Explaining Some Cells in the 'Scenario' Tabs of the LPP Spreadsheets when the constant Elasticity need not be Minus One-Half

by Gabriel A. Lozada

7/15/18

Let q be quantity of water (in acre-feet per year), pop be population, p_r be the retail price per acre-foot (in St. George), and p_{wh} be the WCWCD's wholesale price per acre-foot. Then for some constant "α" we assume the retail water demand curve is isoelastic with elasticity of ϵ :

$$q = \alpha \cdot pop \cdot p_r^{\epsilon} \,. \tag{1}$$

Tab "Revenues and Expenses" Cell **G49** calculates α using 2013 data.

To derive the expression underlying Cell **B8** of the spreadsheets' "scenarios," note that at two different dates i and f (for "initial" and "final"),

$$\frac{q_f}{q_i} = \frac{pop_f \; p_{rf}^{\epsilon}}{pop_i \; p_{ri}^{\epsilon}} \quad .$$

It follows that the condition for $q_f = q_i$ is

$$\begin{aligned} pop_i \; p_{ri}^{\epsilon} &= pop_f \; p_{rf}^{\epsilon} \\ pop_i^{1/\epsilon} \; p_{ri} &= pop_f^{1/\epsilon} \; p_{rf} \\ p_{rf} &= \left(\frac{pop_f}{pop_i}\right)^{-1/\epsilon} p_{ri} \; . \end{aligned}$$

If population grows at an annually-compounded rate of r then $pop_f = pop_i(1+r)^{f-i}$ and the above condition for $q_f = q_i$ becomes

$$p_{rf} = (1+r)^{(i-f)/\epsilon} p_{ri} \,.$$

Markup Pricing

"Constant Markup Pricing" is defined as a constant *percentage* difference between retail and wholesale prices:

$$p_r = (1 + \text{markup}) \cdot p_{wh} \,. \tag{2}$$

It is easy to show that if the retail demand curve is isoelastic, which is the assumption I maintain throughout, then the wholesale demand curve is also isoelastic and that the two demand curves share the same elasticity:

$$\begin{aligned} \frac{\partial \ln q}{\partial \ln p_r} = \epsilon \quad \Longrightarrow \quad \frac{\partial \ln q}{\partial \ln p_{wh}} &= \frac{p_{wh}}{q}\frac{\partial q}{\partial p_{wh}} = \frac{p_{wh}}{q} \cdot \frac{\partial q}{\partial p_r} \cdot \frac{\partial p_r}{\partial p_{wh}} \\ &= \frac{1}{1+\text{markup}} \frac{p_r}{q} \cdot \frac{\partial q}{\partial p_r} \cdot (1 + \text{markup}) = \epsilon \,. \end{aligned}$$

So for some constant "β" the wholesale demand curve can be written as

$$q = \beta \cdot pop \cdot p_{wh}^{\epsilon} \,. \tag{3}$$

From (1),

$$\begin{aligned}
\alpha \cdot pop \cdot p_r^{\epsilon} &= \beta \cdot pop \cdot p_{wh}^{\epsilon} \quad \text{so} \\
\alpha \cdot p_r^{\epsilon} &= \beta \cdot p_{wh}^{\epsilon} \quad \text{and using (2),} \\
\alpha \, (1 + \text{markup})^{\epsilon} p_{wh}^{\epsilon} &= \beta \cdot p_{wh}^{\epsilon} \quad \text{and} \\
\beta &= \alpha \, (1 + \text{markup})^{\epsilon} \,.
\end{aligned}$$

Let the WCWCD's total water sales revenue be denoted "WSR." Then from (3),

$$\begin{aligned}
WSR &= p_{wh} \cdot q \\
&= p_{wh} \cdot \beta \cdot pop \cdot p_{wh}^{\epsilon} \\
&= \beta \cdot pop \cdot p_{wh}^{1+\epsilon} \,. \qquad (4)
\end{aligned}$$

In the "base case" (the case with unchanged prices), denote water sales revenue at a given date by WSR_0. In this base case, the present value of the stream of WSR_0 values plus the District's other revenues (one for every date) turns out to be less than the present value of the District's expenses.

In the "correct case," the present value of the stream of water sales revenue values plus the District's other revenues is equal to the present value of the District's expenses. Denote water sales revenue at a given date in the "correct case" by WSR'. The spreadsheets work by calculating the factor "F" that makes the present value of the stream of $F \cdot WSR_0$ plus the District's other revenues equal to the present value of the District's expenses. I set $WSR' = F \cdot WSR_0$.[1]

Continuing to denote the base case with "0" subscripts and the correct case with $'$ superscripts, we have from (4)

$$\begin{aligned}
WSR_0 &= \beta \cdot pop \cdot (p_{wh\,0})^{1+\epsilon} \\
F \cdot WSR_0 = WSR' &= \beta \cdot pop \cdot (p'_{wh})^{1+\epsilon} \quad \text{so} \\
F \cdot \beta \cdot pop \cdot (p_{wh\,0})^{1+\epsilon} &= \beta \cdot pop \cdot (p'_{wh})^{1+\epsilon} \\
F \cdot (p_{wh\,0})^{1+\epsilon} &= (p'_{wh})^{1+\epsilon} \\
F^{1/(1+\epsilon)} \cdot p_{wh\,0} &= p'_{wh} \,. \qquad (5)
\end{aligned}$$

Equation (5) generates the spreadsheet's **B12** and **B20**. Using (2), (5) implies

$$\begin{aligned}
F^{1/(1+\epsilon)} \cdot p_{r0}/(1 + \text{markup}) &= p'_r/(1 + \text{markup}) \\
F^{1/(1+\epsilon)} \cdot p_{r0} &= p'_r
\end{aligned}$$

[1]Multiplying WSR_0 by F is not the *only* way to ensure that the present value of the stream of water sales revenue values plus the District's other revenues is equal to the present value of the District's expenses. There are an infinite number of policy changes that would achieve that result. It is the simplest to analyze, however.

so **B12** and **B20** report the ratio of old to new for *both* the retail price and the wholesale price.

Then from (3),

$$\begin{aligned} q' &= \beta \cdot pop \cdot (p'_{wh})^{\epsilon} \quad \text{; from (5),} \\ &= \beta \cdot pop \cdot (F^{1/(1+\epsilon)} \cdot p_{wh\,0})^{\epsilon} \\ &= \beta \cdot pop \cdot F^{\epsilon/(1+\epsilon)} (p_{wh\,0})^{\epsilon} \\ &= q_0 \cdot F^{\epsilon/(1+\epsilon)} \,. \end{aligned} \tag{6}$$

Equation (6) generates the spreadsheet's **B13** and **J21** as well as **I11** and **J22**.

Pass-Through Pricing

Instead of the constant markup pricing (2) of the previous section, in this section consider constant pass-through pricing, which is defined as:

$$p_r = p_{wh} + C \tag{7}$$

for some constant "C." The tab "Revenues and Expenses" Cell **G44** calculates $C \approx$ \$182 per acre-foot using 2013 data.

From (7) we now have

$$\begin{aligned} WSR &= p_{wh} \cdot q \\ &= (p_r - C) \cdot q \end{aligned}$$

and from (1)

$$WSR = (p_r - C) \cdot \alpha \; pop \; p_r^{\epsilon} \,. \tag{8}$$

For general values of ϵ there is no analytical way to solve (8) for p_r, so a numerical method had to be used to obtain Cells **K3**, **M10**, and **K20**. Cells **B13** and **J21** follow from the computed value for p_r and from (1).