A Rule of Thumb for Hedging the Purchase of a Fixed Immediate Annuity in the Near Future

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July 2011

Brief Biography. Gabriel A. Lozada, Ph.D., is a professor of economics at the University of Utah. His research interest is how to allocate economic resources over time. In the field of financial planning, his primary interest is portfolio design for retirement savings.

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Executive Summary

- Fixed immediate "life" annuities provide monthly income for the remainder of the purchaser's life. They are becoming important potential sources of retirement income for retirees as defined-benefit pension plans grow rare.
- The monthly income those annuities provide generally falls when the interest rate at the time of purchase falls and rises when it rises.
- This volatility is a source of concern to prospective purchasers, worried they may retire when interest rates are low.
- By holding funds destined to buy such an annuity not 100% in cash, but as a mixture of cash and bonds, a prospective purchaser can partially "lock in" an annuity price. "Hedging" this way is unattractive for anyone convinced interest rates will rise or fall, but is an attractive asset mix for risk-averse younger investors to plan on holding once they get close to buying their annuity.
- A rule of thumb for such hedging is to hold funds destined for purchasing a fixed immediate annuity within the next few quarters as 1/5 in a simple long-term corporate bond fund and the remainder in cash. Prospective purchasers who find hedging unattractive because they think interest rates will rise should hold a much shorter duration bond portfolio than that; those convinced interest rates will fall should hold a longer duration bond portfolio than that.
- The hedging rule of thumb reduces volatility in the annuity income the consumer can afford to buy, but still results in a positive correlation between such an income and unhedged annuities. This correlation can be totally eliminated by holding about one-half to two-thirds in long corporate bonds and the rest in cash, but that results in volatility as large as the unhedged position's.

According to the McKinsey Consumer Retirement Surveys, the proportion of US working-age adults who consider the lack of a guaranteed retirement income an extremely important risk or a very important risk increased from 28% in 2004 to 61% in 2009 [Hunt et al. 2009]. Such worries are well-founded because both private and public employers have been curtailing defined-benefit programs, shifting longevity risk to workers. Besides defined-benefit pensions and Social Security, the only financial instrument that provides guaranteed retirement income is a fixed immediate annuity, and accordingly it has been the focus of increased interest by, among others, the US Departments of Labor [September 2010] and Treasury.[1]

The fixed monthly payout offered by fixed immediate annuities tends to fall as interest rates at the time of purchase fall. This, in addition to the fact that consumers often plan to live off interest from fixed-income investments when they retire, explains why, in the same McKinsey surveys, the number of respondents who thought interest rates were an extremely or very important retirement-planning risk rose from 26% to 62% as interest rates fell. In order to avoid the fate of today's retiring workers, risk-averse younger workers saving for retirement may benefit from being able to hedge the interest-rate risk of annuity purchases, so that the annuity income they will be able to afford when they retire will not depend on whether interest rates then are high or low. Hedging is not always desirable: for example, workers retiring in mid-2011, with interest rates very low, might prefer to leave their assets exposed to interest-rate risk, feeling it worthwhile to take the gamble that interest rates will be soon be higher. Although this paper is not about whether to hedge, but about how to hedge if a consumer wants to do so, by describing hedging portfolios it by implication describes what type of portfolio not to hold if one does not want to hedge.

Because annuities pay less when interest rates are low, a hedging instrument's value has to rise when interest rates fall. Bonds have that property, so the simplest hedge would be to hold wealth in the form of cash and bonds or a bond mutual fund. This paper solves for the best bond mutual fund maturity and bond-to-cash proportion for the hedge. The resultant asset mix

[1]Background information on fixed immediate annuities is available at `www.immediateannuities.com/content_pages/lesson.htm` and at `www.actuarialfoundation.org/consumer/wiser.pdf`. The "guarantee" of a fixed immediate annuity is only as good as the insurance company issuing the policy, with a limited state guarantee fund backup. Fixed immediate annuities, especially if the payments are fixed in *real* terms, are widely recommended at least in theory by economists, including Zvi Bodie and Jeremy Siegel (well known for their opposite opinions on how to best save for retirement) [National Association of Personal Financial Advisors, 2004]. Babbel and Merrill [2007a, p. 11–12] point out that if, as seems likely, there is some "minimum threshold of consumption tolerable to the individual" below which utility is $-\infty$, then the *only* utility-maximizing plan involves buying a "default-free annuity which continues throughout one's lifetime" to cover that minimum threshold.

will mitigate annuity-purchase risk but not eliminate it because the hedge will not be perfect.

