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Resource depletion, national income accounting, and the value of optimal dynamic programs

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Abstract

Under assumptions satisfied by many economic problems, I derive a fundamental new equation for the time rate of change of the optimal value function of any optimal control problem. This is then applied to Hotelling's model of the resource extracting firm. The precise differences between rent, depreciation, and depletion charges are discovered, the flow and stock price appreciation rates are distinguished, and novel characterizations of mine value are derived. Most importantly, the correct contribution of mining to net national product (and to sustainable development) follows. In perfect foresight equilibrium, competitively managed mines can appreciate while being exhausted.

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JEL classification: Q300; D900; C610; O470; C820; M400

"Gross domestic product (GDP) figures are widely used by economists, politicians, and the media. Unfortunately, they are generally used without the caveat that they represent an income that cannot be sustained. Current calculations ignore the degradation of the natural resource base and view the sales of nonrenewable resources entirely as income. A better way must be found to measure the prosperity and progress of mankind."

— Barber B. Conable (President, The World Bank) and Mostafa Tolba (Executive Director, United Nations Environment Programme) (foreword to Ahmad et al., 1989).

Standard practice is to calculate net national product without making a deduction for the depreciation of mines. In effect this places a zero value on a nation's

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wealth of mineral resources because there is no cost associated with losing that wealth. This is incorrect for reasons well understood by Marshall, Pigou, and Hicks: receipts earned by running down capital stocks should not be counted as part of net income. Since the early 1980s, public concern over environmental degradation and resource depletion has grown, and by the late 1980s the seriousness of this error in the national accounts was recognized at the highest levels of the World Bank and the United Nations. In 1994 the U.S. government's Bureau of Economic Analysis (BEA) began publishing its ''integrated economic and environmental satellite accounts'' (Carson et al., 1994a, 1994b), which include the most recent attempt to determine how to properly treat exhaustible resources in the national accounts. Earlier attempts include those of Henry Peskin (1991), Roefie Hueting of the Central Bureau of Statistics of the Netherlands (1991), Arnold Katz of the US Department of Commerce (1990), Salah El Serafy of the World Bank (1989), and a group at the United Nations Statistical Office (see the last chapter of Ahmad et al. (1989)), but the very earliest attempts were made by the U.S. BEA between 1942 and 1947 (Carson et al., 1994a, p. 36, 1994b, p. 51).

These attempts have been ad hoc to some degree because economic theory has had little which could contribute to the analysis. Since Hotelling's (1931) paper, economic analysis of resource exhaustion has been primarily confined to finding the optimal flow of resource from a mine – that is, the optimal flow of services from the economic asset. Only a few authors (e.g. Miller and Upton, 1985) have analyzed the resulting value of the asset. No one has analyzed the equilibrium rate of change of the resulting value of the asset, which is the asset's depreciation and which is the object of the current interest. It has been argued that this neglect is appropriate; Hicks (1946, p. 171), for example, believed that the optimal flow of services was the only question of economic interest. However, even Hicks admitted that income, depreciation, and investment ''are the terms in which one has been used to think.'' Furthermore, the correct calculation of depreciation is of vital importance not only for national income accounting but also for computing taxes; for example, for many years the U.S. tax code has contained a mathematical formula for mine depreciation.

In the absence of guidance from economic theory, the World Bank, UN, and U.S. BEA will adopt one of the ad hoc solutions which have been proposed to the exhaustible resource depreciation problem. It is important that before these choices become finalized, research on a theoretically correct approach to the problem be advanced.

Accordingly, the purpose of this paper is to give an expression for mine depreciation in perfect foresight equilibrium. Section 1 presents a new abstract result in optimal control theory which applies to problems far more general than mine depreciation. This result makes the rest of the paper possible. Section 2 gives quantitative results which connect mine depreciation, mine valuation, and in situ price with the more familiar concepts of rent, depletion rate, and extracted resource price. It provides the first general expression for the time rate of change of the in situ resource price and shows why this differs from the fundamental Hotelling Rule. Section 2 also shows that an outside observer can calculate the value of a mine monopoly without knowing anything about the future and without knowing the current stock size (nor can the current stock size be inferred by such an observer). Section 3 explains a surprising qualitative result on mine depreciation: a competitive firm's mine may *appreciate* in current value terms in equilibrium.

In fact, under certain conditions, whether the stock is being *physically* depleted at a rate faster or slower than the interest rate determines whether a competitive mine depreciates or appreciates; never before has any significance been found in comparing the physical depletion rate to the interest rate. Section 4 brings the previous results together to explain how to calculate mine depreciation in the national income accounts, and how to ensure 'sustainable development.' Taken together these results greatly expand the theory of mine valuation.

1. The value of optimal dynamic programs

Let x be the state variable, u be the control variable, raised dots denote differentiation with respect to time, and boldfaced characters such as $\boldsymbol{x}$ and $\boldsymbol{u}$ denote functions of time from some date τ to another date T, with $0 \leqslant \tau < T \leqslant \infty$. Consider the standard optimal control problem of maximizing $\mathscr{V}(\tau,x_\tau,\boldsymbol{u}) = \int_\tau^T f(x,u,t)\,\mathrm{d}t$ such that $\dot{x} = g(x,u,t)$, x_τ is given, x and u are scalars (this is easily relaxed), and $u_t \in U \subseteq \mathbf{R}^1$. As usual, f, $\partial f/\partial x$, g, and $\partial g/\partial x$ are all continuous in x, u, and t. Letting asterisks denote the optimal solution, let $V(\tau,x_\tau)$ be the 'optimal value function' for this problem:

$$V(\tau,x_\tau) \equiv \mathscr{V}(\tau,x_\tau,\boldsymbol{u}^*).$$

Use the following abbreviations: $\dot{\mathscr{V}}_\tau$ for $\mathrm{d}\mathscr{V}(\tau,x_\tau,\boldsymbol{u})/\mathrm{d}\tau$, $\dot{\mathscr{V}}_\tau^*$ for $\mathrm{d}\mathscr{V}(\tau,x_\tau,\boldsymbol{u}^*)/\mathrm{d}\tau$, and $\dot{V}_\tau$ for $\mathrm{d}V(\tau,x_\tau)/\mathrm{d}\tau$. Clearly $\dot{V}_\tau \equiv \dot{\mathscr{V}}_\tau^*$. Let λ be the adjoint variable corresponding to the constraint $\dot{x} = g$, and let $H = f + \lambda g$ be the Hamiltonian. Finally, let $\partial H/\partial t\,|_*$ (respectively $\mathrm{d}H/\mathrm{d}t\,|_*$) be the function formed by calculating $\partial H(x,u,t)/\partial t$ (respectively $\mathrm{d}H(x(t),u(t),t)/\mathrm{d}t$) and then evaluating at the optimal solution.

$\dot{V}_\tau$ is closely related to depreciation, so we require a formula for it. From the Fundamental Theorem of Calculus, $\dot{V}_\tau$ always exists and is equal to $f(x^*,u^*,\tau)$, but much more useful is the following new result.

