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Modeling spatial dimensions of housing prices in Milwaukee, WI

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Abstract. In this study we investigate spatial dimensions of housing-market dynamics in the City of Milwaukee by modeling the determinants of housing prices. From the 2003 Master Property data file of the city, two sets of owner-occupied single-family houses were randomly selected (one to construct the models, and the other to rest the models). Besides conventional housing attributes, remote-sensing information, in particular the fractions of soil and impervious surface representing degraded neighborhood environment conditions, is added to improve the model. Spatial regression and geographically weighted regression approaches are employed to examine spatial dependence and heterogeneity. Results reveal that these spatial models tend to perform better, especially in terms of model performance and predictive accuracy, than the ordinary least squares estimates.

1 Introduction

The hedonic housing price model is a powerful econometric tool for capturing important determinants of prices/housing values regarding structural and locational (neighborhood) attributes, and has been widely used in housing and urban studies. Serving as a "joint-envelope of a family of value equations (consumers' preferences) and another family of offer functions (suppliers' technologies)" (Rosen, 1974, page 44), the hedonic model establishes a formal relationship between housing values/prices and a set of housing attributes (the quantity and qualities embodied in housing). Usually, housing attributes contain not only structural attributes such as floor size, but also locational or neighborhood conditions, such as proximity to certain public facilities. The model is appealing in that the implicit price of various housing attributes can be estimated from the model. Traditionally, the regression is calibrated through the ordinary least squares (OLS) estimator, under the general assumption of independent observation. However, despite the mature OLS technology and its wide application in examining the relationships between housing prices and attributes (for a review see Can, 1992), the full potential of the hedonic model remains to be exploited (Ekeland et al, 2004), and locational attributes in particular have drawn inadequate attention (Orford, 2002).

During the late 1980s and early 1990s, largely due to the advancement in spatial statistics and spatial econometrics (eg Anselin, 1988; Cliff and Ord, 1981; Griffith, 1988; Upton and Fingleton, 1985), studies on the hedonic model explicitly took into account the inherent spatial characteristics of housing data—namely, spatial autocorrelation and heterogeneity. Pioneered by Dubin (1988) and Can (1990; 1992), a large body of literature has emerged to address the spatial effects or to apply spatial techniques in modeling housing prices (eg Basu and Thibodeau, 1998; Bowen et al, 2001; Dubin, 1998; Fotheringham et al, 2002; Kelejian and Prucha, 1998; Militino et al, 2004; Pace et al, 1998). Can (1990, page 254; 1992) advances the concept that "the influence of various housing attributes on housing prices is characterized by spatial variability", and applies Cassetti's (1972; 1986) expansion method in her modeling scheme, trying to capture possible spatially varying influences of various housing attributes on housing prices. Through the construction of a neighborhood quality index based on nine substantive neighborhood characteristics and a principal component analysis technique, her spatial expansion model produces quite attractive results when compared with the OLS counterpart (Can, 1990; 1992).

In this study we intend to achieve three objectives, using the City of Milwaukee's master property dataset. First, we examine the influences of spatial autocorrelation on housing-market modeling by applying two types of spatial autoregressive models. Second, we intend to advance the understanding of spatial variable relationships between housing attributes and housing prices/values by applying a newly developed spatial data analysis technique, the geographically weighted regression (Fotheringham et al, 2002). Third, to assess the performance and predictability of the hedonic models incorporating spatial information, we investigate the prediction accuracy of the models using a testing dataset that is different from the dataset used to construct the models. After this introduction the next section reviews the research background of the hedonic model and studies addressing spatial effects. The third section discusses the study area and data. Section 4 presents specifications of the OLS model, spatial autoregressive models, and the geographically weighted regression. Analytical results are discussed in section 5, and the paper concludes with a summary and future research foci.

2 Theoretical background

2.1 The hedonic price model

The hedonic price model is based on the hedonic hypothesis that goods are valued for their utility-bearing attributes or characteristics. In his classical text, Rosen (1974) states that the hedonic price model is determined by a set of choices made by consumers and producers under the market clearing conditions. Although a house is a special commodity with bundles of attributes which cannot be traded separately, it has multiple qualities, such as lot size, improvement, neighborhood, accessibility, proximity externalities, land used, and time (eg Basu and Thibodeau, 1998). Housing attributes have traditionally been divided into structural, and neighborhood or locational attributes (Can, 1992; Orford, 2002). Housing structural attributes refer to the structure of the property, such as lot size and improvement. Neighborhood or locational attributes include all externalities associated with the geographic location of the house, such as accessibility, proximity externalities, environmental amenities, and land-use information.

Under such generalization, and at market equilibrium, a hedonic model can be formally expressed as:

$$\mathbf{P}(\mathbf{H}) = \mathrm{f}(S, N) + \varepsilon \, , \tag{1}$$

where $\mathbf{P}(\mathbf{H})$ is a matrix containing housing prices/values, $\mathrm{f}(S, N)$ is a functional form with structural (S) and neighborhood (N) attributes, and ε is the residual term. In a linear form the marginal prices of various housing attributes are hence identified as the partial differentiation of corresponding attributes. This marginal price is usually referred to as the hedonic or implicit price of the attributes (Rosen, 1974). This notion also constitutes the theoretical background of the hedonic regression technique.

An extensive literature has emerged to test and to expand the hedonic model. The research focuses mainly on identifying the variables to represent housing attributes and the correct functional forms (Can, 1992; Mulligan et al, 2002). However, the full potential of the hedonic model remains to be exploited (Ekeland et al, 2004). Given the traditional neglect of locational externalities by econometric models and the methodological constraints in incorporating spatial information, studies on locational effects have drawn inadequate attention (Orford, 2002).

2.2 Spatial effects

It is well known that, when analyzing geographical phenomena and cross-sectional data, geographic location plays an important role in the occurrence of spatial effects—that is, spatial autocorrelation and heterogeneity. As defined in Anselin (2001), spatial autocorrelation is referred to as the 'coincidence of value similarity with locational similarity'. In housing markets it means that houses at nearby locations tend to have similar prices. This indeed describes how the metropolitan housing markets operate. First, with a nearby location, homeowners tend to follow their neighbors' improvement activities, which result in similar dwelling size, vintage, designs, and other structural characteristics. Second, spatial autocorrelation arises from the shared locational amenities of houses in the nearby location and neighborhood (Basu and Thibodeau, 1998; Militino et al, 2004), such as school districts, police stations, green space, transportation nodes, shopping centers, and other facilities. Last, realtors or property assessors tend to evaluate the value of houses by referring to the neighborhood conditions, an activity which also results in similar housing values in the nearby locations.

