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Econ. 5250
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NOTES FOR CHAPTERS 4 AND 6

Chapter 4
A. Profit and Marginal Profit

Figure 1 below shows how the firm's profit-maximizing level of output Q^{π} is determined.

B. Marginal Net Private Benefit, MNPB

What is the value to the consumer of an arbitrary level of output $\hat{Q}$? What is the marginal value to the consumer of such a $\hat{Q}$?

1. Traditional answer: Refer to Figure 2. Auctioning Q_1 would result in willing buyers paying the area under ab. (This is an example of "willingness and ability to pay," WATP) Auctioning from Q_1 to Q_2 would result in willing buyers paying the area under cd. Auctioning from Q_2 to Q_3 would result in willing buyers paying the area under ef. Auctioning from Q_3 to $\hat{Q}$ would result in willing buyers paying the area under gh. So the answer is: the area under $abcdefgh$. The *marginal* value of going from Q_3 to $\hat{Q}$ is the area under gh. In the limit as the number of auctions goes to infinity and the size of each auction goes to zero, the payment area would approach the area under mh. This is the gross consumer surplus. In this limit, the *marginal* value of going from Q_3 to $\hat{Q}$ is the area under fh.

2. The traditional answer obtained a different way: Here we value $\hat{Q}$ by going down from $\hat{Q}$ to zero, instead of up from zero to $\hat{Q}$. If quantity is reduced from $\hat{Q}$ to Q_3, willingness to accept ("WTA") compensation is the area under fi. Further reducing quantity, from Q_3 to Q_2, has a willingness to accept of dj. Further reducing quantity, from Q_2 to Q_1, has a willingness to accept of bk. Further reducing quantity, from Q_1 to zero, has a willingness to accept of ml. So the answer is: the area under $ifjdkblm$. The marginal value of going from Q_3 to $\hat{Q}$ is the area under fi. In the limit as the number of auctions goes to infinity and the size of each auction goes to zero, the payment area would approach the area under mh, which again is gross consumer surplus. In this limit, the marginal value of going from Q_3 to $\hat{Q}$ is the area under fh.

Refer to Figure 3. Gross consumer surplus (think of an infinite number of auctions) is the area under the demand curve ab. Net consumer surplus

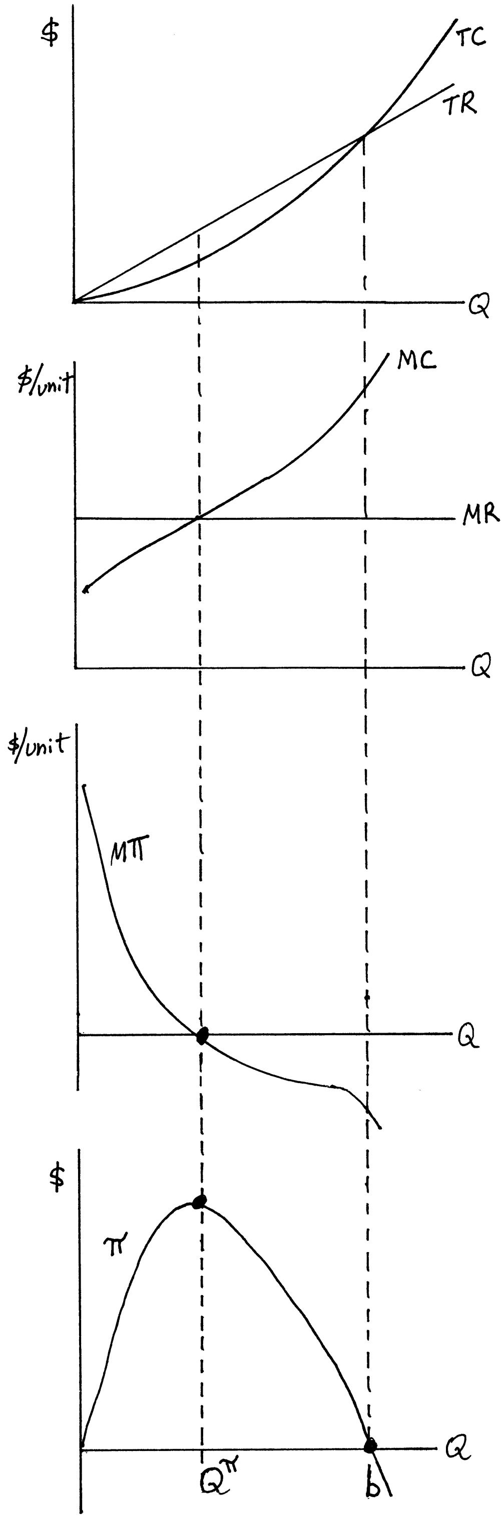

Figure 1. Total Cost and Total Revenue; Marginal Cost and Marginal Revenue; Marginal Profit; and Total Profit.