Investors must resolve two annuity hedging problems: a long-run problem of how to invest conservatively many years before an annuity purchase, and a short-run problem of what sort of asset mix would make a consumer comfortable buying an annuity immediately even when he fully understands the potential rewards and risks of keeping his current asset mix intact and delaying his purchase for up to a year. Analysis of the long-run and the short-run problems require somewhat different techniques, and this paper only deals with the short-run problem.

1. Prior Work

There is a large body of work on imperfect hedging, called "cross-hedging," but very little concerning hedging an annuity purchase. Certainly the hesitancy to purchase annuities when interest rates are low is well known; for example, `morningstar.com` recently carried an interview in which Harold Evensky (president of Evensky & Katz Wealth Management) said:

> Because rates are historically low, we don't feel the pressure to recommend it [fixed immediate annuities] right now, but I think within the next few years as interest rates get more historically normal that it will become an extraordinarily important part of everyone's planning process. [Benz 2010]

However, hedging possibilities themselves have rarely been discussed. Cairns et al. [2006] simply remark that "the plan member's preference for a pension at retirement over a cash lump sum needs to be matched by a switch to long-dated bonds before retirement... rather than cash." Babbel and Merrill [2007b, p. 10] point out that "your accumulated assets need to be invested in something during the interim while awaiting the time to purchase a life annuity... the value erosion that typically accompanies rising interest rates may offset part or all of the gain that one hopes to garner by delaying the annuitization decision." Koijen et al. [2009] directly address hedging of annuity purchases, but their paper does not use actual annuity prices as this paper will, instead assuming annuities are (exactly) "fairly priced." They find that the intertemporal-utility-maximizing "hedging strategy holds long positions in 3-year nominal bonds and stocks, while 10-year nominal bonds and cash are shorted" (p. 23). This paper's simpler approach does not find the utility-maximizing percent of assets to be invested in annuities but takes that as given; analyzes only a one-year time horizon; and, to focus on easily-implemented strategies, it uses only two instruments.

Another line of research that resembles this paper's problem is asset-liability matching and liability-driven investing (see for example Brown and

Jones [2011], Moore [2004], Vanguard [2007], and Amenc et al. [2009]), but annuity purchases have not been analyzed in that framework.

2. Two Kinds of Hedging

At the start of the period, the consumer is assumed to have earmarked certain funds to purchase the fixed immediate annuity, and holds these funds in the form of cash and a bond mutual fund. Each month, the value of the bond fund fluctuates and the price of an annuity fluctuates. At the start of each month a calculation is made of how large a monthly annuity payout that month's wealth could buy given that month's annuity price (ignoring tax implications of selling the bonds). This will be referred to as the consumer's "affordable annuity income," sometimes abbreviated "affordable income."[2]

A consumer may have either of two goals for hedging:

Volatility Goal: minimizing volatility (or variability) of affordable annuity income; or

Correlation Goal: reducing correlation of affordable annuity income with other assets.

The Volatility Goal reflects the older meaning of the term "hedge": it focuses on risk minimization, on trying to "lock in" a future annuity income. The Correlation Goal is the newer meaning of the term "hedge," as in "hedge fund;" it accepts risk, in the hopes of greater return, as long as the resulting changes in affordable annuity income are "market neutral"—"annuity-market neutral" in this paper, which is roughly the same as "bond-market neutral." Achieving bond market neutrality would be like having zero duration overall, in that the overall assets-plus-liabilities portfolio would not systematically vary with interest rates, though it would be volatile for other reasons.[3]

A perfect hedge would achieve the Volatility Goal with a volatility of zero and therefore it would achieve the Correlation Goal with a correlation of zero. Imperfect hedging (cross-hedging), as in this paper, entails a tradeoff between the Volatility and Correlation Goals. To give a hypothetical example of the trade-off which imperfect hedging requires, suppose that over a period of five months, interest rates changed as follows:

[2]Using this "affordable annuity income" framework is equivalent to assuming that the value of \$1 at date t for a consumer who will purchase an annuity at the future date T is inversely proportional to the annuity price at t. However, it is also inversely proportional to the present value at t of \$1 at T. This paper ignores the latter effect because it considers periods of only one year, and during that year, the annuity might be purchased at any time, including at the very beginning.

