

Finding Answers to Homework Problems

The odd-numbered homework problems are all answered at the end of your textbook. In addition, I have written answers to the even-numbered homework problems and to some of the odd-numbered problems as well. However, some of the homework problems' numbers have changed since I wrote the answer sheets. The following table gives the correspondence between the numbers in your edition of the textbook and the numbers in my answer sheets. For example, to find the answer to Chapter 3's Question 4, look in my answer sheets for "Chapter 4 Question 3," which is what that question used to be called.

All lines marked "*" in the table correspond to a question whose answer is given in my answer sheets. The other symbols mean the following:

RQ: Review Question (instead of 'Problem')

†: Typo: change " $Y = 50?$ " to " $Y = \sqrt{50}?$ " or " $Y^2 = 50$ ".

††: Suppose the price of clothing equals one and the price of other goods equals one.

†††: Typo: change " $299P'$ " to " $200P'$ ".

¶: Comment on part 7b (in the old numbering system) in my handout.

Current Textbook		Old Edition	
App.Ch.1	5	Chap. 2	5
	6		7*
	10†		8*
Chap. 2	1	Chap. 3	1
	2		2*
	5		6*
	7		8*
Chap. 3	4††	Chap. 4	3*
	7		9
Chap. 4	1	Chap. 5	1
	4†††		2*
	5		3
	6abc		4abc*
Chap. 5	RQ5	Chap. 7	4*
	1		1
	2		2*
	3		3
Chap. 6	1	Chap. 8	1
	2		10*
	4		4*
	5		5
	7¶		7
Chap. 7	RQ3cde	Chap. 9	4*
	1		1
	6 (monopoly)		8*

...table continues →

Chap. 8	2	Chap. 11	2*
	3¶¶		3
	6§		6*
	7		7
Chap. 10	1	Chap. 12	1
	2b		2b*
	3		3
	4ab		4ab*
	5		5
	6a		6*
Chap. 13	1	Chap. 14	3
	2		4*
	3		5
	6		6*
	7		7
	8		9
Chap. 14	5¶¶¶	Chap. 16	2*
	8		3*

¶¶: Part b to be discussed in class.

§: The first term on the right-hand side
of the first equation in this problem
should read $q^3/300$, not $q^3/300$.

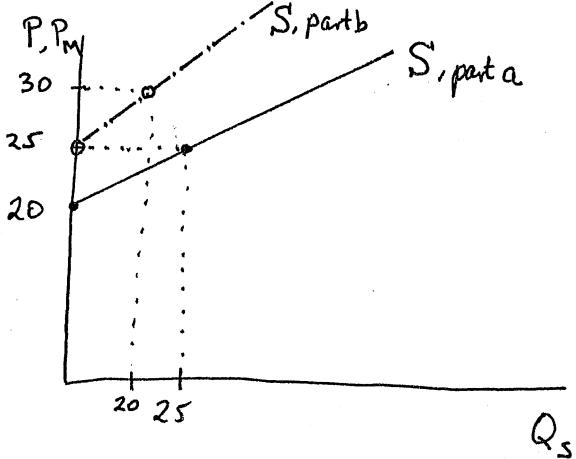
¶¶¶: In old answer, the second interest
rate was 9% instead of 8%.

2.4 $Q_s = -100 + 5P$, $Q_s > 0$

a) We need two points in order to graph the straight line $Q_s = -100 + 5P$.

If $Q_s = 0$ then $5P = 100$, or $P = 20$. So one point is $(Q_s = 0, P = 20)$.

If $P = 25$ then $Q_s = -100 + 125 = 25$, which gives another point.



At $P = 20$ producers start supplying flounder to the market.

b) $P_M = 1.25P$. So $P = \frac{P_M}{1.25}$, and

$$Q_s = -100 + 5P = -100 + 5\left(\frac{P_M}{1.25}\right) = -100 + 4P_M.$$

$$Q_s = 0 \text{ at } 100 = 4P_M \Rightarrow P_M = 25.$$

$$P_M = 30 \Rightarrow Q_s = -100 + 120 = 20.$$

Since $Q_s = 4P_M - 100$, start supplying flounder at $P_M = 25$.

2.7.

$$T = \frac{1}{100} I^2$$

T : thousands of dollars of tax liability

I : thousands of dollars of income.

a) income of \$10,000 $\Rightarrow I = 10 \Rightarrow T = \frac{100}{100} = 1 \Rightarrow$ taxes = \$1000

income of \$30,000 $\Rightarrow I = 30 \Rightarrow T = \frac{900}{100} = 9 \Rightarrow$ taxes = \$9000

income of \$50,000 $\Rightarrow I = 50 \Rightarrow T = \frac{2500}{100} = 25 \Rightarrow$ taxes = \$25,000.

Average tax on \$10,000 is $\frac{1000}{10,000} = 10\%$

Average tax on \$30,000 is $\frac{9000}{30,000} = 30\%$

Average tax on ~~\$25,000~~ is $\frac{25,000}{50,000} = 50\%$.

For what income does tax liability equal total income?

$$\downarrow \begin{matrix} T \\ I \end{matrix}$$

We set $T = I$. But $T = \frac{1}{100} I^2$, so

$$\frac{1}{100} I^2 = I$$

$$I = 100 \Rightarrow \text{income of } \underline{\$100,000}.$$

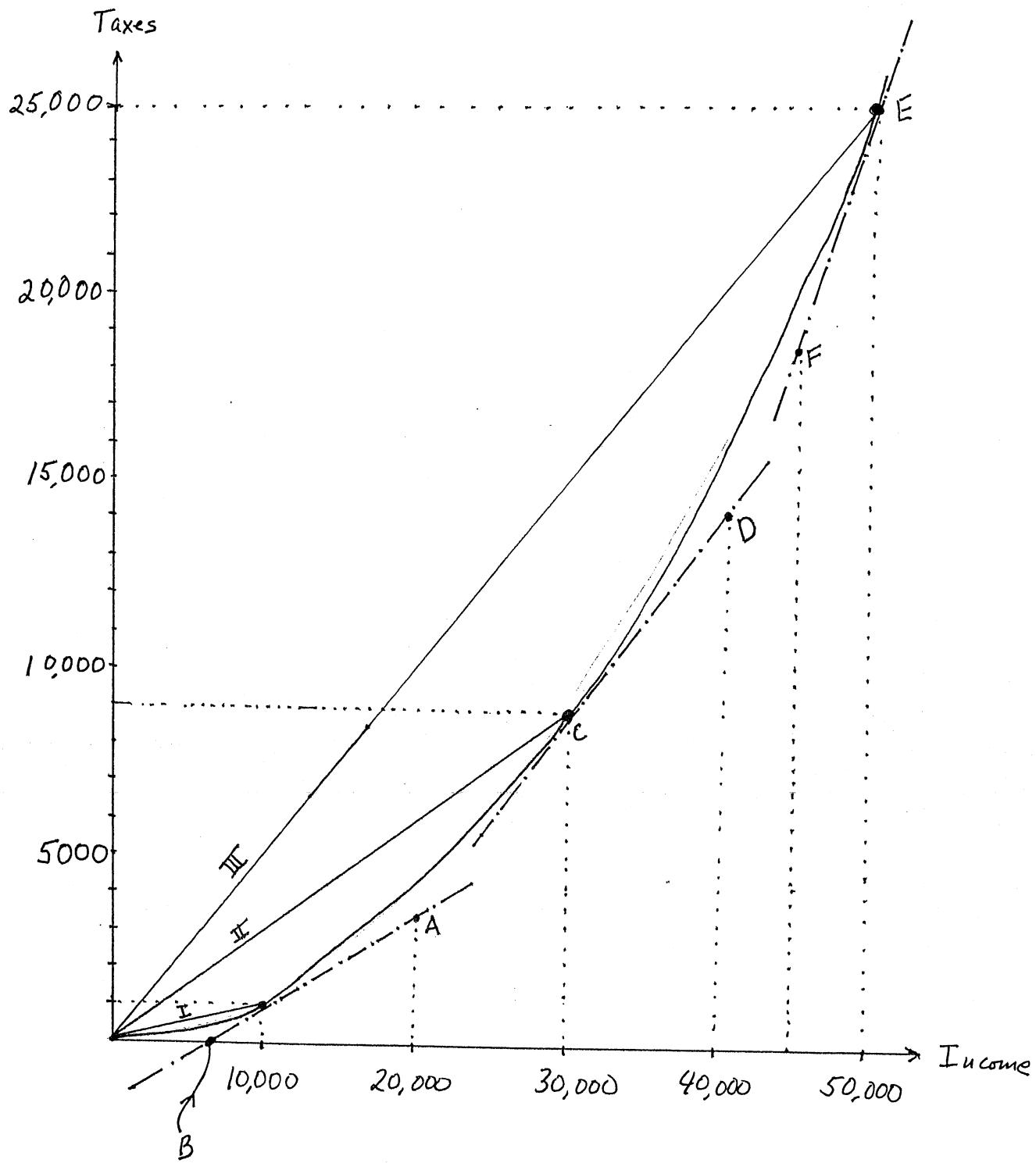
b) Next page for graph.

Slope of the tangent line for income of \$10,000: A is at $(20,000, 3500)$

roughly. B is at $(6500, 0)$. So $\frac{\Delta y}{\Delta x} = \frac{3500 - 0}{20,000 - 6500} \approx 26\%$.

(over two pages \rightarrow)

$\begin{matrix} \leftarrow \text{income} \\ \downarrow \text{taxes} \end{matrix}$



Slope of tangent line at income of \$30,000: C is at (30000, 9000). D is roughly at (40000, 14000).

$$S_0 \frac{\Delta Y}{\Delta X} = \frac{14,000 - 9000}{40,000 - 30,000} \approx 50\%.$$

Slope of tangent line at income of \$50,000: E is at (50,000, 25000). F is roughly at (45000, 19000).

$$S_0 \frac{\Delta Y}{\Delta X} = \frac{25000 - 19000}{50000 - 45000} \approx 120\%.$$

The book uses calculus to get the exact answers. $T = \frac{1}{100} I^2 \Rightarrow \frac{dT}{dI} = \frac{1}{50} I$. But

~~$I = 10,000 \Rightarrow \frac{I}{50} =$~~ our data uses taxes in dollars and income in dollars, not thousands of dollars. Let 't' be taxes in dollars

(so $t = 1000 T$) and let "i" be income in dollars (so $i = 1000 I$). Then

$$T = \frac{1}{100} I^2 \Rightarrow \frac{t}{1000} = \frac{1}{100} \left(\frac{i}{1000} \right)^2$$

$$\Rightarrow t = \frac{i^2}{100000}, \text{ and } \frac{dt}{di} = \frac{i}{50000}. \text{ We then get}$$

i	10,000	30,000	50,000
$\frac{di}{dt}$	20%	60%	100%

just like the book.

The average tax rates are the slopes of the lines labelled I, II, and III on the graph. Take line II, for example. One point on it is $(0, 0)$, and another is point C, (30000, 9000). So $\frac{\Delta Y}{\Delta X} = \frac{0 - 9000}{0 - 30000} = 30\%$, as we said two pages ago.

Work out the slopes of lines I and III in the same way.

$$2.8 \quad Y = \sqrt{XZ}$$

$$X=4 \Rightarrow Z=4$$

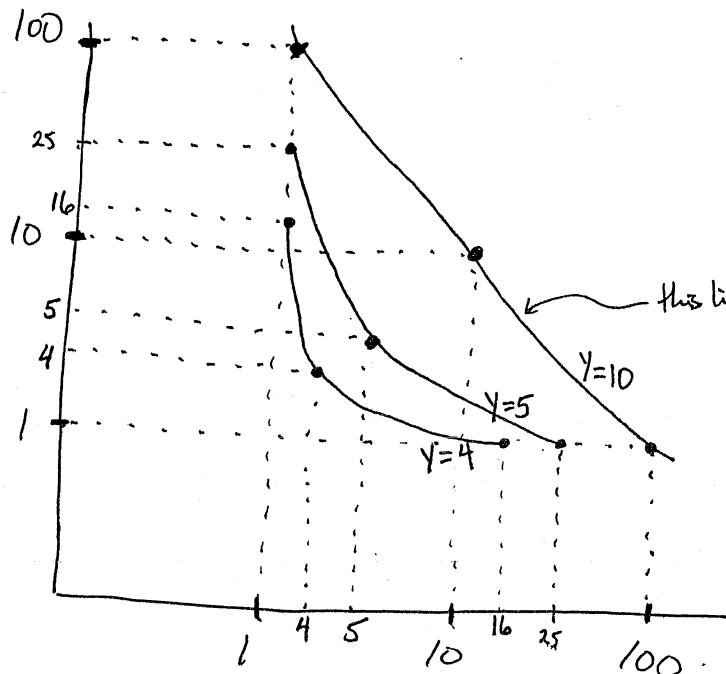
$$X=16 \Rightarrow Z=1$$

$$X=1 \Rightarrow Z=16$$

$$\text{If } Y=4 \text{ then } 4 = \sqrt{XZ} \Rightarrow 16 = XZ \Rightarrow Z = 16/X.$$

$$\text{If } Y=5 \text{ then } 5 = \sqrt{XZ} \Rightarrow 25 = XZ \Rightarrow Z = 25/X.$$

$$\text{If } Y=10 \text{ then } 10 = \sqrt{XZ} \Rightarrow 100 = XZ \Rightarrow Z = 100/X.$$



this line should be curved like the others, but the scale I used to draw with was not linear (it goes 1, 10, 100, not 1, 2, 3), so the shape is distorted.