The existence of spatial autocorrelation in housing prices/values violates the standard assumptions of independence of observations in a traditional OLS regression estimator. The OLS estimation becomes biased and/or inconsistent; hence the estimated coefficients (the implicit or hedonic prices of the attributes) might be incorrect or misleading (Anselin, 1988). Spatial statistical and econometric techniques such as spatial autoregressive analysis and geostatistical models have been developed to address this concern (Anselin, 1988; Bowen et al, 2001; Can, 1990; 1992; Dubin, 1998; Kelejian and Prucha, 1998; Militino et al, 2004). By explicitly incorporating the spatial autocorrelation information in model construction, these models tend to eliminate the spatial effects on the coefficients. This research will focus on applying spatial autoregressive techniques and incorporating the spatial autocorrelation in modeling housing hedonic prices. In particular, we will consider both the spatial lag and error autoregressive models, and examine their performance and predictive accuracy.

Spatial heterogeneity, on the other hand, refers to a nonstationary process over space. Simply put, it describes a housing-market operational process in which the same set of housing attributes may yield different housing prices in different parts of the studied region (Bailey and Gatrell, 1995; Fik et al, 2003; Fotheringham et al, 2002; Théirault et al, 2003). As observed by many scholars (Adair et al, 1996; Can, 1990; Goodman and Thibodeau, 1998; Maclennean and Tu, 1996; Orford, 2000; Watkins, 2001), the traditional hedonic price model is based upon the theory of a unitary housing market functioning in instantaneous equilibrium. However, modern metropolitan housing markets are very likely composed of many submarkets, and can be characterized by functional disequilibrium and segmentation (Case and Mayer, 1996; Goodman and Thibodeau, 1998; Knox, 1995; Straszheim, 1975), as the supply and demand of the housing bundle tend to be inelastic (Adair et al, 1996). Some of the housing attributes, such as building areas, environmental amenities, and neighborhoods, may not be substitutable. For instance, Schnare and Struyk (1976) argue that housing-market segmentation occurs when demands for a particular structural or neighborhood characteristic are shared by a relatively large number of households. In addition, as argued by Can (1990), urban neighborhood structures are quite diversified, and such diversity has a significant impact on consumers' valuation of housing attributes on housing prices, which also leads to market segmentation. A direct consequence of market segmentation is a nonstationary housing market. As a result, a stationary model ignores the operational processes and structures that can lead to the disequilibrium in housing supply demand (Orford, 2000), which in turn can lead to biased or misleading parameter estimates of the hedonic model.

Although researchers in general agree with the existence of urban housing submarkets, empirical studies differ regarding how submarkets are specified, and hence how spatial heterogeneity is treated. On the methodological side, a multilevel (or hierarchical) modeling technique is often employed (Goodman and Thibodeau, 1998; Jones and Bullen, 1994; Orford, 2000; 2002). In such analyses, although the analysts are aware of the spatial heterogeneity in the housing market, it is assumed that the exact pattern of heterogeneity is known. Hence a discrete set of boundaries is implicitly imposed to identify submarkets. These empirical analyses have yielded a large body of insightful results, which are generally reported to perform better than the OLS estimator in terms of both data fitting and prediction. However, from a methodological point of view, the multilevel modeling scheme might seem to be too arbitrary in delineating housing submarkets with distinct boundaries, since spatial heterogeneity in the housing market more likely results from a continuous process (Fotheringham et al, 2002). To address the spatial heterogeneity in the housing market, Can (1990) adopts an expansion method in which regression parameters drift on a set of substantive variables that are deemed to account for the extent of spatial variation. The expansion method, though important in capturing spatial heterogeneity directly in the model, does have some limitations. One is that the form of the expansion equations needs to be assumed a priori. Though many forms are feasible, linear forms are often used in the literature. However, since the underlying spatial process generating the spatial heterogeneity is unknown, the assumption of a linear contextual drift seems arbitrary. A second limitation results from the use of the substantive variables to account for spatial variation, which might not be readily available, although Can (1990) uses nine variables and a principal component analysis to generate a composite neighborhood quality index. Furthermore, in practice, it is hard to tell whether those substantive variables well represent the spatial variation.

In light of the above analysis, this research advances the conceptual idea of spatial nonstationarity in the housing market through a geographically weighted regression (GWR) analysis (Fotheringham et al, 2002). Instead of requiring a priori knowledge of housing market segmentation structure or the exact form of the process that generates spatial heterogeneity, GWR incorporates geographic locations in the analysis following the general 'first law of geography', which states that closer things are more related with one another than things farther away (Tobler, 1970). In so doing, GWR accounts for spatial variation in a more intuitive way.

3 The study area and data

This study uses data from the City of Milwaukee, Wisconsin, which is located on the western shore of Lake Michigan (figure 1). The city grew rapidly during the early 20th century, and formed its current urban shape in the late 1950s. After the completion of the current highway network in the late 1960s and early 1970s, property development in the city stepped into a relatively stable period.