[3]Technically, since zero correlation does not mean independence, zero correlation would still allow nonlinear dependence on interest rates. In this way zero correlation also resembles zero duration, which does not mean "no effect of changing interest rates on portfolio value," merely no linear effect, allowing nonzero nonlinear "convexity" effects.

5.0%, 5.1%, 5.2%, 4.9%, 4.8%.

In response, suppose annuity payouts for a $100,000 premium changed as follows:

Example 1: $700, $720, $740, $680, $660 (average = $700).

If the consumer's $100,000 was invested entirely in cash, these annuity payouts were also the consumer's affordable monthly annuity income. Suppose that investing some of the $100,000 in a bond fund would have allowed affordable monthly income to instead change as

Example 2: $700, $710, $720, $690, $680 (average = $700),

whereas choosing yet a different investment would have changed affordable monthly income to the following during this hypothetical period (where, for the sake of argument, "x" does not have to be zero):

Example 3: $\$700 + x$, $\$600 + x$, $\$755 + x$, $\$780 + x$, $\$665 + x$ (average = $\$700 + x$).

The portfolio underlying Example 2 lowered the volatility of affordable annuity income (keeping it closer to $700) compared to Example 1, but it maintained the tight correlation between affordable annuity income and interest rates, because whenever the latter went up the former went up, and whenever the latter went down the former went down. (The correlation coefficient between interest rates and Examples 1 and 2 both is 1.00.) Example 3 completely eliminates any correlation between interest rates and affordable annuity income: the former went up, up, down, down, whereas the latter went down, up, up, down (and their correlation coefficient is zero); however, Example 3 has very great volatility in affordable annuity income. Consumers who prefer the prospect of Example 2 to the prospect of Example 3 as responses to interest rates of {5.0%, 5.1%, 5.2%, 4.9%, 4.8%} think the volatility goal is more important than the correlation goal; consumers with the opposite preference think the correlation goal is more important than the volatility goal. It is likely that a consumer within a few quarters of buying an annuity will be primarily interested in the Volatility Goal, minimizing risk without thought of further reward; whereas a younger consumer will probably want affordable annuity income to grow, so is willing to accept risk, but may not want to be exposed to the vagaries of the bond market, and therefore will be primarily interested in the Correlation Goal. Because this paper concerns the short term, its main results are in the sections below that design asset mixes to meet the Volatility Goal (Sections 4 and 5). Section 6 offers brief remarks on asset mixes that meet the Correlation Goal.

3. Data: Annuity Prices and Hedging Instruments

The web site `immediateannuities.com` publishes monthly "Comparative Annuity Reports," which give data on the "Single-Premium Immediate Annuity ["SPIA"] Payout Factor for Life with 10 Years Certain;" applying these factors to a $100,000 premium results in a monthly income (unindexed for inflation) this paper will simply call "payouts." The data set gives payouts offered by two to three dozen companies for eight age/gender categories of purchasers living in New Jersey: men and women aged 60, 65, 70 and 75 (abbreviated M60, F60, etc.). For each month, for each age/gender category, the payouts averaged over these companies were used. The data goes from 1/2003 to 7/2010.[4] This paper studies overlapping 12-month periods, of which there are 80 in the data set (the 12 months starting in January 2003, the 12 months starting in February 2003, etc.).[5]