Proposition 1. Assume either that: (i) f and g are twice differentiable in t and that the optimal control u_t^* is continuous for all $t > \tau$; or that (ii) u_t^* is differentiable for all $t > \tau$. Then

$$\dot{V}_\tau = \lambda_\tau^* g_\tau^* + \int_\tau^T \frac{\partial H}{\partial t}\,|_*\,\mathrm{d}t - H_T^*. \tag{1}$$

The proof is in the Appendix. Eq. (1) holds even if the optimal control is not an interior solution; it fails only if u has discontinuities, as in 'bang-bang' solutions. [1]

Each of the terms on the right-hand side of (1) has an important economic interpretation: as τ increases, V changes because the state variables change ($\lambda g = \lambda \dot{x}$), because the problem involves nonautonomous trends in exogenous variables ($\partial H/\partial t$), and because the horizon draws nearer ($H_T = \mathrm{d}V/\mathrm{d}T$). These interpretations will be more fully discussed in the context of mineral extraction. Yet even at this stage it is clear that the $\partial H/\partial t$ term – which measures roughly what Weitzman (1976, fn. 8) meant by "the pure effect of time alone" – has an important economic meaning. Ignoring discounting, a competitive firm's problem is in general nonautonomous ($\partial H/\partial t \neq 0$) because it perceives price as fluctuating *exogenously*: instantaneous profit could in simple cases be written $p_t u - C(u)$, where u is quantity, p_t is price, and C is cost. However, a monopolist facing the same (say, stationary) demand and cost curves would have an autonomous problem ($\partial H/\partial t = 0$) since his future price changes are *endogenous*: instantaneous monopoly profit is in the simplest case $p(u)u - C(u)$, which does not involve t. These differences in the $\partial H/\partial t$ terms of (1) give rise to the many differences between competitive and monopolistic firms in Sections 2 and 3, despite the fact that there complications arising from discounting have to be taken into account.

Calculations of national income and taxation are conventionally done in current value terms, so Proposition 1 must be extended to that case. For any given function ψ_τ let $J(\tau,x_\tau)$ be $\psi_\tau V(\tau,x_\tau)$ and let $\mathscr{J}(\tau,x_\tau,u)$ be $\psi_\tau \mathscr{V}(\tau,x_\tau,u)$. $J(\tau,x_\tau)$ is the optimal value function for the problem of maximizing $\mathscr{J}(\tau,x_\tau,u) = \psi_\tau \int_\tau^T f(x,u,t)\,\mathrm{d}t$ subject to the same constraints as in Proposition 1. In economics, ψ_τ is typically $\mathrm{e}^{r\tau}$, where r is the interest rate. Letting λ and H respectively be the adjoint variable and Hamiltonian corresponding to the present value problem in Proposition 1, one has (suppressing asterisks, as I will usually do in what follows)

$$\dot{J}_\tau = \dot{\psi}_\tau \int_\tau^T f(x,u,t)\,\mathrm{d}t + \psi_\tau \left[\lambda_\tau g_\tau + \int_\tau^T \frac{\partial H}{\partial t}\,\mathrm{d}t - H_T \right]. \tag{2}$$

Proposition 1 and Eq. (2) are fundamental new results concerning the value of optimal dynamic programs. In economics, $-\dot{J}_\tau$ is depreciation (see also footnote 15).

In order to compare these results with older ones, I finish this section by analyzing two special cases.

[1] One can calculate $\dot{V}_\tau$ for *any* τ by solving the single maximization problem at $\tau = 0$, because of the Bellman Principle of Optimality, which optimal control solutions obey. Formally: Let $(\boldsymbol{x}^{0*},\boldsymbol{u}^{0*})$ be the optimal paths corresponding to $V(0,x_0)$. Let $V^0(\tau,x_\tau) = \mathscr{V}(\tau,x_\tau^{0*},\boldsymbol{u}^{0*}) = \int_\tau^T f(x^0,u^0,t)\,\mathrm{d}t$ be the portion of $V(0,x_0)$ contributed by the $(\boldsymbol{x}^{0*},\boldsymbol{u}^{0*})$ program after time τ. Then $\mathrm{d}V^0(\tau,x_\tau)/\mathrm{d}\tau = \mathrm{d}V(\tau,x_\tau)/\mathrm{d}\tau$.

Geometric discounting. To put the results into an economic context, let r be the rate of interest, π be profit, p_t be price, $\psi_\tau = e^{r\tau}$, and let $f(x,u,t) = \pi(x,u,p_t)e^{-rt}$.

Suppose T is free, so that $H_T = 0$. [2] Since $\mathscr{J}_\tau = \int_\tau^T \pi_t \, e^{-r(t-\tau)} \, dt$, elementary differentiation implies

$$r = \left(\dot{\mathscr{J}}_\tau + \pi_\tau\right)/\mathscr{J}_\tau. \tag{3}$$

On the other hand, (2) implies

$$\begin{aligned} \dot{J}_\tau &= r\,e^{r\tau}\int_\tau^T \pi_t \, e^{-rt}\, dt \\ &\quad + e^{r\tau}\left[\lambda_\tau g_\tau + \int_\tau^T\left(-r\pi_t \, e^{-rt} + \frac{\partial \pi}{\partial p}\frac{dp}{dt}\,e^{-rt} + \frac{\partial(\lambda g)}{\partial t}\right)dt\right] \\ &= e^{r\tau}\lambda_\tau \frac{dx}{d\tau} + \int_\tau^T \frac{\partial\pi}{\partial p}\frac{dp}{dt}\,e^{r(\tau-t)}\,dt + \int_\tau^T \lambda_t \frac{\partial g}{\partial t}\,e^{r\tau}\,dt, \end{aligned} \tag{4}$$

since $\partial\lambda/\partial t = 0$ even though $d\lambda/dt \neq 0$. So far in this paper I have never assumed that V (or J) is differentiable in x_τ, but when engaging in nonrigorous, intuitive discussion, that assumption can be useful. In this paper each expression derived assuming that V or J is differentiable in x_τ will be followed by a superscript $^{\#}$ to alert the reader to its heuristic nature. Making this assumption here, $\partial J/\partial x = \partial(V e^{r\tau})/\partial x = \lambda e^{r\tau\,\#}$ and hence (4) implies

$$\dot{J}_\tau = \frac{\partial J}{\partial x}\frac{dx}{d\tau} + \int_\tau^T \frac{\partial \pi}{\partial p}\frac{dp}{dt}\,e^{r(\tau-t)}\,dt^{\#} \quad \text{if } \frac{\partial g}{\partial t} = 0. \tag{5}$$

Eq. (3) is the old and well-known ‘efficiency condition’ that the rate of capital gains plus dividends equals the interest rate. It is far less than a characterization of optimality because it even holds for nonoptimal controls. [3]

Eqs. (4) and (5), by contrast, are formulas for the evolution of the *optimal* value of $\mathscr{J}$ (namely J), so one would expect them to contain additional useful information beyond (3). To explain what this information is, imagine constructing the indirect profit function $\mathscr{J}^*$ as a function of the exogenous variables, which in this case are some initial state variable(s) x (such as the capital stock) and the exogenous path of prices $\boldsymbol{p}$. Intuition leads one to conjecture simply that

$$\dot{\mathscr{J}}^* = \frac{\partial \mathscr{J}^*}{\partial x}\dot{x} + \frac{\partial \mathscr{J}^*}{\partial \boldsymbol{p}}\dot{\boldsymbol{p}}, \tag{6}$$

[2] When T is free, if $T < \infty$ then $H_T = 0$ is a transversality condition which is a necessary condition for optimality. If $T = \infty$, $\lim_{t\to\infty} H_t = 0$ from Michel (1982) and Seierstad and Sydsæter (1987, p. 245) whenever r_t is constant (geometric discounting).