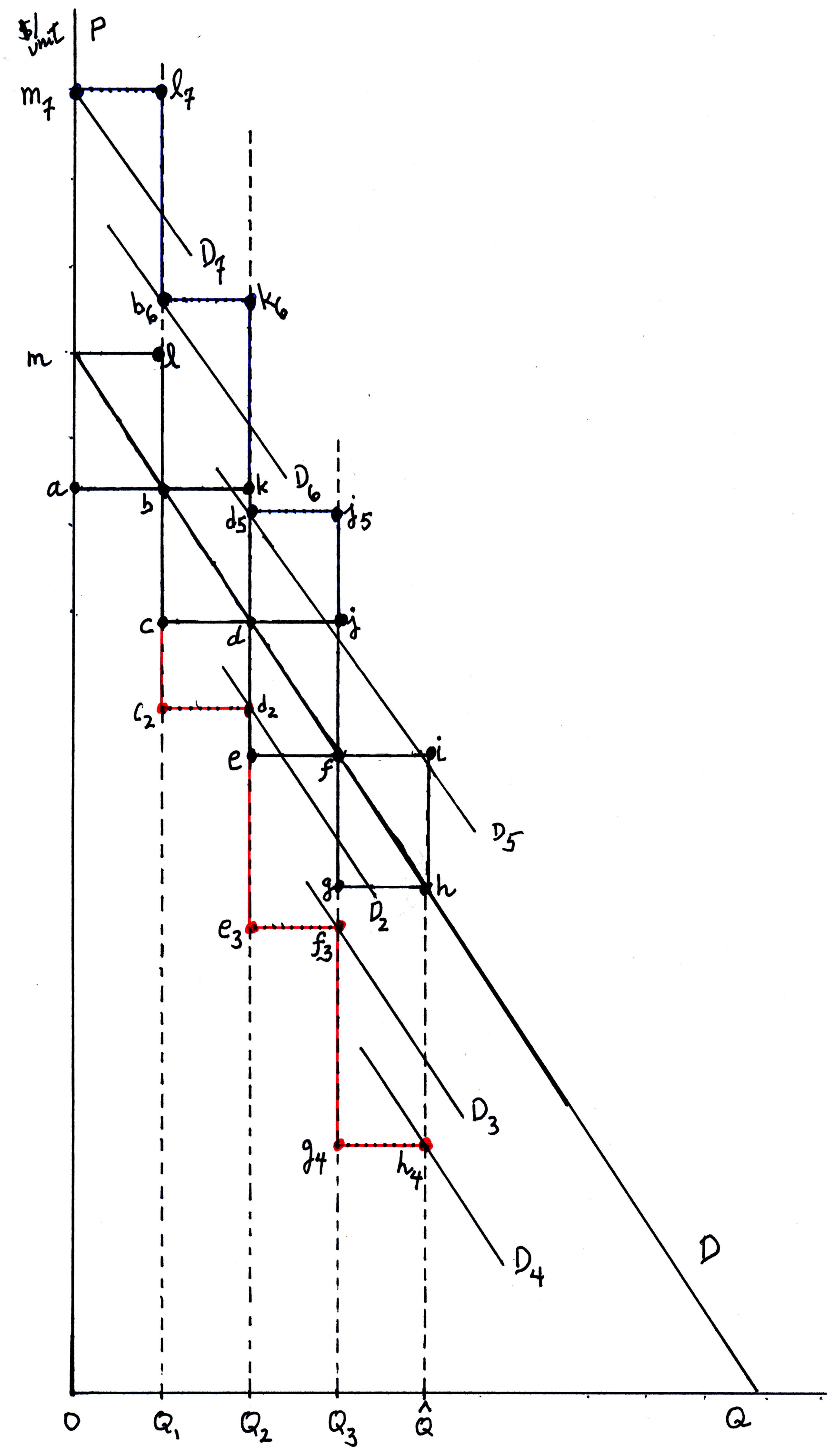

Figure 2. Derivation of consumer surplus.

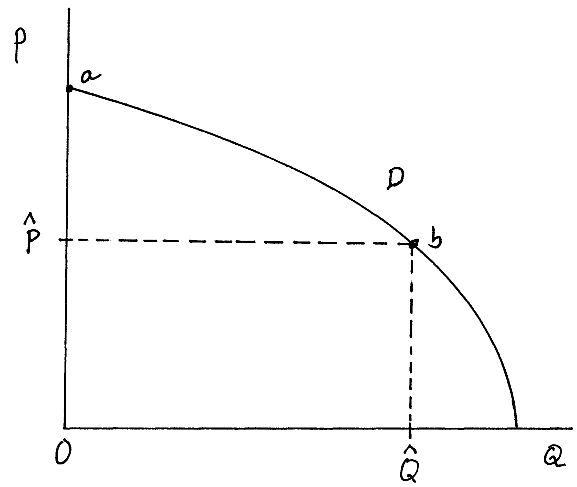

Figure 3. Gross and Net Consumer Surplus.

is this area minus what the consumer pays for $\hat{Q}$, which is the rectangle under $\hat{P}b$. This net consumer surplus is $ab\hat{P}$.

To calculate social surplus, which is the sum of consumer surplus and profit, refer to Figure 4. When quantity of output is q, the gross *marginal* consumer surplus is the line below b, and so the gross (total) consumer surplus is $\int_0^q D(\alpha)\,d\alpha$, and the net consumer surplus is

$$\int_0^q D(\alpha)\,d\alpha - pq\,.$$

The firm's gross value for producing q is its total revenue, which is pq where p is price. In this Figure 4, price is p^*. The firm's net value is its profit, which is total revenue p^*q minus total cost, $C(q)$. The firm's net *marginal* profit is the derivative of its profit with respect to q, so the firm's net marginal profit is $p^* - MC(q)$, which is hf at q in Figure 4.

Society's value is the net consumer surplus plus the firm's profit, which is

$$\Big(\int_0^q D(\alpha)\,d\alpha - pq\Big) + \big(pq - C(q)\big)\,.$$

Society's marginal value is d/dq of society's value; canceling terms, it is

$$\frac{d}{dq}\Big(\int_0^q D(\alpha)\,d\alpha - C(q)\Big) = D(q) - MC(q) \qquad (1)$$

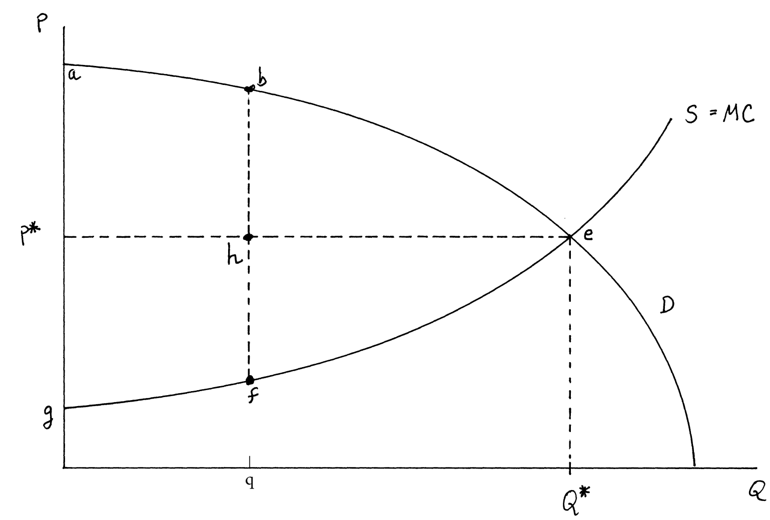

Figure 4. Marginal social surplus, bf, is marginal consumer surplus bh plus marginal profit hf. The social optimum is at Q^* because that is where marginal social surplus is zero. However, it is also where marginal profit is zero, so using the rule "choose Q so that marginal profit is zero" will lead to the correct answer although it is not for the right reason. This is what the textbook does, identifying the textbook's "marginal net private benefit" with marginal profit, in accordance with p. 69 of the textbook.

where to differentiate the integral with respect to its upper limit I used Leibniz' Rule, a simple version of which is $d(\int_0^q D(\alpha)\, d\alpha)/dq = D(q)$. (For a more complete version of Leibniz' Rule, see `https://en.wikipedia.org/wiki/Leibniz_integral_rule`.) From intermediate microeconomics, the supply curve is the marginal cost curve (above the bottom of average variable cost), which is why I have labeled one curve $S = MC$. It follows that society's marginal value at q in Figure 4, which is (1), is bf. Society's total value at q is $abfg$. The quantity that maximizes society's total value is Q^*, because marginal increases in Q benefit society if and only if $D(q) - MC(q) > 0$, which is true for $q \in [0, Q^*)$. Since the competitive equilibrium quantity is also Q^*, this proves a version of Adam Smith's Invisible Hand: the competitive equilibrium, which merely balances quantity demanded and quantity supplied, is "good" in that it is the quantity that maximizes society's total value.