The hedging instruments considered are the following Vanguard mutual funds (listed here with their ticker symbols, years to maturity, and duration): Short-Term Bond Index Fund (VBISX, $2\frac{3}{4}$, $2\frac{1}{2}$), Short-Term Investment-Grade Fund (VFSTX, 3, 2), Intermediate-Term Bond Index Fund (VBIIX, 7, 6), Intermediate-Term Investment-Grade Fund (VFICX, 7, 5), Long-Term Bond Index Fund (VBLTX, 23, 13), and Long-Term Investment-Grade Fund (VWESX, 24, 13).[6] The years to maturity and duration of these Vanguard funds do not change much over time, and the results in this paper should

[4]The "Comparative Annuity Reports" also provide graphs comparing annuity payouts with the yield on Moody's AAA Corporate Bonds, starting in 2003 and updated monthly. The author would like to express his great appreciation to Mr. Hersh Stern of `immediateannuities.com` for providing the raw annuity pricing data in computer-readable form. Similar data for Canada, using Canadian dollars and the Government of Canada 10 year bond yield and presented in tabular form, is available at `http://www.ifid.ca/payout.htm`.

[5]This oversamples the months in the middle of the data set but preserves the historically-interrelated time paths of bond prices and SPIA payouts.

[6]The three actively-managed Vanguard corporate bonds funds were included because although active management introduces idiosyncracies, all-corporate bond funds might be better hedging instruments than bond index funds because SPIA pricing generally varies with the price of the bonds that annuity issuers buy to back their promises, and those issuers may hold more corporate bonds—or bonds that behave like corporates—than bond index funds hold. Some insurance company holdings deviate notably from bond index funds' holdings, as shown by the following comparison of the holdings in Vanguard's long-term bond index fund (as of June 2011) and the 2010 holdings of two annuity issuers, TIAA [2010] and Northwestern Mutual [2010]: Treasury/Agency debt, 44% vs. 9% and 6%; corporate bonds, 34% vs. 39% and 52%; mortgage-backed securities and direct mortgages, 0% vs. 39% and 35%. This shows large differences in holdings of Treasury versus non-Treasury debt, but small differences in the holdings of corporates per se. (Unlike Vanguard's other bond index funds, its Total Bond Market Fund (VBMFX) holds real-estate-related securities (about 31%), so it resembles TIAA's and Northwestern Mutual's portfolios more closely than Vanguard's Intermediate Index fund; however, analysis showed it did a worse job of hedging annuities.)

broadly apply to other simple bond mutual funds that hew to a well-defined style. Mutual fund price data ("Adjusted Close" prices, adjusted for distributions and splits) were obtained for the first day of each relevant month from `finance.yahoo.com`, because the "Comparative Annuity Reports" are published at the beginning of each month.

The hedging instruments (bond funds) in this paper are combined with non-interest-bearing cash, and the consumer does not rebalance during the 12 months.[7]

4. Hedging to Protect Against Volatility

Within each of the 80 twelve-month periods, there are twelve affordable annuity income values. To minimize volatility, the consumer's goal is to have the last eleven vary as little as possible from the first.

The Volatility Goal could be achieved perfectly by a sophisticated trader if put and call options on annuity prices existed or if there were futures contracts for annuities, but there are not, so a hedger has to cross-hedge. A cross-hedge for a sophisticated trader might be interest-rate futures or options, and for the simplest investor would be bond mutual funds; in either case, zero volatility is unattainable because although bond prices move in the opposite direction to annuity payouts most of the time, about a quarter of the time they do not. For example, in April 2007 payouts for every age/gender category went up while the value of all the bond funds also went up. For sixty-five-year-old men, the payouts rose so dramatically then and in February 2008 that even the entire 12-month period starting in April 2007 showed strongly positive correlation between bond values and annuity payouts. These irregularities limit how good a hedge against volatility bond funds will be, although they also mean consumers do not need interest-rate hedges as much as they would if interest rates and annuity payouts moved in lockstep.

