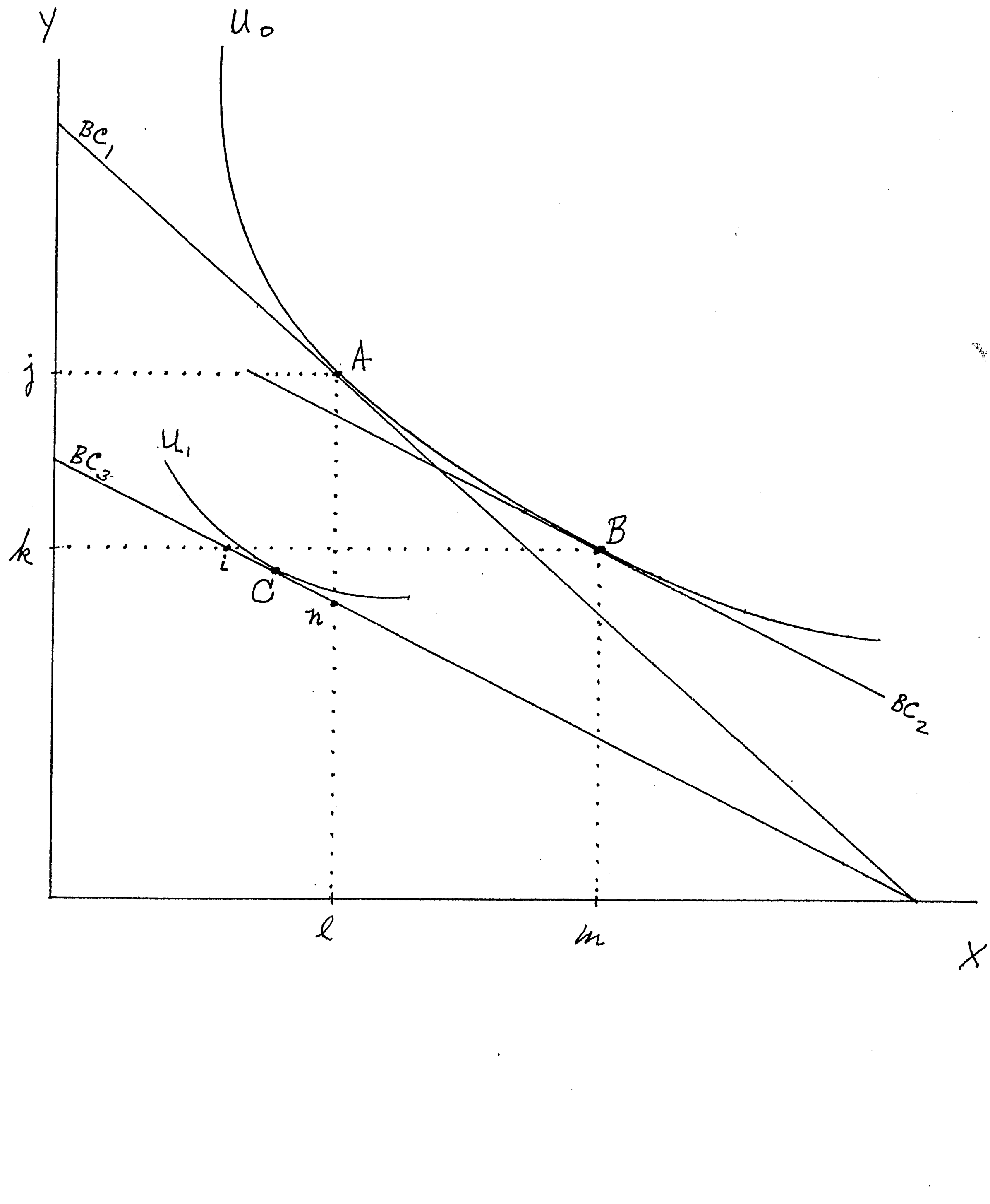
2 points/entry

⑤ The graph for this question is on the next page. The rise in P_y causes the budget constraint to shift from BC_1 to BC_3. U_0 is the original indifference curve and BC_2 is tangent to it and parallel to BC_3, so the movement from A to B is the substitution effect. (4 points)
↑ 3pts.

If X is normal then the drop in "income" from BC_2 to BC_3 means that the final amount of X demanded should be <u>less than "m"</u> (8 points) on the graph. If Y is normal then the final amount of it demanded should be <u>less than "k"</u> (8 points) on the graph. Finally, if X and Y are complements then

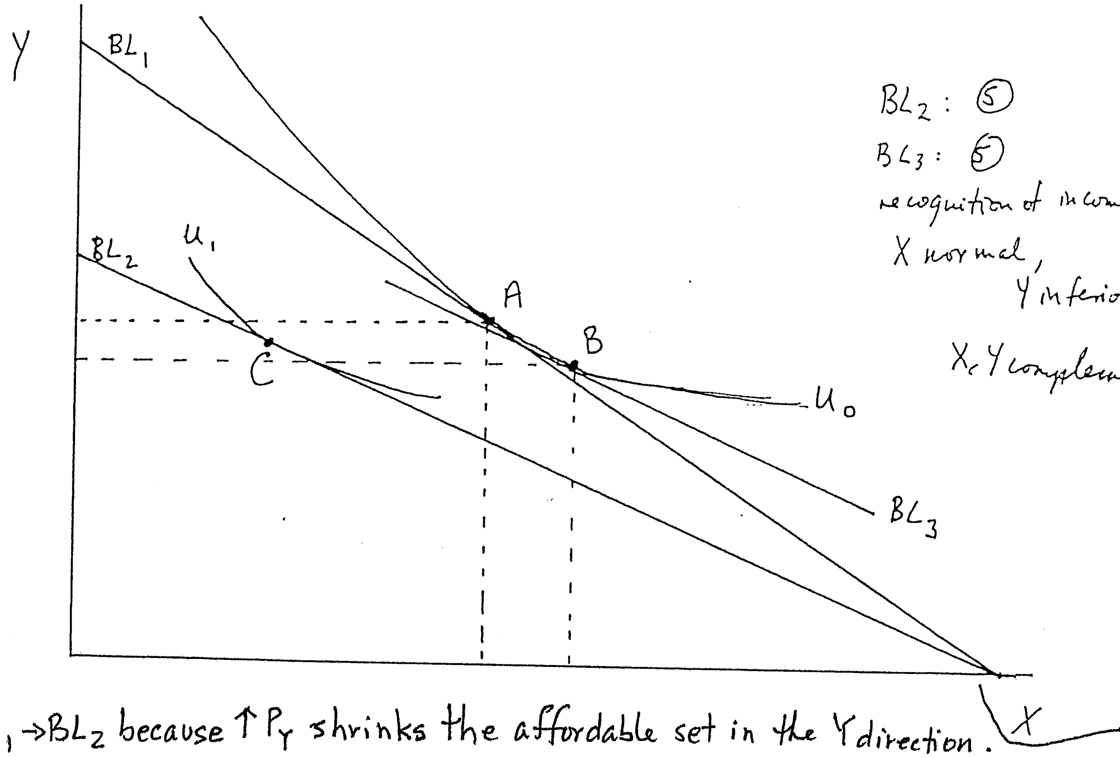
X moves left of "l" and "Y" moves below "j" (8 points)

so that they both decrease. This only leaves the part of BC_3 between "i" and "n" as a possible choice for the final location "C." The movement from B to C is the income effect. (2 points)