The contour lines are convex

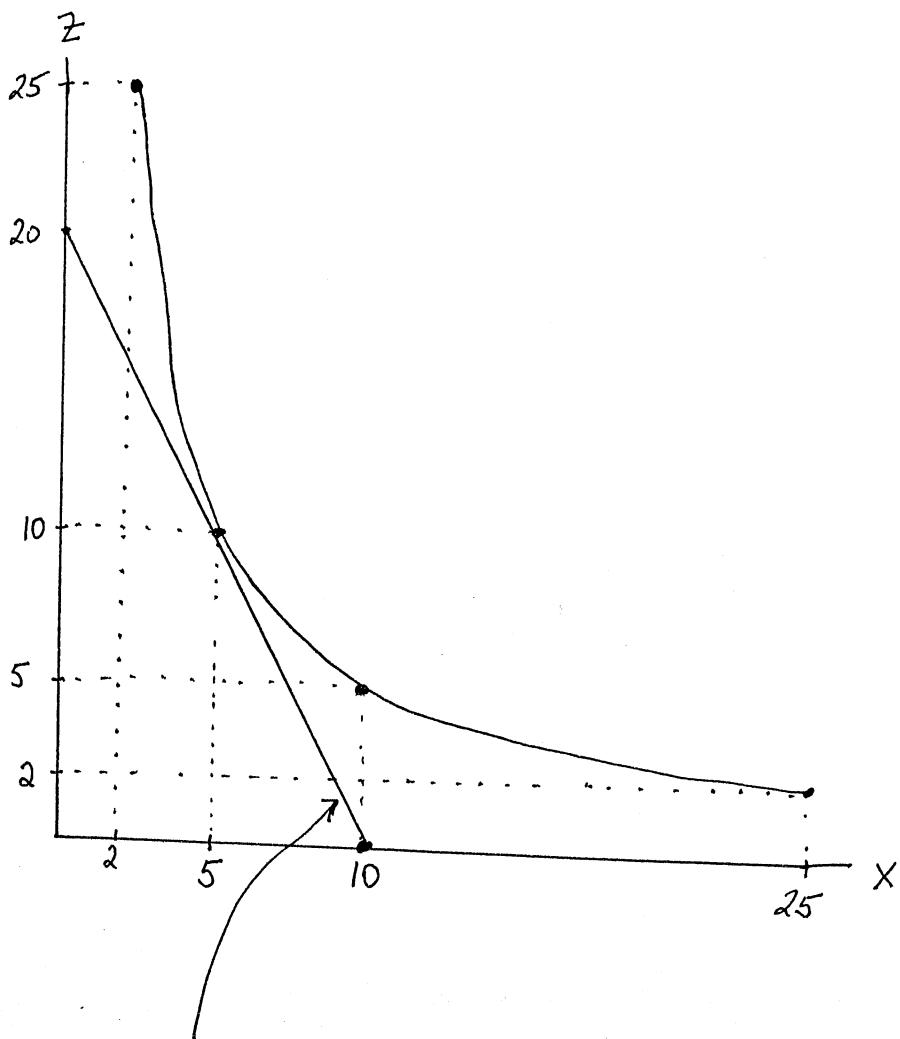
as you can see from $Y=4$, $Y=5$ (and $Y=10$ if I'd drawn it to scale).

$$\text{If } Y=\sqrt{50} \text{ then } \sqrt{50} = \sqrt{XZ} \Rightarrow 50 = XZ \Rightarrow Z = \frac{50}{X}.$$

X	5	10	25	2
$Z = \frac{50}{X}$	10	5	2	25

$$20X + 10Z = 200 \Rightarrow 10Z = 200 - 20X$$

$$\Rightarrow Z = 20 - 2X. \quad (\text{next page} \rightarrow)$$



$$Z = 20 - 2X : X=0 \Rightarrow Z=20$$

$$Z=0 \Rightarrow 2X = 20 \Rightarrow X=10.$$

We need $Z = \frac{50}{X}$ and $Z = 20 - 2X$. So $Z = Z \Rightarrow \frac{50}{X} = 20 - 2X$,

$$50 = 20X - 2X^2,$$

$$2X^2 - 20X + 50 = 0,$$

$$X^2 - 10X + 25 = 0,$$

$$(X-5)^2 = 0 \Rightarrow X=5 \text{ and}$$

$$Z = 20 - 2(5) = 10 \text{ or}$$

$$Z = 50/5 = 10.$$

So the lines are tangent at $X=5$, $Z=10$.

Solutions to Ch. 3's even-numbered problems

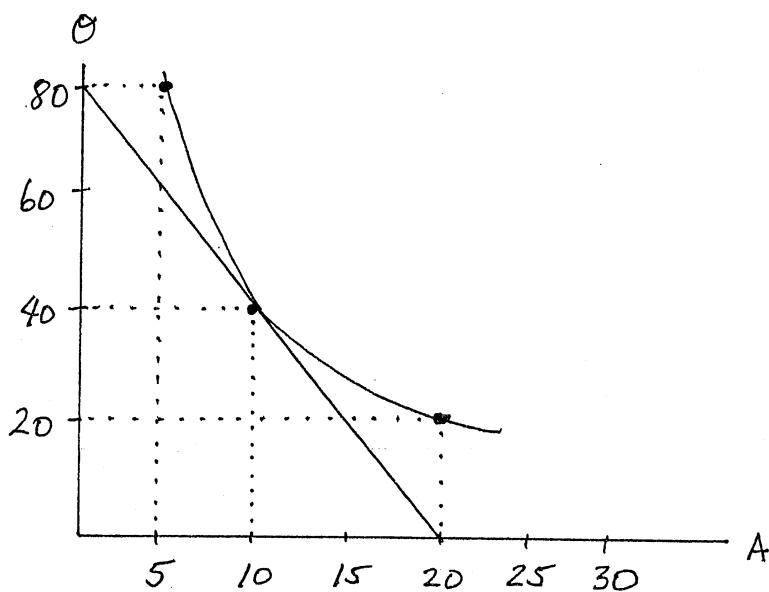
$$\textcircled{2} \quad U = \sqrt{A \cdot O}$$

a) $U = \sqrt{5 \cdot 80} = \sqrt{400} = \boxed{20}$

b) $U = 20 = \sqrt{10 \cdot O} \Rightarrow 400 = 10O \text{ or } \boxed{O = 40}$

c) $U = 20 = \sqrt{20 \cdot O} \Rightarrow 400 = 20O \text{ or } \boxed{O = 20}$

d) {
e) }



From the graph, it's apparent that the person can buy 10 apples and 40 oranges. One can also get this algebraically:

$$\text{indifference curve: } 20 = \sqrt{AO} \quad (1)$$

$$\text{budget constraint: } \frac{4}{10}A + \frac{1}{10}O = 8. \quad (2)$$

$$\text{From (1), } A = \frac{400}{O}. \quad (3)$$

Substituting (3) into (2),

$$\frac{4}{10} \cdot \frac{400}{O} + \frac{1}{10}O = 8 \text{ or}$$

$$\frac{160}{\theta} + \frac{\theta}{10} = 8$$

$$1600 + \theta^2 = 80\theta$$

$$\theta^2 - 80\theta + 1600 = 0$$

$$(\theta - 40)^2 = 0$$

$$\Rightarrow \theta = 40 \text{ and, from (3), } A = \frac{400}{40} = 10.$$

f) If the person buys 1 less apple, he can buy 4 more oranges. This would make $A = 9$ and $\theta = 44$, so $U = \sqrt{9 \cdot 44} = \sqrt{396}$. This is less than the utility at $A = 10$ and $\theta = 40$, which was $U = \sqrt{10 \cdot 40} = \sqrt{400}$.

Buying 1 more apple (that is, setting $A = 11$) also causes utility to fall. This is because, if $A = 11$, $\theta = 36$ (buying 1 more apple forces one to give up 4 oranges), and $U = \sqrt{11 \cdot 36} = \sqrt{396} < \sqrt{400}$.

So $A = 10$, $\theta = 40$ seems to be the utility-maximizing bundle.

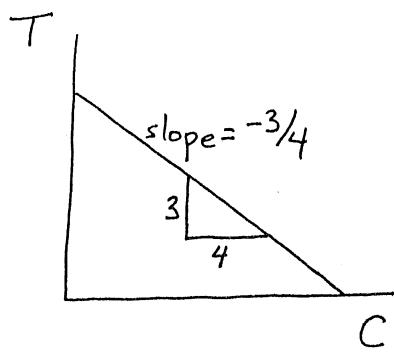
⑥ $V(C, T) = 3C + 4T$

We start by graphing a typical indifference curve:

$$U = 3C + 4T$$

$U - 3C = 4T \Rightarrow T = -\frac{3}{4}C + \frac{1}{4}U$. So a typical indifference curve is a straight line with slope $-\frac{3}{4}$.

(over \rightarrow)



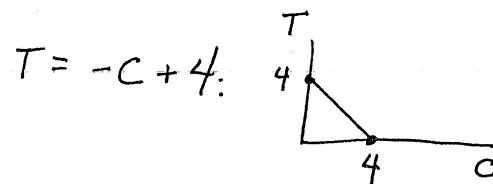
Ms. Caffine's MRS of coffee for tea is minus the slope of the indifference curve, so it is $3/4$.

Another way to think of it: if you take 3 units of tea away from Ms. Caffine, how much

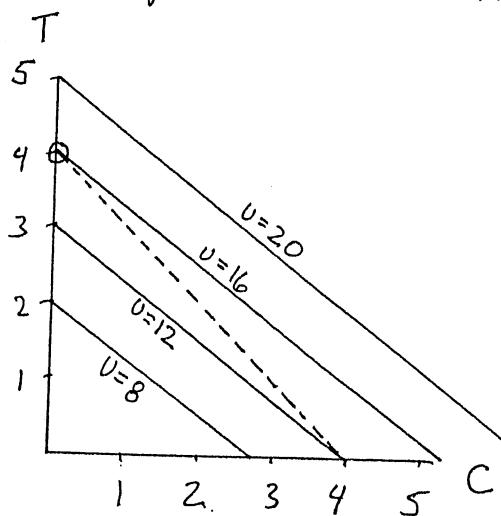
coffee do you have to give her to make her just as well off as she was before?
4 units of coffee.

An important point here is that the MRS of C for T is constant; usually, we think of it as diminishing. So this example does not illustrate our usual case.

$$P_T = P_C = \$3, \quad I = \$12 \Rightarrow \text{budget constraint} \quad 12 = 3C + 3T \quad \text{or}$$



Put the budget constraint together with some indifference curves:



Utility-maximizing point: $C=0, T=4$.

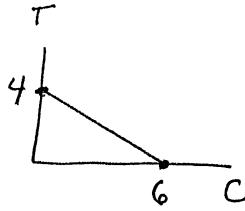
If she had any more money, the budget constraint would take a parallel shift out. You should convince yourself that she'd still spend all her income on tea.

(over →)

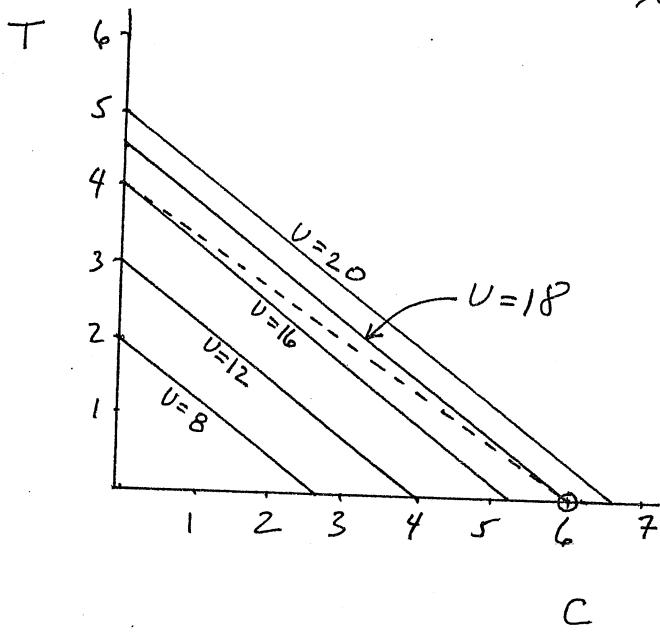
If the price of coffee fell to \$2, the budget constraint would be

$$12 = 2C + 3T \quad \text{or}$$

$$T = -\frac{2}{3}C + 4$$



Putting this together with the indifference curves, we get:

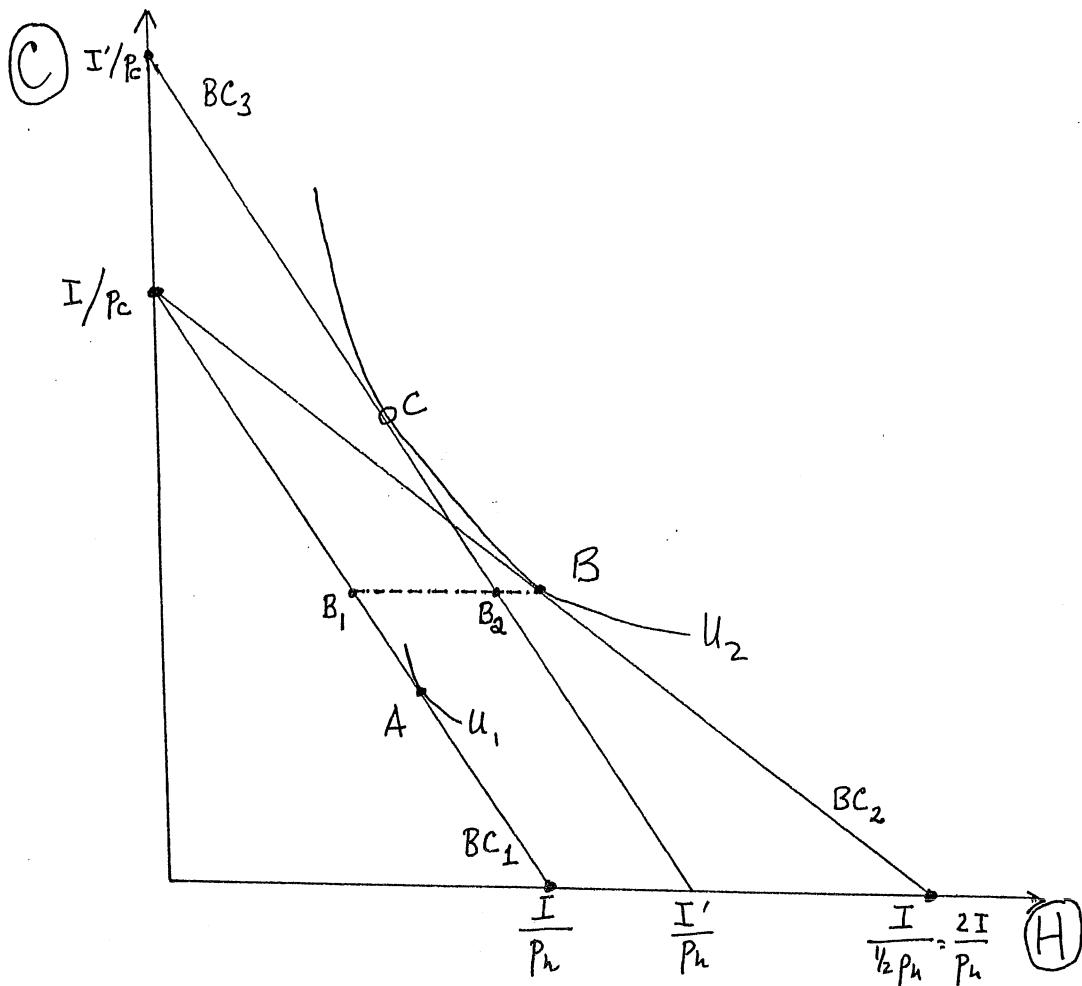


So with these prices, Ms. Caffeine buys only coffee: $C=6, T=0$.

⑧ budget constraint: $I = p_h H + p_c C$. (Think of C being measured in something like pounds or ounces. Nicholson's suggestion that C be measured in dollars will get really confusing if you think about it.)

a) next page; consumer is at point A, with utility U_1 , and budget constraint BC_1 .

b) The budget constraint is now $I = (\frac{1}{2} p_h) H + p_c C$. The affordable set gets bigger in the H direction. On the next page, I've assumed the consumer moves to point B, with utility level $U_2 > U_1$, and budget constraint BC_2 .



- c) BC_3 , which is parallel to BC_1 and tangent to U_2 at point C , represents the situation in which prices for housing and consumption are unchanged from part a, but income has gone up to I' . This boost in income is enough to benefit the consumer as much as the housing subsidy did, since both get the consumer to U_2 . But the government is better off with the income supplement instead of the housing subsidy, because the income supplement only costs $B_1 B_2$, whereas the housing subsidy costs $B_1 B$.

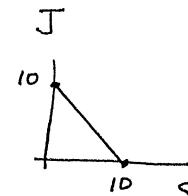
Ch. 4, solutions to even-numbered problems.

(2) a) budget constraint: $I = p_J J + p_S S$

$$200 = 20J + 20S$$

$$10 = J + S \quad \text{or} \quad J = -S + 10.$$

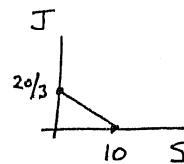
See part c for the rest of the answer.



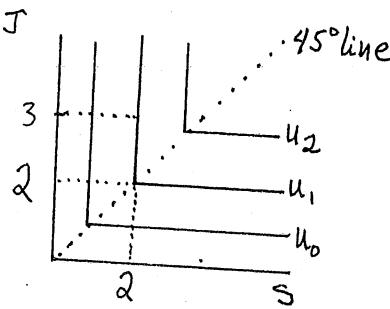
b) budget constraint: $200 = 30J + 20S$

$$20 = 3J + 2S \quad \text{or} \quad J = \frac{-2}{3}S + \frac{20}{3}$$

See part c for the rest of the answer.