[3] One can put the efficiency condition into present value form. Since $\mathscr{V}_\tau = \int_\tau^T \pi_t \, e^{-rt}\, dt$, Leibnitz' Rule implies $\dot{\mathscr{V}}_\tau = -\pi_\tau \, e^{-r\tau}$.

that is, the value of a firm changes because its state variables change and because the exogenous variables it faces change. The second term of (6) is problematic because $\boldsymbol{p}$ is a function, but it is rather easy to see heuristically that (5) is merely the correct form of (6). Since the contribution to $\mathscr{J}^*$ made by *one* future date t is $\pi_t \, \mathrm{e}^{-r(t-\tau)}$, a heuristic guess for the contribution to $[\partial \mathscr{J}/\partial \boldsymbol{p}]\dot{\boldsymbol{p}}$ made by that date t is $[\partial(\pi_t \, \mathrm{e}^{-r(t-\tau)})/\partial p_t] = (\partial \pi_t/\partial p_t)\mathrm{e}^{-r(t-\tau)}\dot{p}_t$; then adding the dates up (i.e., integrating) turns (6) into (5). Hence (5) has a simple economic interpretation because it is merely the correct form of (6), and (5) gives information significantly beyond that contained in the old efficiency condition (3) because (5), (2), and Proposition 1 all deal with the *optimal* evolution of $\mathscr{J}$.

Autonomous problems except for geometric discounting. While, as remarked above, many problems of economic interest are not autonomous – the case of a competitive firm being the most obvious example – other problems to be considered in this paper are autonomous. In such problems, $\psi_\tau = \mathrm{e}^{r\tau}$, g is not a function of t, one can write $f(x,u,t)$ as $\hat{f}(x,u)\mathrm{e}^{-rt}$, and it will be convenient to define $\hat{\lambda}$ by $\lambda_t = \hat{\lambda}_t \mathrm{e}^{-rt}$; $\hat{f} + \hat{\lambda} g$ is the 'current value Hamiltonian' $\hat{H}$. Let x be a vector instead of a scalar (so λx becomes an inner product and $g = \dot{x}$ becomes a gradient); the above propositions are unchanged. Three new results follow:

Proposition 2. For an optimal control problem which is autonomous except for geometric discounting, make the same assumptions as in Propositon 1 and assume that $H_T = 0$ (say because T is free). Then $J_\tau = \hat{\mathrm{H}}_\tau^*/r$.

Proof. By elementary differentiation, $\dot{\mathscr{J}}_\tau^* = -\hat{f}_\tau^* + r\mathscr{J}_\tau^*$ similarly to (3); hence $\dot{J}_\tau = -\hat{f}_\tau^* + rJ_\tau$. Applying (2) yields $\dot{J}_\tau = \hat{\lambda}_\tau^* \dot{x}_\tau^*$ similarly to (4). Combining these expressions yields $J_\tau = (1/r)[\hat{f}_\tau^* + \hat{\lambda}_\tau^* \dot{x}_\tau^*]$. ■

Corollary 1. (Abstract form of Weitzman's interpretation of net national product.) *Under the conditions of Proposition 2,*

$$\int_\tau^\infty \mathrm{e}^{-r(t-\tau)} \hat{f}_t^* \, \mathrm{d}t = \int_\tau^\infty \mathrm{e}^{-r(t-\tau)} \left(\hat{f}_\tau^* + \hat{\lambda}_\tau^* \dot{x}_\tau^* \right) \mathrm{d}t.$$

Proof. $\int_\tau^\infty \mathrm{e}^{-r(t-\tau)} \mathrm{d}t = 1/r$. ■

Corollary 2. (Abstract form of Hartwick's Rule.) *Under the conditions of Proposition 2, if* $\hat{\lambda}_\tau^* \dot{x}_\tau^* = 0$ then $(\mathrm{d}\hat{f}_t^*/\mathrm{d}t)|_{t=\tau} = 0$.

Proof. From Proposition 2, the premise implies $J_\tau = \hat{f}_\tau^*/r$. Since $\dot{J}_\tau = \hat{\lambda}_\tau^* \dot{x}_\tau^*$ from (2), the premise implies $\dot{J}_\tau = 0$. Hence at τ, $\mathrm{d}(\hat{f}_t^*/r)/\mathrm{d}t = 0$. ■

Since we will be concerned with national income accounting, give this a macroeconomic interpretation: $\hat{f}$ is consumption, $\dot{x}$ is investment, and $\hat{\lambda}$ is the shadow price of capital. Then Corollary 2 becomes the well-known 'Hartwick Rule': zero value of investment implies constant consumption. See Solow (1986). Similarly, net national product NNP_τ^* is $\hat{f}_\tau^* + \hat{\lambda}_\tau^* \dot{x}_\tau^*$ (consumption plus the value of net investment), which is in turn equal to $\hat{H}_\tau^*$ (see Weitzman, 1976, p. 159). Then Corollary 1 is just 'Weitzman's Interpretation of Net National Product,' which is also well known (see Solow, 1986, §III; Mäler, 1991, p. 11): NNP or $\hat{H}$ at the *single* date τ measures consumption possibilities for the *entire* future, $t > \tau$.

It is perhaps ironic that the same Proposition 2 and Corollaries 1 and 2 which show that Hartwick's Rule and Weitzman's Interpretation of NNP are abstract mathematical properties rather than mere economic relationships *also* show that for an economist interested, as we are, in perfect competition or other nonautonomous models, Hartwick's Rule and Weitzman's Interpretation of NNP do not apply.