Y
U_0
BC_1
j
A
U_1
BC_3
k
i
C
n
B
BC_2
l
m
X

6



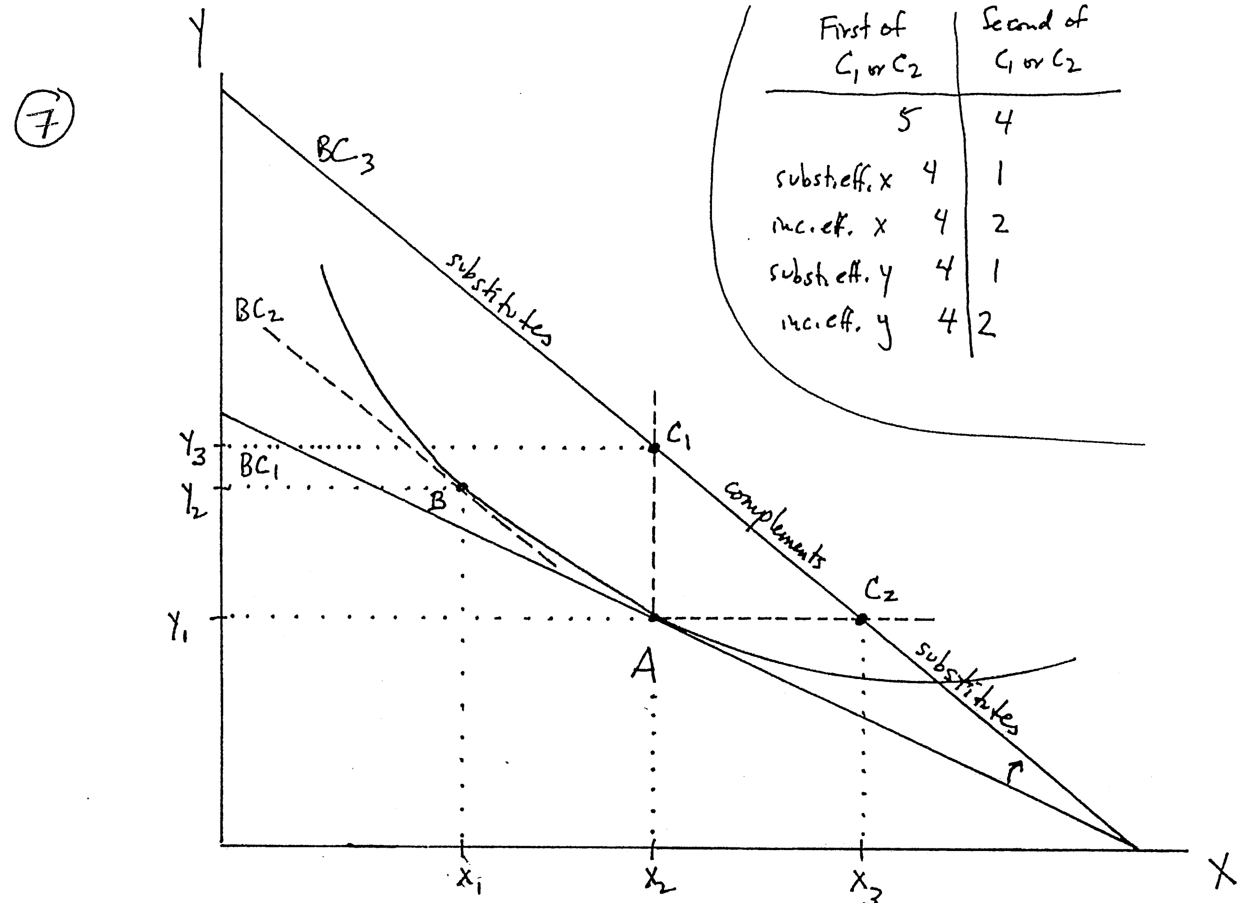
BL_2: 5
BL_3: 5
recognition of income effect: 5
X normal,
Y inferior: 5
X, Y complements: 5

$BL_1 \rightarrow BL_2$ because $\uparrow P_Y$ shrinks the affordable set in the Y direction.

A → C: X↓, Y↓. Since they go in the same direction, A and C are complements.

B → C is the income effect since BL_3 is a parallel translation of BL_2 and BL_3 is tangent to u_0. From B to C, I↓, X↓, Y↑. So X is normal and Y is inferior.

7



	First of C_1 or C_2	Second of C_1 or C_2
	5	4
subst. eff. x	4	1
inc. eff. x	4	2
subst. eff. y	4	1
inc. eff. y	4	2

2 pts

If the price of Y falls, the budget constraint twists out as shown. X and Y are complements if they move in the same direction and substitutes if they move in opposite directions (as measured from the original point A). BC_3, the new budget constraint, can be divided into three sections according to whether X and Y would be complements or substitutes if the final point C were in that section (see the graph). C_1 and C_2 are on the borders of those sections, so they are the points the question refers to.

First suppose C_1 is the final point. $A \to B$ is the substitution effect and $B \to C_1$ is the income effect. So for X, the substitution effect is $X_2 \to X_1$, the income effect is $X_1 \to X_2$, which are equal in magnitude and opposite in sign.

For Y, the substitution effect is $y_1 \to y_2$ and the income effect is $y_2 \to y_3$, which are in the same direction (which one is bigger will vary with your graph).

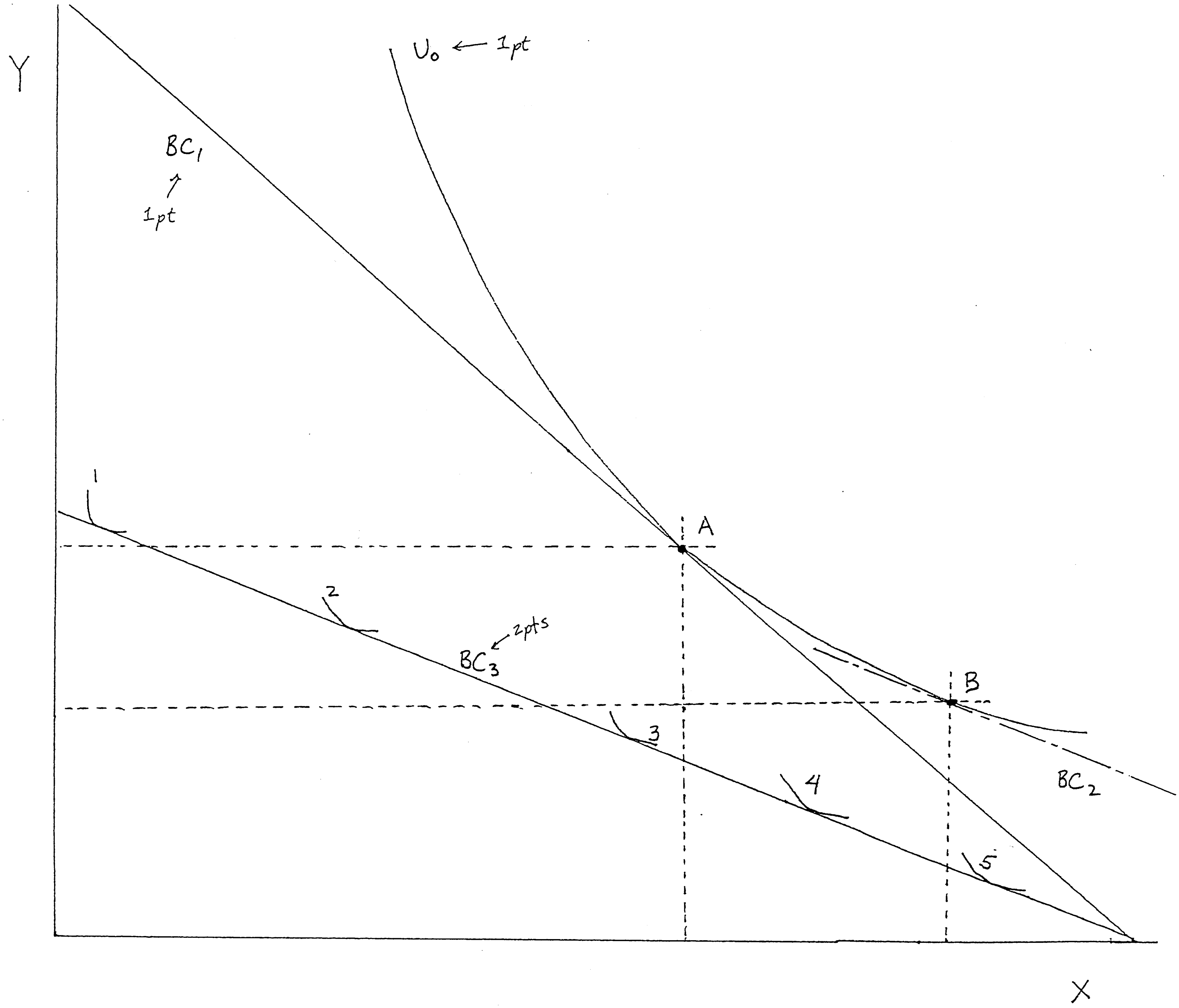
Now suppose C_2 is the final point. Then for X, $x_2 \to x_1$ is the substitution effect and $x_1 \to x_3$ is the income effect : these have opposite signs and the income effect is larger. For Y, $y_1 \to y_2$ is the substitution effect and $y_2 \to y_1$ is the income effect ; these are equal in magnitude and opposite in sign.