3. A more correct answer: In Figure 2, auctioning Q_1 would result in willing buyers paying the area under ab. Paying this makes buyers poorer. Being poorer shifts the demand curve down if the good is "normal" and shifts the demand curve up if the good is "inferior." Assume the good is normal. Then the demand curve shifts down, for example to D_2. Auctioning from Q_1 to Q_2 would result in willing buyers paying the area under c_2d_2. Paying this makes buyers poorer, shifting the demand curve down, for example to D_3. Auctioning from Q_2 to Q_3 would result in willing buyers paying the area under e_3f_3. Paying this makes buyers poorer, shifting the demand curve down, for example to D_4. Auctioning from Q_3 to $\hat{Q}$ would result in willing buyers paying the area under g_4h_4. So the answer is: the area under $abc_2d_2e_3f_3g_4h_4$. This is less than the gross consumer surplus. The marginal value of going from Q_3 to $\hat{Q}$ is the area under g_4h_4.

4. A different but also correct answer: Let's value $\hat{Q}$ by going down from $\hat{Q}$ to zero, instead of up from zero to $\hat{Q}$. If quantity is reduced from $\hat{Q}$ to Q_3, willingness to accept is the area under fi. Accepting this payment makes the person richer, so his demand curve shifts up, for example to D_5. Further reducing quantity, from Q_3 to Q_2, has a willingness to accept of d_5j_5. Accepting this payment makes the person richer, so his demand curve shifts up, for example to D_6. Further reducing quantity, from Q_2 to Q_1, has a willingness to accept of b_6k_6. Accepting this payment makes the person richer, so his demand curve shifts up, for example to D_7. Further reducing quantity, from Q_1 to zero, has a willingness to accept of m_7l_7. So the answer is: the area under $ifj_5d_5k_6b_6l_7m_7$. This is more than the gross consumer surplus. (As in point (2), the marginal value of going from Q_3 to $\hat{Q}$ is still the area under fi. However in point (3), the marginal value of going from Q_3 to $\hat{Q}$ was the area under g_4h_4.)

5. Yet another correct answer: Start from point d (look at Figure 5). Auctioning from Q_2 to Q_3 would result in willing buyers paying the area under ef. Paying this makes buyers poorer, shifting the demand curve down from D, for example to D_2'. Auctioning from Q_3 to $\hat{Q}$ would result in willing buyers paying the area under $g_4'h_4'$. So the answer is: the area under $efg_4'h_4'$. The marginal value of going from Q_3 to $\hat{Q}$ is the area under $g_4'h_4'$.

6. Yet another correct answer: start from point v (near the bottom right of Figure 5). If quantity is reduced from Q_5 to Q_4, willingness to accept is the area under rs. Accepting this payment makes the person richer, so his demand curve shifts up from D, for example to D_5'. Further reducing quantity, from Q_4 to $\hat{Q}$, has a willingness to accept of tu. Accepting this payment makes the person richer, so his demand curve shifts up, for example to D_6'. Further reducing quantity, from $\hat{Q}$ to Q_3, has a willingness to

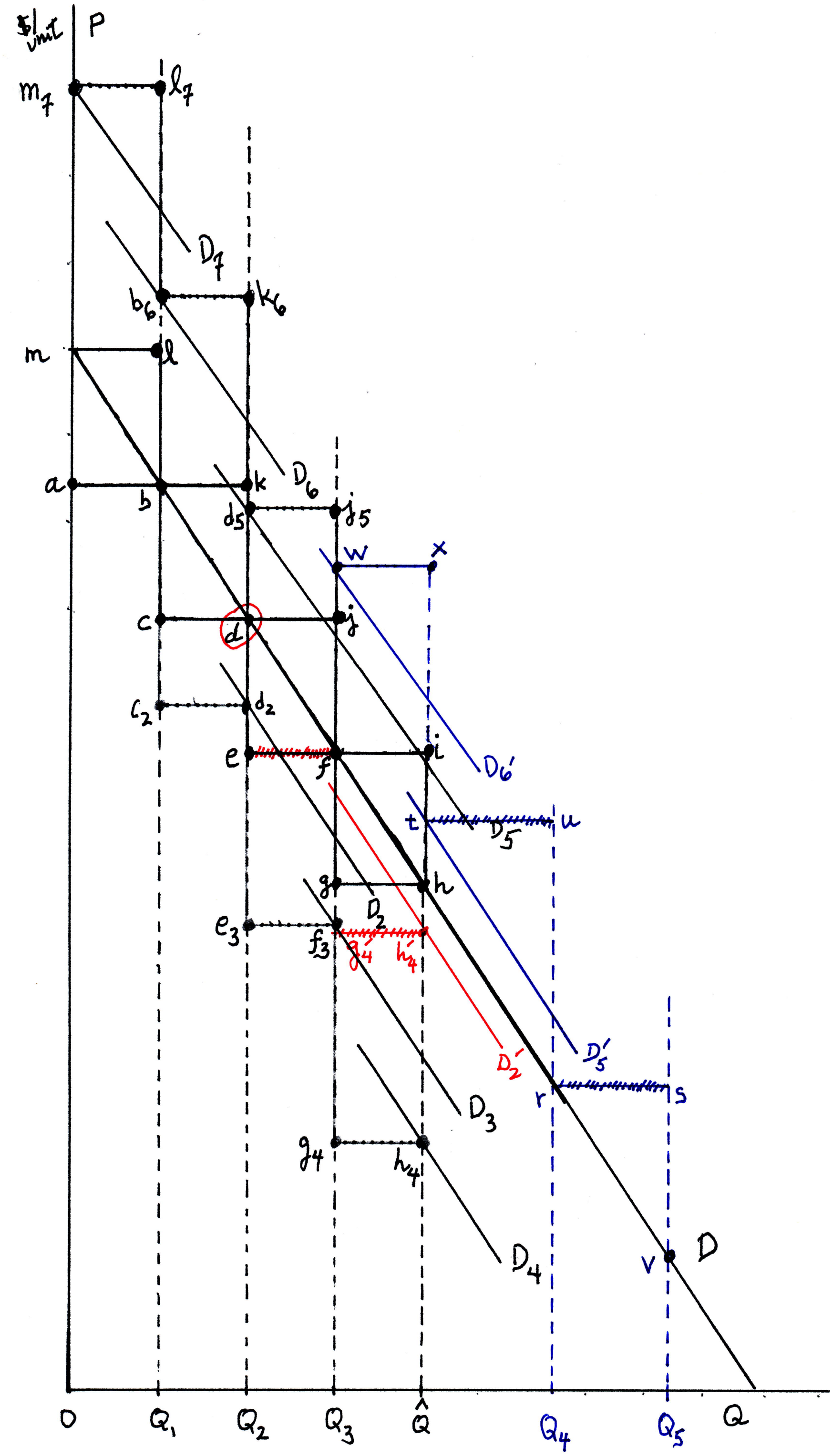

Figure 5. Critique of consumer surplus.