2. Resource extraction: competition, monopoly, social planner

Suppose along with Hotelling (1931) that an industry is composed of profit-maximizing competitive firms who own fixed stocks of an exhaustible natural resource with certainty and are endowed with perfect foresight. Let the resource stock of one firm at time t be x_t and let the control variable be extraction q_t, so $g(x,q) = -q$. One has $x_t \geqslant 0$ for all t. Let $x_0 = S$ and let T be free. A firm's objective function is $f(x,u,t) = \pi_t/\delta_t = [p_t q - C(x,q)]/\delta_t$, where p_t is the exogenously given price path, C is the extraction cost function, $\delta_t = \exp[\int_0^t r_s\,\mathrm{d}s]$, and $\delta_t = \mathrm{e}^{rt}$ if discount rates r_t happen to be constant. [4] To put flows in current value terms, time τ revenues should not be discounted, so $\psi_\tau = \delta_\tau$. The Hamiltonian is $H = \pi_t/\delta_t - \lambda_t q_t$. Letting primes denote differentiation with respect to q and making the standard assumptions $C' > 0$, $C'' \geqslant 0$, and $\partial C/\partial x \leqslant 0$, the optimal solution is the familiar Hotelling Rule $M\Pi_t = \lambda_t \delta_t$ where $M\Pi$ is marginal profit. Most papers conclude by finding the optimal solution; here, the optimal solution is the starting point. The mine value J, its path through time, and the behavior of the 'stock price' $\rho_t = J_t/x_t$ are to be determined.

For this problem, $\dot{\psi}_\tau = r_\tau \delta_\tau$ and $\partial H/\partial t = \partial f/\partial t = \dot{p}_t q \delta_t^{-1} - \pi_t r_t \delta_t^{-1}$. Since T is free, under weak assumptions [5] $H_T = 0$. Eq. (2) implies that the current value of the mine changes according to

$$\dot{J}_\tau = -\lambda_\tau \delta_\tau q_\tau + \int_\tau^T \dot{p}_t q_t \delta_\tau \delta_t^{-1}\,\mathrm{d}t + \textstyle\int_\tau^T \pi_t \delta_\tau \delta_t^{-1}(r_\tau - r_t)\,\mathrm{d}t. \tag{7}$$

Eq. (7) has a very appealing interpretation. The third term captures the fact that if interest rates are, say, falling, then current value increases over time because an

[4] Time-varying interest rates do not lead to Strotz-type time inconsistencies here.

[5] This holds when $T < \infty$. If $T = \infty$, it holds when r_t is constant (footnote 2). Alternatively, when $T = \infty$, $\liminf_{t \to \infty} q_t = 0$ because of the resource constraint; if in addition $\limsup_{t \to \infty} q_t = 0$ then $\lim_{t \to \infty} q_t = 0$ and $\lim_{t \to \infty} H_t = 0$.

era of heavy discounting will end. If, as is usually assumed, interest rates are constant, this term drops out and (7) becomes identical to (5) (since $\partial g/\partial t = 0$). The absolute value of the first term is heuristically the 'depletion charge' because when J is differentiable in x the term represents the loss in current value due to depletion, $(\partial J/\partial x)(\mathrm{d}x/\mathrm{d}\tau)^{\#}$. The second term of (7) gives the appreciation of the mine due to increasing price. Because of the Hotelling Rule, one is used to thinking that $\dot{p}$ will be positive. This opens up the new and important possibility that mine appreciation due to price increases, represented by the second term, might outweigh mine depreciation due to depletion represented by the first term, resulting in $\dot{J} > 0$. Section 3 proves that this can occur.

The rest of this section is written supposing for simplicity that r_t is constant (though more complicated cases can easily be treated with the results given above), and it is devoted to discussing Table 1.

Let G be defined as q times the difference between average profit and marginal profit, and for any function z_t, let an overbar as in $\bar{z}$ denote $\int_\tau^T z_t \,\mathrm{e}^{-r(t-\tau)}\,\mathrm{d}t$. For monopoly, assuming $\pi''(q) \leqslant 0$, so a profit-maximizing plan can exist, will imply $G \geqslant 0$. Similarly, for competition, assuming $C'' \geqslant 0$, so an equilibrium can exist, will imply $G = q(\pi/q - \pi') = qC'(q) - C(q) \geqslant 0$. Rows 1, 2, and the last line of row 5 have been shown by previous authors; the rest of the table is new. For proof of Table 1's results, see the Appendix.

By definition depreciation is $-\dot{J}$. Row 3 definitively characterizes the relation between depletion charges and depreciation, a topic which has interested writers as long ago as Marshall (1920/1962, p. 364) and as recently as Hartwick (1989, pp. 9–11, Appendix II). It is known that user cost times q is a charge for depletion, but row 3 shows that user cost times q is not necessarily equal to depreciation. [6]

Define the depletion rate as $-\dot{x}/x$ and the depreciation rate as $-\dot{J}/J$. Row 4 gives the difference in these two rates, which is the same as the appreciation rate of the 'stock price' $\rho_t = J_t/x_t$. This shows for the first time what $\dot{\rho}/\rho$ looks like in the general case. Many authors have shown that $\dot{\rho}/\rho = r$ in special cases, engendering confusion about the difference between: (i) '$\dot{\rho}/\rho = r$,' which pertains to stocks and which from Table 1 only holds if $C'' = 0$ (so $G \equiv 0$) and the firm is competitive; and (ii) the Hotelling Rule '$\dot{M\Pi}/M\Pi = r$,' which pertains to *flows* and which holds as long as $\partial C/\partial x = 0$, regardless of whether $C'' = 0$ or $C'' > 0$ and regardless of whether the firm is competitive or monopolistic. For example, Dasgupta and Heal (1979, Eq. (6.5)) call $\dot{\rho}/\rho = r$ 'the Hotelling Rule,' but there is nothing in Hotelling's paper about $\dot{\rho}$, only about $\dot{p}$ (i.e., $\dot{M\Pi}$). The Hotelling

[6] Hartwick almost completely determined the relationship between the depletion charge and depreciation for a competitive firm facing a constant price ($\dot{p}_t \equiv 0$). He did not consider the more difficult case of competitive equilibrium ($\dot{p}_t \not\equiv 0$) which I am discussing. See also footnote 7. Hartwick (1990, §2) treats the case of a linear objective function, again in an autonomous problem unlike that of a competitive firm in equilibrium.

Table 1

		Competition	Monopoly
1.	$\dot{M\Pi}/M\Pi$	$(\dot{\lambda}_\tau/\lambda_\tau)+r$ $=r$ [a]	$(\dot{\lambda}_\tau/\lambda_\tau)+r$ $=r$ [a]
2.	'User cost' (or 'rent') $\times q$	depletion charge	depletion charge
3.	Depreciation, $-\dot{J}_\tau=\pi_\tau-rJ_\tau$	depletion charge $-\overline{\dot{p}q}$ = depletion charge $-rJ_\tau+G_\tau$ $=(q_\tau/x_\tau-r)x_\tau M\Pi_\tau-\overline{q\dot{q}C''}$ [a]	depletion charge
4.	Depletion rate − depreciation rate = stock price appreciation rate, $\dot{\rho}_\tau/\rho_\tau$	$\dfrac{r+\overline{q\dot{q}C''}/(x_\tau M\Pi_\tau)+q_\tau\overline{G}/(x_\tau^2 M\Pi_\tau)}{1+\overline{G}/(x_\tau M\Pi_\tau)}$ [a]	$\dfrac{q_\tau\overline{G}/(x_\tau^2 M\Pi_\tau)}{1+\overline{G}/(x_\tau M\Pi_\tau)}$ [a]
5.	Mine value, J_τ	$(G_\tau+\overline{\dot{p}q})/r$ $=x_\tau M\Pi_\tau+\overline{G}$ [a]	G_τ/r $=x_\tau M\Pi_\tau+\overline{G}$ [a]
6.	Stock price − flow price, $\rho_\tau-p_\tau$	$-C'_\tau+\overline{G}/x_\tau$ [a]	$q_\tau(\mathrm{d}p(q_\tau)/\mathrm{d}q)-C'_\tau-\overline{G}/x_\tau$ [a]

[a] Requires $\partial C/\partial x=0$. Note that $\partial C/\partial x=\dot{\lambda}$.

