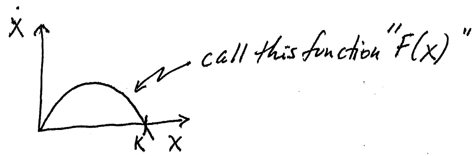
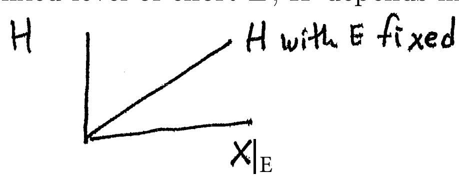
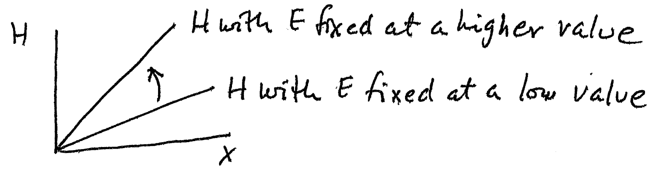
Dr. Lozada
Econ. 5250

For logistic growth, we have 

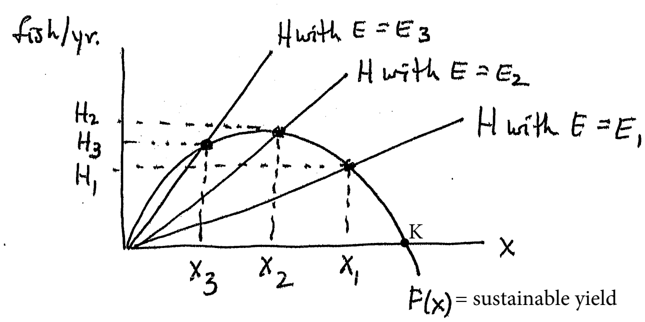
. Harvest "H" depends on effort "E" and on X. With harvesting, $\dot{X} = F(X) - H$. Suppose that for a fixed level of effort E, H depends linearly on X:



If E increases, this line moves up:

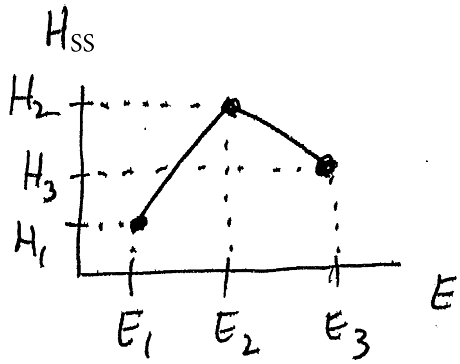


If $H = F(X)$ then X will not change, so there will be a *steady state.* So let's get H and $F(X)$ on one graph, so we can make them equal. (In the graph below, $E_1 < E_2 < E_3$.)



It follows that

if $E = E_1$, steady-state harvest is H_1;
if $E = E_2$, steady-state harvest is H_2; and
if $E = E_3$, steady-state harvest is H_3.

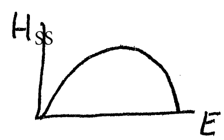
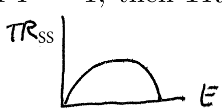
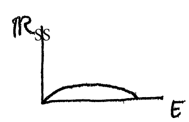
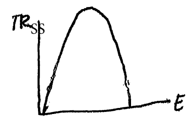
Graphically:  Clearly more effort does not always yield more fish.

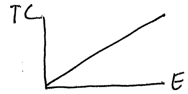
Algebraic Example. Suppose $F(X) = X\,(1 - X)$ and $H = XE^{1/2}$. Find the steady-state relationship between H and E.

Answer: In the steady state, $F(X_{SS}) = H_{SS}$, so

$$\begin{aligned} X_{SS}\,(1 - X_{SS}) &= X_{SS}E_{SS}^{1/2} \\ 1 - X_{SS} &= E_{SS}^{1/2} \\ X_{SS} &= 1 - E_{SS}^{1/2} \end{aligned}$$