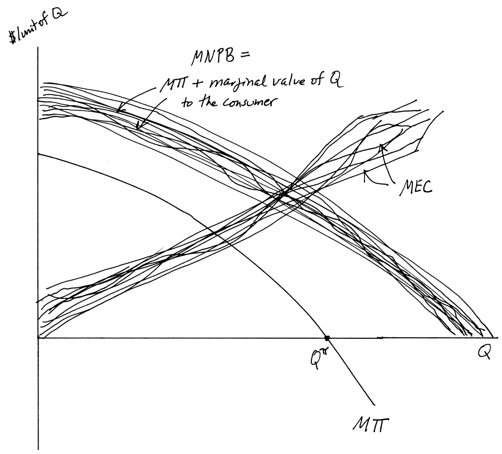

Figure 6. Multiple correct *MNPB* and *MEC* curves.

accept of wx. The marginal value of going between Q_3 and $\hat{Q}$ is the area under wx.

7. Summary: From correct answers 3, 4, 5, and 6, the marginal value of going between Q_3 and $\hat{Q}$ was the area under g_4h_4, fi, $g_4'h_4'$, and wx. All of these are correct answers to the question "what is the marginal value of going between Q_3 and $\hat{Q}$," depending on where the process began. The conclusion is that there are many correct answers to the question "what is the marginal value of going between Q_3 and $\hat{Q}$," not just one. This is illustrated by the *MNPB* lines in Figure 6.

C. Marginal External Cost, MEC

What is the marginal external cost to the pollution victim of moving from pol$_a$ to pol$_b$ in Figure 7? The answer is the WATP and WTA in Figure 7. However, there are many possible values of WATP and many possible values of WTA, depending on what the initial starting point is (that is, depending on the initial number of apples that go along with pol$_a$). So there are many correct answers to the question "what is the marginal value of going between pol$_a$ to pol$_b$," not just one. This is illustrated by the *MEC* lines in Figure 6. That figure shows that there is no single, well-defined optimal level of output, Q^*.

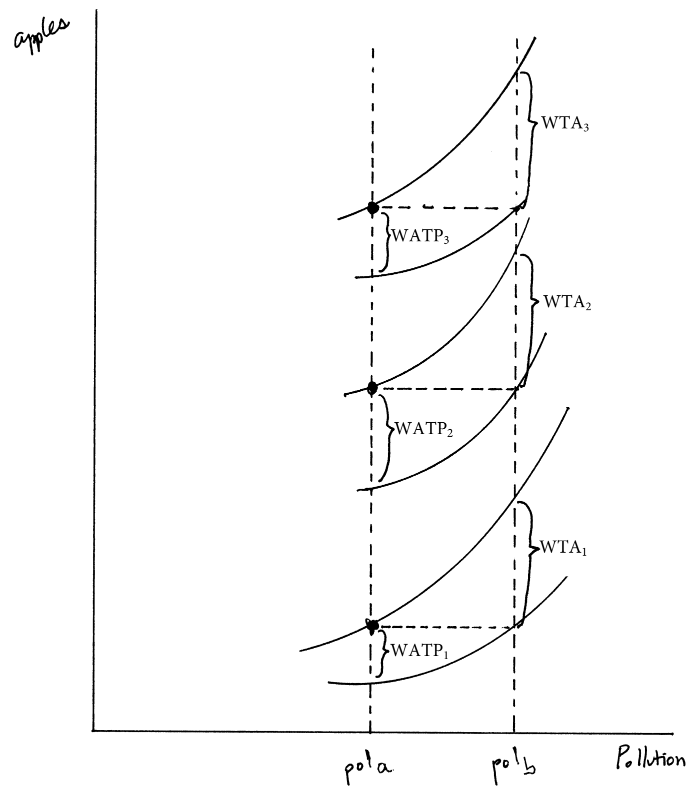

Figure 7. Multiple correct *WATP* and *WTA* values for the move from pol_a to pol_b.

D. Various MEC Curves

What in class I called "Case 1" is illustrated in Figure 8. External cost is labeled *EC* and net private benefit is labeled *NPB*. The socially optimal level of output is Q^*.

What in class I called "Case 2" is illustrated in Figure 9. In it, unlike in Figure 8, *MEC* begins strictly larger than zero; that is, the limit of *EC* as $Q \to 0$ does not asymptote along the Q axis, as it does in Figure 8. Figure 10 also has *MEC* beginning strictly larger than zero, but Q^* is zero in Figure 10, whereas $Q^* > 0$ in Figure 9.

What in class I called "Case 3" is illustrated in Figure 11.

What in class I called "Case 4" is illustrated in Figure 12, which shows two possible locations for *NPB* (one dotted and the other solid) and, therefore, *MNPB* (one dotted and the other solid). Both cases involve *MNPB* curves which intersect the *MEC* curve two times. It is hard to tell from the marginal graph whether or not the second intersection point, which is at Q^π, is optimal or not, but the "total" graph shows that that point is not optimal in either of the two cases illustrated. However, in Figure 13, the right-most of the two intersections of *MNPB* and *MEC* is optimal.

What in class I called "Case 5" is illustrated in Figure 14, which shows two possible locations for *EC* and, therefore, *MEC* (one dotted and the other solid). In the case of the dotted lines, $EC_2 > NPB$ for all Q, so $Q^* = 0$. in the case of the solid lines, $EC_1 < NPB$ at the right-most intersection point between *MNPB* and *MEC*; that would be Q^*.

What in class I called "Case 6" (a positive externality) is illustrated in Figure 15. In the diagram, *EC* is convex; analyzing the case when *EC* is concave is more complicated, and is omitted.

(Chapter 6 comments come after several pages of figures.)