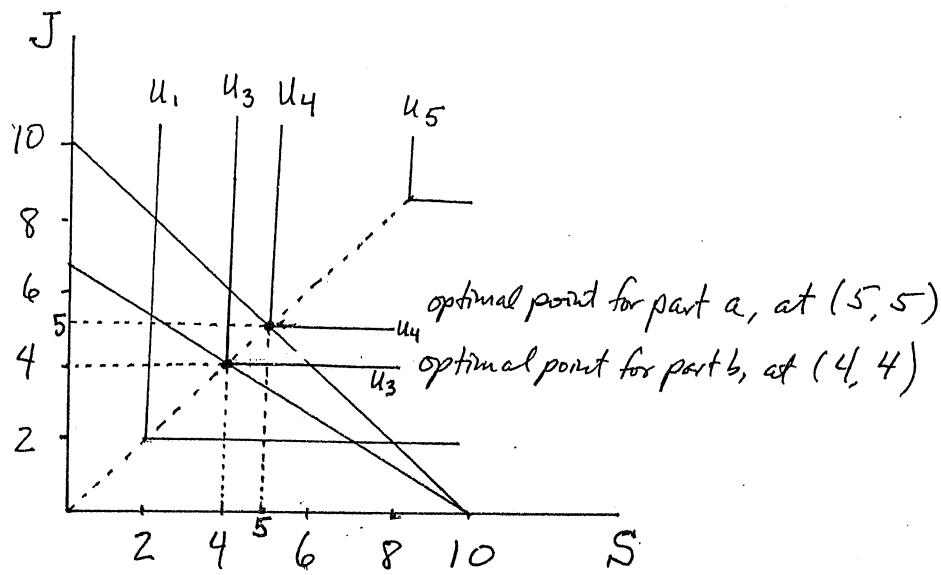
c) Elizabeth's indifference curves look like this.



To understand this, suppose that $J = 2$ and $S = 2$. Then utility is U_1 . Now, give Elizabeth more jeans (say, $J = 3$), but keep $S = 2$. Her utility has not gone

up at all, because for each new pair of jeans, she needs a new sweater, and she hasn't been given a new sweater. So $S = 2, J = 3$ has the same utility as $S = 2, J = 2$. The same is true for $J = 2, S = 3$, and so forth.

Superimposing the budget constraints from (a) and (b) onto the indifference curves, we get the graph which appears on the next page.



Instead of using the graph, one can get the answer analytically as follows. Take part b as an example; part a is similar. You know that Elizabeth will always buy the same number of jeans as sweaters: $J = S$ at the optimum. Using the budget constraint for part b, then, we get $200 = 30J + 20S$

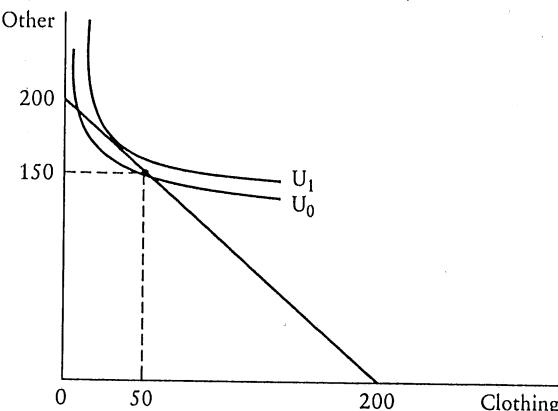
$$= 30J + 20J = 50J \Rightarrow J = 4 \text{ and,}$$

$$\text{since } J = S, S = 4.$$

This is the same answer we got in the graph.

- d) There is no substitution effect here; Elizabeth won't substitute sweaters for jeans or vice versa. So it must be only an income effect.

- 4.3 In the figure, Mr. Wright is required to buy \$50 worth of clothing. Given this constraint, he can only attain utility level U_0 . With an unconstrained choice, U_1 can be attained.



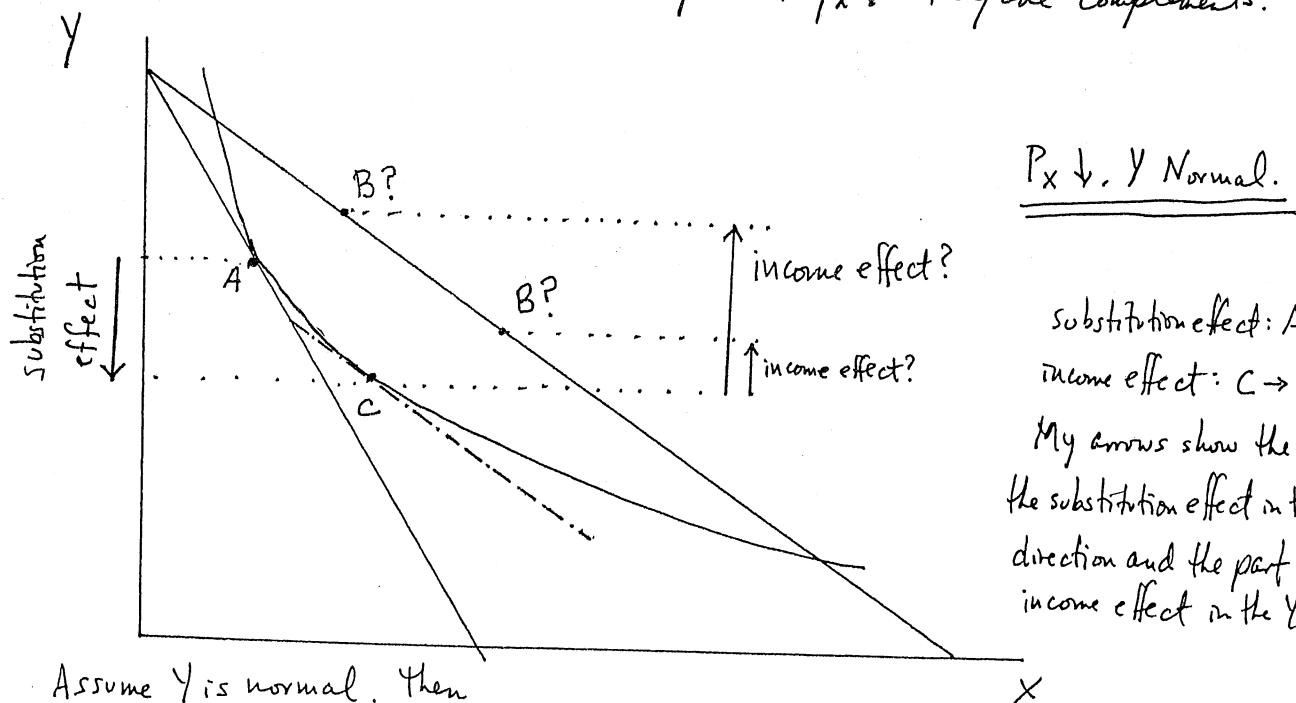
(6) Suppose $P_x \downarrow$. The substitution effect will always go in the direction of buying less Y and more X.

What about the income effect?

If Y is an inferior good, the fall in P_x , which makes you feel richer, will result in buying less Y. This adds to the substitution effect's $\downarrow Y$. So $P_x \downarrow \Rightarrow Y \downarrow$, and X and Y are substitutes.

If Y is a normal good, the fall in P_x , which makes you feel richer, will result in buying more Y. So in this case, the substitution ^{effect} has $Y \downarrow$, but the income effect has $Y \uparrow$.

- if the substitution effect wins out, $Y \downarrow$ in response to $P_x \downarrow$: they are substitutes.
- if the income effect wins out, $Y \uparrow$ in response to $P_x \downarrow$: they are complements.



substitution effect: $A \rightarrow C$

income effect: $C \rightarrow B?$

My arrows show the part of the substitution effect in the Y direction and the part of the income effect in the Y direction.

Assume Y is normal. Then

B is higher than C because Y is a normal good. (Be sure you understand this.)

But we don't know just how high B is. The higher B is, the bigger the income effect on Y is, and the better the chance that $Y \uparrow$ when $P_x \downarrow$ (which implies that X and Y are complements).



Ch. 5 even-numbered problems.

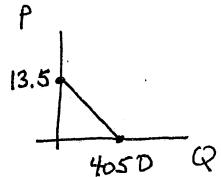
$$\textcircled{2} \quad Q = 1000 + 0.3 I - 300 P + 200 P'$$

a) $Q = 1000 + 0.3(10,000) - 300P + 200(0.25)$

$$= 4050 - 300P$$

$$Q=0 \text{ at } 0 = 4050 - 300P \Rightarrow 300P = 4050 \Rightarrow$$

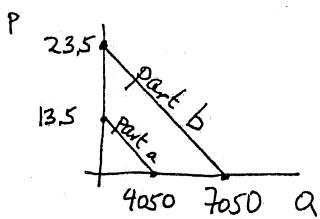
$$P = 13.50$$



b) $Q = 1000 + 0.3(20,000) - 300P + 200(0.25)$

$$= 7050 - 300P$$

$$Q=0 \text{ at } 300P = 7050 \Rightarrow P = 23.50$$

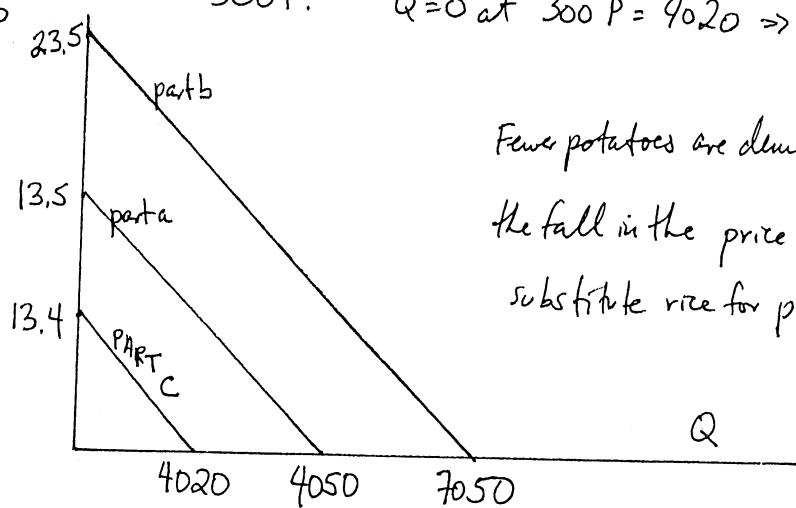


More potatoes are demanded at every price because the demand curve has been shifted to the right by an increase in income.

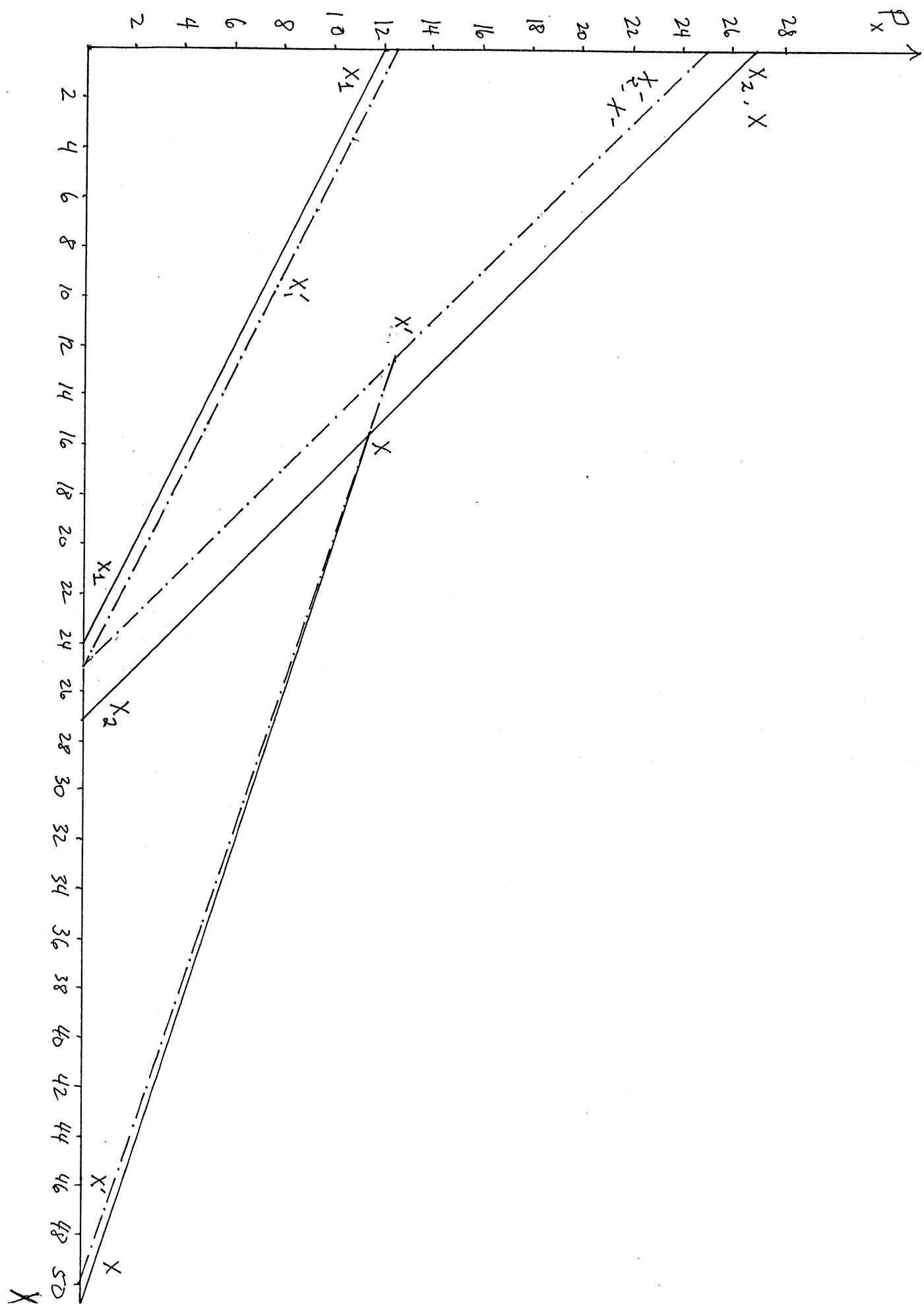
c) $Q = 1000 + 0.3(10,000) - 300P + 200(0.1)$

$$= 4020 - 300P$$

$$Q=0 \text{ at } 300P = 4020 \Rightarrow P = 13.4$$



Fewer potatoes are demanded at any price because the fall in the price of rice is causing consumers to substitute rice for potatoes.



(4)

$$X_1 = 10 - 2P_x + 0.01 I_1 + 0.4 P_y$$

$$X_2 = 5 - P_x + 0.02 I_2 + 0.2 P_y$$

a) $X = X_1 + X_2 = 15 - 3P_x + 0.01 I_1 + 0.02 I_2 + 0.6 P_y$

b) $X_1 = 10 - 2P_x + 0.01(1000) + 0.4(10) = 24 - 2P_x$

$$X_2 = 5 - P_x + 0.02(1000) + 0.2(10) = 27 - P_x.$$

c) For $P_x > 12$, $X = X_2 = 27 - P_x$.

For $P_x < 12$, $X = X_1 + X_2$
 $= 51 - 3P_x$.