Rule is the more fundamental of the two results, because in order to derive $\dot{\rho}/\rho$ in Table 1 we first had to use the Hotelling Rule.

It remains to explain why $\dot{\rho}/\rho = r$ in simple cases. When the firm is competitive and $C'' = 0$, in equilibrium the firm is indifferent between extracting or not. Suppose the firm does not extract. Then $\dot{J}_\tau = r_\tau J_\tau - \pi_\tau = r_\tau J_\tau$ and $\dot{x} = 0$, so $\rho = J/x$ implies $\dot{\rho}/\rho = r$. Solow (1974, p. 2) presented one of the first asset market arbitrage arguments of this sort, showing that if the firm is indifferent between holding all its resource stock and holding another asset then $\dot{\rho}/\rho = r$. However, the important point is that if $C'' > 0$ or if the firm is a monopolist then the firm is *not* indifferent between holding all its resource stock and holding another asset: to achieve asset market equilibrium it *strictly* prefers liquidating some of its resource stock, so that the mine produces dividends as well as capital gains. In these equilibria the simple asset market arbitrage argument fails, $\dot{x} \neq 0$, and consequently $\dot{\rho}/\rho$ does not equal r – but the fundamental Hotelling Rule $M\dot{\Pi}/M\Pi = r$ still holds.

Suppose the market demand curve does not shift through time. Then for a monopolist, price, instead of being an exogenous function of time, is an endogenous function of q. From this fundamental difference it follows that $\partial H/\partial t$ is simply $-\pi_t r_t \delta_t^{-1}$, and for the monopolist the analogue of (7) has only *two* terms, not three:

$$\dot{J}_\tau = -\lambda_\tau \delta_\tau q_\tau + \int_\tau^T \pi_t \delta_\tau \delta_t^{-1} (r_\tau - r_t)\, \mathrm{d}t. \tag{8}$$

If r_t is constant, assumed for the rest of this section, then $\dot{J}$ will always be negative for a monopolist, and $\dot{J}$ will simply equal user cost times $-q$. As reported in Table 1's row 5, $J_\tau = G_\tau / r$. (This is easily recognized as a consequence of Proposition 2: $G = \hat{H}$ because $\pi(q) = \hat{f}$, $\pi'(q) = \hat{\lambda}$, and $-q = \dot{x}$.) The interpretation is that at time τ a mine is, to a monopolist, like a perpetuity paying $\$G_\tau$ at all dates beyond τ. Furthermore, a monopolist's mine value can be calculated by an outside observer knowing nothing about the future and nothing about the current stock size, although the monopolist must know these things.

A monopolist's $\dot{\rho}/\rho$ is listed in row 4 of Table 1.

For a monopolist, $G \neq 0$ and therefore $\dot{\rho}/\rho \neq 0$. Unlike the competitive case, the expression for a monopolist's $\dot{\rho}/\rho$ bears no simple relation to r.

To describe resource extraction by a social planner, let $B(q_t)$ be the benefit of q_t tons of extraction. All the results for the monopolist's problem, except row 6, apply to the social planner's problem by replacing the monopolist's π with $B(q) - C(q)$, $M\Pi$ (user cost) with $B' - C'$, J with $J^{\mathrm{sp}} = \int_\tau^T (B_t - C_t) \mathrm{e}^{-r(t-\tau)}\, \mathrm{d}t$, and G with $G^{\mathrm{sp}} = q[(B - C)/q - (B' - C')]$, understanding that q_τ and x_τ now refer to the optimal values for a social planner. For the rest of this paper I will assume that B is the area under the demand curve. In that case the social planner's choice of q will coincide with that of a competitive industry, and row 6 becomes $\rho_\tau^{\text{sp}} - p_\tau^{\text{sp}} = -C'_\tau + \overline{G^{sp}}/x_\tau^{\text{sp}}$ because $B' = p^{\text{sp}}$. [7]

3. Appreciation of competitively owned mines

Section 2 established the fundamental expressions for mine depreciation; the next question is whether this depreciation is positive or negative. In particular, in Section 2 it was conjectured that *competitively* owned mines can appreciate in *current* value terms while they are being depleted. [8] To confirm this conjecture, suppose throughout this paragraph that r_t is constant and that $T = \infty$. From row 3 of Table 1, $\dot{J}_\tau = rJ_\tau - \pi_\tau$. Since π_τ can be written as $\int_\tau^\infty \pi_\tau r\,\mathrm{e}^{r(\tau-t)}\,\mathrm{d}t$ and since $J_\tau = \int_\tau^\infty \pi_t\,\mathrm{e}^{-r(t-\tau)}\,\mathrm{d}t$, it follows that $\dot{J}_\tau = r\,\mathrm{e}^{r\tau}\int_\tau^\infty(\pi_t - \pi_\tau)\mathrm{e}^{-rt}\,\mathrm{d}t$. Therefore

Proposition 3. Given a fixed time τ, if π_t is monotonically increasing (decreasing, constant) for all $t > \tau$, then $\dot{J}_\tau > 0$ ($\dot{J}_\tau < 0$, $\dot{J}_\tau = 0$).

Corollary. Given a fixed time τ, if π_t is monotonically increasing (decreasing, constant) for all $t > \tau$, then $\dot{J}_t > 0$ ($\dot{J}_t < 0$, $\dot{J}_t = 0$) for all $t > \tau$.

Proof. Suppose Proposition 3's premise holds and let t_1 be any date greater than τ. Then π_t is monotonically increasing (decreasing, constant) for all $t > t_1$. Hence from Proposition 3, $\dot{J}_{t_1} > 0$ ($\dot{J}_{t_1} < 0$, $\dot{J}_{t_1} = 0$). Since t_1 is an arbitrary date greater than τ, the corollary follows. ■

Suppose industry equilibrium obtains with all N firms being identical and behaving identically (say, because $C'' > 0$). Let the inverse demand curve be $\phi(Nq)$. Then the profit of each firm is $\pi = q\phi(Nq) - C(q)$ and $\dot{\pi} = \dot{q}\phi[(1 + (1/\epsilon)) - C'/\phi]$ where ϵ is the elasticity of the demand curve. Suppose $\dot{q} < 0$, as is the case when the demand curve and cost function do not shift with time. [9]

[7] Using the social planner's first-order condition $B'(q_t) - C'(q_t) = \lambda\,\mathrm{e}^{rt}$ (which is the same as the competitive firm's since $B' = p$), Proposition 1 gives $\dot{V}_t = -(B' - C')q_t\mathrm{e}^{-rt} - rV_t$. Eq. (8) gives $\dot{J}_t = -(B' - C')q_t$. Hartwick's work (1989, p. 139) was an extremely important advance because it was the first time an author realized that it might be possible, and that it would be useful, to rigorously construct a formula connecting mine depreciation and user cost. Furthermore, Hartwick's analysis depends on his Proposition I (Hartwick, 1989, p. 139), which states that $\dot{V}_t = -(B' - C')q_t$. Given what was shown in the last paragraph, this pioneering formula is almost correct.