(8) Your graph should look like the one on the following page, with only one of the indifference curves labeled 1 – 5. 1pt BC_1 is the original budget constraint, BC_3 is the final budget constraint, and BC_2 4pts is parallel to BC_3 and tangent to the original indifference curve U_0. The substitution effect 5pts is the movement from A to B ; the income effect 5pts is the movement from B to the appropriate final point on BC_3. X and Y are complements if the final indifference curves are like 2 or 3 (since then the purchases of both goods fall, compared to point A) explanation: 3 pts ; otherwise answer: 4 pts the goods are substitutes. Since the movement from BC_2 to BC_3 is like a fall in income, this movement entails a fall in the consumption of normal goods and a rise in the consumption of inferior goods. Hence:

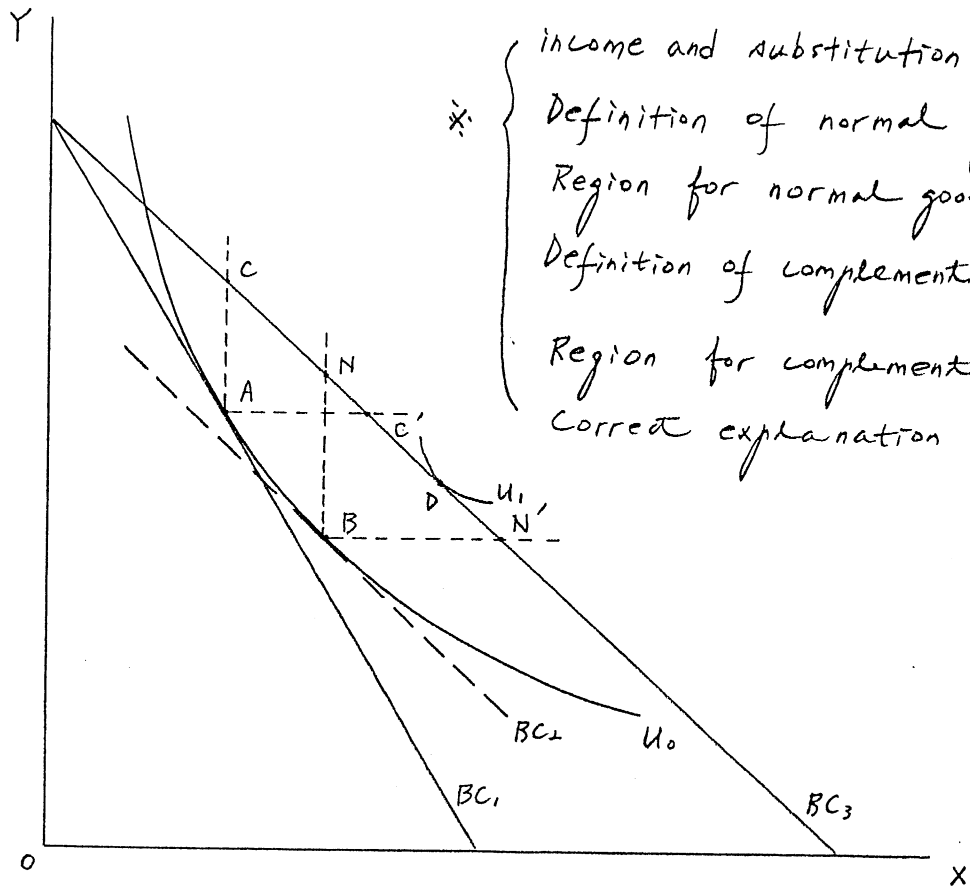
	1	2	3	4	5
X	normal	normal	normal	normal	inferior
Y	inferior	inferior	normal	normal	normal

explanation: 3pts
answer: 4pts

(The comparison is made from point B.)


Y
X
$U_0 \leftarrow$ 1pt
BC_1
1pt
BC_3
2pts
BC_2
A
B
1
2
3
4
5

9

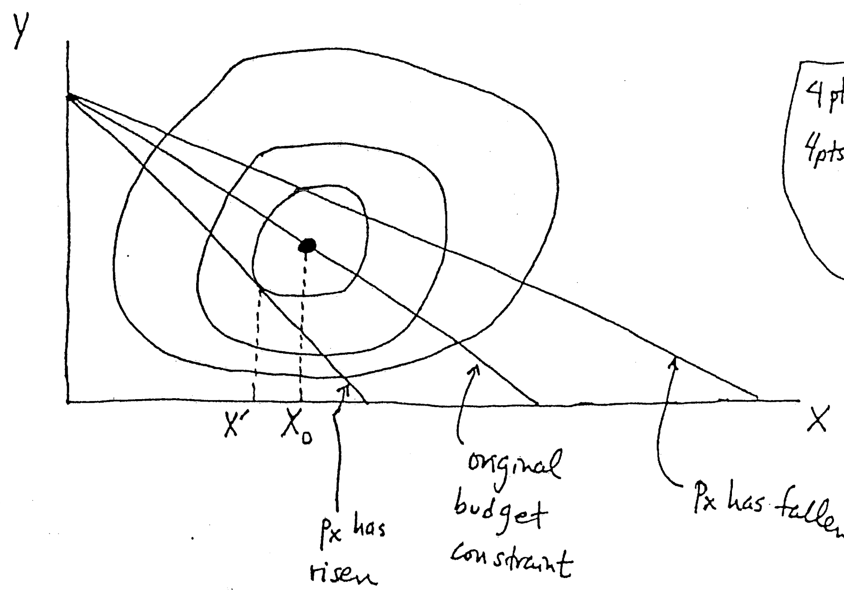


※ income and substitution effects : 5 point
Definition of normal good : 5 points
Region for normal good : 5 points
Definition of complements (or substitutes) : 5 points
Region for complements : 5 points
Correct explanation : 7 points

The initial consumption point is point A where the indifference curve U_0 is tangent to the original budget line BC_1. Suppose that the price of X falls such that the initial budget line BC_1 shifts to BC_3. Now draw a hypothetical budget line which is parallel to BC_3 and tagent to U_0; this touches U_0 at the point B. The income effect is represented by the movement from B to the final point on BC_3. Since both X and Y are normal goods the consumption of X and Y must increase by the movement from B to the final point on BC_3. Hence the final consumption point must be between the two points marked NN' by the normal good requirement.