For the graph on the next page, the line $X_1 X_1$ is the demand for person 1, the line $X_2 X_2$ is the demand for person 2, and the line XXX , which has a kink in it, is the market demand curve.

d) $X_1 = 10 - 2P_x + 0.01(1100) + 0.4(10) = 25 - 2P_x$, labelled X'_1

$$X_2 = 5 - P_x + 0.02(900) + 0.2(10) = 25 - P_x$$
, labelled X'_2

For $P_x > 12 \frac{1}{2}$, $X = X_2 = 25 - P_x$

For $P_x < 12 \frac{1}{2}$, $X = X_1 + X_2 = 50 - 3P_x$.

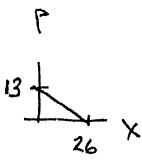
} labelled $X' X' X'$. The problem in the book doesn't ask for this, but I

decided to do it, since I'm going to omit part (c). Otherwise the graph will

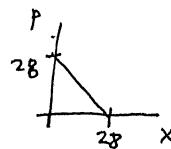
(e) get too messy. The algebraic answers for part e are, however,

(on the assumption that $I_1 = I_2 = 1000$),

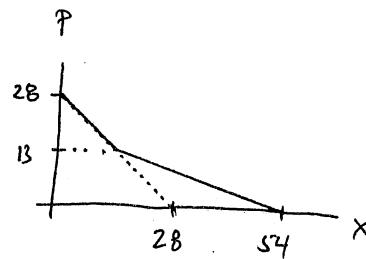
$$X_1 = 10 - 2P_x + 0.01(1000) + 0.4(15) = 26 - 2P_x$$



$$X_2 = 5 - P_x + 0.02(1000) + 0.2(15) = 28 - P_x$$



For $P_x > 13$, $X = X_2 = 28 - P_x$.



For $P_x < 13$, $X = X_1 + X_2 = 54 - 3P_x$.

If a good is a luxury then $\frac{\% \Delta Q^D}{\% \Delta I} > 1$. This means that if income increases by 10%, the quantity purchased of the good increases by more than 10%. Expenditure on the good is its price times its quantity; if quantity goes up by more than 10% and price is unchanged, then expenditure on the good goes up by more than 10%. Conclusion: if a good is a luxury, a 10% rise in income will result in more than a 10% rise in expenditures on the good.

Now suppose that there are only two goods in the economy, and both are luxuries. Also suppose that income rises by 10%. Then expenditures on both goods rise by more than 10%. This means that the consumer's total expenditure for both of the goods in the economy combined rises by more than 10%. This is impossible, since the consumer's income only increased by 10%, and the consumer cannot spend more than he has. Conclusion: assuming that there are only two goods in the economy and both are luxuries leads to an impossible result. So in a two-good economy, both goods cannot be luxuries.

Let 'o' be oil, 'g' be natural gas, and 'c' be coal.

Let P_{OIL} = price of 1 gallon of fuel oil,

P_{GAS} = price of 1000 ft.³ of natural gas, and

P_{COAL} = price of 50 lbs. of coal.

Given: $P_o = P_g = P_c$. This is true for every year, because these prices always move together.

Base Year Quantities: Let the base year be year 1 and the current year be year 2.

$$\text{Price Index} = \frac{P_o^2 Q_o^1 + P_g^2 Q_g^1 + P_c^2 Q_c^1}{P_o^1 Q_o^1 + P_g^1 Q_g^1 + P_c^1 Q_c^1} = \frac{P_o^2}{P_o^1} \frac{Q_o^1 + Q_g^1 + Q_c^1}{Q_o^1 + Q_g^1 + Q_c^1} = \frac{P_o^2}{P_o^1}$$

Current Year Quantities:

$$\text{Price Index} = \frac{P_o^2 Q_o^2 + P_g^2 Q_g^2 + P_c^2 Q_c^2}{P_o^1 Q_o^2 + P_g^1 Q_g^2 + P_c^1 Q_c^2} = \frac{P_o^2}{P_o^1} \frac{Q_o^2 + Q_g^2 + Q_c^2}{Q_o^2 + Q_g^2 + Q_c^2} = \frac{P_o^2}{P_o^1}, \text{the same as.}$$

This works because in this problem, $P_o^1 = P_g^1 = P_c^1$ and $P_o^2 = P_g^2 = P_c^2$.

Chapter 7, Problems 2 and 4

2) $Q = 2K + L$

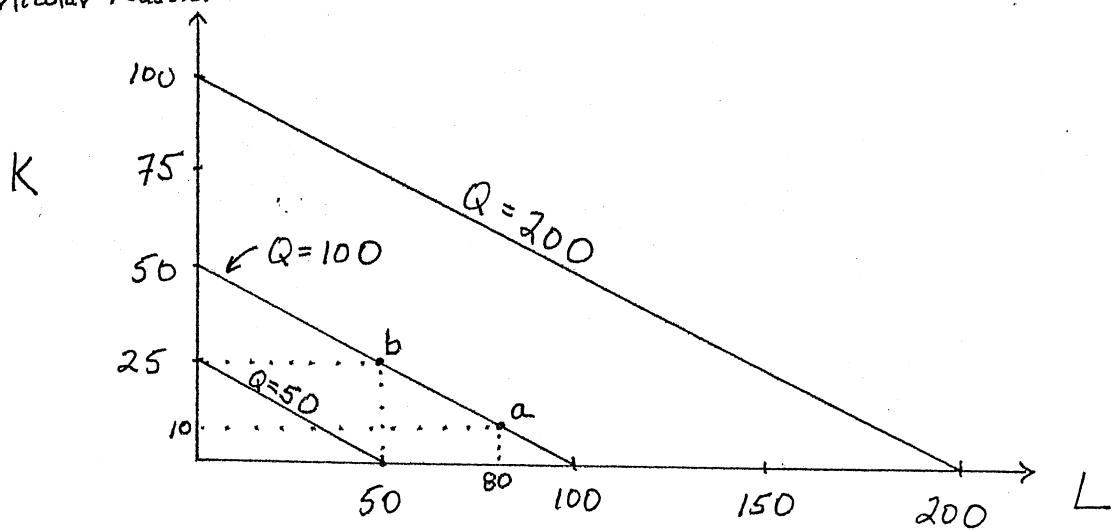
a. $K = 10$ and $Q = 100 \Rightarrow 100 = 2(10) + L \Rightarrow L = 80$

b. $K = 25$ and $Q = 100 \Rightarrow 100 = 2(25) + L \Rightarrow L = 50$.

c. If $Q = 100$ then $100 = 2K + L$ or

$L = -2K + 100$ or $K = -\frac{1}{2}L + 50$. The last form is most convenient

if you decide to put K on the vertical axis and L on the horizontal axis, which is done here, for no particular reason.



$RTS = -(\text{slope of isoquant})$. Since $K = -\frac{1}{2}L + 50$, $RTS = +\frac{1}{2}$. (If you put L on the vertical axis and K on the horizontal axis, you would get

$RTS = +2$ from $L = -2K + 100$. This is also OK.)

RTS is the same because this isoquant is a straight line. Usually, isoquants are convex, causing RTS to be diminishing as you go from left to right.

d. If $Q = 50$ then $50 = 2K + L$ or $K = \frac{1}{2}L + 25$.

If $Q = 100$ then $100 = 2K + L$ or $K = \frac{1}{2}L + 100$.

The entire isoquant map is a family of straight lines with slope $-\frac{1}{2}$. Their general form is $K = \frac{1}{2}L + \frac{Q}{2}$. (If you put L on the vertical axis and K on the horizontal axis, you would get $L = -2K + Q$: isoquants are straight lines with slope -2 .)

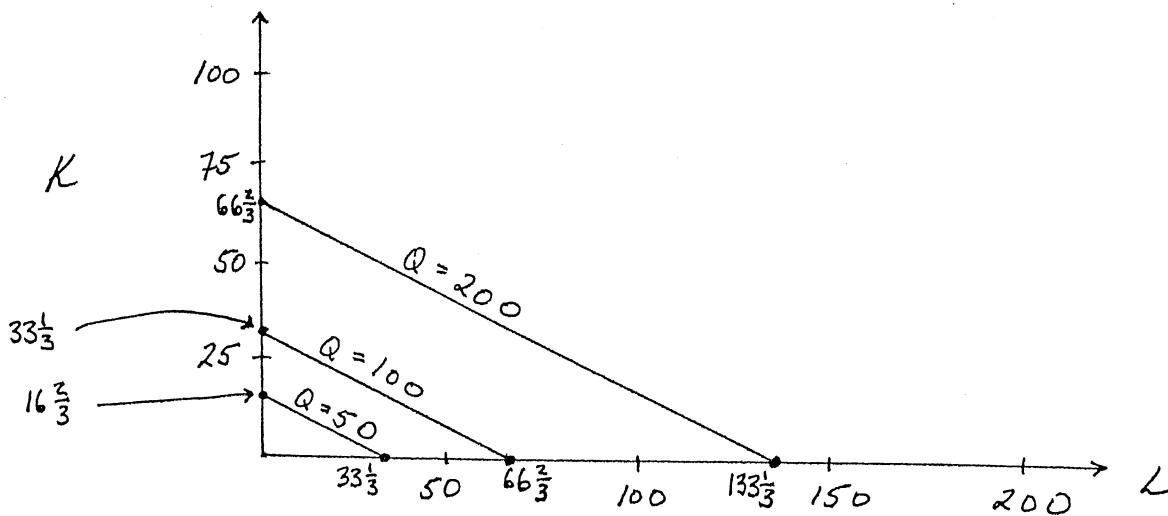
$$e. Q = 3K + 1.5L$$

$$K = 10, Q = 100 \Rightarrow 100 = 30 + 1.5L \Rightarrow L = 46\frac{2}{3}.$$

$$K = 25, Q = 100 \Rightarrow 100 = 75 + 1.5L \Rightarrow L = 16\frac{2}{3}.$$

In general: $3K = -1.5L + Q$,

$$K = \frac{1}{2}L + \frac{Q}{3}.$$



The isoquants are still straight lines with slope $-\frac{1}{2}$. However, now less L and K are required to get to a given level of Q . This 'technical progress' causes the isoquants

to shift inwards. This might be puzzling until you think about it: when a production process becomes more efficient, you need less inputs for the same level of output.

4) Diminishing Returns to Scale: when ALL inputs double, output less than doubles

Diminishing Marginal Productivity (= Law of Diminishing Returns): when only ONE input increases, with all other inputs remaining fixed, then eventually the extra output you get becomes less and less.

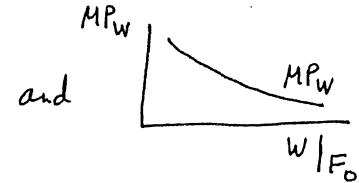
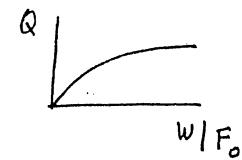
Example. $Q = \sqrt{WF}$.

This has constant returns to scale: doubling W and doubling F yields

$$\text{new } Q = \sqrt{(2W)(2F)} = \sqrt{4WF} = 2\sqrt{WF} = 2 \times (\text{old } Q).$$

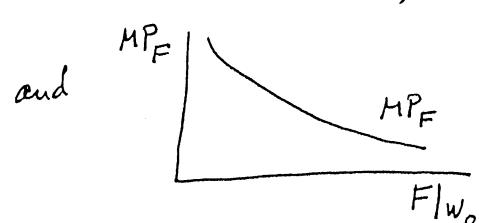
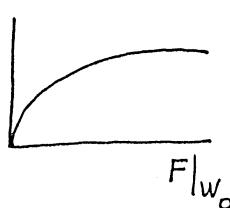
This also has diminishing MP of W keeping F fixed: for $F = F_0$,

$$Q = (\sqrt{F_0}) \sqrt{W}, \text{ so}$$



Finally, this has diminishing MP of F keeping W fixed: for $W = W_0$,

$$Q = (\sqrt{W_0}) \sqrt{F}, \text{ so}$$

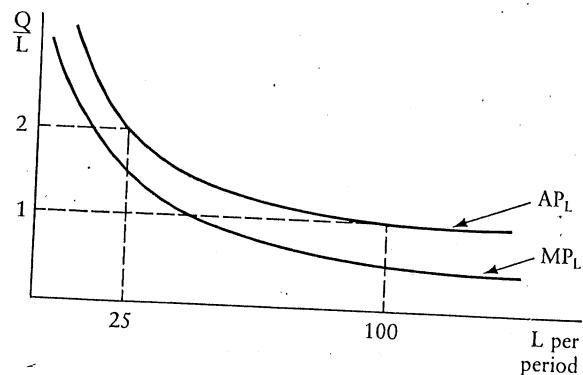


7.7

$$Q = K^{1/2}L^{1/2}$$

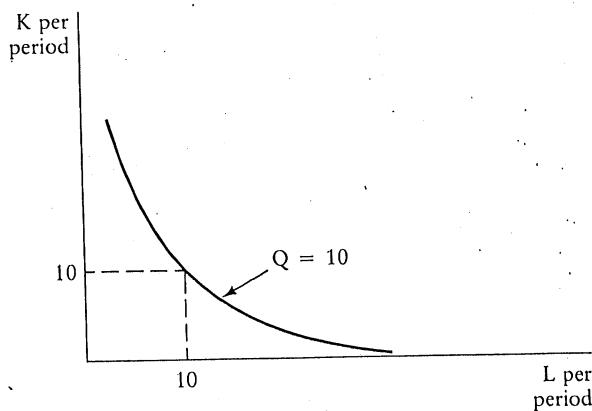
a. $AP_L = \frac{Q}{L} = \left(\frac{K}{L}\right)^{1/2}$, $AP_K = \left(\frac{L}{K}\right)^{1/2}$.

b. $K = 100$, $AP_L = \frac{10}{\sqrt{L}}$.



- c. It is unusual that MP_L never actually reaches zero.

d.



e. $RTS = \frac{MP_L}{MP_K} = \frac{AP_L}{AP_K} = \frac{(K/L)^{1/2}}{(L/K)^{1/2}} = \frac{K}{L}$.

$K = 10$, $L = 10$, $RTS = 1$.

$K = 25$, $L = 4$, $RTS = 6.25$.

$K = 4$, $L = 25$, $RTS = .16$.

Yes, it does exhibit diminishing RTS.