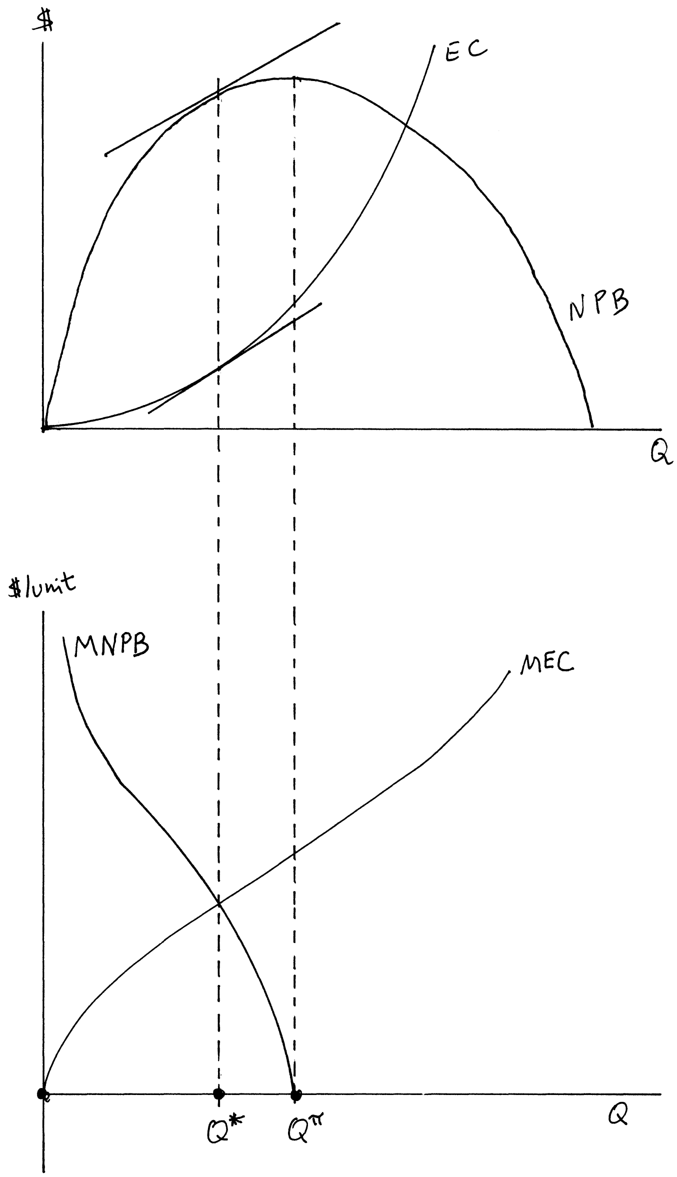

Figure 8. Case 1, in which at Q = 0, MEC = 0.

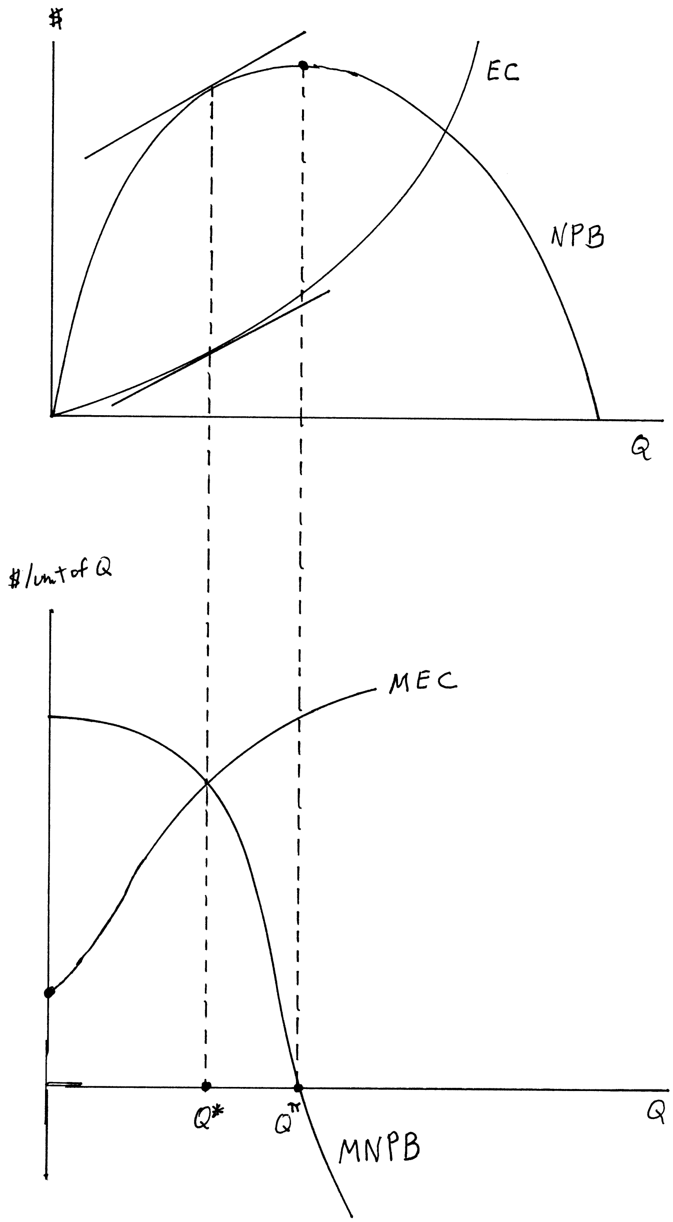

Figure 9. Case 2, in which at $Q = 0$, $MEC > 0$.

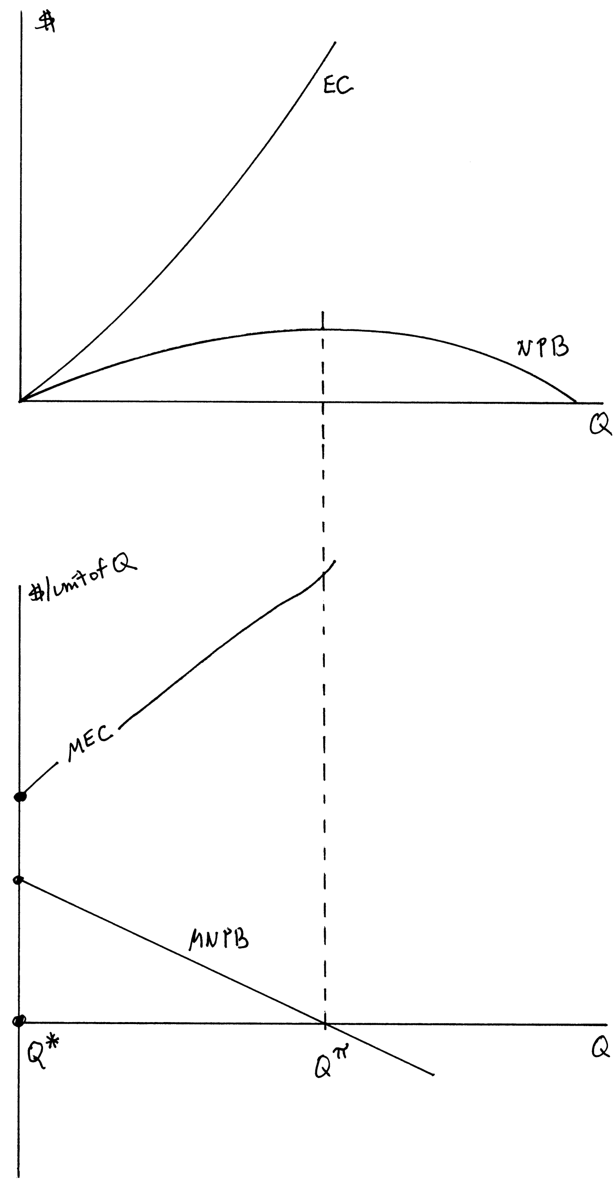

Figure 10. Another version of Case 2 (at Q = 0, MEC > 0).