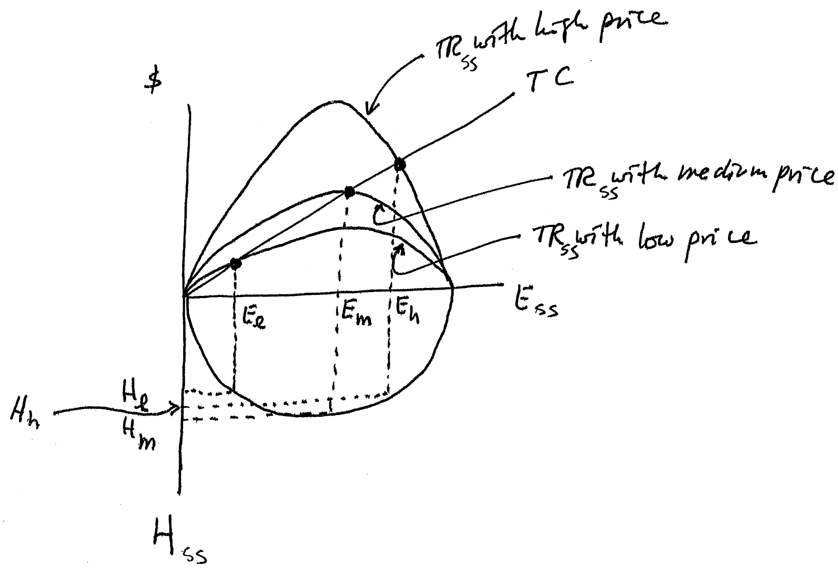
and therefore $H_{SS} = X_{SS}E_{SS}^{1/2} = (1 - E_{SS}^{1/2})E_{SS}^{1/2} = E_{SS}^{1/2} - E_{SS}$.

In any case, we have in general something like  . Total revenue is price times quantity produced, namely PH. We'd like to graph TR versus E assuming a competitive industry (that is, an industry whose firms all take price P as given). If $P = 1$, then $TR = (1)\,H = H$, so "TR vs. E" looks just like "H vs. E":  . If $P = 1/10$, then $TR = (1/10) * H$, so the graph would look like  . If $P = 5$, then $TR = 5H$, so the graph would look like 

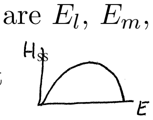
. As for total cost, suppose 

.

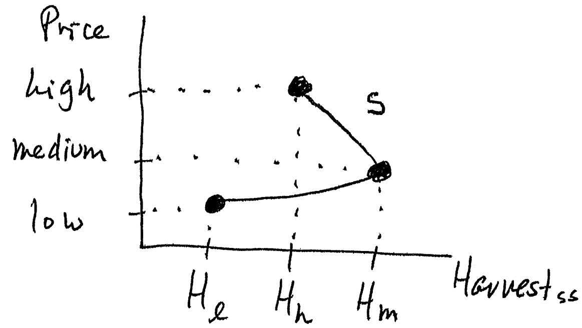
Put the TC graph together with the three TR graphs (for low, medium, and high prices):



1. Competitive, Open-Access Fishery

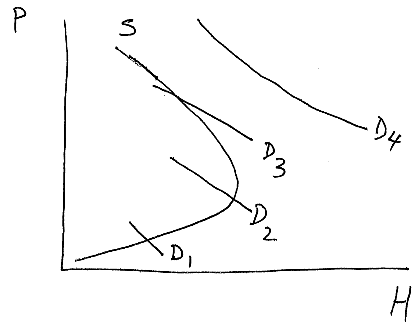
First consider the top graph. Suppose no one owns the fish—an "open access" fishery—and there is no government regulation. Then $TR = TC$, because if $TR < TC$, firms would leave the industry, and if $TR > TC$, firms would enter the industry. I've marked the "$TR = TC$" places in the top graph with "•." For low, medium, and high prices, the equilibrium effort levels are E_l, E_m, and E_h. These imply through the bottom graph—which is just 

from page 2 flipped upside down—harvest levels of H_l, H_m, and H_h. Since from the graph $H_l < H_h < H_m$, we have:

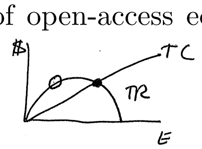


Thus we have a backward-bending supply curve.

Suppose demand for this fish rises from D_1 to D_2 to D_3 to D_4:



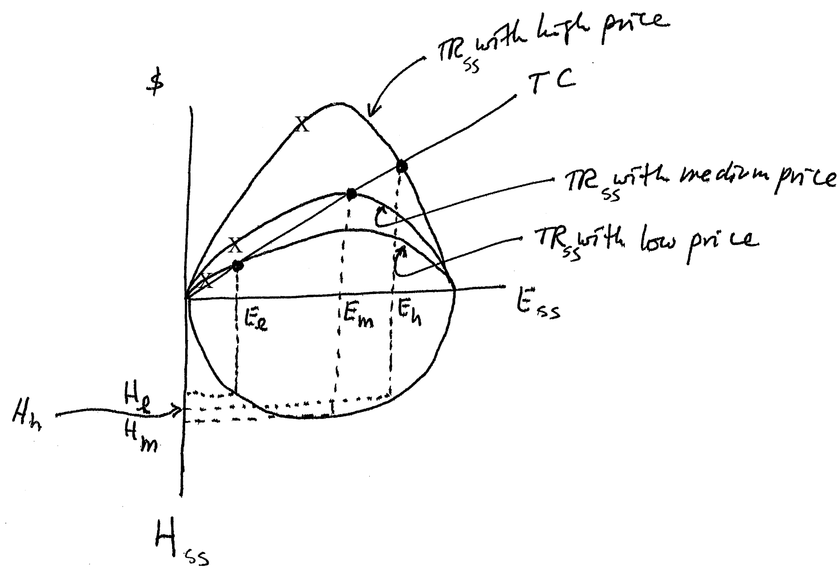
From D_1 to D_2, $P \uparrow$ and $H \uparrow$. From D_2 to D_3, $P \uparrow$ and $H \downarrow$: as more people like this fish, the steady-state harvest of it falls! If demand further rises to D_4, supply can never equal demand, and there is no steady-state equilibrium.

