

This exam has 67 points. There are seven questions on the exam; you should work all of them. Each question 9 points each, except the first two questions, which are worth 11 points each.

Put your answers to the exam in a blue book or on blank sheets of paper.

Answer the questions using as much precision and detail as the time allows. Correct answers which are unsupported by explanations will not be awarded points. Therefore, even if you think something is “obvious,” do not omit it. If you omit anything, you will not get credit for it. You get credit for nothing which does not explicitly appear in your answer. If you have questions about the adequacy of an explanation of yours during the exam, ask me.

**Answer all of the following questions.**

1. [11 points] Below is a long excerpt from a handout we used in fisheries economics. Explain how each of its equations (1)–(11) would change if we instead were trying to derive the Hotelling Rule. If an equation would not change, say that in your answer.

The profit of each firm is

$$\Pi(H_t, X_t) = TR_t(H_t) - TC(H_t, X_t) \quad (1)$$

where  $H$  is the harvest,  $X$  is [definition omitted on this Final Exam so as not to give away the answer to Question 2(a)(i)],  $TR$  is the total revenue, and  $TC$  is the total cost, all at time  $t$ . The objective of the firm is to

$$\max \sum_{t=0}^{\infty} \frac{\Pi_t}{(1+\delta)^t} \quad \text{s.t.} \quad (2)$$

$$X_{t+1} - X_t = F(X_t) - H_t \quad (3)$$

where  $F$  is the natural excess of births over deaths. (3) represents an infinite number of constraints on (2). Using  $k_1, k_2, \dots$ , to denote the Lagrange multipliers, the Lagrangian is

$$\begin{aligned} L = \Pi_0 + \cdots + & \frac{\Pi_6(H_6, X_6)}{(1+\delta)^6} + \frac{\Pi_7(H_7, X_7)}{(1+\delta)^7} + \frac{\Pi_8(H_8, X_8)}{(1+\delta)^8} \\ & + \frac{\Pi_9(H_9, X_9)}{(1+\delta)^9} + \frac{\Pi_{10}(H_{10}, X_{10})}{(1+\delta)^{10}} + \cdots \\ & + k_1(X_1 - X_0 - F(X_0) + H_0) + \cdots + k_6(X_6 - X_5 - F(X_5) + H_5) \\ & + k_7(X_7 - X_6 - F(X_6) + H_6) + k_8(X_8 - X_7 - F(X_7) + H_7) \\ & + k_9(X_9 - X_8 - F(X_8) + H_8) + k_{10}(X_{10} - X_9 - F(X_9) + H_9) + \cdots. \end{aligned} \quad (4)$$

We wish to maximize this with respect to  $X_t$  and  $H_t$  for all  $t$ . For example,

$$\frac{\partial L}{\partial X_8} = 0 = \frac{\partial \Pi_8 / \partial X_8}{(1+\delta)^8} + k_8 + k_9(-1 - F'(X_8)) \quad (5)$$

$$\frac{\partial L}{\partial H_8} = 0 = \frac{\partial \Pi_8 / \partial H_8}{(1+\delta)^8} + k_9 \quad (6)$$

$$\frac{\partial L}{\partial H_7} = 0 = \frac{\partial \Pi_7 / \partial H_7}{(1+\delta)^7} + k_8. \quad (7)$$

(6) and (7) can easily be solved for  $k_9$  and  $k_8$ . Substituting these values into (5) yields

$$0 = \frac{\partial \Pi_8 / \partial X_8}{(1+\delta)^8} - \frac{\partial \Pi_7 / \partial H_7}{(1+\delta)^7} + \frac{\partial \Pi_8 / \partial H_8}{(1+\delta)^8} [1 + F'(X_8)], \quad (8)$$

so

$$0 = \frac{\partial \Pi_8}{\partial X_8} - (1 + \delta) \frac{\partial \Pi_7}{\partial H_7} + [1 + F'(X_8)] \frac{\partial \Pi_8}{\partial H_8}. \quad (9)$$