[7]Using a money market fund instead of non-interest-bearing cash would give incorrect hedging bond percentages. For example, suppose that during the next 12 months, a new retiree firmly believes that an SPIA payout will be constant. It is natural to say that in this case the retiree "does not need to be hedged" and does not need to invest in the hedging asset; he can invest completely in cash. However, if "cash" means an interest-bearing instrument, his affordable annuity income will rise over the year, and a hedging algorithm will detect this deviation and try to find a money-losing investment to counteract it. This would be entirely appropriate for a hedge against volatility if consumers were observed putting annuity purchases off into the future because they know their money market accounts will be larger then—in other words, it would be appropriate if consumers delayed annuity purchases because current interest rates are high. However, all available evidence (including the McKinsey surveys and the Evensky quote given earlier) suggests exactly the opposite: consumers delay annuity purchases when interest rates are low. The only way to be consistent with this behavioral evidence is to model "cash" as being non-interest-bearing.

	Payouts				
	Average	Range	AvAnMSDI	AvAnMxDI	AvAnAv
Male 60	585	86	14.6	25	587
Female 60	555	83	14.7	25	557
Male 65	636	87	14.7	25	638
Female 65	600	86	14.2	24	602
Male 70	702	92	14.7	25	704
Female 70	659	90	14.3	24	661
Male 75	776	94	15.0	26	779
Female 75	737	93	14.3	24	740

Table 1. All figures in dollars per month, rounded to the nearest dollar. "AvAnMSDI" is "Average Annual Mean Squared Deviation from Initial;" "AvAnMxDI" is "Average Annual Maximum Deviation from Initial;" "AvAnAv" is "Average Annual Average." Numbers next to genders denote years of age.

Since zero volatility is unattainable, it is necessary to measure how close to zero volatility different imperfect hedges are. There are many possible ways to measure how much the last eleven payouts "vary from the first payout;" this paper uses

$$\sqrt{\sum_{i=1}^{12} \frac{(a_i - a_1)^2}{12}} \quad , \tag{1}$$

the (annual) "mean squared deviation from a_1." This is related to the standard deviation, but standard deviation measures deviations from the mean of the a_i's, not from the initial level a_1, which is the target of the hedge.[8]

Because Equation (1) pertains to one twelve-month period, there are 80 values of it that have to be combined into one summary measure of volatility. There are many possible ways to do this; this paper uses the average of the 80 numbers, which will be called the "Average Annual Mean Squared Deviation from Initial" ("AvAnMSDI").

5. Results for Hedging to Protect Against Volatility

Table 1 presents statistics on annuity payouts. Besides AvAnMSDI, Table 1 reports another measure of volatility, the "Average Annual Maximum Deviation from Initial," "AvAnMxDI." "AvAnAv" is the "Average Annual Average," that is, the average of the 80 annual averages; it differs slightly from the "Average" column because AvAnAv oversamples months in the middle of the data set.

[8]For different risk measures see Hoe et al. [2010], Huang [2010], and Polak [2010].

Affordable Income with Table 3's Optimal Mixes of Bonds & Cash

	AvAnMSDI						AvAnMxDI	AvAnAv
	Short Index	Short Corp.	Inter. Index	Inter. Corp.	Long Index	Long Corp.	Long Corp.	Long Corp.
Male 60	14.4	13.8	12.8	12.7*	12.3**	11.5***	21	591
Female 60	14.5	14.0	13.0*	13.0*	12.4**	11.7***	21	561
Male 65	14.6	13.9	13.1	12.8*	12.5**	11.7***	21	642
Female 65	14.1	13.6	12.9	12.6*	12.3**	11.5***	20	606
Male 70	14.5	13.6	12.9*	12.4**	12.4**	11.4***	20	708
Female 70	14.2	13.7	13.0	12.6*	12.5**	11.5***	20	665
Male 75	14.6	13.5	12.8	12.2**	12.6*	11.7***	21	783
Female 75	14.2	13.5	12.8*	12.1**	13.3	11.2***	20	744

Table 2. All figures in dollars per month, rounded to the nearest dollar. Abbreviations as in Table 1. Three asterisks denote the minimum AvAnMSDI in each row, two asterisks denote the second-lowest, and one denotes the third-lowest.

Table 2 gives the minimized AvAnMSDI volatility resulting from Table 3's optimal start-of-year percentage of bonds (with the rest in cash), for each of the eight age/gender categories and each of the six kinds of bond funds. Table 2 shows that for all eight age/gender categories, the best AvAnMSDI volatility is obtained hedging with long-term corporate bonds, with those bonds making up (from Table 3) between 18% and 24% of the asset mix. The second-best hedging was usually obtained from the Long-Term Bond Index, but for a few age/gender categories from Intermediate Corporates .