[8] Mines cannot appreciate in *present* value terms because $\dot{V}_\tau = -\pi_\tau\,\mathrm{e}^{-r\tau} < 0$ – see footnote 15 – nor, from (8), can a *monopolist's* $\dot{J}$ be positive when r is constant. Accountants keep books in *current* value terms.

[9] If $C'' = 0$ (item 2 below) this discussion goes through without assuming identical behavior of firms if one replaces π with industry profit $\Sigma_i \pi^i$, replaces q with industry output $\Sigma_i q_i$, and replaces J with $\Sigma_i J^i$ (i is the index for firms).

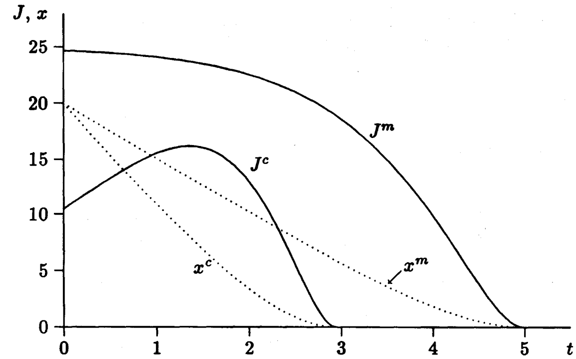

Fig. 1.

1. If the demand curve is inelastic for all sufficiently small q, then $1+(1/\epsilon)<0$, $\dot{\pi}_t>0$, and $\dot{J}_t>0$ from Proposition 3, for all correspondingly large t.
2. Suppose $C \equiv 0$. If, for all sufficiently small q, the demand curve is (inelastic, unitary elastic, elastic), then ($\dot{J}_t>0$, $\dot{J}_t=0$, $\dot{J}_t<0$) for all correspondingly large t.

It follows that $\dot{J}$ will be strictly positive forever for competitive mining firms facing everywhere-inelastic demand curves. That happens because total revenue increases as depletion proceeds. If average costs are constant then, because from Table 1 $\dot{J}$ for a competitive firm is $(r-q/x))x\,M\Pi$, whether J rises or falls depends on whether the stock is being *physically* depleted at a rate that is less than or greater than the interest rate – an unprecedented but very intuitively appealing result.

Comparing across market structures, a slower rate of depletion can even correspond to a faster rate of depreciation. With a linear demand curve, a competitive industry will have a *faster* rate of depletion than a monopolist beginning with the same stock size, demand curve, and cost function. However, there are examples, like that in Fig. 1, where the competitive industry is experiencing *appreciation* at small values of t (when it produces on the inelastic part of the demand curve), while the monopolist is experiencing depreciation then. Fig. 1 compares the competitive and monopoly cases for the problem with $r=1$, $C \equiv 0$, $S=20$, and demand curve $p=10-Q$. [10]

While equilibrium marginal profit always increases at the rate of interest, equilibrium *total* profit correspondingly decreases (assuming convexity). For some mines not only is profit falling, but capital gains are negative. In spite of

[10] For the competitive industry, $T \approx 2.947$, $Q_t=10-10\mathrm{e}^{(t-T)}$, $J_t=100\mathrm{e}^{t-T}[T-t-1+\mathrm{e}^{t-T}]$, and $x_t=S-10t+10\mathrm{e}^{t-T}-10\mathrm{e}^{-T}$. For the monopolist, $T \approx 4.993$, $Q_t=5-5\mathrm{e}^{t-T}$, $J_t=25[1+\mathrm{e}^{2(t-T)}-2\mathrm{e}^{t-T}]$, and $x_t=S-5t+5\mathrm{e}^{t-T}-5\mathrm{e}^{-T}$.

this, the definition of mine value J ensures that the rate of return on such mines $((\dot{J}+\pi)/J)$ always equals r.

4. Mine depreciation in the national income accounts

Section 1 provided the mathematical basis for the analysis of mine depreciation which was carried out in Section 2 and qualitatively analyzed in Section 3. Here I treat the implications of these results for national income accounting.

National income accounting may be criticized for the following reason. Except when the economy is 'autonomous except for geometric discounting,' the first corollary of Proposition 2 implies that NNP in general fails to be "the stationary equivalent of future consumption [or utility], and this is its primary welfare interpretation" (Weitzman, 1976, p. 160). The methods I describe for adjusting NNP for mine depreciation are equally valid for adjusting any similar welfare measure for mine depreciation.

The contribution of a mine to gross national product is measured either by factor payments, $C+\pi$, or equivalently by the value of output, pq. To find the contribution of a mine to net national product, depreciation has to be subtracted from both sides of the balance sheet. One question is whether depreciation should be measured from the social planner's point of view, in which case it would equal $q_t^{\mathrm{sp}}[p_t^{\mathrm{sp}}-C'(q_t^{\mathrm{sp}})]>0$ as explained at the end of Section 2, or whether depreciation should be measured from a firm's point of view, in which case from Table 1 it would equal $q_t[p_t-C'(q_t)]$ *minus* the adjustment for price increases, $\overline{\dot{p}q}$ (assuming competition). The correct viewpoint for national income accounting is that of the social planner: future price increases are the result of increasing scarcity, which may make firms better off but does not make society better off. The depreciation deduction in the national accounts should therefore be $q_t^{\mathrm{sp}}[p_t^{\mathrm{sp}}-C'(q_t^{\mathrm{sp}})]$, which under ideal conditions would equal competitive firms' $q_t M\Pi_t$. If the observed market q_t is not equal to q_t^{sp}, then a reasonable depreciation charge would be $q_t[p_t^{\mathrm{sp}}-C'(q_t^{\mathrm{sp}})]$, which exceeds or falls short of the social planner's charge as q_t exceeds or falls short of q_t^{sp}.

Traditional national income accounting sets mine depreciation not at $q_t[p_t^{\mathrm{sp}}-C'(q_t^{\mathrm{sp}})]$ but at zero. [11] Carson et al. (1994b, pp. 54–57) report that the U.S. Commerce Department's new 'Integrated Economic and Environmental Satellite Accounts' value mineral resources in five different ways. 'Current Rent Method I' sets mine depreciation at π. [12] This overestimates depreciation by $\pi - qM\Pi = G$;

[11] That would be correct only if there were no economically exhaustible resources, so that extraction should be pushed all the way to the point when price (marginal benefit) equals marginal cost.