From (1),  $\Pi_8 = TR_t(H_8) - TC(H_8, X_8)$ , so  $\frac{\partial \Pi_8}{\partial X_8} = -\frac{\partial TC}{\partial X_8}$ ; call this  $-C_{X_8}$  for short. By definition,  $\frac{\partial \Pi_7}{\partial H_7} = M\Pi_7$  and  $\frac{\partial \Pi_8}{\partial H_8} = M\Pi_8$ . Also, let  $F'(X_8)$  be abbreviated by  $F'_8$ . Then substituting these results into (9) yields

$$0 = -C_{X_8} - (1 + \delta)M\Pi_7 + [1 + F'_8] M\Pi_8 \quad (10)$$

which can be rewritten as

$$(1 + \delta)M\Pi_7 = [1 + F'_8] M\Pi_8 - C_{X_8} \quad (11)$$

or as

$$(1 + \delta)M\Pi_7 = [1 + F'_8] M\Pi_8 + \frac{\partial \Pi_8}{\partial X_8}. \quad (12)$$

2. [11 points] We have spent significant time in class discussing consequences of the equation

$$\dot{X} = rX \left(1 - \frac{X}{K}\right). \quad (13)$$

- (a) In this equation, what is:
  - i.  $X$ ?
  - ii.  $\dot{X}$ ?
  - iii.  $\dot{X}/X$ ? (This does not appear explicitly in the equation, but what would its interpretation be?)
  - iv.  $K$ ?
  - v.  $r$ ?
 Especially for the last four, do not merely give the name of the variable: explain why the name is appropriate. For the last one ( $r$ ), explain the relationship between  $r$  and  $\dot{X}/X$ .
- (b) Draw a graph of  $\dot{X}$  versus  $X$  for (13).
- (c) Draw a graph of  $X$  versus time  $t$  for (13). You need not explain its derivation, but give some idea of why your answer makes sense.
- (d) Give a realistic example of a fisheries growth function  $F(X)$  which does not obey (13). Explain why the difference between the  $F(X)$  you gave and (13) is important.
- (e) What is “logistic growth”?

3. [9 points] Concerning exhaustible resources, contrast the viewpoints of the schools of thought which your textbook calls the “Malthusian” and the “Ricardian” school.
4. [9 points]
  - (a) Draw a simple graph (with only two curves) showing how a social planner would compute the optimal Pigouvian tax on the *output* of a good whose production causes pollution. Explain, as always.
  - (b) Draw a simple graph (with only two curves) showing how a social planner would compute the optimal Pigouvian tax on *pollution*. Explain, as always.
5. [9 points] Discuss “hot spots” in the context of tradeable pollution permits. (Don’t merely define “hot spots.” Thoroughly discuss them.)
6. [9 points] Discuss “starting point bias.”
7. [9 points] Pollution laws have sometimes mandated the use of “Best Available Control Technology.” What is that?

Answers to Final Exam, Econ 5250/6250,

Fall 2013

①

(1) is

$$\Pi(H_t, X_t) = TR_t(H_t) - TC(H_t, X_t)$$

Fish:  $\uparrow$  harvest  $\downarrow$  stock size

Exhaustible Resource:  $\uparrow$   
 $Q_t$ ,  
 quantity extracted  
 at time t

$X_t$ , amount of resource in the  
 ground at time t

Equation

2

Change for Exhaustible Resource

none

3

$$F(X_t) \equiv 0$$

4

all the F's are zero

5

$$F' = 0$$

6

none

7

none

8

$$F' = 0$$

9

$$F' = 0$$

10

$$F'_8 = 0$$

11

$$F'_8 = 0$$

12

$$F'_8 = 0$$

So from (12), for an  
 exhaustible resource,

$$M\pi_8 =$$

$$(1+\delta)M\pi_7 - \frac{\partial \pi_8}{\partial X_8},$$

which is the Hotelling  
 Rule when  $\partial \pi_8 / \partial X_8$  is  
 not zero. When that is

zero, (12) just gives

$$M\pi_8 = (1+\delta)M\pi_7,$$

a more familiar version of  
 the Hotelling Rule.