Comparing Table 1 with Table 2, hedging with long corporates also reduced AvAnMxDI volatility. Hedging against volatility is not supposed to affect average outcomes, and fortunately the AvAnAv columns of the two tables do not differ by more than $4.

The implications of these results for a 70-year-old man are that by holding 20% of his assets in long corporates with the rest in cash, the AvAnMSDI volatility of his affordable annuity income will fall from $14.7/mo to $11.4/mo; the AvAnMxDI volatility will fall from $25/mo to $20/mo; and the average annuity income (AvAnAv) will be roughly unchanged ($704 versus $708.).

Figure 1 shows how much AvAnMSDI volatility would increase if a 70-year-old man used a nonoptimal instrument or a nonoptimal bond percentage. Volatility does go up when using a nonoptimal instrument, but the effect is not large as long as the "proportion not in cash" is approximately 1/10 to 1/3 and one avoids using the Short Index. For age/gender categories other than 70-year-old men, the corresponding graphs look similar.

	Optimal Mixes for Table 2, % not in cash					
	Short Index	Short Corp.	Inter. Index	Inter. Corp.	Long Index	Long Corp.
Male 60	21	30	32	26	23	23
Female 60	23	29	34	26	25	24
Male 65	14	27	29	23	21	21
Female 65	8	23	28	23	21	22
Male 70	14	27	27	23	19	20
Female 70	6	22	26	22	19	21
Male 75	16	28	25	23	17	18
Female 75	8	24	25	23	18	20

Table 3. Percent of bonds which, with the rest in cash, minimizes the "AvAn-MSDI" volatility of affordable monthly annuity income. Rounded to the nearest percent.

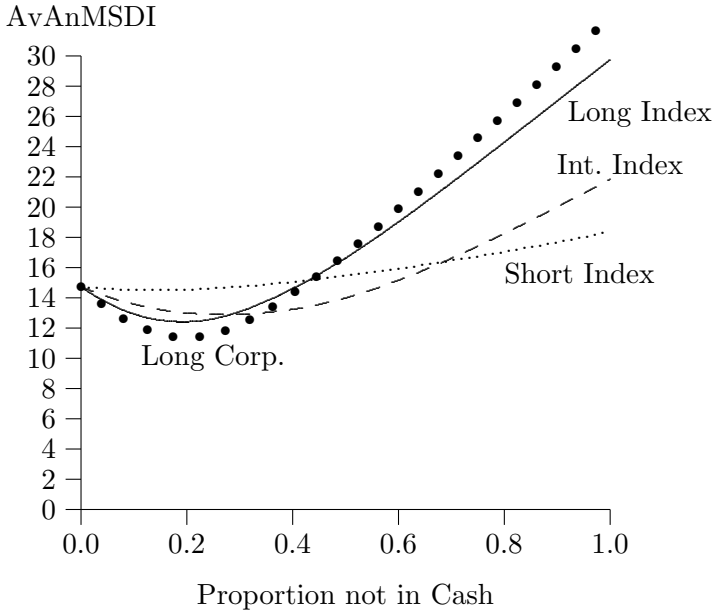

Figure 1. "Average Annual Mean Squared Deviation from Initial" monthly annuity income for a 70-year-old man, using one of the listed bond funds for hedging; in dollars per month.