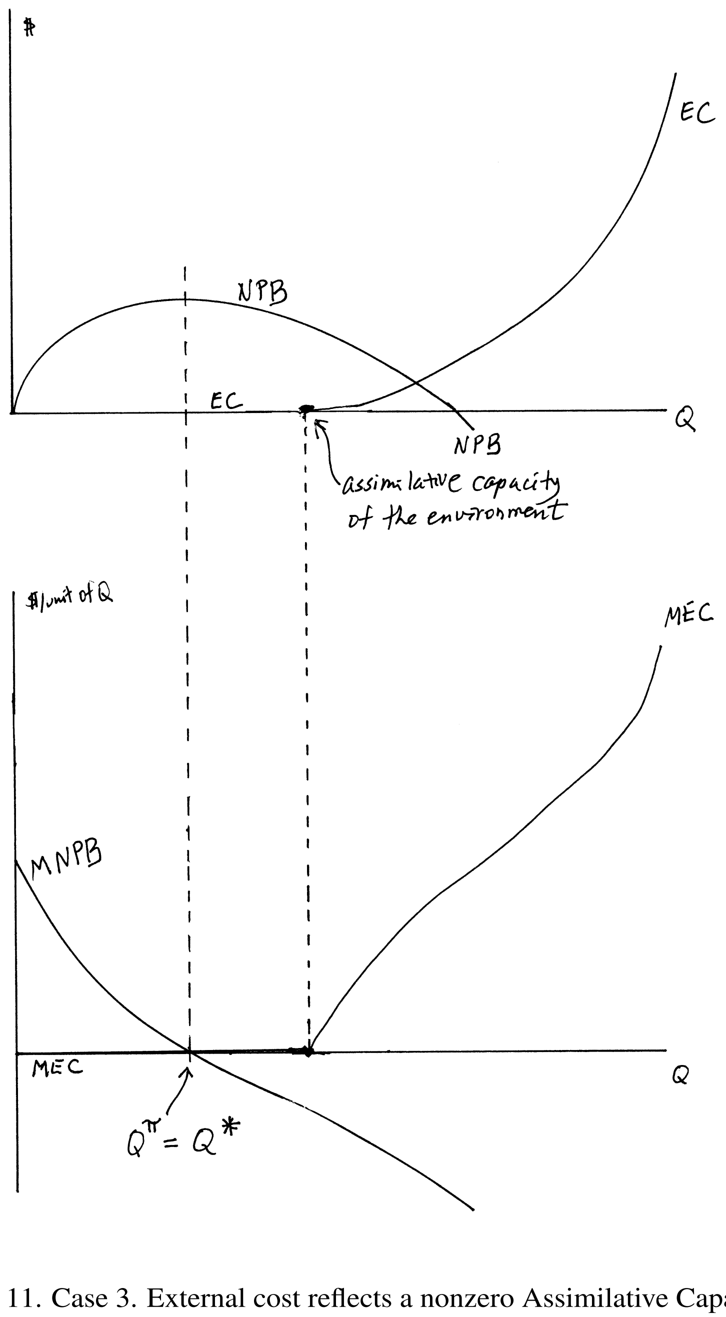

Figure 11. Case 3. External cost reflects a nonzero Assimilative Capacity.

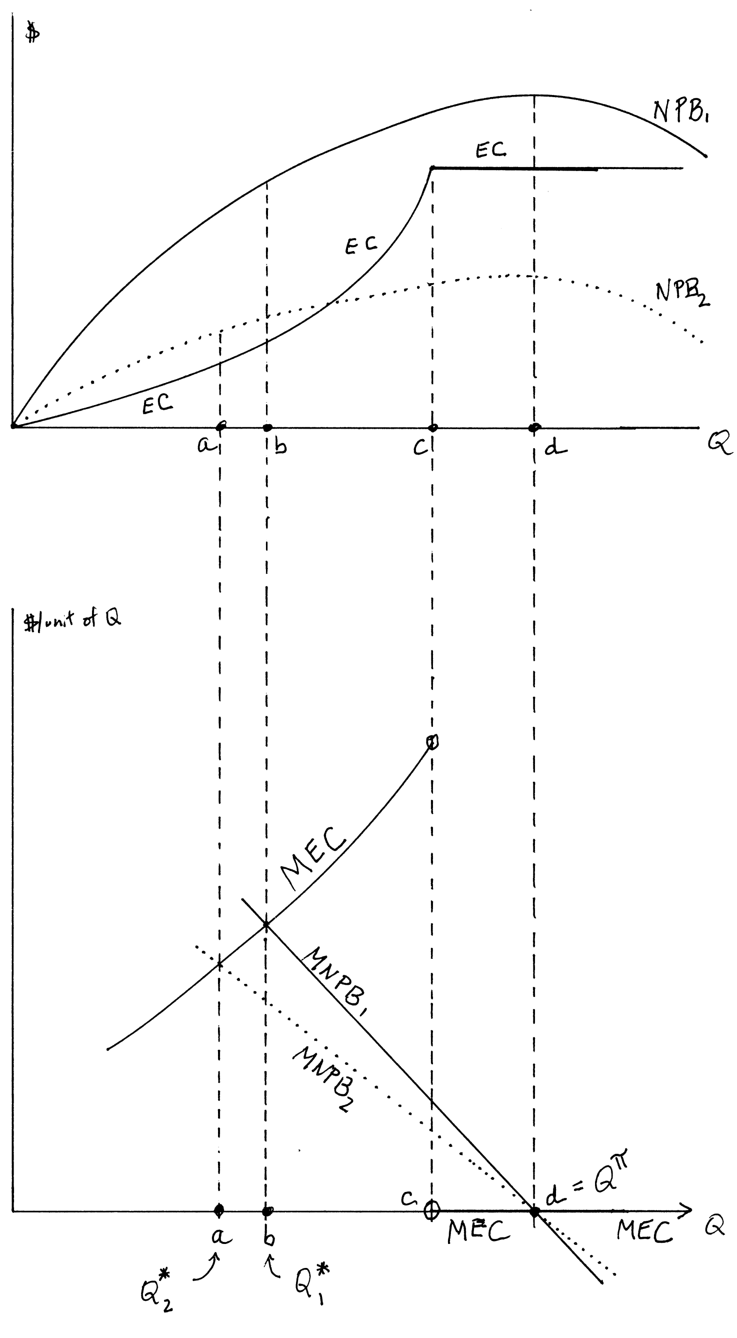

Figure 12. Case 4. External cost has a maximum level.