2

a)  $X$  is "stock size," the amount of fish (either their number or their biomass)

$K$ : the carrying capacity of the environment. If  $X < K$ ,  $X$  moves towards  $K$  (since then  $\dot{X} > 0$ ); if  $X > K$ ,  $X$  moves towards  $K$  (since then  $\dot{X} < 0$ ); if  $X = K$ ,  $X$  stays at  $K$  (since then  $\dot{X} = 0$ ).

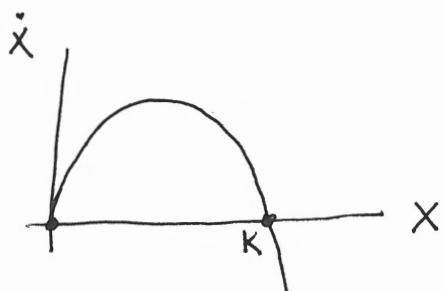
$r$ : the intrinsic growth rate. From (13),

$$\frac{\dot{X}}{X} = r \left(1 - \frac{X}{K}\right) \Leftrightarrow r \left(1 - \frac{X}{K}\right) = r$$

so that  $\dot{X}/X \leq r$ , and  $\dot{X}/X = r$  only when  $X = 0$ .

Hence  $r$  is the maximum proportional growth rate.

b)



$$\dot{X} = rX\left(1 - \frac{X}{K}\right) = rX - \frac{r}{K}X^2,$$

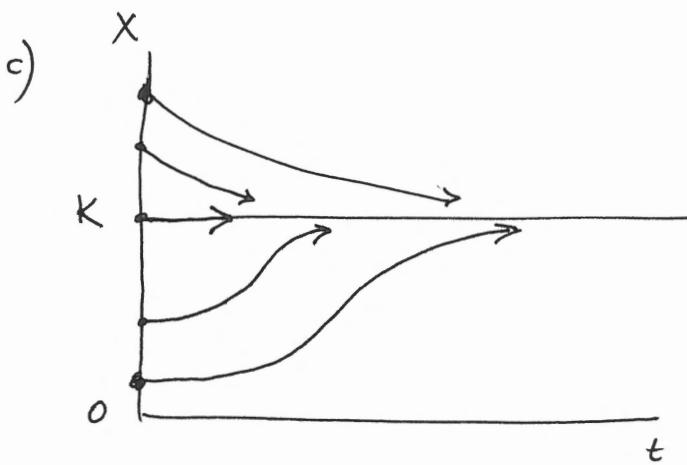
a parabola.

$$x \Rightarrow \dot{x} = 0$$

$$x = k \Rightarrow x' = 0$$

$$0 < x < k \Rightarrow \dot{x} > 0$$

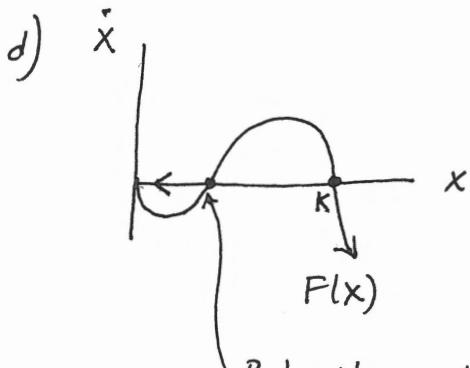
$$x > k \Rightarrow \dot{x} < 0$$



A few sample time paths  
for  $X_t$ .

the answer to,  
As<sub>1</sub>(b) states, if  $X < K$   
then  $\dot{X} > 0$  so  $X$  is rising,

whereas if  $X > K$  then  $\dot{X} < 0$  so  $X$  is falling. From the graph in (b)'s answer,  $\dot{X}$  is close to zero — so  $X$  versus  $t$  is close to horizontal — when  $X$  is close to zero or to  $K$ , but not when  $X$  is near the middle between 0 and  $K$ . This results in the "S" shaped curves below  $K$  in this graph.