Two types of data are elected for this study. The first is housing structural attribute data that are extracted from the 2003 Master Property (MPROP) data file of the city. The MPROP data file has around 160 000 entries of all real properties within the city boundary. Each entry contains more than 80 various attributes, including a

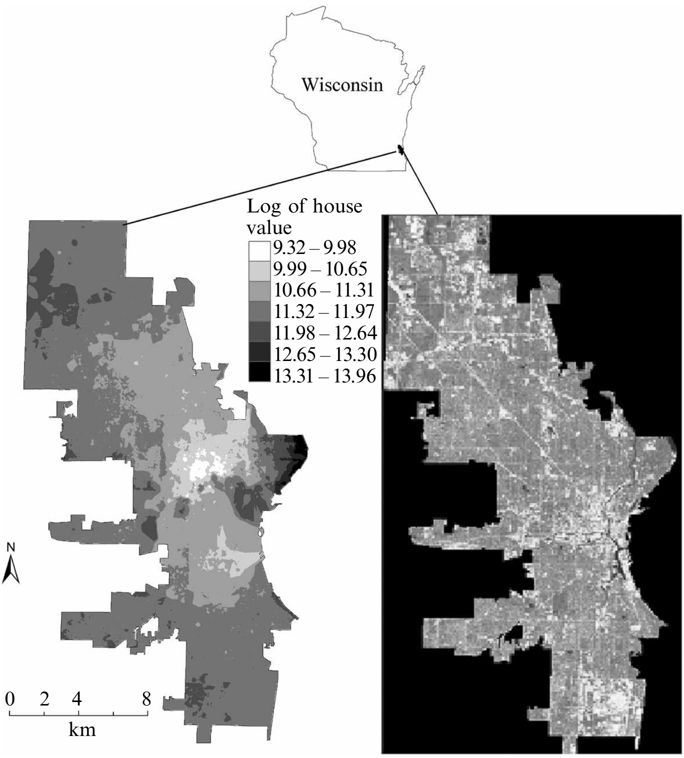

Figure 1. Location of the City of Milwaukee, and spatial distribution of house values.

house's location, assessed value, and housing structural characteristics (Kim, 2003; Luo and Wei, 2004; Yu and Wu, 2006). In this study we only focus our analyses on the owner-occupied single-family houses, which have 68 728 records.

Although it is very attractive to utilize all 68 728 records in building the hedonic price model, the computation costs for both spatial autoregressive regression and GWR models are too prohibitive [actually, in an attempt to include 10% of the records in our model, the GWR code written in R (http://www.r-project.org) failed to run on a PC with double 3.0 GHz CPU, and 2 GB of RAM]. In addition, since this research focuses on incorporating spatial effects in the hedonic price model and comparing the performance of various modeling techniques, we adopt a random sampling procedure with the selection of two sets of samples covering the entire geographic area of the city. Balancing computational complexity and modeling accuracy, we select a set of 1821 random samples for model construction, and another nonoverlapping set of 1822 random samples to evaluate the models' performance and predictive capability. This sampling strategy allows us to have 99% confidence that the sample means will be within a 3% marginal error from the population (Lenth, 2001). The subset operation is accomplished using the ArcGIS's Geostatistical Analyst (ESRI, Redlands, CA) extension to ensure that the sample covers the entire study area.

In addition, we notice from the MPROP data file that only assessed housing values, instead of sale prices, are recorded. Nonetheless, according to Wisconsin law, the assessed value and the market value of a house cannot vary by more than 10%. Such a difference might still pose a problem in estimating the hedonic models. However, as pointed out by Case (1978), the assessment quality is highly related to assessing frequency. In the City of Milwaukee the property valuation is conducted annually in order to reduce assessment errors. According to the City of Milwaukee's 2004 Plan and Budget Summary (Soika and Czarnezki, 2004), the coefficient of dispersion, an important measurement of assessment performance, is estimated to be at a 9% level for 2003 (Soika and Czarnezki, 2004, page 46), which is well within the excellent equity per industrial standards. We hence deem it reasonable to approximate the housing market values by the assessed values. A surface created from the logarithm-transformed housing values is presented in figure 1, which indicates that the housing market in Milwaukee has clear spatial patterns. Specifically, houses in the suburban areas and the lakeside (central east side of the city) tend to be more expensive, whilst housing values on the west side of the Milwaukee River are amongst the lowest (figure 1).

Aside from the assessed housing value, five housing structural attributes were identified and retrieved from the MPROP data file. In particular, we chose one dummy variable, AirCd, which indicates whether central air conditioners are present; one discrete variable, FirePlc, which indicates how many fireplaces are in the house; and three continuous variables—floor size (FlSize), number of bathrooms (NofBath), and house age (HsAge)—to construct the hedonic model. Other variables, including the number of bedrooms, number of stories, lot area, and garage type are also considered in our preliminary analysis. However, we discovered serious multicollinearity problems between the number of bathrooms and the number of bedrooms, and among the building area, lot area, and number of stories. The garage type, on the other hand, does not seem to have a significant impact on housing value. Intuitively, and based on the literature (Kim, 2003; Yu and Wu, 2006), AirCd, FirePlc, FlSize, and NofBath are hypothesized to be positively associated with the housing value, whereas HsAge is hypothesized to be negatively associated.

Recent literature has expanded traditional neighborhood socioeconomic attributes to include neighborhood environmental attributes in the hedonic model (Decker et al, 2005; Geoghegan et al, 1997; Kestens et al, 2004). The second type of data in this study was generated from a Landsat ETM+ image (see figure 1) acquired on 9 July 2001, representing neighborhood environmental conditions. This image was downloaded from the Wisconsin View project website (http://www.wisconsinview.com). It has a 30 m resolution in the visible and near-infrared bands. Three environmental characteristics—the fractions of vegetation, impervious surface, and soil for each pixel—were generated using the normalized spectral mixture analysis method (Wu, 2004). In the preliminary data analysis it was found that all three remote-sensing-generated environmental characteristics project significant impacts on housing values. However, the combined effect of soil and impervious surface (SoilImp), which generally represents deteriorated neighborhood environmental conditions, yields the best model performance and is elected in the final model. It is termed the neighborhood environmental deterioration index in this study, and is also hypothesized to be negatively associated with housing value. Table 1 reports the summarization descriptive statistics of both the population and the sample records.

4 Model specification, spatial regression, and geographically weighted regression

4.1 Model specification

There are a variety of model specifications of $f(S, N)$ in equation (1) in the literature according to various purposes and studying regions. The most often used specifications in the literature include linear, semilogarithmic, and log–log specifications [for discussion on various specifications of $f(S, N)$ see Basu and Thibodeau (1998) and Kim (2003)]. Empirically searching over alternative specifications using the Box–Cox transformation is also suggested from a statistical validation point of view (Halvorsen and Pollakowski, 1981). Although linear specification of $f(S, N)$ has the appealing characteristics that the estimated coefficients can be interpreted as the corresponding

Table 1. Descriptive statistics for the housing data.