The concludes our analysis of open-access equilibrium, except for one point: since this occurs at "$\bullet$" in 

, where profit equals zero, the industry is unable to take advantage of the opportunity to produce at "$\circ$," where $TR \gg TC$, so profit is positive. The point "$\circ$" is better for the firms and better for the fish (less $E \Rightarrow$ more fish), but the open access externality—namely that the harvest of Firm 1 affects X, so it increases the cost of Firm 2's harvesting—leads to the worse outcome "$\bullet$."

2. Competitive, Private-Property, Net-Present-Value-Maximizing Fishery

One might think that the competitive, private property solution could be obtained where short-run profit is maximized, for example, approximately the points marked "X" in this diagram:



However, a proper analysis requires acknowledging the intertemporal aspects of the problem, even if we choose to concentrate on the steady state.

The profit of each firm is

$$\Pi(H_t, X_t) = TR_t(H_t) - TC(E(H_t, X_t)) \tag{1}$$

where H is the harvest, X is the stock size, TR is the total revenue, TC is the total cost, and E is fishing effort, all at time t. The objective of the firm is to

$$\max \sum_{t=0}^{\infty} \frac{\Pi_t}{(1+\delta)^t} \quad \text{s.t.} \tag{2}$$

$$X_{t+1} - X_t = F(X_t) - H_t \tag{3}$$

where F is the natural excess of births over deaths. (3) represents an infinite number of constraints on (2). Using k_1, k_2, ..., to denote the Lagrange multipliers, the Lagrangian is

$$
\begin{aligned}
L = \mathit{\Pi}_0 + \cdots + \frac{\mathit{\Pi}_6(H_6, X_6)}{(1+\delta)^6} + \frac{\mathit{\Pi}_7(H_7, X_7)}{(1+\delta)^7} + \frac{\mathit{\Pi}_8(H_8, X_8)}{(1+\delta)^8} \\
+ \frac{\mathit{\Pi}_9(H_9, X_9)}{(1+\delta)^9} + \frac{\mathit{\Pi}_{10}(H_{10}, X_{10})}{(1+\delta)^{10}} + \cdots \\
+ k_1(X_1 - X_0 - F(X_0) + H_0) + \cdots + k_6(X_6 - X_5 - F(X_5) + H_5) \\
+ k_7(X_7 - X_6 - F(X_6) + H_6) + k_8(X_8 - X_7 - F(X_7) + H_7) \\
+ k_9(X_9 - X_8 - F(X_8) + H_8) + k_{10}(X_{10} - X_9 - F(X_9) + H_9) + \cdots .
\end{aligned} \tag{4}
$$

We wish to maximize this with respect to X_t and H_t for all t. For example,

$$0 = \frac{\partial L}{\partial X_8} = \frac{\partial \mathit{\Pi}_8/\partial X_8}{(1+\delta)^8} + k_8 + k_9(-1 - F'(X_8)) \tag{5}$$

$$0 = \frac{\partial L}{\partial H_8} = \frac{\partial \mathit{\Pi}_8/\partial H_8}{(1+\delta)^8} + k_9 \tag{6}$$

$$0 = \frac{\partial L}{\partial H_7} = \frac{\partial \mathit{\Pi}_7/\partial H_7}{(1+\delta)^7} + k_8. \tag{7}$$

(6) and (7) can easily be solved for k_9 and k_8. Substituting these values into (5) yields

$$0 = \frac{\partial \mathit{\Pi}_8/\partial X_8}{(1+\delta)^8} - \frac{\partial \mathit{\Pi}_7/\partial H_7}{(1+\delta)^7} + \frac{\partial \mathit{\Pi}_8/\partial H_8}{(1+\delta)^8}\left[1 + F'(X_8)\right], \tag{8}$$

so

$$0 = \frac{\partial \mathit{\Pi}_8}{\partial X_8} - (1+\delta)\frac{\partial \mathit{\Pi}_7}{\partial H_7} + \left[1 + F'(X_8)\right]\frac{\partial \mathit{\Pi}_8}{\partial H_8}. \tag{9}$$