④ $Q = 2\sqrt{H}$ (There is only one input to this production function.)

$$P_H = \$8$$

a) Total Cost = $P_H \cdot H$. But $P_H = 8$ and, since $Q^2 = 4H$, $H = \frac{1}{4}Q^2$. So

$$TC = 8 \cdot \frac{1}{4}Q^2$$

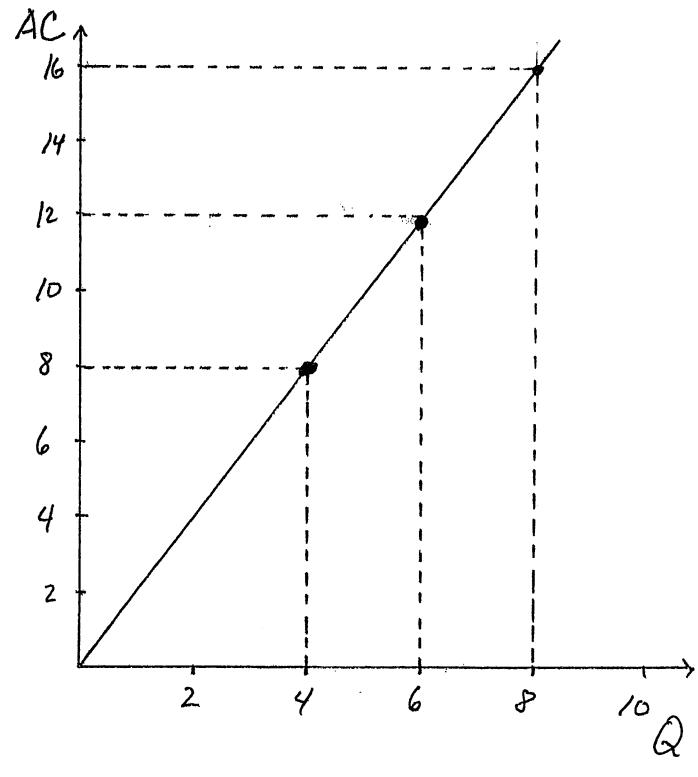
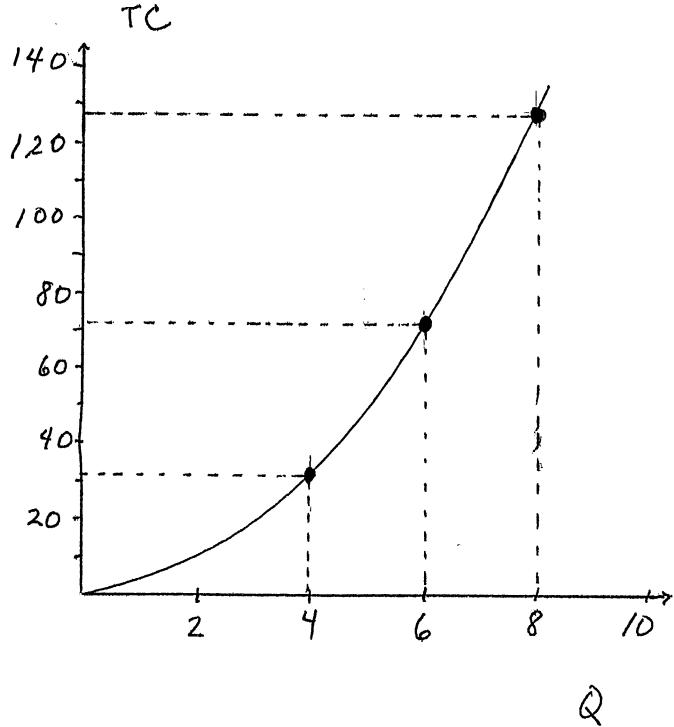
$$TC(Q) = 2Q^2.$$

b) $Q=4 \Rightarrow TC = 2(4^2) = 32$ and $AC = \frac{TC}{Q} = \frac{32}{4} = 8$

$$Q=6 \Rightarrow TC = 2(6^2) = 72 \text{ and } AC = \frac{TC}{Q} = \frac{72}{6} = 12$$

$$Q=8 \Rightarrow TC = 2(8^2) = 128 \text{ and } AC = \frac{TC}{Q} = \frac{128}{8} = 16.$$

You should notice that in general, $AC(Q) = \frac{TC(Q)}{Q} = \frac{2Q^2}{Q} = 2Q$.



7b) Comment: the book got MC here by using calculus. However, you can find the MC of the 100th widget from the TC function. For example, $MC(100) \approx \frac{TC(101) - TC(100)}{101 - 100}$, or $MC(100) \approx \frac{TC(90) - TC(100)}{90 - 100}$, or any such approximate formula which uses $MC = \frac{\Delta TC}{\Delta Q}$, will be OK. Similar formulas will do for $MC(125)$, $MC(200)$, etc.

⑧ 2 workers + 1 machine \rightarrow 10 units of output.

$$V = \$1, w = \$3$$

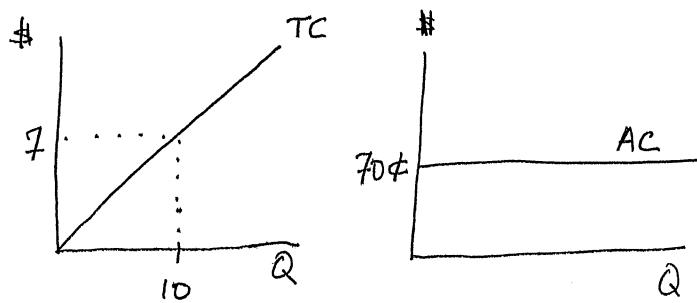
a) 2 workers at \$3 each = \$6

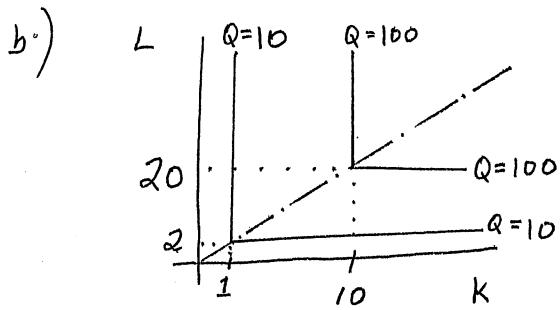
1 machine at \$1 each = \$1.

So TC of 10 units of output is \$7, and AC at 10 units of output is $\$ \frac{7}{10} = \0.70 .

Since the inputs are used in fixed proportions, there are no substitution possibilities.

Now, let's assume Constant Returns to Scale. (I think this is what Nicholson wants.) Then AC is constant and TC is a straight line.





$$\begin{aligned} TC &= rK + wL \\ &= (\$1)K + (\$3)L = 10 + 3L. \end{aligned}$$

$K = 10$. What is the relation between L and Q ? Well, if $Q = 100$, $L = 20$:

2 workers to one machine. If $Q < 100$, there are too many machines.

For example, if $Q = 60$, only 6 machines will be used, along with 12 workers. In general, if $Q < 100$, $\frac{Q}{10}$ machines will be used, along with $2 \times \frac{Q}{10} = \frac{Q}{5}$ workers. So if $Q < 100$, $L = \frac{Q}{5}$. Therefore

$$\begin{aligned} TC &= 10 + 3\left(\frac{Q}{5}\right) \\ &= 10 + \frac{3}{5}Q \quad \text{for } Q < 100. \end{aligned}$$

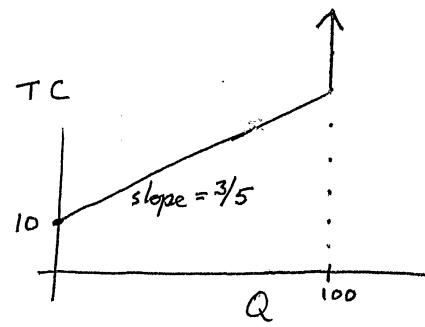
For $Q > 100$, production is impossible, since you only have 10 machines.

For $Q = 100$, $TC = 10 + \frac{3}{5}Q$ works, because it gives $TC = 10 + 60 = 70$,

we said already that if $Q = 100$, $L = 20$, so $TC = 10 + 3(20) = 70$.

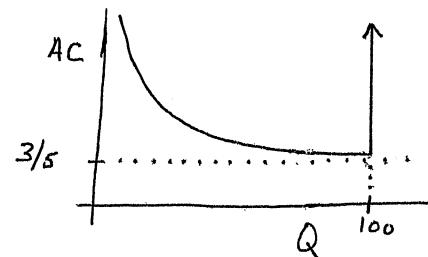
So our answer is:

$$TC = \begin{cases} 10 + \frac{3}{5}Q & \text{for } Q \leq 100 \\ \infty & \text{for } Q > 100 \end{cases}$$



Since $AC = TC/Q$, we easily get

$$AC = \begin{cases} \frac{10}{Q} + \frac{3}{5} & \text{for } Q \leq 100 \\ \infty & \text{for } Q > 100 \end{cases}$$



Notice that I have written TC and AC as functions of Q , not of L or K or anything else. This is important : cost curves are always functions of Q .

$$MC = \frac{\Delta TC}{\Delta Q}.$$

$$10^{\text{th}} \text{ unit: } MC = \frac{TC(11 \text{ units}) - TC(10 \text{ units})}{11 - 10} = \frac{\left[10 + \frac{3}{5}(11)\right] - \left[10 + \frac{3}{5}(10)\right]}{11 - 10}$$

$$= \frac{\frac{3}{5}(1)}{1} = \frac{3}{5} = 0.6. \quad \text{Instead of 11, I could have chosen 12 or 13 or 9 or 8 or something like that.}$$

$$25^{\text{th}} \text{ unit: } MC = \frac{TC(28 \text{ units}) - TC(25 \text{ units})}{28 - 25} = \frac{\frac{3}{5}(3)}{3} = \frac{3}{5}. \quad \text{I chose 28 more or less at random.}$$

$$50^{\text{th}} \text{ unit: } MC = \frac{TC(49 \text{ units}) - TC(50 \text{ units})}{49 - 50} = \frac{\frac{3}{5}(-1)}{-1} = \frac{3}{5}. \quad \text{continued} \rightarrow$$

100th unit: this is sort of a trick question.

$$MC = \frac{TC(99 \text{ units}) - TC(100 \text{ units})}{99 - 100} = \frac{\left[10 + \frac{3}{5}(99)\right] - \left[10 + \frac{3}{5}(100)\right]}{99 - 100}$$

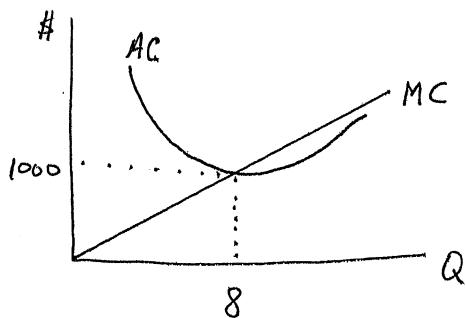
$$= \frac{\frac{3}{5}(-1)}{-1} = \frac{3}{5}, \text{ but}$$

$$MC = \frac{TC(101 \text{ units}) - TC(100 \text{ units})}{101 - 100} = \frac{\infty - TC(100 \text{ units})}{1} = \infty.$$

So is the MC equal to $\frac{3}{5}$ or equal to ∞ ? Well, neither, or both - it's a little vague.

By the way, you should be able to get MC just from the slope of the TC function, graphed at the top of p. 4

(10)



When AC is at its lowest, $AC = MC$.

At $Q = 8$, the problem says that AC is at its lowest, and it says that $AC = 1000$.

Hence at $Q = 8$, $MC = 1000$.

So what is MC at $Q = 9$? $MC(Q)$ has the form $MC = mQ + b$.

↓ ↓
 slope intercept

$b=0$. Two points on MC are $(0, 0)$ and $(8, 1000)$, so MC has a slope of

$$\frac{1000 - 0}{8 - 0} = 125. \text{ Hence } MC = 125Q, \text{ so if } Q = 9, MC = 125 \times 9 \\ = \$1125.$$

Chapter 9, Homework Solutions.

- ④ a: A lump-sum profit tax, which I think is the same as a lump-sum tax on a firm, would affect neither MC nor MR. This is because lump-sum amounts (taxes or subsidies) do not change when Q changes, whereas both MC and MR measure how something changes when Q changes.

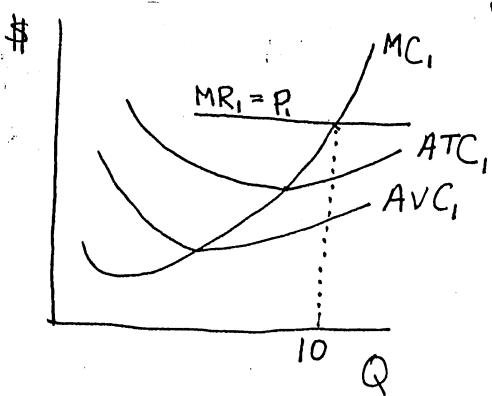
$$\pi(Q) = R(Q) - C(Q) - (\text{lump sum tax}) ;$$

$$\frac{\Delta \pi}{\Delta Q} = \frac{\Delta R}{\Delta Q} - \frac{\Delta C}{\Delta Q} - \frac{\Delta(\text{lump sum tax})}{\Delta Q}$$

$\downarrow \quad \downarrow \quad \downarrow = 0$
 $M\pi \quad MR \quad MC$

Since lump sum taxes can't
change.

But this is not the whole story. Suppose before the tax is imposed we have

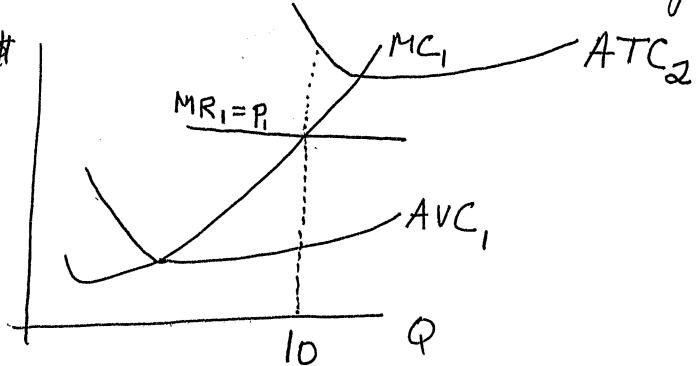


this graph. Then $Q = 10$, profits are positive, and everything is OK. Now impose the tax.

MR and MC do not change, as we said before.

AVC does not change either, because variable costs do not change. (A lump sum tax is not a variable cost since it does not vary with Q .) But ATC does go up. Now, that does not matter in the short run: in the short run, as long as $P > AVC$ you are OK. So in the

short run, $Q = 10$ still. In the long run, though, if ATC rose enough, the graph might look like this:



Now, $P < ATC$, so $\pi < 0$. Therefore, if ATC rose by this much, the firm will not produce $Q = 10$ still; instead,

in the long run it will produce $Q = 0$ (shut down).

b: A proportional tax on profits is something like:

$$\pi_t = (1 - \text{tax rate}) [R(Q) - C(Q)];$$

before the tax,

$$\pi = R(Q) - C(Q).$$

Suppose before the tax, $Q = 18$. In other words, $Q = 18$ makes $R(Q) - C(Q)$ the biggest. Now, what Q will you choose after the tax is imposed? Well, after the tax is imposed you still want to make $[R(Q) - C(Q)]$ as big as you can; the $(1 - \text{tax rate})$ term you cannot do anything about. And to make $[R(Q) - C(Q)]$ as big as you can, you would still pick $Q = 18$, because we said two sentences ago that $Q = 18$ makes $R(Q) - C(Q)$ the biggest. So a proportional tax on profits does not affect the output of a firm.

C: A tax assessed on each unit of output is like a sales tax. For instance, if

the firm is perfectly competitive then

$$\pi = P \cdot Q - C(Q)$$

before the tax, and

$$\pi = (P - \text{tax rate}) \cdot Q - C(Q)$$

after the tax. MC is therefore not affected, but MR goes down from P to $(P - \text{tax rate})$ in this example. If MC stays the same but MR drops, the firm's choice of Q will certainly go down (unless it had decided to shut down even before the tax was imposed, in which case Q couldn't go down because it was already zero).

⑧ This problem is important.

First note that "Universal Widget" is a monopolist. The key here is that MR in Australia should be the same as in Lapland, because if MR were higher in one place than in the other, you should be selling more where MR is higher and less where MR is lower.

$$MR_{\text{Australia}} = 50 - Q_{\text{Australia}} . \text{ Let } A = \text{Australia} .$$

$$MR_{\text{Lapland}} = 25 - 2Q_{\text{Lapland}} . \text{ Let } L = \text{Lapland} .$$

We want $MR_A = MR_L$. That means that

$$50 - Q_A = 25 - 2Q_L$$

$$\boxed{25 + 2Q_L = Q_A} \quad (1)$$

So, as long as we stick with (1), we have $MR_A = MR_L$. Now what?

Well now, we need to set $MR = MC$. Which MR ? It doesn't matter, since with (1), they are equal. Let's take Australia's. That gets us

$$50 - Q_A = 0.50 Q. \quad (2)$$

What is Q ? Q is the total production at Universal Widget's only plant, in Golch, Nevada. So

$$Q = Q_A + Q_L. \quad (3)$$

(Q is split up between the Australian and hapland markets.)