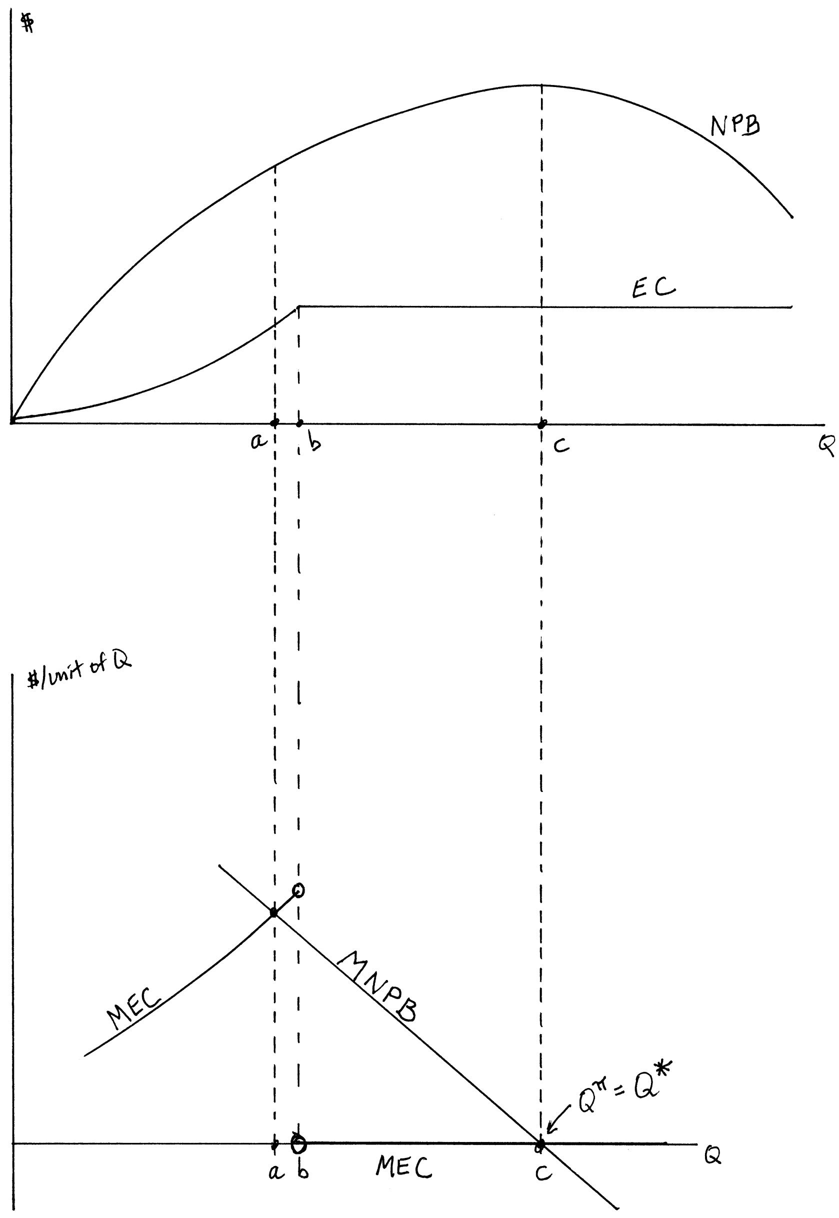

Figure 13. Another version of Case 4, external cost has a maximum level.

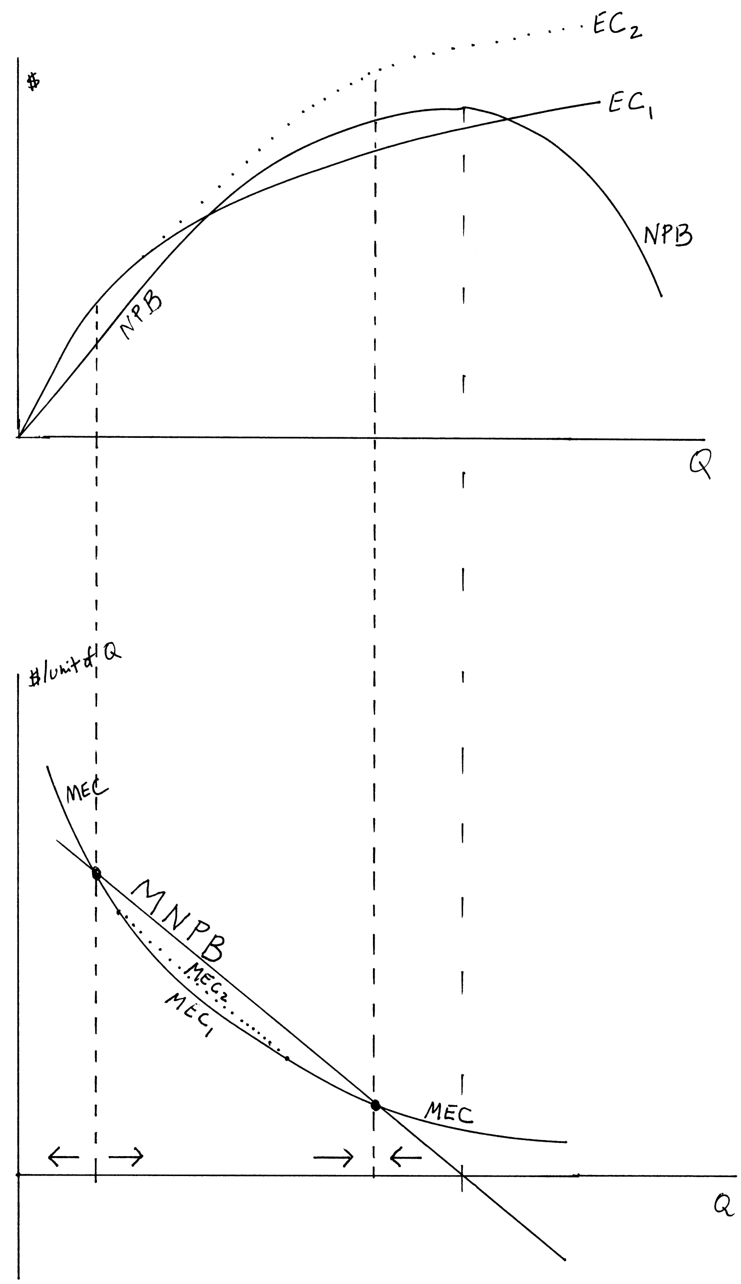

Figure 14. Case 5. *EC* is concave, instead of its usual convex shape.