	Mean/mode	Standard deviation	Minimum	Maximum
Population (68 728 records)				
House price	106 442	55 227.78	11 000	1 249 100
Floor size	1 260	428.28	413	9 154
House age	60.17	20.52	1	168
Fireplace	0 *	0.45	0	8
Air conditioner	1*	0.50	0	1
Number of bathrooms	1.32	0.46	1	7.5
Soil and impervious surface	0.61	0.12	0	1
Sample (modeling records—1 821)				
House price	105 492	52 358.29	11 100	789 900
Floor size	1 261	404.84	444	5 977
House age	60	20.61	2	139
Fireplace	0 *	0.45	0	5
Air conditioner	1 *	0.50	0	1
Number of bathrooms	1.32	0.44	1	5.5
Soil and impervious surface	0.61	0.11	0.14	1
Sample (testing records—1 822)				
House price	105 156	58 732.92	13 400	924 000
Floor size	1 254	425.64	600	5 419
House age	60	21.07	2	155
Fireplace	0 *	0.43	0	4
Air conditioner	1 *	0.50	0	1
Number of bathrooms	1.33	0.48	1	6
Soil and impervious surface	0.61	0.12	0	1

* These numbers are mode instead of mean of the variable.

housing attributes' marginal (implicit) prices, Rosen's (1974) theory suggests that, as the house is an untied bundle of attributes, the price function is most likely nonlinear.

In addition, in case studies using similar datasets, Kim (2003) and Yu and Wu (2006) suggest a log–log with dummy variable specification of $f(S, N)$. According to Kim (2003), such specification yields the best model performance. Since the major objective of this research is to advance the conceptual framework that incorporating spatial effects in the hedonic price model might improve model performance, a similar log–log with dummy variable specification of $f(S, N)$ is adopted here. In particular, with the data extracted from the MPROP file and the remote sensing imagery, the hedonic price model is specified as:

$$\begin{aligned}\mathbf{P(H)}^* &= \beta_0 + \beta_1 \mathsf{FlSize}^* + \beta_2 \mathsf{NofBath}^* + \beta_3 \mathsf{HsAge}^* \\ &\quad + \beta_4 \mathsf{FirPlc} + \beta_5 \mathsf{AirCd} + \beta_6 \mathsf{SoilImp} + \varepsilon \ , \end{aligned} \tag{2}$$

where $\mathbf{P(H)}^*$, FlSize^*, $\mathsf{NofBath}^*$, and HsAge^* are the respective log-transformed values.

4.2 Spatial autoregressive regression

Two types of spatial autoregressive regression technique were employed to incorporate spatial autocorrelation in model construction, namely, the lag [or substantive as suggested in Anselin and Rey (1991)] and error (or nuisance) autoregressive specifications. The substantive autoregressive model takes the form:

$$\boldsymbol{y} = \rho \mathbf{W} \boldsymbol{y} + \boldsymbol{\beta} \mathbf{X} + \varepsilon \ , \tag{3}$$

where $\boldsymbol{y}$ is a vector with elements being the observed housing values (in its log-transformed form), $\boldsymbol{\beta}$ is a vector of parameters including the constant, $\mathbf{X}$ is a matrix with all the housing attributes [as symbolized in equation (2)], $\mathbf{W}$ is a spatial weight matrix that defines the spatial linkage among the houses, ρ is the coefficient of the spatial lag, and ε is an independent and identically distributed (iid) error term. From equation (3) the substantive autoregressive specification resembles a regular regression equation with an added neighborhood variable, $\mathbf{W}\boldsymbol{y}$, which represents the influences of neighboring houses on the observed house.

The nuisance autoregressive specification, on the other hand, deems the autocorrelation to be in the error term, and takes the form:

$$\boldsymbol{y} = \boldsymbol{\beta}\mathbf{X} + \varepsilon \ , \tag{4}$$

$$\varepsilon = \lambda\mathbf{W}\varepsilon + \mu \ , \tag{5}$$

where μ is an iid error, λ is the coefficient of the error, and the other symbols are defined above.

Since there exists the spatial autoregressive term $\rho\mathbf{W}\boldsymbol{y}$ and $\lambda\mathbf{W}\varepsilon$, the OLS estimator is no longer applicable. The maximum likelihood estimator is usually suggested as an effective asymptotic alternative (Anselin, 1988). Consequently, the conventional OLS goodness-of-fit – adjusted R^2 will no longer be applicable; instead, likelihood-based goodness-of-fit measures, mainly the Akaike information criterion (AIC, Akaike, 1974), will be used to compare the models' goodness-of-fit for the data.

Also worth noting here is the importance of the weight matrix $\mathbf{W}$ in equations (3) and (4). Although different $\mathbf{W}$s can be specified for the lag and error, respectively, we use the same spatial weight matrix for both equations. The definition of the weight matrix needs to be justified according to the contextual settings of the study region and objectives under investigation (Anselin, 1988). In general, critical distances are widely used to define the spatial neighboring relationships among point locations. In this procedure points that fall within a certain critical distance are considered as neighbors to one another. Geostatistical procedures using the empirical semivariogram are adopted in the literature to define the critical distances (Bowen et al, 2001). However, such procedures require a weak stationarity assumption that might not be met in urban housing data. In this study we chose a few critical distances to generate the weight matrix, namely, 2.5 km, 3.5 km, and 4.5 km. The choice of the critical distance is based on the observation that, in our sample size, 2.5 km is roughly the smallest distance that can guarantee each data point has at least one neighbor (it is to be noted, though, since our sample size is only approximately 3% of the total records, that, in the City of Milwaukee, even within a 2.5 km radius there might still exist significant heterogeneity; owing both to the main purpose of the current project and the computation cost, we intend to reserve the exploration of this possibility for our future endeavors). The characteristics of each weight matrix generated by those critical distances, with both the modeling sample (1821 records) and the testing sample (1822 records), are reported in table 2. All the weight matrixes are row standardized. The one which performs the best will be used to construct the final models.

4.3 Geographically weighted regression

The GWR technique is a newly developed GIS and spatial data analysis method specifically dealing with spatial heterogeneity among regressed relationships. It has recently received increased attention among scholars (eg Brunsdon et al, 1996; 1999; Fortheringham and Brunsdon, 1999; Fortheringham et al, 1997; 1998; 2002; Huang and Leung, 2002; Leung et al, 2000a; 2000b; Páez et al, 2002a; 2002b; Yu and Wu, 2004). GWR develops the idea of Cassetti's (1972; 1986) expansion regression method in

Table 2. Characteristics of alternative spatial weight matrixes.