Substituting (3) into (2) gets

$$\boxed{50 - Q_A = 0.50 (Q_A + Q_L).} \quad (4)$$

(1) and (4) is a system of two equations in two unknowns Q_A and Q_L , so we should be able to get numbers out for Q_A and Q_L .

To do this, substitute (1) into (4) :

$$50 - (25 + 2Q_L) = \frac{1}{2} [(25 + 2Q_L) + Q_L]$$

$$25 - 2Q_L = \frac{1}{2} (25 + 3Q_L)$$

$$50 - 4Q_L = 25 + 3Q_L$$

$$25 = 7Q_L$$

$$\left. \begin{array}{l} Q_L = 25/7 \\ Q_L = 3\frac{4}{7} \end{array} \right\} \text{From (1), then, } Q_A = 25 + 2\left(\frac{25}{7}\right)$$

$$\boxed{Q_A = 32\frac{1}{7}.}$$

Ch. 11, even-numbered problems.

(2) a) $Q_s = -1000 + 2000P$. When will Q_s be ≥ 0 ?

$$-1000 + 2000P \geq 0$$

$$2000P \geq 1000$$

$$P \geq 0.50.$$

You can easily check that if $P = 0.50$, $Q_s = 0$. So the price of flounder will have to be 50¢/lb. before any is supplied to the Cape May market.

b) $Q_d = 1600 - 600P$

$Q_s = -1000 + 2000P$

$Q_d = Q_s$

TYPO: CHANGE $Q_d' = 1,600 - 600P$
 TO $Q_d = 1,600 - 600P$
 in the problem's first sentence.

3 equations in 3 unknowns Q_d , Q_s , and P . We solve this by substitution.

Substitute the third equation into the first:

$$\left. \begin{array}{l} Q_s = 1600 - 600P \\ Q_s = -1000 + 2000P \end{array} \right\} \text{2 equations in 2 unknowns } Q_s \text{ and } P.$$

Substitute the last equation, $Q_s = -1000 + 2000P$, into the second-to-last equation, $Q_s = 1600 - 600P$:

$$-1000 + 2000P = 1600 - 600P \quad \text{or} \quad (\text{over} \rightarrow)$$

$$2600P = 2600$$

$$\boxed{P = 1.}$$

Now we could also get $Q_s = -1000 + 2000P$

$$Q_s = -1000 + 2000(1)$$

$$Q_s = 1000$$

and from $Q_d = Q_s$,

$$Q_d = 1000.$$

(You can check this by seeing that $Q_d = 1600 - 600P$ will also give $Q_d = 1000$ for $P = 1.$)

c) $Q'_d = 2200 - 600P$

$$Q_s = -1000 + 2000P.$$

We do this a little quicker than in part b. For equilibrium, $Q'_d = Q_s$.

Substituting for Q'_d and Q_s , we get

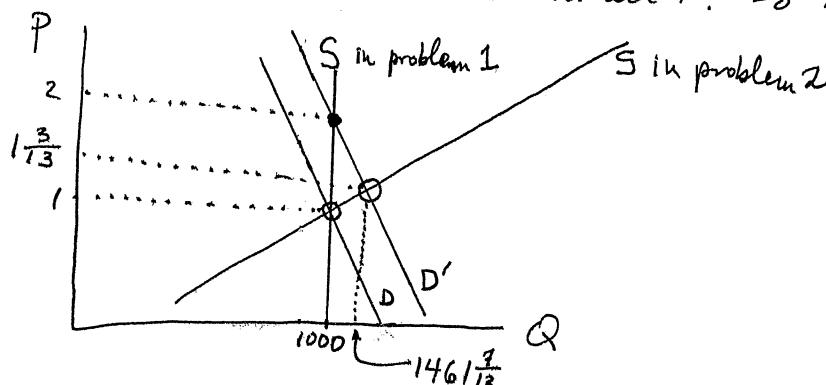
$$2200 - 600P = -1000 + 2000P$$

$$3200 = 2600P$$

$$P = \frac{32}{26} = \frac{16}{13} = 1\frac{3}{13} \approx 1.23.$$

d) $Q'_d > Q_d$ because $2200 - 600P > 1600 - 600P$ for all P . So in

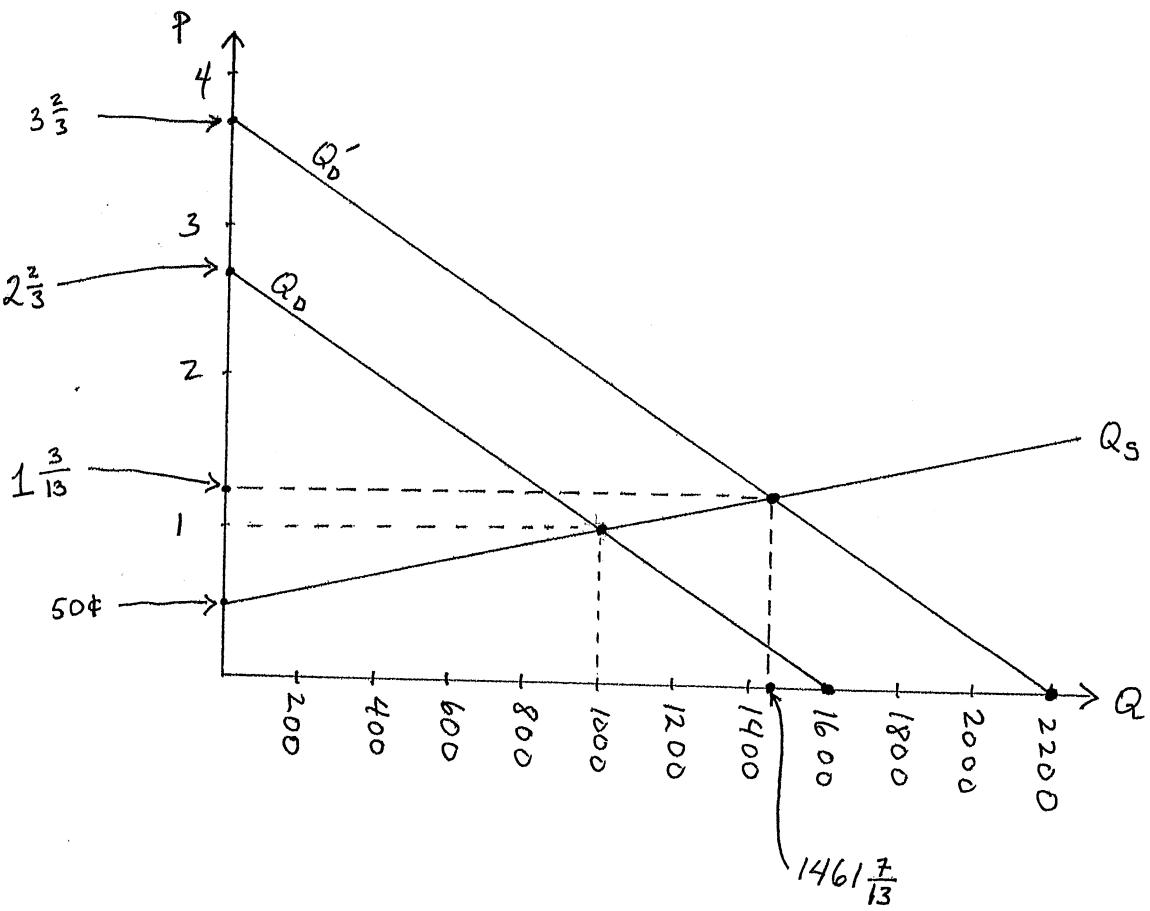
a graph we have



The vertical distance between the two black dots represents the price rise in problem 1, while the vertical distance between the two small circles represents the price rise in problem 2. The second is smaller than the first because in the second case, the rise in demand causes quantity supplied, which is flexible, to increase. In the first case quantity supplied is rigid, so no adjustment can be made in Q and all the adjustment has to come in P.

c) First let's get one more number, Q^* (the equilibrium quantity) of part c.

We had $P^* = \frac{16}{13}$, so $Q^* = Q_s = -1000 + 2000 P^* = -1000 + 2000 \left(\frac{16}{13}\right) = 1461\frac{7}{13}$ lbs.



(6)

$$N = 100$$

$$SRTC = \frac{1}{300} q^3 + 0.2 q^2 + 4q + 10$$

$$SRMC = 0.01 q^2 + 0.4q + 4.$$

a) In the short run, $P = SRMC$:

$$P = 0.01 q^2 + 0.4q + 4$$

$100P = q^2 + 40q + 400$, which just happens to be a perfect square:

$$100P = (q + 20)^2$$

$$10\sqrt{P} = q + 20$$

$$\boxed{q = 10\sqrt{P} - 20.} \quad (\text{Of course, if } P < 4, q = 0.)$$

You may want to check that $P > AVC$:

$$AVC = \frac{1}{300} q^2 + 0.2q + 4$$

$$P = \frac{1}{100} q^2 + 0.4q + 4.$$

You should know how to get this. The fixed part of $SRTC$ is the "+10", so the rest is the variable part:

$$SRVC = \frac{1}{300} q^3 + 0.2 q^2 + 4q.$$

$AVC = \frac{VC}{q}$, so dividing this by q gives you AVC .

Clearly, in this problem $P > AVC$ for all $q > 0$, so the shut-down rule does not interfere with the supply curve we derived above.

b) $Q = 100q$

$$= 1000\sqrt{P} - 2000.$$

$$C. Q_D = -200P + 8000$$

$Q_S = 1000\sqrt{P} - 2000$ from part b.

$$Q_D = Q_S \Rightarrow -200P + 8000 = 1000\sqrt{P} - 2000$$

$$0 = 200P + 1000\sqrt{P} - 10000$$

$$0 = 2P + 10\sqrt{P} - 100$$

$$0 = P + 5\sqrt{P} - 50$$

$$\text{Let } x = \sqrt{P}. \text{ Then } 0 = x^2 + 5x - 50$$

$$= (x+10)(x-5). \quad [\text{You really have to remember high-school algebra for this problem.}]$$

$$\Rightarrow x = -10 \text{ or } x = 5$$

$$\Rightarrow \sqrt{P} = -10 \text{ or } \sqrt{P} = 5. \quad \text{But the square root of}$$

a real number can never be negative, so $\sqrt{P} = -10$ is impossible, and $\sqrt{P} = 5$ is the answer:

$$\boxed{P = 25.}$$

$$\text{Then } Q = Q_D = -200(25) + 8000$$

$$\boxed{Q = 3000.}$$

$$d. Q_D = -200P + 10,000$$

$$Q_S = 1000\sqrt{P} - 2000$$

$$Q_D = Q_S \Rightarrow -200P + 10,000 = 1000\sqrt{P} - 2000$$

$$0 = 200P + 1000\sqrt{P} - 12,000$$

$$0 = P + 5\sqrt{P} - 60$$

$$\text{Let } x = \sqrt{P}. \text{ Then } 0 = x^2 + 5x - 60.$$

Here we need the quadratic formula,

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{25 + 240}}{2} \\ &= \frac{-5 \pm \sqrt{265}}{2}. \end{aligned}$$

As we said before, $x < 0$ is impossible in our problem. So

$$x = \frac{-5 + \sqrt{265}}{2}$$

and

$$P = x^2 = \left(\frac{-5 + \sqrt{265}}{2}\right)^2 \doteq 31.8029.$$

$$\text{Then } Q = Q_D = -200 \left(\frac{-5 + \sqrt{265}}{2}\right)^2 + 10,000 \doteq 3639.41,$$

$$q = \frac{Q}{100} = 36.3941.$$

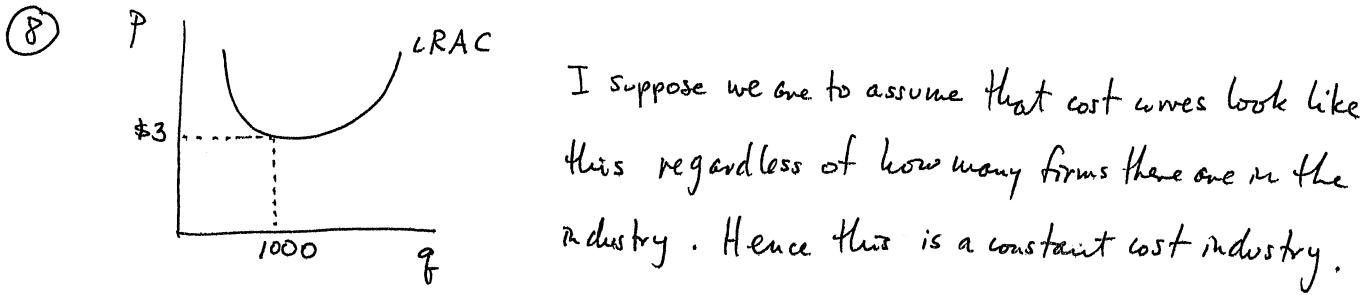
$$\text{TR of 1 firm} = Pg = 31.8029 \times 36.3941 = \$1157.44$$

over →

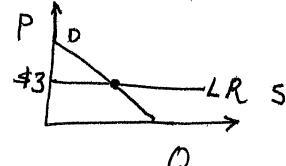
$$\begin{aligned} TC \text{ of 1 firm.} &= \frac{1}{300} (36.3941)^3 + 0.2(36.3941)^2 + 4(36.3941) + 10 \\ &= \$581.17 \end{aligned}$$

$$\text{So rr of 1 firm} = \$1157.44 - \$581.17$$

$$= \underline{\$576.27}.$$



a) The LR Supply curve is flat at \$3/bushel because the only possible LR equilibrium price is \$3/bu.



$$Q_d = 2,600,000 - 200,000P. \text{ At } \underline{P=3},$$

$$Q^* = 2,600,000 - 200,000(3)$$

$$= \underline{2,000,000}.$$

$Q^* = Nq^*$ where N is the number of firms. $q^* = 1000$ since $P^* = 3$. Hence

$$N = \frac{Q^*}{q^*} = \frac{2,000,000}{1,000} = \underline{2000 \text{ firms}} \text{ ("farms").}$$

Over →

b) $Q_D = 3,200,000 - 200,000 P.$

$$\left. \begin{array}{l} q = 1000 \text{ from part(a)} \\ N = 2000 \text{ from part (a)} \end{array} \right\} \Rightarrow Q = 2,000,000.$$

$$\text{So } 2,000,000 = 3,200,000 - 200,000 P$$

$$20 = 32 - 2P$$

$$2P = 12 \Rightarrow P = \$6/bu.$$

For $q = 1000$, $\angle RAC = \$3/bu.$ So

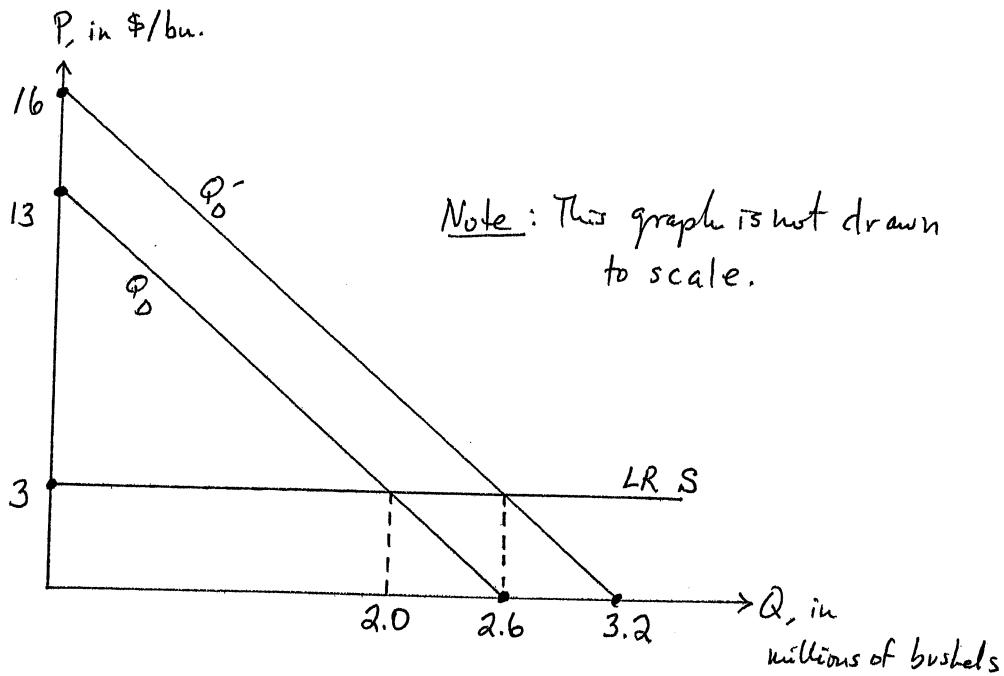
$$\begin{aligned} \pi &= TR - TC = Pq - TC = Pq - (AC \cdot q) \\ &= (P - AC)q = (6 - 3)q = 3q = 3(1000) \\ &= \$3000 \text{ for a typical firm.} \end{aligned}$$

c) Again, as in part a, in LR equilibrium $P = \$3/bu.$ With the demand curve in part b, then,

$$\begin{aligned} Q^* &= 3,200,000 - 200,000(3) \\ &= 2,600,000 bu. \end{aligned}$$

$$N = \frac{Q^*}{q^*} = \frac{2,600,000}{1000} = \underline{2600 \text{ farms.}} \quad (q^* = 1000 \text{ since } P^* = 3.)$$

d)



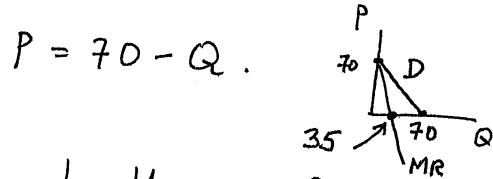
- ⑩ This problem was worked in class.