Complements are goods which both increase or both decrease with a price change. It is measured by the movement from the original point A to the final point on BC3. For X and Y to be complements, the final consumption point on BC3 must be between the two points marked C and C' in this case.
We need to show a situation in which both X and Y are normal but not complements. Any final point between N and N' is normal but only the points between N and C' are complements. Even though the points between C' and N' are normal, they are not complements (for example, the point D).

(10)



4 pts: bliss point on original B.C.
4 pts: shapes of other indifference curves (need only be shown in the affordable set)

(4pts) The large dot lying on the original budget constraint is the bliss point, which is the combination of X and Y that makes the consumer as happy as he can possibly be.

(3pts) a) The "price ratio" (3pts) is $\frac{P_X}{P_Y}$, the opposite of the slope of the budget constraint. The "marginal rate of substitution" is the MRS of X for Y on this graph, and the usual condition for utility maximization is that $\frac{P_X}{P_Y}$ = MRS of X for Y.

Also OK is P_Y/P_X, MRS of Y for X, and usually $\frac{P_Y}{P_X}$ = MRS of Y for X

over →

Originally, the consumer is at his bliss point. There is no marginal rate of substitution here, so MRS of X for $Y \neq P_x/P_y$.

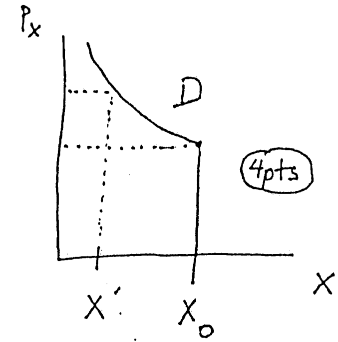
4pts

5pts

b) When P_x falls, X remains at X_0 because X_0 is the best the consumer can do.

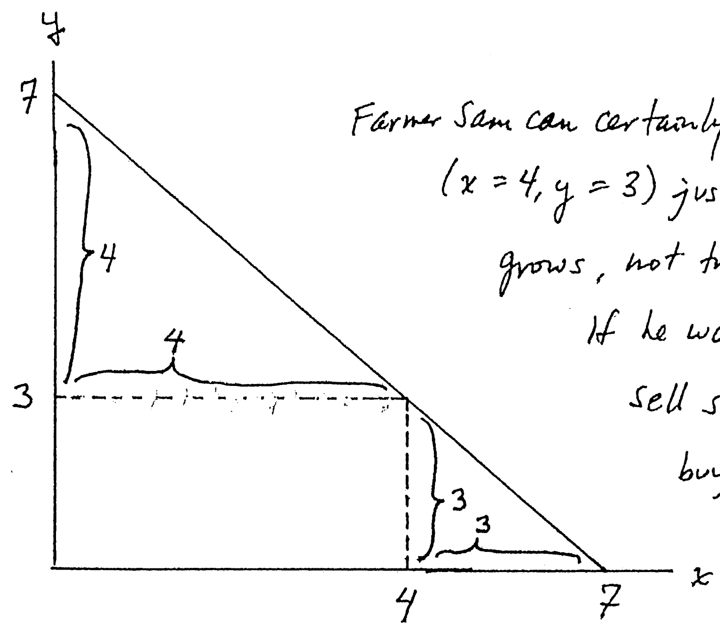
4pts

When P_x rises, X falls, for example to X' in the graph on p. 2. So:



The demand curve falls until X_0 can be afforded, then the demand curve becomes vertical.

(11) a)

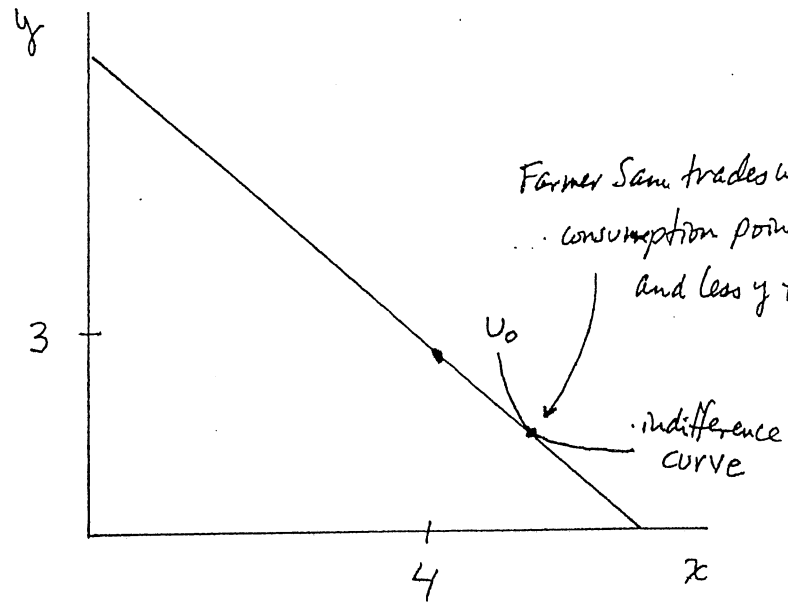


Farmer Sam can certainly consume the bundle ($x = 4, y = 3$) just by eating what he grows, not trading with anyone.

If he wants to trade, he has to sell some of one good in order to buy more of the other. This moves him away from his (4, 3) point, either down and to the right or up and to the left. The slope of the budget constraint is $-P_x/P_y = -1/1 = -1$. The intercepts of the budget constraint are (0, 7) and (7, 0). These are determined by starting at (4, 3) and then moving -3 in the y direction (and hence $+3$ in the x direction, leading to (7, 0)) or moving -4 in the x direction (and hence $+4$ in the y direction, leading to (0, 7)).

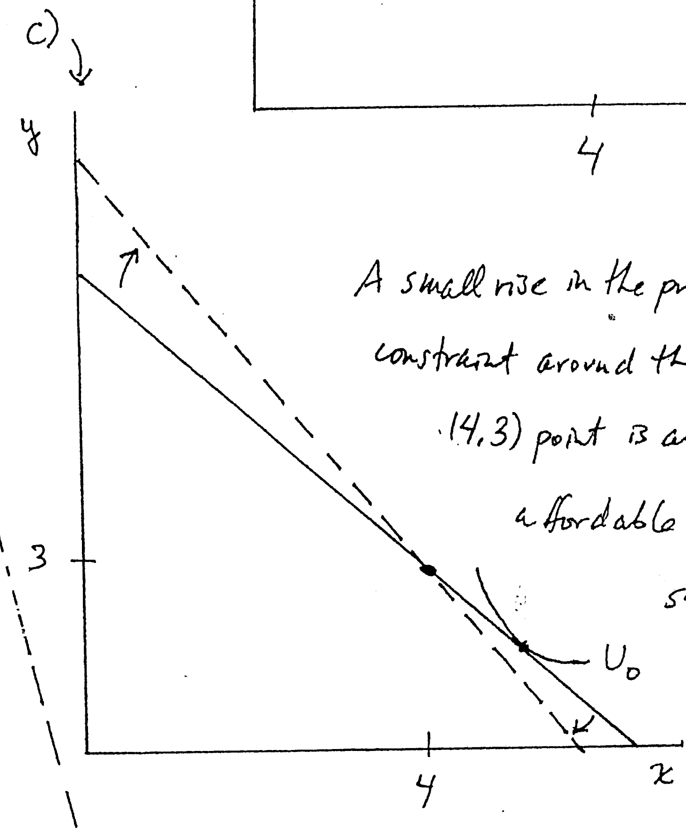
a) 12 points
b) 6 "
c) 8 " (4 for budget constraint, 4 for conclusion)
d) 8 "

b)