From (1), $\mathit{\Pi}_8 = TR_8(H_8) - TC(H_8, X_8)$, so $\frac{\partial \mathit{\Pi}_8}{\partial X_8} = -\frac{\partial TC}{\partial X_8}$; call this $-C'_{X8}$ for short. By definition, $\frac{\partial \mathit{\Pi}_7}{\partial H_7} = M\mathit{\Pi}_7$ and $\frac{\partial \mathit{\Pi}_8}{\partial H_8} = M\mathit{\Pi}_8$. Also, let $F'(X_8)$ be abbreviated by F'_8. Then substituting these results into (9) yields

$$0 = -C'_{X8} - (1+\delta)M\mathit{\Pi}_7 + [1 + F'_8]\,M\mathit{\Pi}_8 \tag{10}$$

which can be rewritten as

$$(1+\delta)M\mathit{\Pi}_7 = [1 + F'_8]\,M\mathit{\Pi}_8 - C'_{X8} \tag{11}$$

or as

$$(1+\delta)M\mathit{\Pi}_7 = [1 + F'_8]\,M\mathit{\Pi}_8 + \frac{\partial \mathit{\Pi}_8}{\partial X_8}. \tag{12}$$

If, in (12), there is a steady state, then this equation becomes

$$\boxed{(1+\delta)M\mathit{\Pi} = [1 + F']\,M\mathit{\Pi} + \tfrac{\partial \mathit{\Pi}}{\partial X}\,,} \tag{13}$$

which simplifies to

$$\delta\, M\Pi = F'\, M\Pi + \frac{\partial \Pi}{\partial X} \tag{14}$$

or

$$\delta = F' + \frac{1}{M\Pi}\frac{\partial \Pi}{\partial X}\,. \tag{15}$$

Finally, to show that this is consistent with what your textbook has, recall that by definition, $C'_X = \partial\, TC/\partial X$. Your book, in (16.13), assumes that $TC = c(X)H$. (Your book uses C instead of c, but I think c is less confusing.) Maintaining this assumption, $C'_X = \partial\big(c(x)H\big)/\partial X = c'(X)H$. In a steady state, $X_{t+1} = X_t$, so from (3), in a steady state, $F(X) = H$. Making this substitution results in

$$C'_X = c'(X)F(x)\,. \tag{16}$$

In addition, in your book, equation (16.13) has $\pi = PH - c(X)H$, so

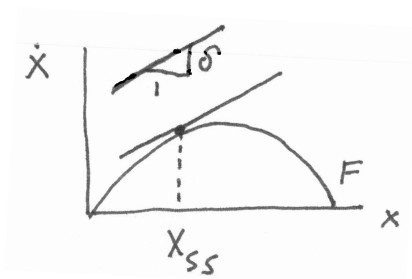
$$M\Pi = \frac{\partial \pi}{\partial H} = P - c(X)\,. \tag{17}$$

Substitute (16) and (17) into (15), remembering that $\partial\Pi/\partial X = -C'_X$:

$$\delta = F' + \frac{-c'(X)F(X)}{P - c(X)}\,. \tag{18}$$

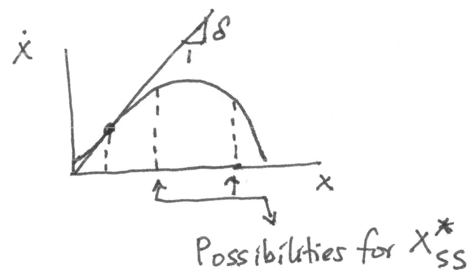
This is (16.16) of your textbook.

Steady state with $\partial TC/\partial X = 0$ ("schooling"): (15) $\Rightarrow \delta = F'(X)$.



If δ is too large (if $\delta > r$), $X^*_{SS} = 0$ (extinction).

Steady state with $\partial TC/\partial X < 0$ ("search"): (15) $\Rightarrow F' = \delta + (C'_X/M\Pi)$.



Comparative Statics:

$P\uparrow \Rightarrow M\Pi \uparrow\Rightarrow \delta + C'_X/M\Pi$ increases and moves closer to $\delta \Rightarrow x^*_{SS}\downarrow$, H^* probably $\downarrow$ (a completely backward-bending SS supply curve) but H^* could $\uparrow$ at first.

$\partial TC/\partial H$ ("MC") $\uparrow \Rightarrow M\Pi\downarrow$, the opposite results from $P\uparrow$.

$\partial TC/\partial X$ more negative $\Rightarrow f' = \delta + C'_X/M\Pi$ moves further below $\delta \Rightarrow X^*_{SS}\uparrow$.

Note: if $C'_X = 0$, none of these hold since all that matters is $\delta = F'(X)$ (unless this means $\pi < 0$).