Ch. 12 Problems

(2)

$$Q = 70 - P$$

$MR = 70 - 2Q$. ← You should be able to figure out this particular MR curve yourself, since the demand curve is linear here. The demand curve is



The MR curve has the same P intercept as the D curve, but it is twice as steep. The D curve has a slope of -1 , so the MR curve has a slope of -2 . This leads to

$$MR = 70 - 2Q,$$

as in the problem statement.

In general, if the demand curve takes the form $P = -mQ + b$, then the MR curve has the form $MR = -2mQ + b$.

Part (b) was the only one which was assigned :

$$TC(Q) = 0.25Q^2 - 5Q + 300$$

$$MC(Q) = 0.5Q - 5.$$

Set $MR = MC$:

$$70 - 2Q = 0.5Q - 5$$

$$75 = 2.5Q$$

$$Q = 30.$$

Now get P from the demand curve.

$$\begin{aligned}P &= 70 - Q = 70 - 30 \\&= 40.\end{aligned}$$

Finally, for profit we have

$$\begin{aligned}\pi &= TR - TC = PQ - [0.25Q^2 - 5Q + 300] \\&= 40(30) - \left[\frac{1}{4}(30)^2 - 5(30) + 300 \right] \\&= 1200 - \frac{900}{4} + 150 - 300 \\&= 825.\end{aligned}$$

Since $TC(Q=0) = 300$, we must be in the short run with $FC = 300$.

So we have to check whether or not the firm will shut down.

π if produce: +\$825 (from above)

π if shut down: -\$300.

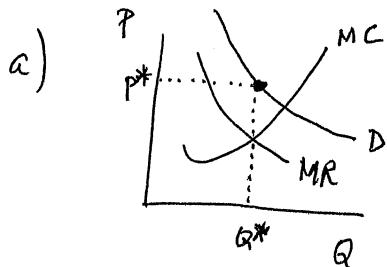
Surely the firm will produce. You can also check that $P > AVC$, since $P = 40$ and

$$AVC = \frac{VC}{Q} = \frac{0.25Q^2 - 5Q}{Q} = 0.25Q - 5 = 0.25(30) - 5 = 2.50.$$

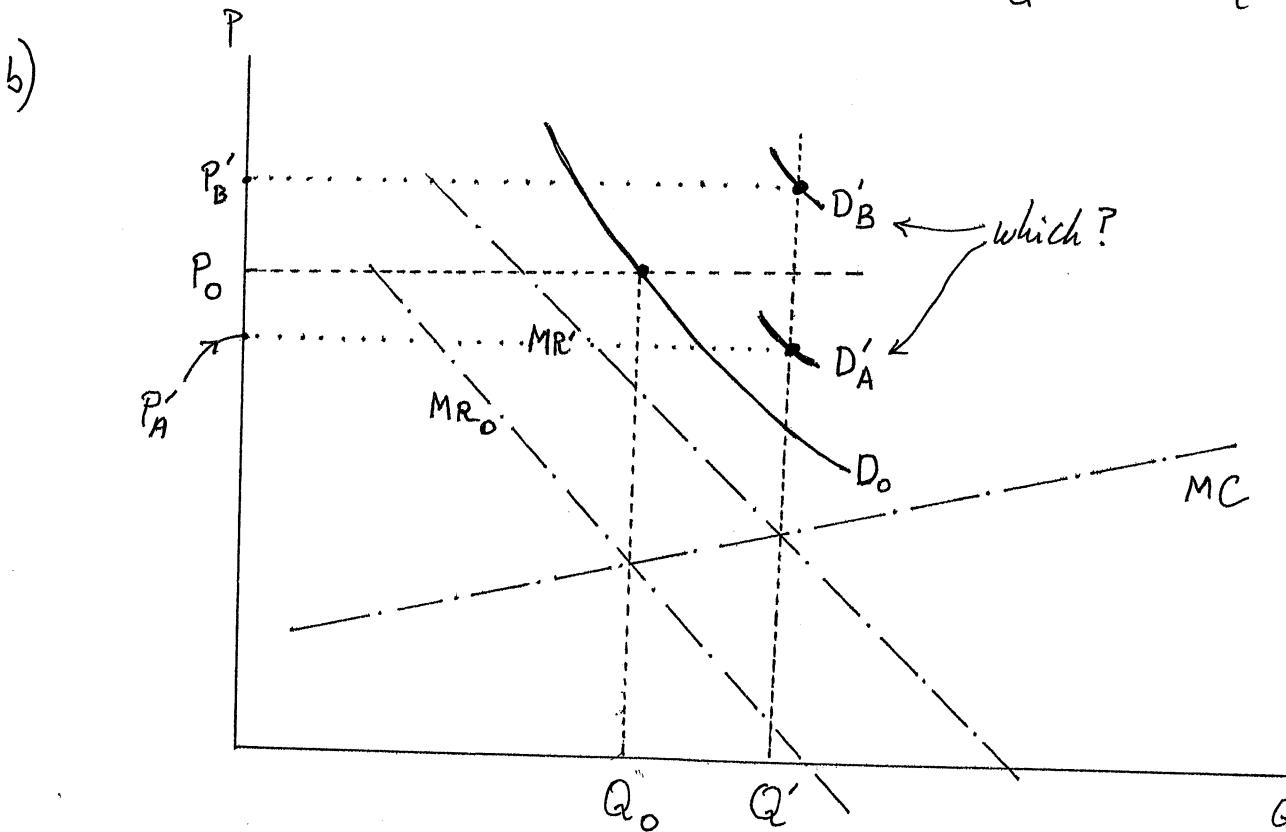
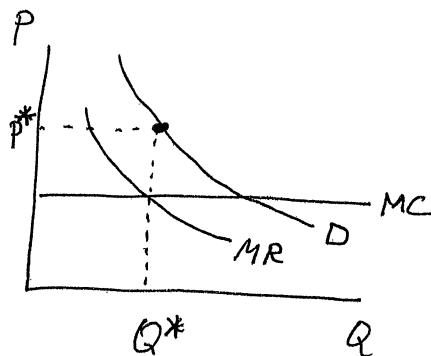
Once you think about it, if $\pi > 0$ then the firm will certainly produce. It is
 $\underbrace{\text{when the firm produces}}$

only when $\pi < 0$ that you have to worry about whether or not the firm will produce in the short run.

④ [Only parts (a) and (b) were assigned.]



or, for the CRS case,



The original D curve is D_0 , which we pretend gives rise to MR_0 , although without having any numbers one cannot calculate where MR_0 is, other than to say that it is below D_0 . Suppose the new MR curve is MR' . If

4

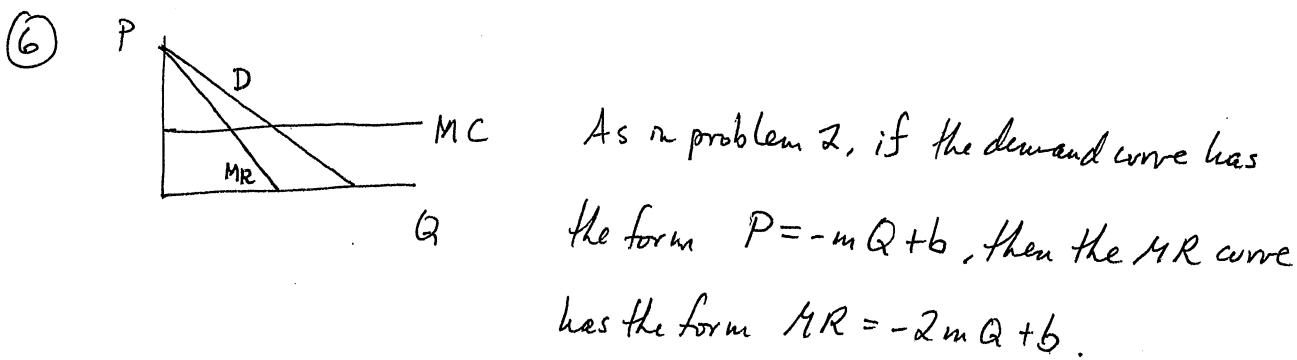
The demand curve corresponding to MR' is D_A' , then the new equilibrium price would be below P_0 , at P_A' . On the other hand, if it is D_B' which corresponds to MR' , then the new equilibrium price is P_B' , which is above P_0 .

Conclusion: A rise in demand may lead either to a rise or to a fall in the price of a product produced by a monopoly.

This conclusion is related to the fact that a monopolist has no supply curve. To understand what this means, remember what a supply curve is: for any price which the firm might face, it tells you how much the firm will produce.

The point is that a monopolist picks his own price, so it makes no sense to ask, "If the monopolist faced such-and-such a price, how much would he produce?" A monopolist has no supply curve, he has a supply point, namely the single combination of Q^* and P^* where he will choose to be. Two examples of supply "points" (Q^*, P^*) are given in the graphs to part(a) of this question.

I should add that the MR curves and the MC curve in part(b) are not necessarily straight lines; the graph just turned out a little neater when I drew them that way. Also, the MC curve might be flat - it really doesn't matter.



$$Q_1 = 55 - P_1 \Rightarrow P_1 = 55 - Q_1 \Rightarrow MR_1 = 55 - 2Q_1$$

$$Q_2 = 70 - 2P_2 \Rightarrow P_2 = 35 - \frac{1}{2}Q_2 \Rightarrow MR_2 = 35 - Q_2$$

MR should be the same in both markets. Hence

$$55 - 2Q_1 = 35 - Q_2$$

$$Q_2 = 35 - (55 - 2Q_1)$$

$$Q_2 = 2Q_1 - 20.$$

Therefore

$$MR_1 = 55 - 2Q_1$$

$$MR_2 = 35 - (2Q_1 - 20) = 55 - 2Q_1$$

Also, $MR = MC$ (either MR_1 or MR_2 will do, since they're equal) :

$$55 - 2Q_1 = 5$$

$$50 = 2Q_1 \Rightarrow \underline{Q_1 = 25} \text{ and}$$

$$Q_2 = 2(25) - 20 \Rightarrow \underline{Q_2 = 30}$$

Now we get prices in each market from the respective demand curves :

$$P_1 = 55 - Q_1 = 55 - 25 = \underline{30}$$

$$P_2 = 35 - \frac{1}{2}Q_2 = 35 - \frac{1}{2}(30) = \underline{20} .$$

$$\Pi = TR - TC$$

$$= (TR_1 + TR_2) - TC = (TR_1 + TR_2) - (AC)(\overset{\text{by "Q" I mean } Q_1 + Q_2}{Q})$$

$$= (P_1 Q_1 + P_2 Q_2) - AC \cdot Q$$

$$= (30 \cdot 25 + 20 \cdot 30) - 5(25 + 30)$$

$$= \underline{1075}.$$

[omit the rest of the problem.]

Econ. 323

Ch. 14 Homework

(4) $Q = 10,000 \sqrt{L}$

$P = \$0.01$ (competitive)

$MP_L = 5000/\sqrt{L}$

a. $w = MP_L P$

$= P \cdot MP_L$

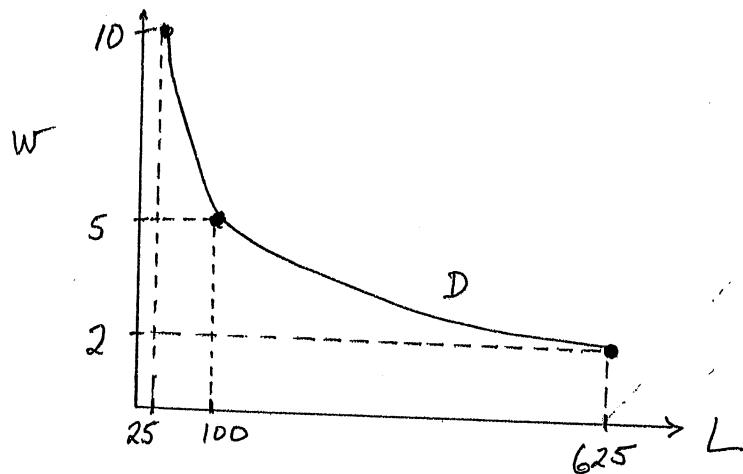
$= 0.01 \left(\frac{5000}{\sqrt{L}} \right) = \frac{50}{\sqrt{L}} \Rightarrow \sqrt{L} = \frac{50}{w},$

$L = \left(\frac{50}{w} \right)^2.$

$w = 10 \Rightarrow L = \left(\frac{50}{10} \right)^2 = 25.$

$w = 5 \Rightarrow L = \left(\frac{50}{5} \right)^2 = 100.$

$w = 2 \Rightarrow L = \left(\frac{50}{2} \right)^2 = 625.$



or \rightarrow

b.

$$ME_L = MP_L \cdot MR \Rightarrow$$

$w = MP_L \cdot P$ because Sticky Gums is competitive in both the output market (yielding $MR = P$) and the input market (yielding $ME_L = w$).