Characteristics	2.5 km		3.5 km		4.5 km	
	modeling	testing	modeling	testing	modeling	testing
Number of points	1 821	1 822	1 821	1 822	1 821	1 822
Number of nonzero links	284 958	282 212	486 822	485 012	719 734	717 966
Percentage of nonzero weights	8.59	8.50	14.68	14.61	21.70	21.63
Average number of links	156.48	154.89	267.37	266.20	395.24	394.05

spatial terms. However, differing from Can's (1990; 1992) treatment of spatial terms, GWR allows regression coefficients to vary across space without explicitly specifying a determinant form on which the relationship drifts. Within the framework of GWR, the traditional log–log specified hedonic model expressed in equation (2) can be rewritten as:

$$\begin{aligned} P_i(\mathbf{H}) \;=\; & \beta_{i0} + \beta_{i1}\,\mathsf{FlSize}_i^* + \beta_{i2}\,\mathsf{NofBath}_i^* + \beta_{i3}\,\mathsf{HsAge}_i^* \\ & + \beta_{i4}\,\mathsf{FirePlc}_i + \beta_{i5}\,\mathsf{AirCd}_i + \beta_{i6}\,\mathsf{SoilImp} + \varepsilon_i \ , \end{aligned} \tag{6}$$

where the subscript i represents specific geographical locations, and other symbols are defined as in equation (2). Instead of being fixed, all the $\beta_{ij}(j = 0, 1, \ldots, 5)$ are now spatially varying.

Calibration of the GWR model follows a local weighted least squares approach (Fortheringham et al, 2002). When calibrating coefficients at location i, GWR assigns weights through a weighting scheme (mechanism) to data at locations according to their *spatial* proximity to location i. These weights ensure that near locations impose more influence on the calibration than locations farther away. The weights are usually obtained through a spatial kernel function. Two types of spatial kernels are often used—that is, fixed and adaptive kernels. In a fixed kernel function an optimal spatial kernel (bandwidth) will be obtained and applied over the study area. This approach is usually less computationally intensive. However, as pointed out by Páez et al (2002a; 2002b) and Fortheringham et al (2002), the fixed kernel approach can produce large local estimation variance in areas where data are sparse, and may mask subtle local variations in areas where data are dense. On the other hand, the adaptive kernel function seeks a certain number of nearest neighbors in order to adapt the spatial kernel to ensure a constant size of local samples. This kernel might present a more reasonable means in representing the degree of spatial heterogeneity in the study area. In this study the adaptive kernel function was employed.

To obtain an optimal size of nearest neighbors for the adaptive kernel, a common approach is to minimize the AIC (Akaike, 1974) of the GWR model (Fotheringham et al, 2002). The AIC of a GWR model is defined following the works of Hurvich et al (1998):

$$X_{\mathrm{AIC}} \;=\; 2n\ln(\widehat{\sigma}) + n\ln(2\pi) + n\left[\frac{n+\mathrm{tr}(\mathbf{S})}{n-2-\mathrm{tr}(\mathbf{S})}\right] , \tag{7}$$

where n is the total number of observations, $\widehat{\sigma}$ is the maximum likelihood estimated standard deviation of the error term, and tr($\mathbf{S}$) is the trace of the hat matrix $\mathbf{S}$ of the GWR, which is defined through:

$$\hat{\boldsymbol{y}} \;=\; \mathbf{S}\boldsymbol{y} \ , \tag{8}$$

where $\boldsymbol{y}$ and $\hat{\boldsymbol{y}}$ are the vectors of the dependent variable and the GWR estimated values. The AIC has a general appeal, in that it can be used to assess whether GWR provides a better fit than a global model (be it the OLS or a spatial autoregressive model), taking into account the reduced degrees of freedom.

4.4 Performance comparison and accuracy assessment

After constructing the models and fitting the modeling samples to the OLS, spatial autoregressive, and GWR models, one major task of this work is to compare the performance of the three models and assess their predicting accuracy using the testing samples. In terms of model performance, as discussed above, the conventional OLS goodness-of-fit criterion, adjusted R^2, will no longer be applicable in the spatial autoregressive models. Instead, the AIC is used as a goodness-of-fit performance indicator. As a rule of thumb (Fotheringham et al, 2002), a decrease of AIC by 3 indicates a significant improvement of the model performance.

To assess model prediction accuracy, we have first to make predictions. In our current study, our predictions are ad hoc predictions to compare the spatial models with OLS estimates. Prediction accuracy assessment on modeling samples is quite straightforward. This is also the case when using testing samples in the OLS and spatial substantive autoregressive models; we only need to apply the testing samples using the estimated model coefficients to obtain the predicted housing values. However, for the spatial nuisance autoregressive model and the GWR model, the use of testing samples for prediction is worth some elaboration. Recall, in equations (4) and (5), in the spatial nuisance autoregressive model, the spatial autocorrelation is expressed in the error term. However, for the testing samples, the error terms are unobservable during the prediction, hence the spatial influence incorporated in λ is not able to participate in the prediction (Bivand, 2005). For the GWR model, according to Fotheringham et al (2002), it is possible to use the modeling samples to obtain spatially varying coefficients on the testing samples' locations. However, quite unfortunately, the codes we are using written in R (Bivand and Yu, 2005) have not implemented such functionality. Neither is the function available in the latest GWR 3.0 software package (Fotheringham et al, 2002). The current scenario only allows us to take an intermediate coefficient surface interpolation before the actual prediction can take place. In the current study two interpolations were carried out using the ArcGIS spatial analyst extension, namely, the inversed distance weighting (IDW) and ordinary Kriging interpolation. After the coefficient surfaces were created, the corresponding coefficients were projected back to the testing samples for prediction.