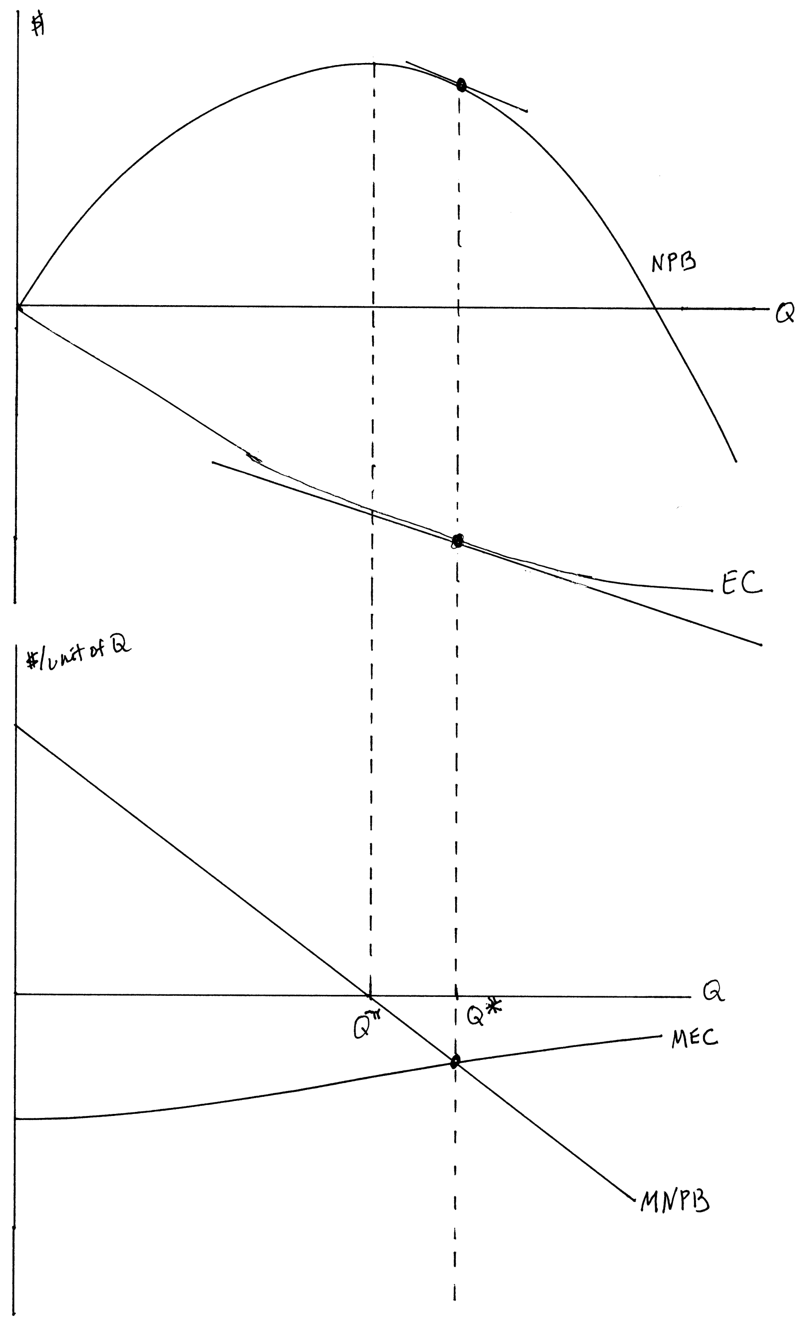

Figure 15. Case 6. A positive externality.

Chapter 6

Here we describe the socially-optimal way of jointly choosing both a level of output and a level of pollution abatement.

Marginal social surplus ignoring pollution is the difference between the value of output q, which is measured by the (inverse) demand curve, $D^{-1}(q)$, and the marginal cost of producing that output, $MC(q, A)$ where A is pollution abatement. Total social surplus ignoring pollution therefore is

$$\int_0^Q [D^{-1}(q) - MC(q, A)]\, dq\,. \tag{2}$$

Let gross pollution be pol(Q). Net pollution (that is, the pollution leaving the factory) then would be pol(Q) − A. The external cost of pollution is a function of net pollution; call this extc(pol(Q) − A).

Society would want to maximize "Total social surplus ignoring pollution" minus "the external cost of pollution" by choosing Q and A jointly:

$$\max_{Q,A} \left[\int_0^Q [D^{-1}(q) - MC(q, A)]\, dq - \mathrm{extc}\big(\mathrm{pol}(Q) - A\big) \right]. \tag{3}$$

The two first-order conditions for this problem are that the partial derivatives of the objective function with respect to Q and with respect to A should be set equal to zero.

The first-order condition with respect to A is:

$$0 = \frac{\partial}{\partial A} \left[\int_0^Q [D^{-1}(q) - MC(q, A)]\, dq - \mathrm{extc}\big(\mathrm{pol}(Q) - A\big) \right]$$

$$= \int_0^Q \left[0 - \frac{\partial MC(q, A)}{\partial A} \right] dq - \frac{\partial\, \mathrm{extc}}{\partial\, \text{net pollution}}(-1) \tag{4}$$

$$= -\frac{\partial}{\partial A} \int_0^Q MC(q, A)\, dq - MEC\,(-1) \tag{5}$$

$$= -\frac{\partial}{\partial A} \text{Total Cost}(Q, A) + MEC \tag{6}$$

$$= -MAC(Q, A) + MEC\,, \text{ therefore} \tag{7}$$

$$MAC(Q, A) = MEC(Q, A)\,. \tag{8}$$

In equation (4) the last term comes from the Chain Rule, noting that the derivative of net pollution pol(Q) − A with respect to A is negative one. Equation (8) has this interpretation: abatement should occur until its marginal cost, the left-hand side of (8), is equal to its marginal benefit, and the marginal benefit of abatement is equal to abatement's marginal reduction in external cost, which is the right-hand side of (8).

The first-order condition with respect to Q is:

$$\begin{aligned}
0 &= \frac{\partial}{\partial Q}\left[\int_0^Q [D^{-1}(q) - MC(q, A)]\, dq - \text{extc}\big(\text{pol}(Q) - A\big)\right] \\
&= \left[D^{-1}(Q) - MC(Q, A)\right] - \frac{\partial\, \text{extc}}{\partial\, \text{net pollution}} \frac{d\, \text{pol}(Q)}{dQ} \qquad (9) \\
&= \left[D^{-1}(Q) - MC(Q, A)\right] - MEC\, \frac{d\, \text{pol}(Q)}{dQ} \qquad (10)
\end{aligned}$$

$$MEC\, \frac{d\, \text{pol}(Q)}{dQ} = D^{-1}(Q) - MC(Q, A)\,. \qquad (11)$$

The right-hand side is marginal profit. The left-hand side can be re-expanded as:

$$\frac{\partial\, \text{extc}}{\partial\, \text{net pollution}}\, \frac{d\, \text{pol}(Q)}{dQ} = M\Pi\,. \qquad (12)$$

Equation (12) has this interpretation: production should occur until its marginal profit, the right-hand side of (12), is equal to its marginal pollution cost, which is the left-hand side of (12); the latter measures how Q increases pollution and then how that increased pollution (keeping abatement constant) affects external cost.

A flaw in this argument is that it uses social surplus, whereas a correct measure of social value is dual in nature, reflecting both willingness and ability to pay (WATP) and willingness to accept (WTA). Fixing this flaw is beyond the scope of this course.