Below the point  $A$ ,  $\dot{X} < 0$ , so  $X$  will go to zero: the species is doomed. This "point of no return" does not exist in (b).

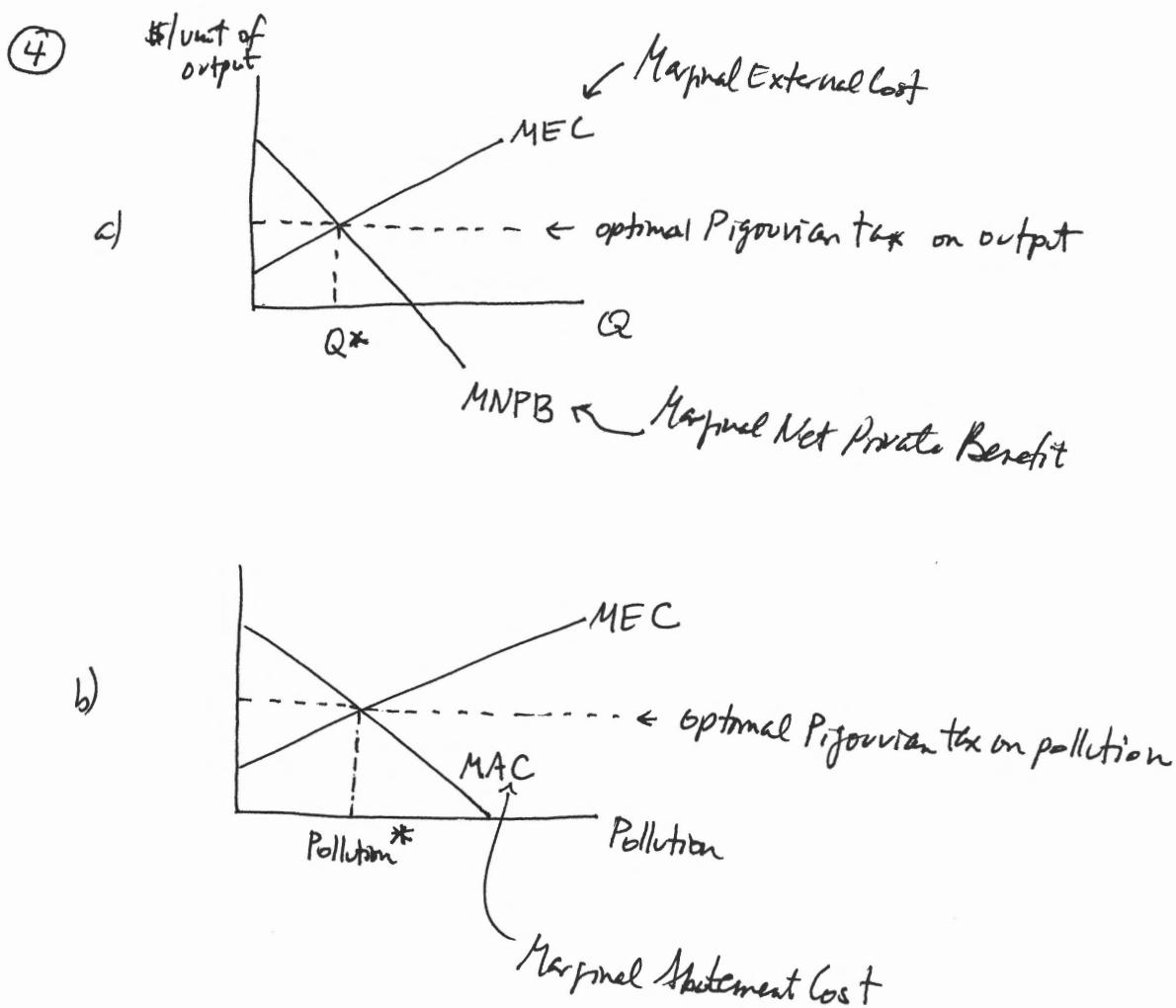
In (b), the species is not doomed unless  $X = 0$ . Here, doom occurs at a strictly positive value of  $X$ .