Two particular statistics are employed in this study when assessing the prediction accuracy. They are the root mean squared error (RMSE) and the relative error (RE). The RMSE takes the form:

$$X_{\mathrm{RMSE}} = \frac{1}{n}\left[\sum_{i=1}^{n}(y_i - \hat{y}_i)^2\right]^{1/2} , \tag{9}$$

where n is the number of observations, y_i is the dependent variable at observation i and $\hat{y}_i$ is the estimated/predicted value of y_i at observation i. RMSE measures the absolute prediction errors of the models. On the other hand, RE measures the relative improvements of the model predictions over the global mean, and takes the form:

$$X_{\mathrm{RE}} = \frac{\sum_{i=1}^{n}|y_i - \hat{y}_i|}{\sum_{i=1}^{n}|y_i - \bar{y}|} , \tag{10}$$

where all the labels are as defined above, with $\bar{y}$ representing the global mean of the dependent variable.

5 Findings and interpretation

The computations were carried out in R (http://www.r-project.org). The SPDEP (Bivand, 2005) and SPGWR (Bivand and Yu, 2005) packages were used for the spatial autoregressive and GWR models, respectively. For the spatial autoregressive models, we found that, when using 2.5 km as the critical distance to construct the weight matrix, the substantive and nuisance models gave the best results (the lowest AICs). Hence only results from these two models are reported. For the GWR model, the optimal AIC score was reached with its ninety-two nearest neighbors. In addition, the coefficient nonstationarity test (Leung et al, 2000a) indicates that all the coefficients of the GWR model significantly vary over space (at more than a 99% confidence level). During the interpolation we found that the simple IDW and ordinary Kriging methods generate very similar results in terms of the accuracy assessment statistics (RMSE and RE); hence only results from the IDW procedure are reported.

The model results, model performance statistics (AIC), and prediction accuracy statistics (RMSE and RE) for both modeling and testing samples are reported in table 3, table 4, and figure 2. The OLS and spatial autoregressive models results are reported in table 3, and the GWR results are reported in figure 2. The GWR coefficient surfaces were created using the IDW interpolation method with 30 m resolution. In addition, only the areas that are pseudosignificant are mapped. The pseudosignificance is determined using the pseudo-t test of the GWR coefficients (pseudo-t tests need to be used with caution, as the tests are not independent of one another). [The tests here only give a general indication of the possible local misspecifications of our model. See Fotheringham et al (2002) for detailed technique discussion.]

From figure 2 and tables 3 and 4, a few interesting observations emerge. First, apparently, from table 3, both the OLS and the spatial autoregressive models indicate that all the housing and neighborhood attributes are significantly related to housing values in the City of Milwaukee. Moreover, the signs of the attributes agree with the hypotheses. Quite intuitively, the model results suggest that more recently built, larger houses, with relatively amenable neighborhood environmental conditions, having fireplaces, air conditioners, and more bathrooms, tend to be more expensive. In addition, the tests for ρ and λ in table 3 indicate that there exists significant spatial autocorrelation among housing values and the error terms. The salient difference between the nonspatial OLS and the spatial autoregressive models lies in the magnitude of the coefficients. A quick examination of table 3 suggests that the nonspatial OLS model tends to overestimate most of the coefficients (the only exception is in the nuisance spatial autoregressive model, where the coefficient of floor size is underestimated by the OLS). We contend that such overestimation is likely a result of the existence of spatial autocorrelation among neighboring housing values. In particular, except for the floor size, which remains relatively stable in all three models, the OLS model overestimates the coefficients of the other five house attributes, from 8.6% to 70% compared with the two spatial autoregressive models (table 3). Note that such autocorrelation of neighboring housing values is a result of similar housing attributes in the neighborhood. When the spatial information has been explicitly included in the spatial autoregressive models, it appears that the entangled spatial dependence among the housing attributes is separated. As such, the estimates for the spatial autoregressive model tend to be lower than for the OLS model and might better represent the real value in the existence of spatial autocorrelation.

Table 3. Modeling results for the ordinary least squares (OLS) and spatial autoregressive models (on modeling samples).

	Estimate [a]	Standard error	t/z value	Pr(>|t|/|z|)
Ordinary least squares				
(Intercept)	8.668	0.243	35.580	0.000
Floor size	0.565	0.036	15.792	0.000
House age	−0.279	0.0212	−13.167	0.000
Fireplace	0.149	0.1186	8.029	0.000
Air conditioner	0.170	0.016	10.877	0.000
Number of bathrooms	0.158	0.032	4.894	0.000
Soil and impervious surface	−0.367	0.066	−5.558	0.003
$F_{6,1814} = 234.1$, p-value $= 0.000$				
Log likelihood $= -478.937$				
Substantive spatial autoregressive model (weight matrix constructed on critical distance = 2.5 km)				
(Intercept)	−3.051	0.138	−22.13	0.000
Floor size	0.554 (98.1)	0.019	28.500	0.000
House age	−0.163 (58.4)	0.012	−13.820	0.000
Fireplace	0.074 (49.7)	0.010	7.412	0.000
Air conditioner	0.069 (40.6)	0.008	8.230	0.000
Number of bathrooms	0.086 (54.4)	0.017	4.968	0.000
Soil and impervious surface	−0.110 (30.0)	0.035	−3.101	0.002
$\rho = 0.980$, LR test value $= 2216.10$, p-value $= 0.000$				
Log likelihood $= 629.136$				
Nuisance spatial autoregressive model (weight matrix constructed on critical distance = 2.5 km)				
(Intercept)	8.977	1.359	6.605	0.000
Floor size	0.630 (111.5)	0.019	32.584	0.000
House age	−0.255 (91.4)	0.014	−17.706	0.000
Fireplace	0.067 (45.0)	0.010	6.991	0.000
Air conditioner	0.059 (34.7)	0.008	7.155	0.000
Number of bathrooms	0.100 (63.3)	0.016	6.062	0.000
Soil and impervious surface	−0.119 (32.4)	0.035	−3.364	0.001
$\lambda = 0.997$, LR test value $= 2395$, p-value $= 0.000$				
Log likelihood $= 718.579$				

[a] Numbers in parentheses are the percentages of the corresponding autoregressive model estimated coefficients when compared with those of the OLS.

Table 4. Modeling assessments for the three types of models; OLS—ordinary least squares; SA(lag)—lag, or substantive, spatial autoregressive model; SA(err)—error, or nuisance, spatial autoregressive model; GWR—geographically weighted regression.