$10 = \frac{5000}{\sqrt{L}} \cdot P$. But since $Q = 10,000\sqrt{L}$, we get $\frac{1}{\sqrt{L}} = \frac{10,000}{Q}$ and

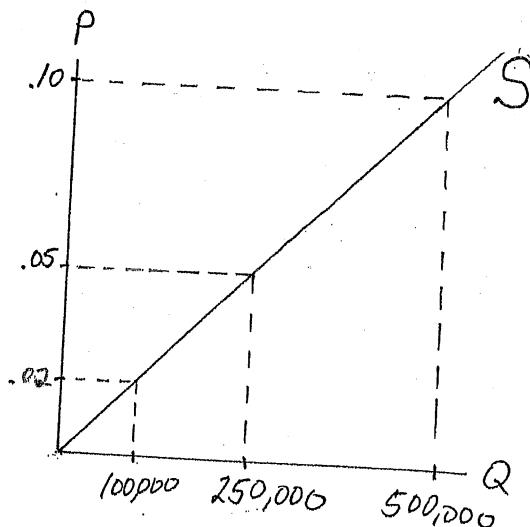
$$10 = \frac{5000}{Q} \cdot \frac{10,000}{1} P,$$

$$Q = 5,000,000 P.$$

$$P = \$0.10 \Rightarrow Q = 500,000$$

$$P = \$0.05 \Rightarrow Q = 250,000$$

$$P = \$0.02 \Rightarrow Q = 100,000.$$



$$\textcircled{6} \quad L = 100w \quad (\text{supply curve for labor})$$

$$ME_L = L/50$$

$$L = 1000 - 100 MRP_L \Rightarrow MRP_L = \frac{1000 - L}{100}$$

this is just $MP_L \cdot MR$.

a) $ME_L = MRP_L$.

$$\frac{L}{50} = \frac{1000 - L}{100}$$

$$2L = 1000 - L$$

$L = \frac{1000}{3} = 333\frac{1}{3}$. Then we get the wage from the supply curve

for labor : $L = 100w \Rightarrow w = \frac{L}{100} = 3\frac{1}{3}$.

$$MRP_L = \frac{1000 - L}{100} = \frac{1000 - 333\frac{1}{3}}{100} = 6\frac{2}{3} . \text{ So the wage rate is half the } MRP_L ;$$

one could say that workers' wage is only half of their (marginal) contribution to firm revenues (MRP).

- b) This is a rather odd question. One's first response to the question "what wage will it pay?" is that "it'll pay whatever the market wage is": that is, that the market will give the firm a wage it has to pay. Then we would set that wage equal to the marginal contribution of an extra worker to revenue, $MRP_L (= MP_L \cdot MR)$, and solve for L . This would give us

w : given by market

L : from $w = MRP_L$

$$w = \frac{1000 - L}{100} \Rightarrow L^* = 1000 - 100w \quad (\text{compare this with the third equation on the previous page}).$$

The book, I think, wants to know when this L^* for our one firm will equal all the labor supplied at that wage. In other words: what if the firm is a monopsonist (so its labor demanded = the market labor demand), but the firm acts as if it were a competitive buyer of labor.

$$\left. \begin{array}{l} \text{Labor Demanded by our} \\ \text{one firm } (L^*) \end{array} \right\} = \left. \begin{array}{l} \text{Labor Demand by} \\ \text{Entire Market} \end{array} \right\} = 1000 - 100w$$

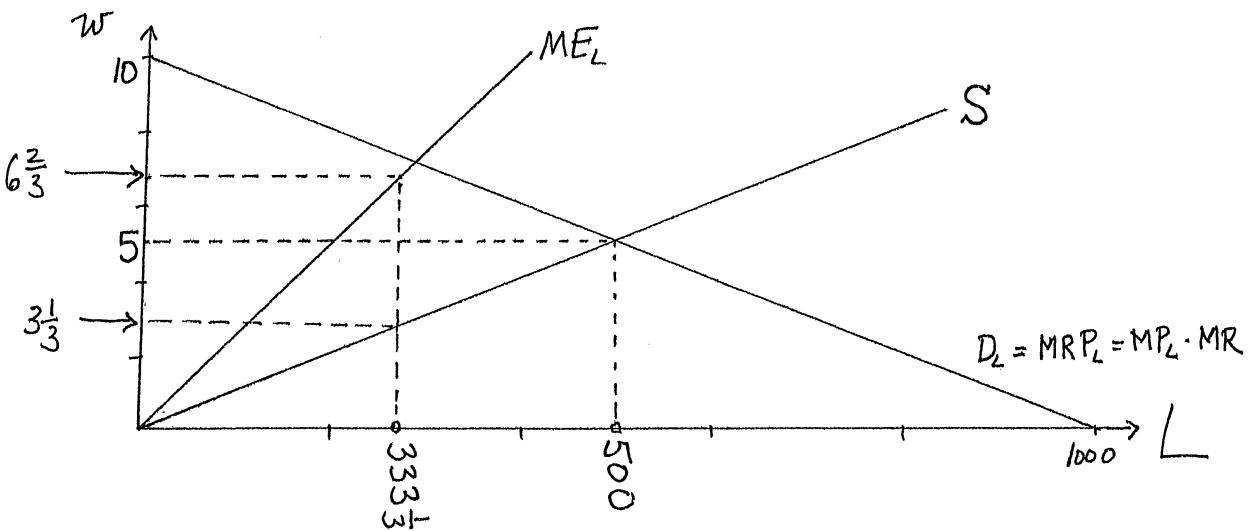
$$\text{Labor Supplied to the Entire Market} = 100w$$

$$\text{Equilibrium: } 100w = 1000 - 100w$$

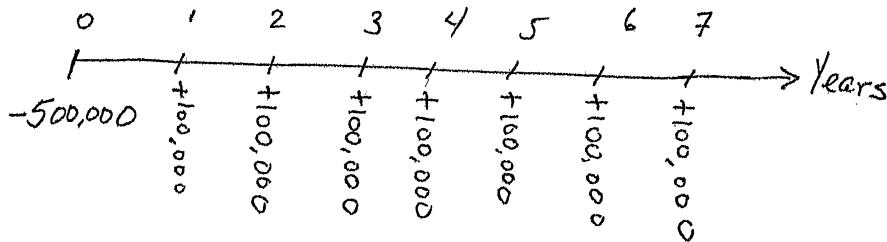
$$2w = 10$$

$$\underline{w = 5}, \quad L = 1000 - 100(5) \Rightarrow \underline{L = 500}.$$

c)



(2)



PV of cost: - \$500,000

$$\text{PV of extra revenues: } \frac{100,000}{(1+r)} + \frac{100,000}{(1+r)^2} + \frac{100,000}{(1+r)^3} + \frac{100,000}{(1+r)^4} + \frac{100,000}{(1+r)^5}$$

$$+ \frac{100,000}{(1+r)^6} + \frac{100,000}{(1+r)^7}$$

$$= 100,000 \left[\frac{1}{1+r} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \frac{1}{(1+r)^4} + \frac{1}{(1+r)^5} + \frac{1}{(1+r)^6} + \frac{1}{(1+r)^7} \right].$$

Helpful Algebra Fact:

$$1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \dots + \frac{1}{(1+r)^n} = \frac{1+r}{r} \left[1 - \frac{1}{(1+r)^{n+1}} \right].$$

Proof of Helpful Algebra Fact:

Let

$$S = 1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \dots + \frac{1}{(1+r)^n}.$$

$$\text{Then } \left(\frac{1}{1+r}\right) S = \frac{1}{1+r} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} + \dots + \frac{1}{(1+r)^n} + \frac{1}{(1+r)^{n+1}}.$$

$$\text{Subtracting, } S - \left(\frac{1}{1+r}\right) S = 1 - \frac{1}{(1+r)^{n+1}}.$$

$$\text{Then } \left(1 - \frac{1}{1+r}\right) S = 1 - \frac{1}{(1+r)^{n+1}}.$$

Since $1 - \frac{1}{1+r} = \frac{1+r}{1+r} - \frac{1}{1+r} = \frac{1+r-1}{1+r} = \frac{r}{1+r}$, we get

$$\frac{r}{1+r} S = 1 - \frac{1}{(1+r)^{n+1}}$$

$$S = \frac{1+r}{r} \left[1 - \frac{1}{(1+r)^{n+1}} \right].$$

If you look at the definition of S , you can see that this is exactly what we wished to prove. ■

Using this helpful algebra fact, we get

$$\begin{aligned} \frac{1}{1+r} + \frac{1}{(1+r)^2} + \cdots + \frac{1}{(1+r)^7} &= \frac{1}{1+r} \left[1 + \frac{1}{1+r} + \cdots + \frac{1}{(1+r)^6} \right] \\ &= \frac{1}{1+r} \left[\frac{1+r}{r} \left[1 - \frac{1}{(1+r)^7} \right] \right] \\ &= \frac{1}{r} \left[1 - \frac{1}{(1+r)^7} \right]. \end{aligned}$$

So

$$\text{PV of extra revenues} = 100,000 \cdot \frac{1}{r} \left[1 - \frac{1}{(1+r)^7} \right].$$

$$\begin{aligned} \text{If } r = 10\% = 0.10 \text{ then PV of extra revenues} &= 100,000 \cdot \frac{1}{0.1} \left[1 - \frac{1}{(1.1)^7} \right] \\ &= \$486,842 \end{aligned}$$

In this case the PV of the entire project is $-500,000 + 486,842 < 0$, so the company should not purchase the trucks.

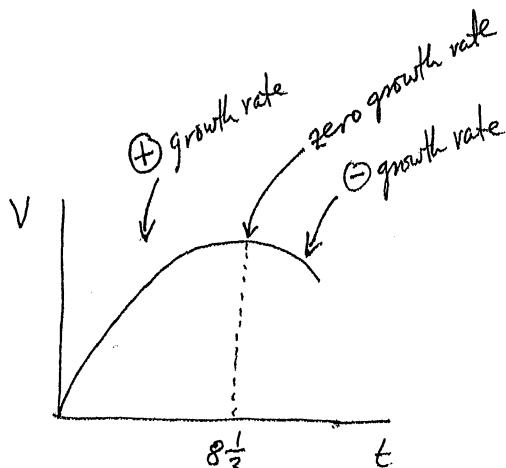
$$\text{If } r = 9\% = 0.09 \text{ then PV of extra revenues} = 100,000 \cdot \frac{1}{0.09} \left[1 - \frac{1}{(1.09)^7} \right] \\ = \$503,295.$$

In this case the PV of the entire project is $-500,000 + 503,295 > 0$, so the company should purchase the trucks.

(8)

$$V = 100t - 6t^2$$

proportional growth rate in V is $\frac{100 - 12t}{V}$



V is at a maximum when the growth rate is zero. This occurs at

$$\frac{100 - 12t}{V} = 0 \Rightarrow t = \frac{100}{12} = \frac{8}{3}.$$

Note: the book has a typo here.
It gives this equation as
 $100 - 12t/V$, which is
 $100 - \frac{12t}{V}$, which is incorrect.

At $t = 8\frac{1}{3}$, V grows at a zero rate. This means that at times close to $8\frac{1}{3}$ but less than $8\frac{1}{3}$ (for example $8\frac{1}{6}$), the growth rate of V is close to zero. At such a time, scotch is a bad investment; it would be better to sell the scotch and invest the money in a bank rather than to hold on to the scotch when its value is appreciating so slowly.

On the other hand, when t is small (for example $t = \frac{1}{10}$), then V is small and $100 - 12t$ is positive, so the growth rate of V , $\frac{100 - 12t}{V}$, is large (it is $9.94 = 994\%$ for $t = \frac{1}{10}$). So in the beginning, scotch is a good investment.

Scotch turns from a good investment into a bad one when the growth rate of V equals the interest rate : before, the growth rate of V was larger than the interest rate, and after, the growth rate of V is less than the interest rate. The growth rate of V equals the interest rate when

$$r = \frac{100 - 12t}{V} = \frac{100 - 12t}{100t - 6t^2}$$

$$(100r)t - (6r)t^2 = 100 - 12t \quad (\text{or} \Rightarrow)$$

$$0 = (6r)t^2 - (100r + 12)t + 100,$$

$$t = \frac{100r + 12 \pm \sqrt{(100r + 12)^2 - 400(6r)}}{12r}$$

If $r = 5\% = 0.05$, then

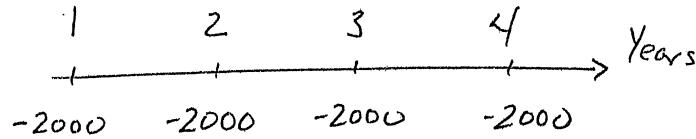
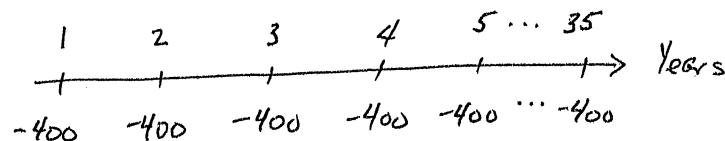
$$t = 6\frac{2}{3} \text{ or } 50.$$

→ impossible since we already know that we want to sell the scotch before $t = 8\frac{1}{3}$.

So $t = 6\frac{2}{3}$ years must be the answer.

Ch. 16, Problems 3&6.

(3)

Whole life:Term:

↙ from the helpful algebra fact
in the Ch. 16 homework

$$\text{Cost of Whole life: } 2000 \left[1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} \right] = 2000 \cdot \frac{1+r}{r} \left[1 - \frac{1}{(1+r)^4} \right].$$

$$\text{Cost of Term: } 400 \left[1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \cdots + \frac{1}{(1+r)^{34}} \right] = 400 \cdot \frac{1+r}{r} \left[1 - \frac{1}{(1+r)^{35}} \right]$$

If $r = 10\% = 0.10$, then

$$\text{cost of whole life} = 2000 \cdot \frac{1.1}{0.1} \left[1 - \frac{1}{(1.1)^4} \right] = 6974$$

$$\text{cost of term} = 400 \cdot \frac{1.1}{0.1} \left[1 - \frac{1}{(1.1)^{35}} \right] = 4243$$

So it's better to buy term insurance.

In this problem, I didn't discount the first payment (of \$2000 or of \$400). This means I'm assuming that the first payment is due immediately, at the start of the first year, instead of at the end of the first year. If one assumes payments are due at the end of each year, the answers change to

$$\frac{6974}{1.1} = 6340 \text{ and } \frac{4243}{1.1} = 3858 \text{ (if you carry 4243.43 to two decimal places)}$$

(instead of none). This is what the book gets. It's OK if you can't see immediately why changing payments from the beginning of the year to the end of the year changes the present value by exactly $\frac{1}{1+r}$, but you should be able to figure out PV the long way for either case.

Can you see why whole life would be better at very low interest rates and term would be better at very high interest rates?

$$⑥ \text{ Astroflite: } -3800 + \frac{1000}{1+r} + \frac{1000}{(1+r)^2} + \frac{1000}{(1+r)^3} + \frac{1000}{(1+r)^4}$$

$$\text{Jack Nickless: } -5000 + \frac{1400}{1+r} + \frac{1400}{(1+r)^2} + \frac{1400}{(1+r)^3} + \frac{1400}{(1+r)^4}$$

t payoff at
end of 1st year

t payoff at end
of 4th year

I assume the business ends at the start of the fifth year, although
maybe the author means for it to end at the end of the fifth year.

$$\begin{aligned}
 \text{Astroflite's PV: } & -3800 + 1000 \left(\frac{1}{1+r} \right) \left[1 + \frac{1}{1+r} + \frac{1}{(1+r)^2} + \frac{1}{(1+r)^3} \right] \\
 & = -3800 + 1000 \cdot \frac{1}{1+r} \cdot \frac{1+r}{r} \left[1 - \frac{1}{(1+r)^4} \right] \\
 & = -3800 + \frac{1000}{r} \left[1 - \frac{1}{(1+r)^4} \right]. \quad \text{Similarly}
 \end{aligned}$$

$$\text{Jack Nickless's PV: } -5000 + \frac{1400}{r} \left[1 - \frac{1}{(1+r)^4} \right]$$

Conclusion from the table:

Neither name pays off.

PV	Astroflite J. Nickless	
10%	-630	-562
15%	-945	-1003