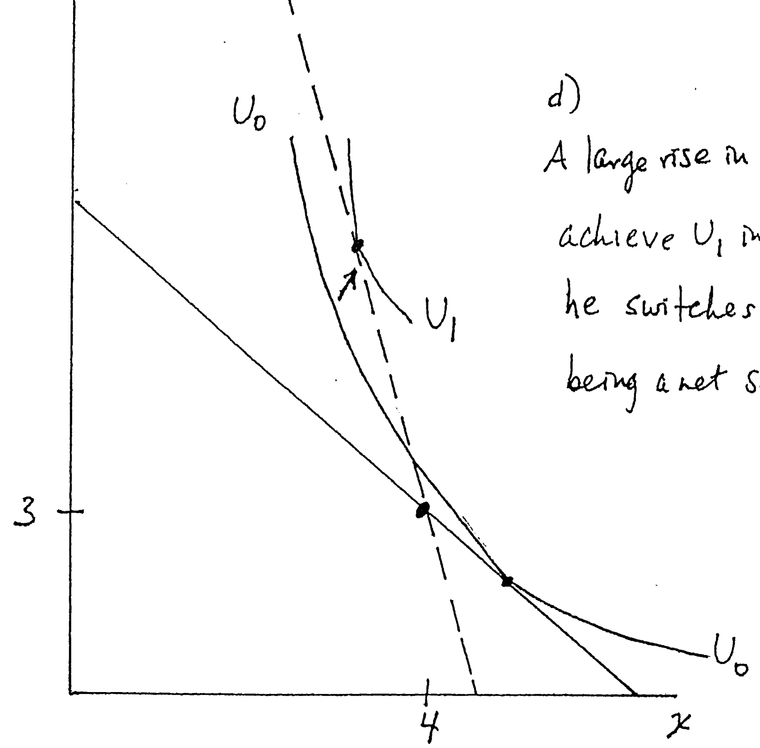
Farmer Sam trades with others, consumption point has more x and less y than (4,3)

c)



A small rise in the price of x twists the budget constraint around the (4,3) point because the (4,3) point is always in Farmer Sam's affordable set. The new budget constraint (the dashed line) does not enable Farmer Sam to get to U_0 any more.

d)



d)
A large rise in p_x may enable the consumer to achieve U_1 instead of just U_0. This happens when he switches from being a net buyer of x to being a net seller of x.

(12) a) If the consumer buys less than 1 unit of X, the affordable set is unchanged.
" " " " more " " " " " " " " expands. Its slope becomes $\frac{-P_x}{P_Y} = -\frac{0.5}{1} = \frac{-1}{2}$ instead of $-\frac{P_x}{P_Y} = -\frac{1}{1} = -1$. If the consumer spends all his income on X, he can buy 3 units, since the first unit costs \$1 and the remaining units cost 50¢ each. So on Figure 1, the new budget constraint is the line 'abc'. (8pts)

b) i) The new budget constraint is shown by the points 'abcde' on Figure 2. Between points 'b' and 'f', the affordable set shifts 1 unit in the X direction (to 'cd') because of the 1 free unit of X; at 'f', the affordable set shifts 2 units in the X direction because of the 2 free units of X.

(12pts)

ii) U_0 and U_1 form one of the many possible pairs of old and new indifference curves in this question. The optimal points are labeled I and II. For different consumers, U_0 might be tangent between 'a' and 'b' (in which case $U_0 = U_1$), or point II might be at 'e' (this would be the case when the indifference curves are steep straight lines, or very steep curves). (2pts)

iii) In my graph, $Y\uparrow$ since II is higher than I. Your graph will look different, and you may have $Y\downarrow$ or Y unchanged. ← (2pts)

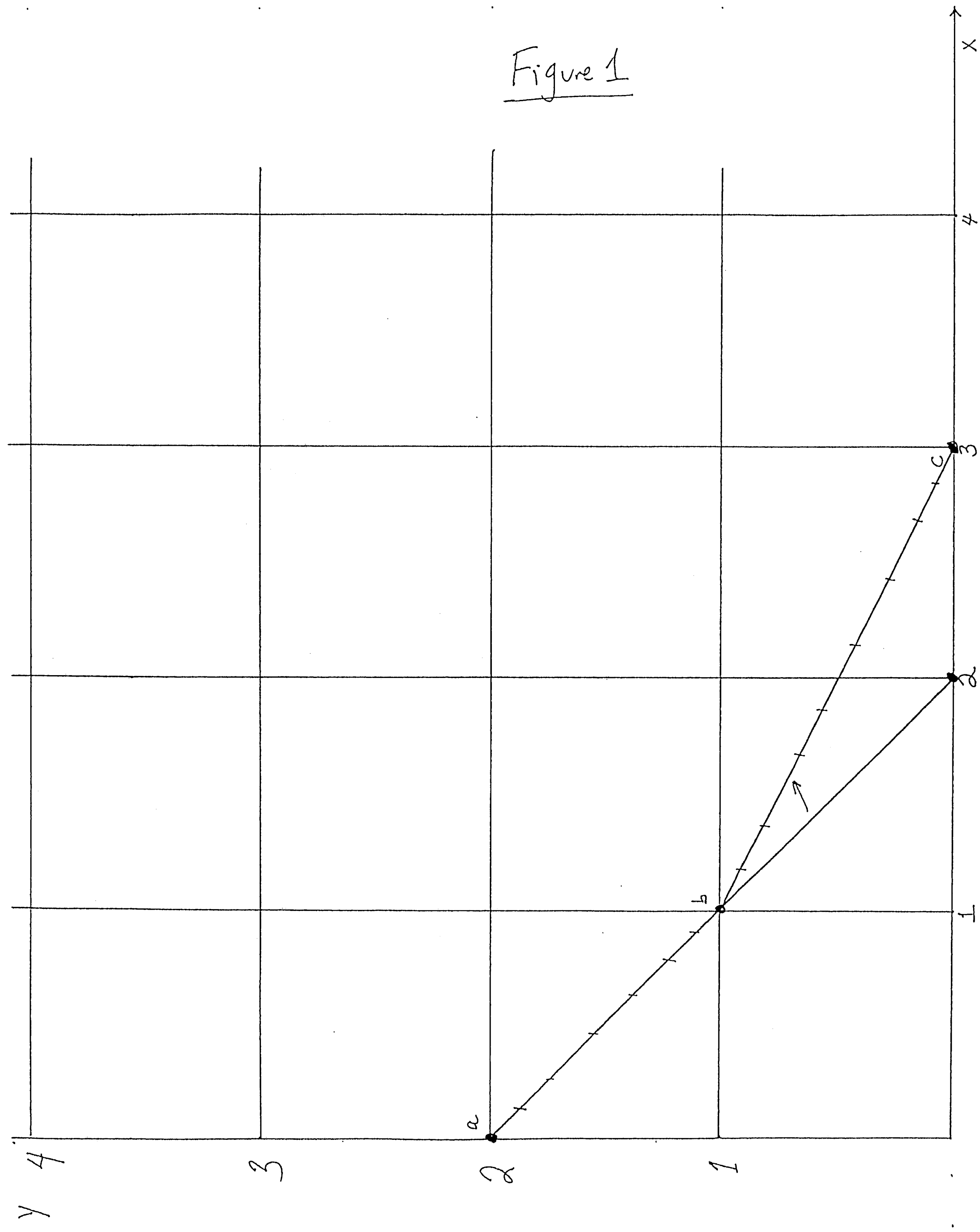
iv) $Y\uparrow$, P_Y is unchanged, so expenditures $P_Y Y$ go up in my graph. ← (4pts)

v) In my graph, $X\uparrow$ since II is farther right than I. ← (2pts)

vi) Income is unchanged, and since from (iv) expenditures on Y went up in my graph, expenditures on X must fall so that expenditures in total do not exceed the unchanged income. (4pts)

In my example, then, the merchant collects less money from the consumer after the "two-for-one" offer than before.