	OLS	SA(lag)	SA(err)	GWR
Adjusted R^2	0.435	–	–	0.923
Modeling samples				
AIC	973.87	−1240.30	−1419.20	−1869.31
RMSE	0.00738	0.00397	0.00377	0.00272
RE	0.774	0.383	0.358	0.250
Testing samples				
RMSE	0.00734	0.00403	0.0233	0.00332
RE	0.766	0.375	2.932	0.296

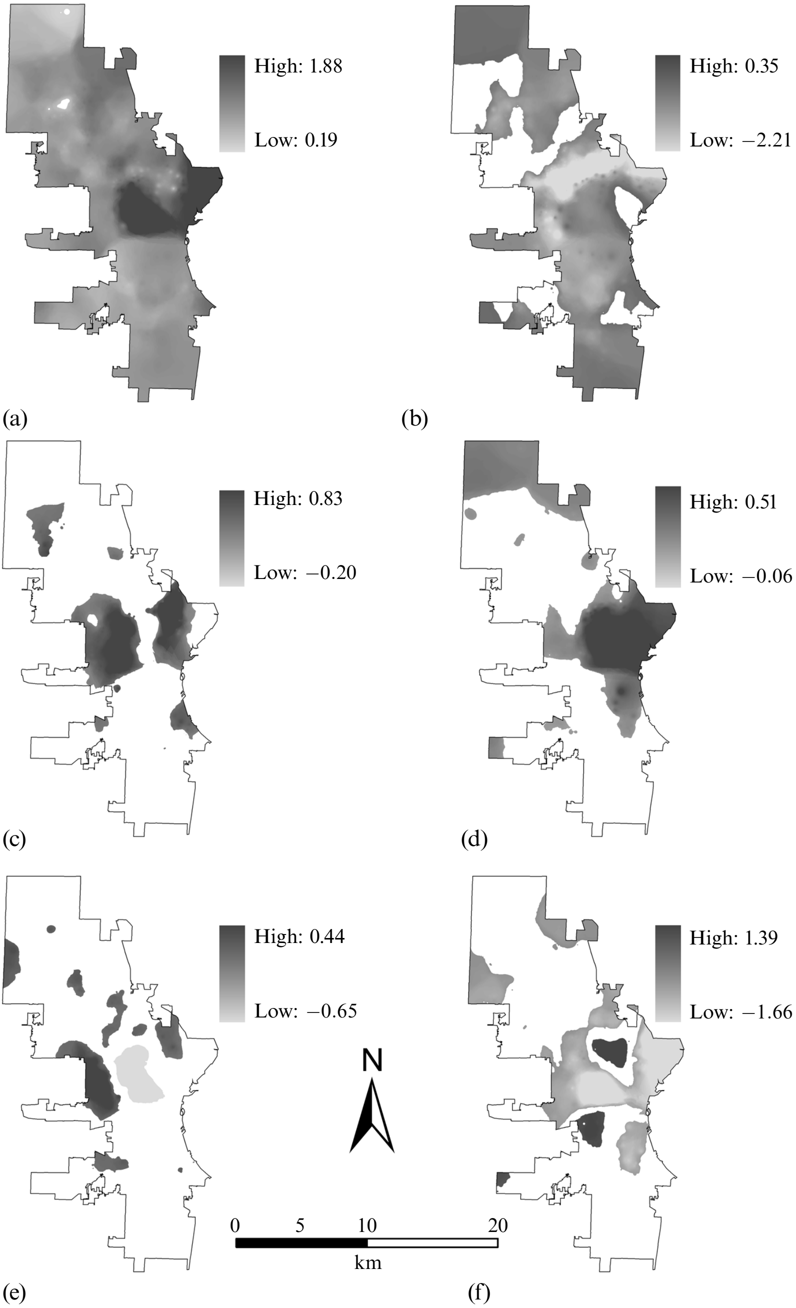

Figure 2. Surface of geographically weighted regression coefficients, only pseudosignificant areas are shown. Surfaces for (a) floor size; (b) house age; (c) fire place; (d) air conditioner; (e) number of bathrooms; (f) soil and impervious surface.

Second, from the AIC statistics in table 4, it is apparent that all the spatial models (autoregressive and GWR) fit the data much better than the nonspatial OLS model. Among the three spatial models, it seems that the GWR model fits the data the best, whilst the nuisance model performs slightly better than the substantive model. In addition, the adjusted R^2 of the OLS model is 0.435, indicating that only about

43.5% of the variation of housing values in the modeling samples is explained. This is to be expected, since some housing attributes are not included in our model, such as lot area, garage, number of rooms, and the like, due to the consideration of variable vector orthogonality. In addition, as is apparent in our model specification, we did not include any specific neighborhood characteristics, such as racial composition, median income and so forth. This implies that the model is potentially misspecified. However, these neighborhood characteristics are indeed related with locations, which can be explicitly incorporated into the modeling structure in our GWR models. Such misspecification can be partially corrected. The adjusted R^2 of the GWR model, which is about 92.3%, supports this argument.

Third, the GWR coefficient surfaces (figure 2) clearly support our postulation that the influence of housing attributes on housing values is not spatially invariant. This finding is also consistent with Can's (1990; 1992) argument. However, advancing Can's work, the GWR pseudosignificant tests on local coefficients reveal more interesting findings. As in figure 2, it is apparent that not all the housing attributes are significantly related to housing values everywhere, as suggested by the OLS or spatial autoregressive models. In fact, except for floor size and house age, the other four structural and neighborhood attributes are only significant in specific areas. In particular, having fireplaces is significant for housing values only in the central parts of the city [figure 2(c)]; air conditioners are the significant determinant for housing values only in the central and north parts of the city [figure 2(d)]; the number of bathrooms influences housing values only in the central west part, and quite anti-intuitively—this variable has a potentially negative influence on housing values on the west side of the Milwaukee River in the central part of the city [figure 2(e)]; and the remote sensing extracted neighborhood information matters mostly in the north suburban areas and in the central part of the city [figure 2(f)]. Moreover, from the range of the coefficient values, except for the floor size, we observe that the coefficients of all attributes have both negative and positive values. This indicates that the real situation in the urban housing market might be much more complicated than a global statement that hypothesizes house attributes to be related to housing values in a unidirectional fashion, though some of the anti-intuitive coefficient values are not necessarily pseudosignificant (figure 2).