[12] In Carson et al.'s notation, $DEPL = RR = TR - COE - (rNS + DEP)$, meaning "depletion equals resource rent equals total revenue minus variable cost minus flow cost of manmade capital (namely the rental rate of capital times the net stock of capital valued at replacement cost, plus the depreciation of manmade capital)."

if the cost function is strictly convex then $G > 0$ and if it is linear then $G = 0$. Of the other four methods of calculating mine depreciation, 'Current Rent Method II' is similar to 'Current Rent Method I,' the 'Transactions Price Method' is similar to the U.S. Treasury Department's method described in the next paragraph, and the remaining two methods have no simple relation to the techniques studied in this paper.

In both Hotelling's time (1931, §14) and now (Miller and Upton, 1985, p. 12), the U.S. Treasury Department's Internal Revenue Service allows the owner of a *newly purchased* mine to take a 'depletion allowance' (really a depreciation allowance) for income tax purposes of $(J/x)q = \rho q$. [13] The correct depreciation allowance $-\dot{J}_\tau$ is $\pi_\tau - rJ_\tau = qM\Pi - \overline{\dot{p}q}$. Using row 5 of Table 1, if $\partial C/\partial x = 0$ then this means that the permitted depreciation allowance is too large by $\overline{\dot{p}q} + \overline{G}q/x = rJ - G + \overline{G}q/x$, which is positive in equilibrium and which simplifies to rJ under constant average costs. Determining whether the Treasury's method overstates or understates the allowance for mines purchased in the past would take us too far afield.

It should be pointed out that since (assuming ideal conditions) correct depreciation from a social planner's viewpoint is $qM\Pi$, if $\partial C/\partial x = 0$ then the tax code's initial permitted allowance of $q\rho$ overstates 'depreciation from the *planner's* viewpoint' only by $\overline{G}q/x$, which is zero (meaning $M\Pi = \rho$) when average costs are constant.

Having found the correct depreciation charge for a mine also theoretically solves the problem of how to ensure that the income of a mining-based economy is forever constant (i.e., 'sustainable'). Laying aside money equal to the depreciation charge $-\dot{J}$ in a sinking fund earning interest at a rate of r would result in maintaining a constant flow of income. To prove this, note that because interest earned by the sinking fund is withdrawn the moment it is earned, the value of the sinking fund at date τ is $\int_0^\tau -\dot{J}_t \, dt = J_0 - J_\tau$. Earnings from the sinking fund are therefore $r[J_0 - J_\tau]$, earnings from the mine are $\pi_\tau + \dot{J}_\tau = rJ_\tau$, so total earnings equal the constant rJ_0 for all τ. [14] Even if the mining industry is made up of perfectly competitive firms extracting just as a social planner would (so $\dot{J}$ for the social planner is equal to $qM\Pi$), the sinking fund accumulated by firms will be smaller than what would be accumulated by a social planner. This is because $-\dot{J}$ for firms is $\overline{\dot{p}q}$ smaller than $-\dot{J}$ for the social planner, who is trying to keep producer *plus* consumer surplus, not just producer surplus, intact. Clearly 'sus-

[13] See also Hartwick (1989, p. 116).

[14] In the context of macroeconomic growth models, the original 'Hartwick's Rule' states that one should invest all resource rents to keep consumption constant (see Hartwick, 1977). The rule in this paper is not identical to Hartwick's Rule because it recommends investing depreciation (which is $-\dot{J}$), not q times rent (i.e. q times user cost), into the sinking fund; for a planner the rules coincide, but for competitive firms they do not. Hartwick's Rule, even in its abstract form derived in Section 1, only applies to autonomous problems, and as pointed out earlier, a competitive firm's problem is not autonomous.

tainable development' is impossible unless there exist investments whose rate of return is at least equal to the rate of discount r. If no such investments exist, the firm is discounting at a rate higher than the opportunity cost of funds.

Two controversies remain to be discussed. First, even not counting the income from the sinking fund, deducting depreciation from profits will *not*, contrary to some influential national income statisticians, leave a firm with zero net income (or zero taxable income), because the firm makes the going rate of return: $\pi - (-\dot{J}) = rJ > 0$. Since $\pi - (-\dot{J}) = \overline{\dot{p}q} + G \geqslant \overline{\dot{p}q} > 0$ for a competitive firm and $\pi - (-\dot{J}) = G > 0$ for a monopolist, only if $G = 0$, firms are competitive, and depreciation is improperly calculated by including capital losses $-q_\tau M\Pi_\tau$ but not capital gains $\overline{\dot{p}q}$ is it true that "the depreciation approach ... would wipe out from the net product the entire proceeds from natural resource sales," as El Serafy and Lutz put it (1989, p. 4) (see also Hotelling, 1931, pp. 170–171).[15]

Finally, there is controversy concerning the applicability of these results to situations in which the mineral deposits are heterogeneous. In such cases, $\partial C/\partial x < 0$, so as the stock shrinks, worse deposits are mined and costs rise. I claim that, with the above-noted exception of the formulas for initial U.S. Treasury Department overestimation of mine depreciation, all of the results of this section hold even if the mineral deposits are heterogeneous. In particular, contrary to some authors, the depletion charge consists of q times the *entire* user cost ('resource rent'), not q times just a part of user cost. It is true that "if exhaustible resources are heterogeneous in quality, part of the rent is a dynamic or Hotelling rent and part is a differential rent" (Hartwick, 1989, p. 116; see Hartwick, 1982). However, both kinds of rent user costs enter into the depletion charge: the loss of deposits earning large differential rents is just as great a diminution of wealth as loss of deposits earning large Hotelling (or 'scarcity') rents.

While the focus of this paper has been on mineral depletion, unanticipated mineral discoveries should be counted in national income accounts as additional investments (negative depreciation) since they entail an unanticipated upward jump in V (see Mäler's (1991, pp. 12–13) concurring opinion). However, a complete analysis of depreciation in a model that allows for investment in discovering new resources remains to be accomplished.

Appendix

Proof of Proposition 1. By definition, $H = f + \lambda g$, so $H_\tau^* = f_\tau^* + \lambda_\tau^* g_\tau^*$ and thus $-f_\tau^* = \lambda_\tau^* g_\tau^* - H_\tau^*$. By Leibnitz' Rule, $\dot{\mathscr{V}}_\tau = -f_\tau$ for any control, and so $\dot{\mathscr{V}}_\tau^* = -f_\tau^*$. Therefore $\dot{\mathscr{V}}_\tau^* = \lambda_\tau^* g_\tau^* - H_\tau^*$. Recalling that $\dot{\mathscr{V}}_\tau^* = \dot{V}_\tau$, one has

[15] 'Present value depreciation' could be defined as $-\dot{V}_\tau = \pi_\tau e^{-r\tau}$ (see footnote 3). Deducting 'present value depreciation' from the present value of profit would indeed wipe out from net product the entire proceeds from natural resource sales. But natural resources are not unique in this regard. Any capital good yields 'present value dividends' of $\pi_\tau e^{-r\tau}$ and has 'present value depreciation' of $-\dot{V}_\tau = \pi_\tau e^{-r\tau}$, so it yields a 'present value net income' of zero to its owner.