Figure 1



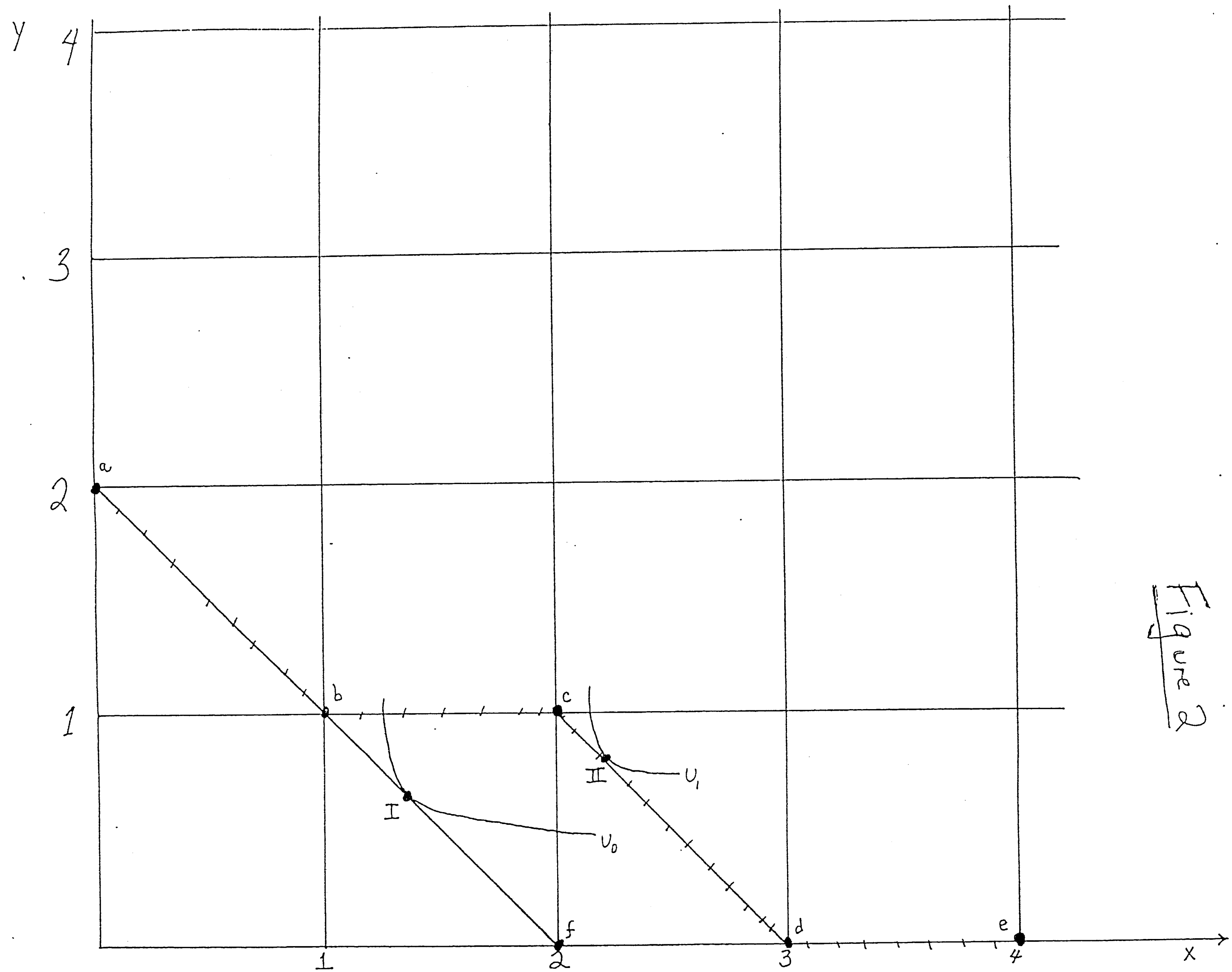
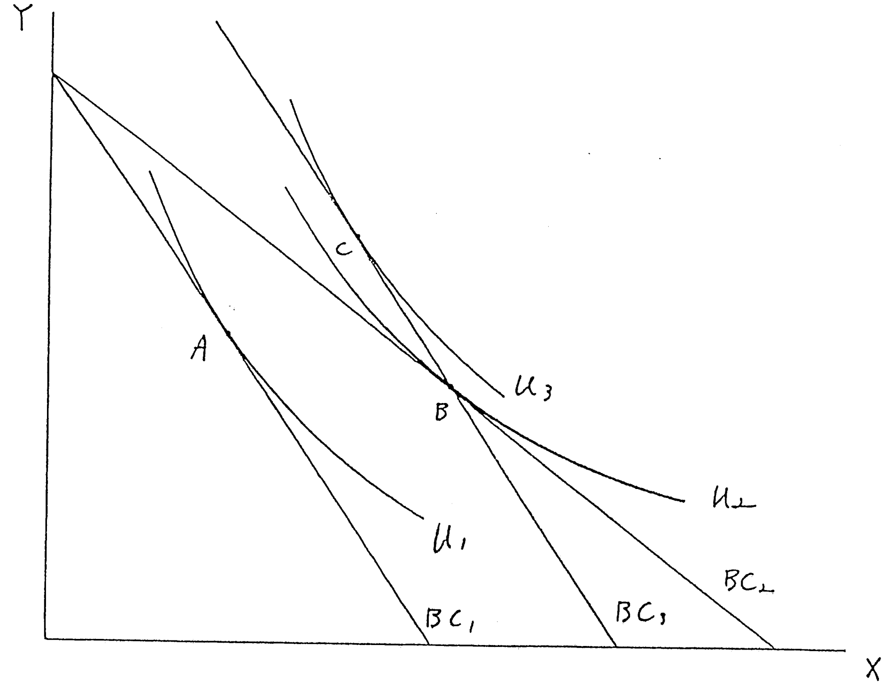