Fourth, in terms of prediction accuracy, the RMSE and RE statistics indicate that, for the modeling samples, GWR performs the best, with the two spatial autoregressive models closely following behind. Compared with the OLS model, the spatial models improve the prediction accuracy by more than 50%. This result reinforces the argument that many of the unobservable or unincluded housing value/price determinants are strongly related to location. Hence, although the spatial models do not explicitly include more determinants other than the location information, these determinants are implicitly built into the spatial models. GWR, with its recognition of spatial heterogeneity as a natural process underlying the urban housing market, stands out to be the best predictive model for the modeling samples.

The testing samples show that the GWR model still stands out as the model which performed the best, although its predictive accuracy drops faster than that of the substantive spatial autoregressive model. For the testing samples the GWR model now only improves—in comparison with the OLS model—by 54.8% and 61.4%, according to RMSE and RE, respectively (in the modeling samples, the GWR model improves by 63.2% and 67.7% in terms of RMSE and RE, respectively, relative to the OLS model). The substantive spatial autoregressive is the model which performed the second best. It now improves—in comparison with the OLS model—by 45.0% and 51.0% by the standards of RMSE and RE, respectively (in the modeling samples, its improvement in terms of RMSE and RE is 46.2% and 50.5%, respectively). This result suggests a very interesting fact regarding the data and the methodology. The substantive autoregressive model, although it takes into account the neighborhood interinfluential effects into the model construction, is essentially a global model that assumes a stationary process in the urban housing market. The estimates for the coefficients of the exogenous housing value determinants, as well as the spatial autoregressive term (the spatial lag), are assumed to be stationary across space. Under such an assumption, replacement of the testing dataset in the estimated model would be legitimate and straightforward; hence, the change from the modeling samples to the testing samples results in relatively small changes in prediction accuracy. However, for the GWR model, we already mentioned that a further surface interpolation had to be carried out before the prediction could take place. This is because the GWR's estimates for the coefficients are location specific. Theoretically, it is possible to obtain coefficients at locations different from the observations (Fotheringham et al, 2002). Owing to availability of software functionality, our current study has to take an interpolation to serve as the medium. Although various interpolation methods are applied for better performance, they inevitably bring further unobservable errors into the prediction process. This result also suggests that, in our current scenario, although the GWR recognition of spatial heterogeneity is close to reality, the mechanism (the spatial kernel function) it uses to represent the heterogeneity is tuned towards the dataset that establishes the model. When a different dataset is applied, using the mechanism, the local subtlety of spatial heterogeneity in the new dataset amounts to large errors.

In addition, from table 4, it is starkly evident that the nuisance spatial model provides the worst prediction when the testing samples are applied. The RMSE and RE statistics are more than three times those of the OLS model. This is unavoidable since, when the testing samples are applied to the nuisance spatial autoregressive model, the error term of the testing samples is simply unobservable (Bivand, 2005). As the nuisance model assumes that spatial autocorrelation occurs in the error term, in the application of the testing samples, the spatial effects are practically excluded from the prediction process. Hence we observe the worst prediction.

6 Conclusions

The hedonic model is a powerful tool for understanding housing-market dynamics. The geographic attributes of houses differentiate them from other commodities in typical markets. The existence of spatial autocorrelation among housing values, due to their geographic proximity, violates the independence assumption of the standard ordinary least squares modeling technique. Establishing a model that describes the market equilibrium of this commodity requires us to incorporate the inherent spatial information. Fueled by the power of rapid development of GIS and spatial analysis techniques, recent studies have advanced the traditional hedonic housing model by explicitly including spatial information into model construction. Using the master property dataset of the City of Milwaukee, this study examines two types of spatial modeling schemes in housing hedonic studies in the grand framework of GIS and spatial analysis. In particular, two alternatives of spatial autoregressive models and a geographically weighted regression model are established in order to investigate the effects of spatial autocorrelation and heterogeneity on model performance and prediction accuracy. In summary, three interesting findings are presented.

First, it is found that, when spatial information is ignored in establishing the housing hedonic model, the model tends to overestimate the importance of structural and neighborhood attributes on housing values. We argue that such overestimation firstly points to the existence of spatial autocorrelation among the neighboring houses.

When such spatial autocorrelation exists, the OLS's estimates of the coefficients might actually take into account the spatial information that is entangled with the housing attributes, hence exaggerating their importance. Moreover, such overestimation also suggests that important locational attributes determining housing values might be missed.

Second, by using the GWR modeling scheme, we find that the relationships between housing values and attributes are not invariant over space, which agrees with Can's (1990; 1992) findings. This finding is also in accordance with the theoretical argument that a stationary housing market is likely untenable. This suggests that urban housing markets might consist of various local submarkets. More importantly, the GWR further reveals that it is not necessary for all the housing attributes to be significantly related to housing values everywhere in the study region. In addition, according to the value ranges of the GWR model's coefficients, it is possible that the same housing attribute can add to housing values in one region, but might negatively impact housing values in a different area.

Third, in terms of predictive accuracy, in general the spatial models perform better than the OLS model, except for the case when using the nuisance spatial autoregressive model, which practically excludes spatial information in the prediction. However, although the GWR model performs the best with the modeling samples, its prediction accuracy drops relatively faster when the testing samples are fed in. Two factors might contribute to such a drop in predicting accuracy of the GWR model in our current scenario when we use an interpolation as the medium for prediction. On one hand, when using the GWR model for prediction on testing samples, the interpolation procedure inevitably introduces new errors, which the model itself cannot remove. On the other hand, when calibrating the GWR model, although the mechanism (the spatial kernel function) determining the spatial variation of the relationship is quite flexible, it is tuned towards the best fit of the modeling samples. Different samples will introduce different and unobservable varying mechanisms (spatial kernel functions), which also bring new errors when the testing samples are used in prediction. One possible improvement of the predicting performance of the GWR model might be to calibrate the model using the modeling samples but at the locations of the testing samples, hence avoiding the interpolation procedure. Another possible improvement we can consider with an interpolation is to increase the size of the modeling samples, which can potentially incorporate a more subtle spatial variation mechanism into the model. However, since the computation cost increases fairly rapidly as the sample size increases, we feel a future investigation might provide more detailed information, which is beyond the scope of the current study.

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