e) Growth obeying (b).

③

Malthusian : concerned about running out of resources. Worried that population growth will rapidly deplete resources. Concern also about increasing pollution, and unsustainability in general.

Ricardian : emphasizes that resources come in many qualities, so when good quality stocks are depleted, there are always poorer quality stocks still available. Hence resources will become more expensive, but we won't run out of any.



(a)

(b) →

If  $Q > Q^*$ , after tax  
marginal net private benefit  
is  $MNPB - \text{tax} < 0$ , so  
it's not worthwhile to produce  
where  $Q > Q^*$ . Where  $Q < Q^*$ ,  
 $MNPB - \text{tax} > 0$  so it is  
worthwhile to produce.



Proof that the firm will  
set  $Q = Q^*$

If  $Q > Q^*$ , marginal social  
benefit  $MNPB - MEC < 0$ ,  
so society doesn't want to  
have these units produced.  
Where  $Q < Q^*$ ,  
 $MNPB - \text{tax} > 0$ , so  
society does want these  
units produced.



Location of  $Q^*$

(b) Proof that the firm will set  $\text{Pollution} = \text{Pollution}^*$ :

If  $\text{pollution} > \text{Pollution}^*$ , firms have to pay the tax, whereas if they abated they'd only have to pay MAC, which is less than the tax. So they'd abate.

If  $\text{pollution} < \text{Pollution}^*$ , MAC > tax, so firms would rather just pay the tax than abate.

### Location of Pollution\*:

If  $\text{pollution} > \text{Pollution}^*$ , society would prefer to abate, thus paying MAC, rather than not abate and pay MEC. If  $\text{pollution} < \text{Pollution}^*$ , society would prefer not to abate, thus paying MEC, rather than abate and pay MAC since here  $\text{MAC} > \text{MEC}$ .

(5) "Hot spots" refer to the possibility that most of the purchasers of a pollution permit might be located close to each other. If the pollutant has bad local effects — such as ground-level ozone, and unlike greenhouse gases — then this clustering together could cause socially-bad high levels of pollution in the local area around these polluters. The number of pollution permits might have been optimal if the polluters were widely dispersed, but since they aren't, there is too much pollution in a small area.

Optional : "Hot spot" originally referred to an area which was highly radioactive.

Optional : A solution to "hot spot" problems would be to divide the area into smaller regions, then issue region-specific permits, to prevent clustering.

⑥

In using Contingent Valuation to elicit preferences, a researcher might present a respondent with a multiple-choice answer, such as:

- a) < \$5
- b) \$5 - \$10
- c) > \$10.

Presumably similar respondents might be presented with:

- a) < \$10
- b) \$10 - \$20
- c) > \$20.

Often, respondents presented with the first set of choices report lower valuations than those presented with the second set of choices. This is "starting point bias." It happens when respondents are unsure of their preference, and use the choices presented to them as "hints" to the set of "reasonable" preferences.

Starting Point Bias does not exist with "open-ended" (fill-in-the-blank) questions.

⑦ "Best Available Control Technology" was never interpreted literally.

In practice, it was always interpreted as "good" pollution control technologies that were not "too" expensive. The definition of "good" and "too expensive" were subjective. This could easily cause controversy, with industry's interpretation of those ideas being very different from environmental groups' interpretation of them.