Figure 2

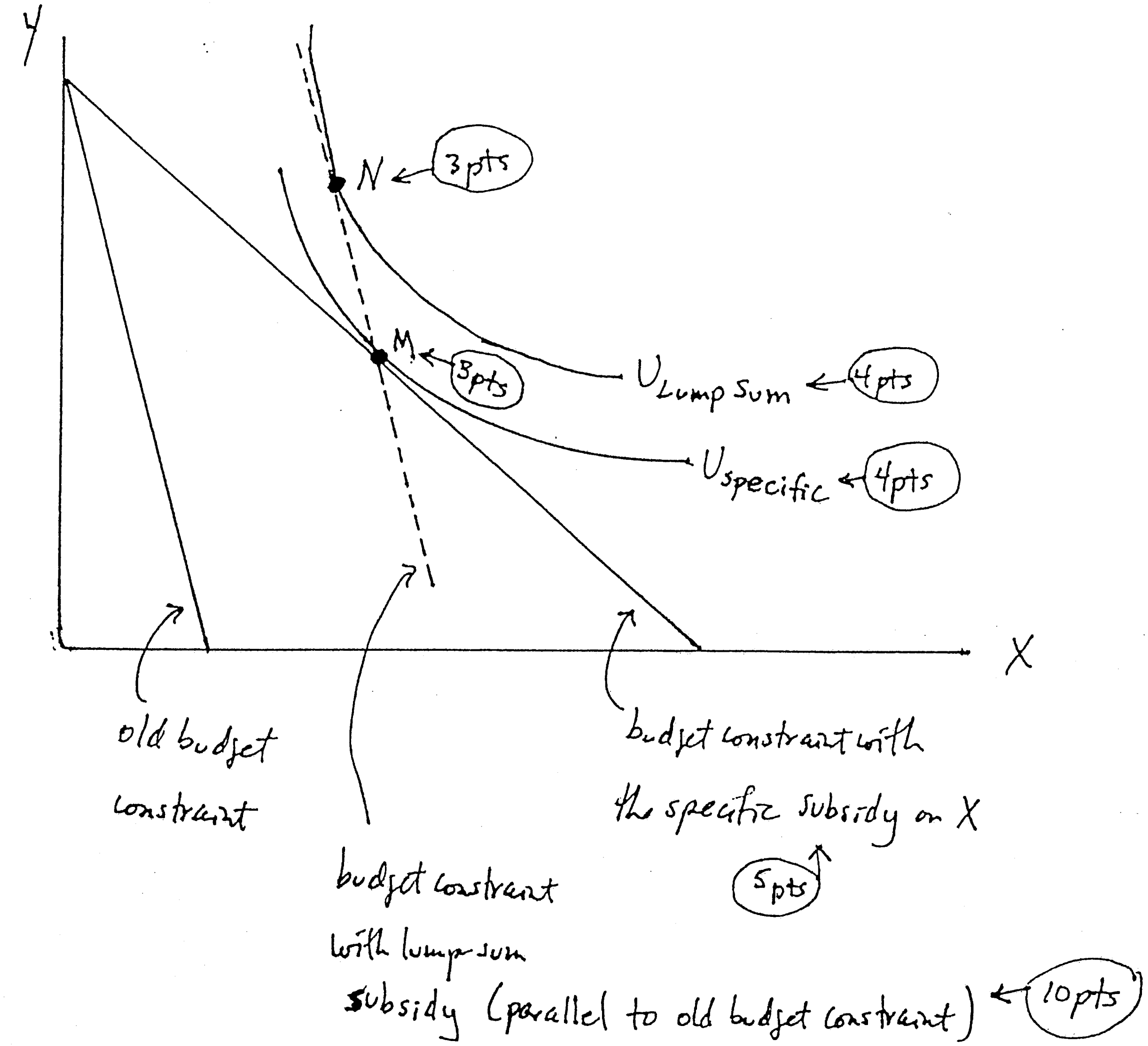
13



A subsidy on one good (X) shifts the budget line from BC_1 to BC_2, ← 5 points
and the consumer can reach the indifference curve U_2. ← 5 points
On the other hand, lump-sum subsidy will result in a parallel shift of budget line from BC_1 to BC_3. ← 5 points

The position of BC_3 is determined by the requirement ← 5 points that the government spends same on the lump-sum subsidy as on the subsidy on only one good. This results in BC_3 passing through point B. ← 5 points On BC_3, U_3 can be reached, so the consumer is better off with the lump-sum subsidy, so that is what should be done. ← 8 points

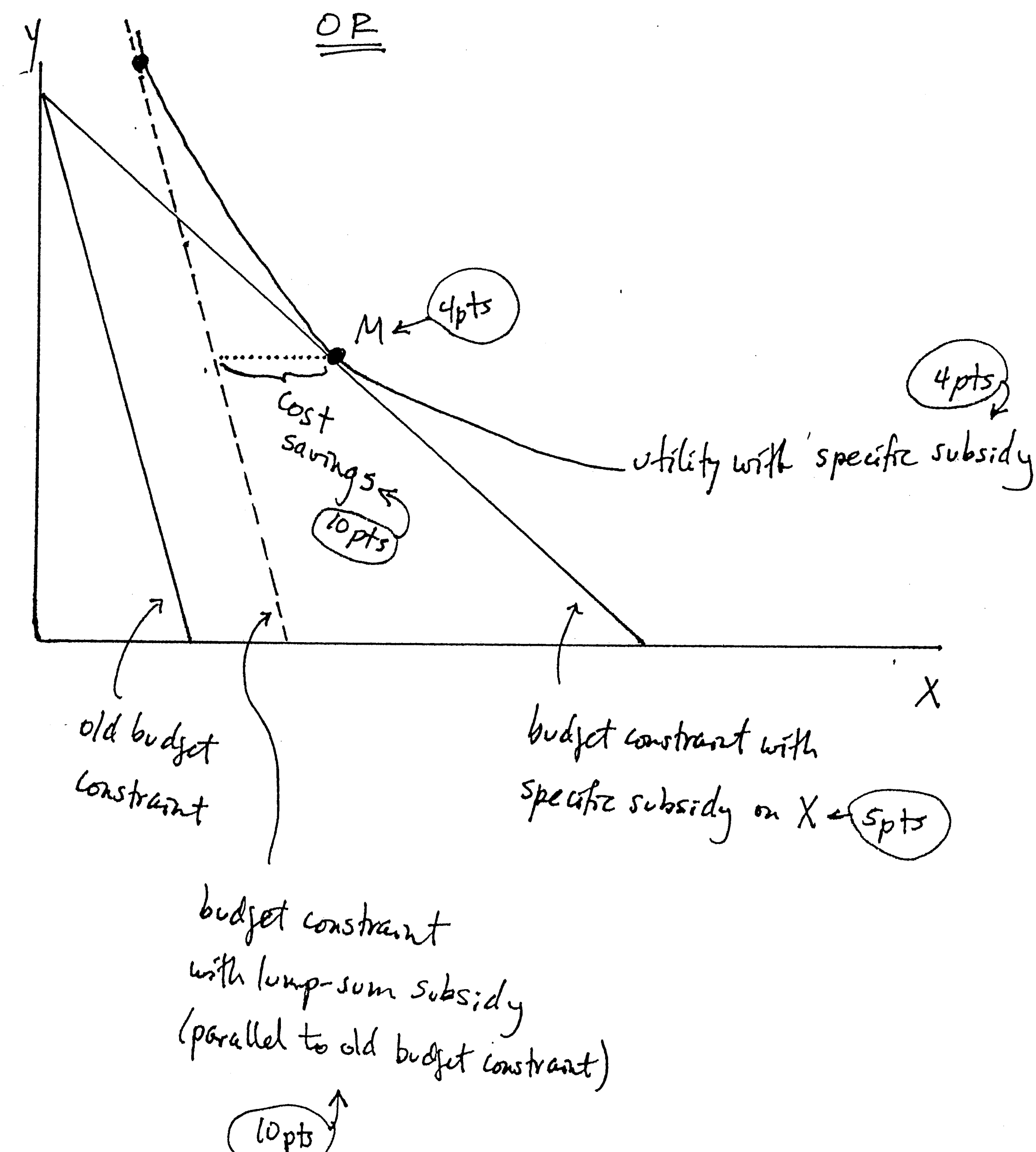
(14) (i)



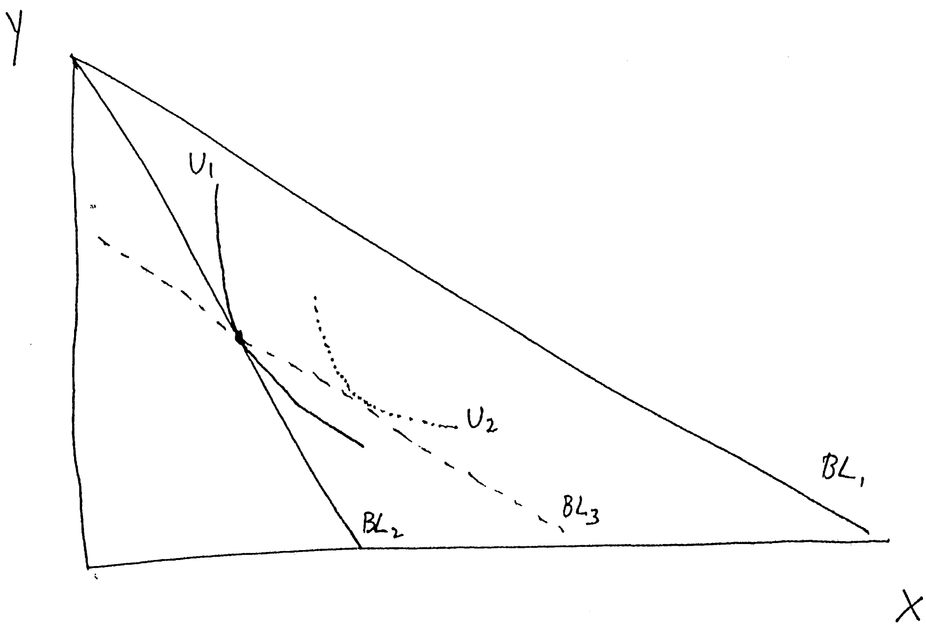
Point M is the chosen point with the specific subsidy. The key to the answer is that the budget constraint with the lump-sum subsidy has to go through

Point M in order for the government to spend the same amount of money on both programs. Superiority of the lump-sum subsidy is shown because point N is preferred to point M. 4pts

(ii)



(15)



X: taxed good

BL1: before sales tax (budget line)

BL2: after " " " "

5 pts.

