

Environmental & Natural Resource Economics
August 2006

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Field Exam

You need to answer all four questions on this exam.

You have until 5 PM to finish, though it probably will not take you that long. Answer the questions using as much precision and detail as the time allows.

The questions are equally weighted.

Answer all of the following four questions.

1. (a) Derive the Hotelling Rule for a monopolist who owns a fixed supply “ S ” of an exhaustible resource.

Hint: If one wishes to maximize

$$\int_0^T g(x_t, u_t, t) dt$$

subject to

$$\dot{x}_t = f(x_t, u_t, t)$$

with x_0 given, Optimal Control Theory specifies that, if one defines the Hamiltonian as $H = g + \lambda f$, one has

$$u^* \text{ maximizes } H \text{ given } x_t^* \text{ and } t$$

and

$$\dot{\lambda}_t^* = -\frac{\partial H^*}{\partial x}.$$

- (b) Explain why part (a) is much easier than deriving the Hotelling Rule for a competitive industry. (You do not have to derive the Hotelling Rule for a competitive industry, merely explain what makes such a derivation difficult.)
2. Comment on the attached first page of an article entitled “Universe Bounded” by Gary Gardner (Worldwatch, September/October 2006).
3. For private-property competition in a fishery, in class we showed that

$$\dot{h}_t = \frac{[\delta - F'(x_t)][\phi(h_t) - c(x_t)] + c'(x_t) F(x_t)}{\phi'(h_t)}.$$

(You will recall that ϕ is the demand curve, c is marginal cost, δ is the discount rate, F is the excess of births over natural deaths, x is stock size, and h is harvest.) Suppose for simplicity that it costs nothing to fish, and that $\delta - F'(x_t)$ is always positive.

- (a) What is the time path of profit for this industry? (Hint: sketch the phase diagram first.)
- (b) What is the industry’s profit at the steady-state point?

- (c) How do you think the steady-state behavior of this industry differs from what it would be if it were an open-access fishery instead of a private-property fishery? (Hint: answering this does not require a new mathematical derivation.)
4. Attached to your exam are pages 88, 89, 90, and 91 from your textbook. Please expand on the last two sentences before section 3.5 starts (the sentences beginning, “Ideally, the mixed system. . .”). What “ideal” situation are the authors imagining? In what sense is it “ideal”? Could it really achieve “the social optimum,” and if so, under what circumstances?