**Unit 03 Example**
Notations:
FV=future value
P = lump sum principle
Pp=amount of periodical payments
r = interest rate for a certain period
n = number of periods

Example: annual r=12%, 2 years or 24 months

1. Future Value Factor
$FVF= (1+r)^n$

Annual compounding: $FVF= (1+r)^n =(1+0.12)^2=1.12^2=1.2544$

*Monthly compounding: r=12%/12=1%=0.01*

$FVF= (1+r)^n =(1+0.01)^{24}=1.01^{24}=1.269735$

2. Future Value Factor Sum (BOM)

$$FVFS = (1+r)^n + (1+r)^{n-1} + ... + (1+r)^1 = \frac{(1+r)^{n+1}-1}{r} - 1$$

Annual compounding:

$$\text{FVFS} = (1+0.12)^2 + (1+0.12)^1 = \frac{(1+0.12)^{2+1}-1}{0.12} - 1 = \frac{(1.12)^3-1}{0.12} - 1$$
$$= \frac{1.404928-1}{0.12} - 1 = \frac{0.404928}{0.12} - 1 = 3.3744 - 1 = 2.3744$$

Monthly compounding: monthly r=12%/12=1%=0.01, n=24 months

$$\text{FVFS} = (1+0.01)^{24} + ... + (1+0.01)^1 = \frac{(1+0.01)^{24+1}-1}{0.01} - 1 = \frac{(1.01)^{25}-1}{0.01} - 1$$
$$= \frac{1.282432-1}{0.01} - 1 = \frac{0.282432}{0.01} - 1 = 28.2432 - 1 = 27.2432$$

Note these above two computations are very different because with the annual compounding question we are assuming only two payments were saved, one per year. The monthly compounding one assumes that 24 payments were made, one per month for 2 years.

3. Future Value Factor Sum (EOM)

$$FVFS = (1+r)^{n-1} + (1+r)^{n-2} \ldots + (1+r)^{0} = \frac{(1+r)^{n} - 1}{r}$$

Annual compounding

$$\text{FVFS} = (1+0.12)^{2-1} + (1+0.12)^{0} = \frac{(1+0.12)^{2} - 1}{0.12} = \frac{(1.12)^{2} - 1}{0.12}$$

$$= \frac{1.2544 - 1}{0.12} = \frac{0.2544}{0.12} = 2.12$$

Monthly compounding: monthly r=12%/12=1%=0.01, n=24 months

$$\text{FVFS} = (1+0.01)^{24-1} \ldots + (1+0.01)^{0} = \frac{(1+0.01)^{24} - 1}{0.01} = \frac{(1.01)^{24} - 1}{0.01}$$

$$= \frac{1.269735 - 1}{0.01} = \frac{0.269735}{0.01} = 26.97346$$

**Unit 04 Example**

Notations:
PV=present value
P = lump sum amount
Pp=amount of equal periodical payment
r = discount rate for a certain period (often we use interest rate to approximate discount rate)
n = number of periods

Example: annual r=12%, 2 years or 24 months

1. Present Value Factor

$$PVF = \frac{1}{(1+r)^n}$$

Annual compounding: r=12%, n=2 years

$$PVF = \frac{1}{(1+r)^n} = \frac{1}{(1+12\%)^2} = \frac{1}{(1+0.12)^2} = \frac{1}{1.12^2} = \frac{1}{1.2544} = 0.797194$$

*Monthly compounding: r=12%/12=1%=0.01, n=24 months*

$$PVF = \frac{1}{(1+r)^n} = \frac{1}{(1+1\%)^{24}} = \frac{1}{(1+0.01)^{24}} = \frac{1}{1.01^{24}} = \frac{1}{1.269735} = 0.787566$$

2. Present Value Factor Sum (BOM)

$$\mathrm{PVFS} = \frac{1}{(1+\mathrm{r})^0} + \frac{1}{(1+\mathrm{r})^1} + \ldots + \frac{1}{(1+\mathrm{r})^{n-1}} = 1 + \frac{1-\frac{1}{(1+\mathrm{r})^{n-1}}}{\mathrm{r}}$$

Annual compounding: r=12%, n=2 years

$$\mathrm{PVFS} = \frac{1}{(1+12\%)^0} + \frac{1}{(1+12\%)^{2-1}} = 1 + \frac{1-\frac{1}{(1+12\%)^{2-1}}}{12\%} = 1 + \frac{1-\frac{1}{(1+0.12)^{2-1}}}{0.12} = 1 + \frac{1-\frac{1}{(1.12)^1}}{0.12}$$

$$= 1 + \frac{1-\frac{1}{1.12}}{0.12} = 1 + \frac{1-0.892857}{0.12} = 1 + \frac{0.107143}{0.12} = 1 + 0.892857 = 1.892857$$

*Monthly compounding: r=12%/12=1%=0.01, n=24 months*

$$PVFS = \frac{1}{(1+1\%)^0} + \ldots + \frac{1}{(1+1\%)^{24-1}} = + \frac{1-\frac{1}{(1+1\%)^{24-1}}}{1\%} = 1 + \frac{1-\frac{1}{(1+0.01)^{24-1}}}{0.01} = 1 + \frac{1-\frac{1}{(1.01)^{23}}}{0.01}$$

$$= 1 + \frac{1-\frac{1}{1.257163}}{0.01} = 1 + \frac{1-0.795442}{0.01} = 1 + \frac{0.2045582}{0.01} = 1 + 20.45582 = 21.45582$$

Note these above two computations are very different because with the annual compounding question we are assuming only two payments will be given, one per year. The monthly compounding one assumes that 24 payments will be given, one per month for 2 years.

3. Present Value Factor Sum (EOM)

$$PVFS = \frac{1}{(1+r)^1} + \ldots + \frac{1}{(1+r)^n} = \frac{1-\frac{1}{(1+r)^n}}{r}$$

Annual compounding: r=12%, n=2 years

$$PVFS = \frac{1}{(1+12\%)^1} + \frac{1}{(1+12\%)^2} = \frac{1-\frac{1}{(1+12\%)^2}}{12\%} = \frac{1-\frac{1}{(1+0.12)^2}}{0.12} = \frac{1-\frac{1}{1.12^2}}{0.12} = \frac{1-\frac{1}{1.2544}}{0.12}$$

$$= \frac{1-0.797194}{0.12} = \frac{0.202806}{0.12} = 1.690051$$

*Monthly compounding: r=12%/12=1%=0.01, n=24 months*

$$PVFS = \frac{1}{(1+1\%)^1} + \ldots + \frac{1}{(1+1\%)^{24}} = \frac{1-\frac{1}{(1+1\%)^{24}}}{1\%} = \frac{1-\frac{1}{(1+0.01)^{24}}}{0.01} = \frac{1-\frac{1}{1.01^{24}}}{0.01} = \frac{1-\frac{1}{1.269735}}{0.01}$$

$$= \frac{1-0.787566}{0.01} = \frac{0.212434}{0.01} = 21.2434$$

Note these above two computations are very different because with the annual compounding question we are assuming only two payments will be given, one per year. The monthly compounding one assumes that 24 payments will be given, one per month for 2 years.