$\dot{V}_\tau = \lambda_\tau^* g_\tau^* - H_\tau^*$. It only remains to show that $-H_\tau^* = \int_\tau^T \partial H/\partial t\,|_* \,\mathrm{d}t - H_T^*$.

Clearly $\mathrm{d}H/\mathrm{d}t = (\partial H/\partial x)\dot{x} + (\partial H/\partial u)\dot{u} + (\partial H/\partial \lambda)\dot{\lambda} + \partial H/\partial t$. Evaluate this at the optimal solution. If u_t^* is differentiable for all $t > \tau$, use the canonical equation $\partial H/\partial x\,|_* = -\dot{\lambda}^*$, the definitions $\dot{x} = g$ and $\partial H/\partial \lambda = g$, and the fact that maximizing the Hamiltonian implies either $\partial H/\partial u\,|_* = 0$ (an interior solution) or $\dot{u}^* = 0$ (a boundary solution with U having no time dependence and u being continuous). The result is that $\mathrm{d}H/\mathrm{d}t\,|_* = \partial H/\partial t\,|_*$ (see Intriligator, 1971, p. 350). If u_t^* is not differentiable for all $t > \tau$, the conclusion $\mathrm{d}H/\mathrm{d}t\,|_* = \partial H/\partial t\,|_*$ still follows under condition (i) of Proposition 1 (see Seierstad and Sydsæter, 1987, p. 86 note 3). In both cases, then, $H_T^* - H_\tau^* = \int_\tau^T \partial H/\partial t\,|_* \,\mathrm{d}t$. Solve for $-H_\tau^*$. ■

Proof of Table 1.

- Row 1: Differentiate the Hotelling Rule $M\Pi_\tau = \lambda_\tau \delta_\tau$ with respect to time.
- Row 2: Substitute the Hotelling Rule $M\Pi_\tau = \lambda_\tau \delta_\tau$ into the definition of the depletion charge $(\lambda_\tau \delta_\tau q_\tau)$ and use the traditional definition of 'user cost' or 'rent,' which is $M\Pi_\tau$.
- Row 5, first line for competition: From $\dot{J}_\tau = -\pi_\tau + rJ_\tau$ (see (3)), (7), and the Hotelling Rule $M\Pi_\tau = \lambda_\tau \delta_\tau$, one has $rJ = \pi - qM\Pi + \overline{\dot{p}q}$. Use the definition of G.
- Row 5, second line for competition: (This is an extension of Miller and Upton's 'Hotelling Valuation Principle' (1985, II).) Because for competition $G = qC'(q) - C(q)$ as mentioned in the text, $C(q) = qC'(q) - G$. Also, from the Hotelling Rule $M\Pi_t = \lambda\delta_t$ one has $M\Pi_t = M\Pi_\tau(\delta_t/\delta_\tau)$. These relationships enable one to write $J_\tau = \int_\tau^T [p_t q_t - q_t C'(q_t)] \delta_\tau \delta t^{-1}\,\mathrm{d}t + \int_\tau^T G_t \delta_\tau \delta_t^{-1}\,\mathrm{d}t$ $= \int_\tau^T M\Pi_t q_t \delta_\tau \delta_t^{-1}\,\mathrm{d}t + \overline{G} = \int_\tau^T [(M\Pi_\tau/\delta_\tau)\delta_t] q_t \delta_\tau \delta_t^{-1}\,\mathrm{d}t + \overline{G} = M\Pi_\tau \int_\tau^T q_t\,\mathrm{d}t$ $+ \overline{G}$.
- Row 3, left-most column: Since (3) holds for any $\mathscr{J}$, it must hold for that corresponding to the optimal path, which is J.
- Row 3, first line for competition: Eq. (7), the Hotelling Rule, and row 2.
- Row 3, second line for competition: Row 5, first entry.
- Row 3, third line for competition: Differentiating the Hotelling Rule $p_t = \lambda \mathrm{e}^{rt} + C'_t$ yields $\dot{p}_t = \lambda r\,\mathrm{e}^{rt} + \dot{q}_t C''_t$. This and $\int_\tau^T q_t\,\mathrm{d}t = x_\tau$ imply $\int_\tau^T \dot{p}_t q_t \mathrm{e}^{-rt}\,\mathrm{d}t = \lambda r x_\tau + \int_\tau^T q_t \dot{q}_t C''_t\,\mathrm{e}^{-rt}\,\mathrm{d}t$ and $\overline{\dot{p}q} = \lambda r x_\tau\,\mathrm{e}^{r\tau} + \overline{q\dot{q}C''} = M\Pi_\tau r x_\tau + \overline{q\dot{q}C''}$. Substitute this into the following expression derived from row 2 and row 3's first entry: $-\dot{J}_\tau = q_\tau M\Pi_\tau - \overline{\dot{p}q}$.
- Row 4, left-most column: Since the depletion rate is $-\dot{x}/x$, the depreciation rate is $-\dot{J}/J$, and ρ_t is J_t/x_t by definition, the difference between the depletion rate and the depreciation rate is $(-\dot{x}/x) - (-\dot{J}/J) = \dot{\rho}/\rho$.
- Row 4, competition: The numerator of the first term on the right-hand side of the following expression comes from row 3's third entry, while the denominator of that term comes from row 5's second entry:

$$\frac{\dot{J}}{J} - \frac{\dot{x}}{x} = \frac{xrM\Pi - qM\Pi + \overline{q\dot{q}C''}}{xM\Pi + \overline{G}} - \frac{\dot{x}}{x}.$$

Simplify and use $q = -\dot{x}$.

- Row 6, competition: Combine row 5's second entry (i.e., $\rho_\tau = J_\tau / x_\tau = M\Pi_\tau + \overline{G}/x_\tau$) with $M\Pi_\tau = p_\tau - C'_\tau$.
- Row 5, first line for monopoly: Replace (7) with (8) in the proof of the analogous result for competition.
- Row 5, second line for monopoly: Since $\pi = G + qM\Pi$, $J_\tau = \int_\tau^T [q_t M\Pi_t] \delta_\tau \delta_t^{-1}\,\mathrm{d}t + \int_\tau^T [G_t] \delta_\tau \delta_t^{-1}\,\mathrm{d}t$ as in the competitive case.
- Row 3, monopoly: Eq. (8), the Hotelling Rule, and row 2.
- Row 4, monopoly: The numerator of the first term on the right-hand side of the following expression comes from rows 2 and 3, while the denominator of that term comes from row 5's second entry:

$$\frac{\dot{J}}{J} - \frac{\dot{x}}{x} = \frac{-qM\Pi}{xM\Pi + \overline{G}} - \frac{\dot{x}}{x}.$$

  Simplify and use $q = -\dot{x}$.
- Row 6, monopoly: Combine row 5's second entry (i.e., $\rho_\tau = J_\tau / x_\tau = M\Pi_\tau + \overline{G}/x_\tau$) with $M\Pi = MR - MC = p'(q) \cdot q + p - C'$.

■

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