U1: indifference curve after sales tax

BL3: budget line with lump-sum tax instead of sales tax

10 pts.

U2: "better" indifference curve than U1, with lump-sum tax

5 pts

over →

(Instead of a "sales tax," the precise term is a "specific tax," as mentioned in the question.)

The algebraic part of the question is:

(a) is the specific tax budget constraint (supposing the tax is on X)

(b) states that (X_S, Y_S) is on the specific tax budget constraint

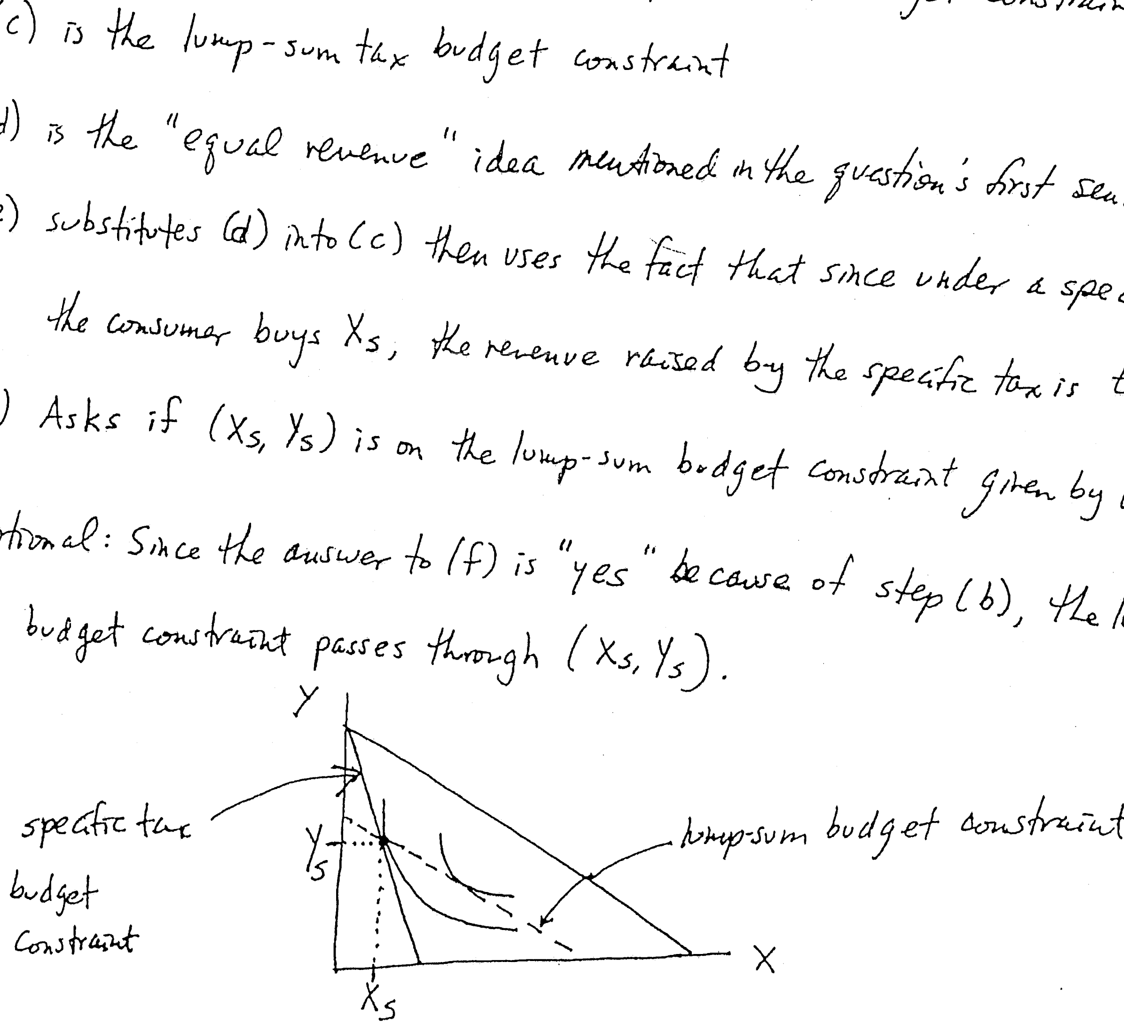
(c) is the lump-sum tax budget constraint

(d) is the "equal revenue" idea mentioned in the question's first sentence

(e) substitutes (d) into (c) then uses the fact that since under a specific tax the consumer buys X_S, the revenue raised by the specific tax is tX_S.

(f) Asks if (X_S, Y_S) is on the lump-sum budget constraint given by (e).

Optional: Since the answer to (f) is "yes" because of step (b), the lump-sum budget constraint passes through (X_S, Y_S).



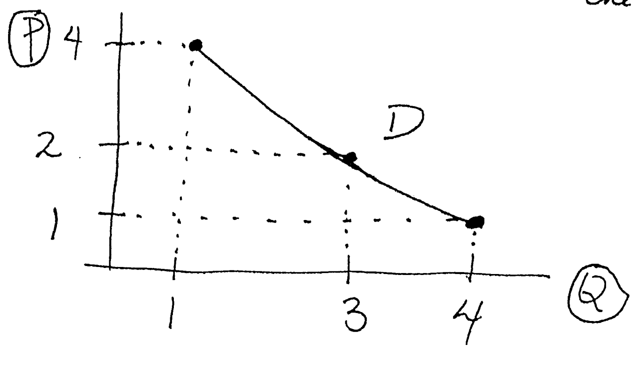
(16) Income = \$10

P_y = \$2

Hence the point $(X=0, Y=5)$ is on the budget constraint (see Fig. 1).

We want to know how the optimal amount of X demanded varies as P_x changes, since this is what a demand curve tells us. The slope of the budget constraint is $\frac{-P_x}{P_y} = \frac{-P_x}{2}$, so budget constraints of different slopes correspond to different values of P_x.

Budget Constraint	X Demanded	P_x
1	1	slope of B.C. $= \frac{-P_x}{2} = \frac{-5}{2\frac{1}{2}} = -2$, so $P_x = 4$. Check: $(X=2\frac{1}{2}, Y=0)$ costs $2\frac{1}{2}(4)+0 = \$10$.
2	3	slope of B.C. $= \frac{-P_x}{2} = \frac{-5}{5} = -1$, so $P_x = 2$. Check: $(X=5, Y=0)$ costs $5(2)+0 = \$10$.
3	4	slope of B.C. $= \frac{-P_x}{2} = \frac{-5}{10} = \frac{-1}{2}$, so $P_x = 1$. Check: $(X=10, Y=0)$ costs $10(1)+0 = \$10$.



One point on the D curve correct: 20 points

The other two points correct: 10 points each

Figure 2