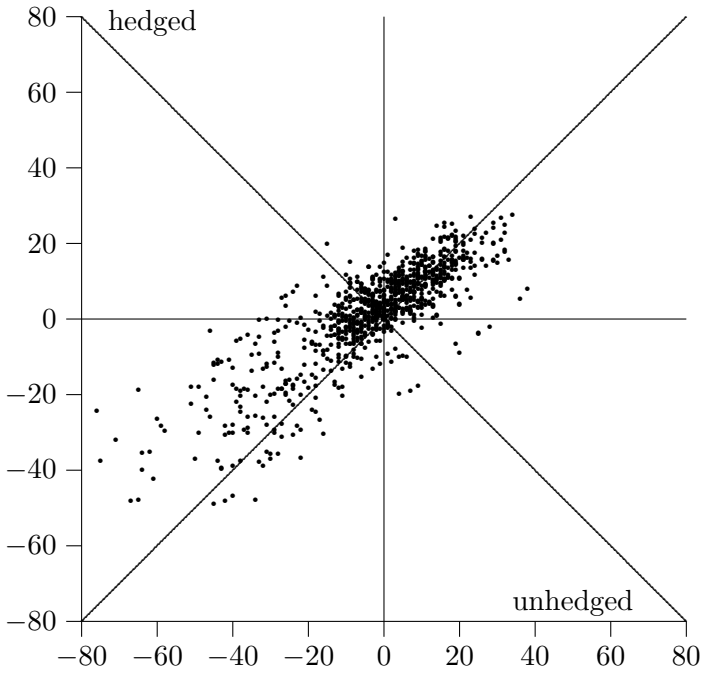

Figure 2. Unhedged (100% cash) vs. hedged (4/5 cash, 1/5 long-term corporate bond fund) deviations from the initial monthly annuity income for a 70-year-old man, measured in dollars.

Summarizing Tables 1–3, a rough rule of thumb for all the age/gender categories would be to hold about a fifth of assets in long corporates and the rest in cash.

Instead of showing the result of this advice using a summarizing measure like AvAnMSDI, Figure 2 shows it using a scatter diagram of the underlying raw monthly volatility; it gives 880 comparisons (80 twelve-month periods times 11 "later than initial" months) of unhedged vs. hedged deviations from initial annuity income for a 70-year-old man whose hedging asset mix is 1/5 long corporates. In this figure, hedging decreased volatility for all the points (months) vertically between the two 45° lines. A perfect hedge would have all the points on the horizontal axis. There are months, especially in Quadrant 1, in which the hedged affordable income's deviation was higher than the unhedged's. However, there are points near the left-hand edge of the graph with unhedged deviations between −\$60 and −\$80, whereas there are no points near the top or bottom edge of the graph, meaning hedging has eliminated that extreme volatility.

The correlation coefficient in Figure 2 is rather high (0.83), showing that this section's hedging-against-volatility asset mixes do not eliminate correlation between the hedged and unhedged series.

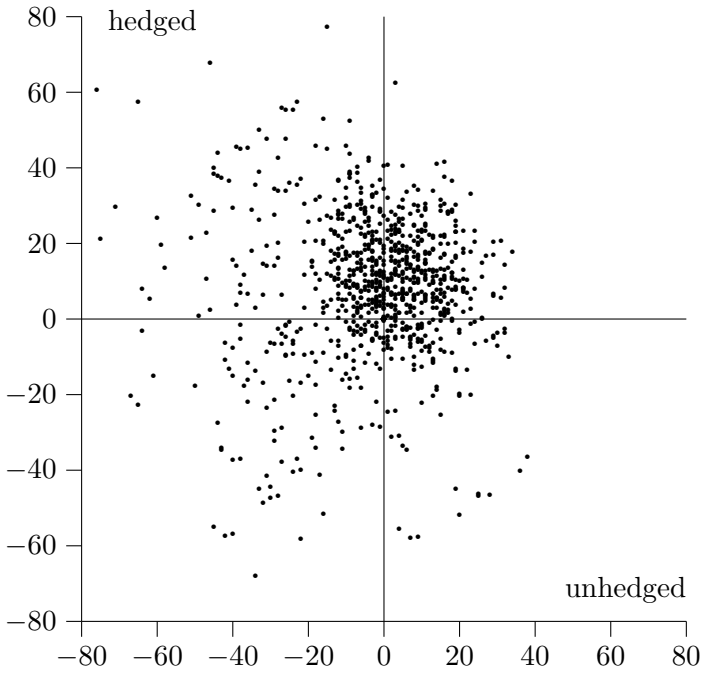

Figure 3. Unhedged (100% cash) vs. hedged (54% long-term corporate bond fund, the remainder in cash) deviations from the initial monthly annuity income for a 70-year-old man, measured in dollars. The correlation in this figure is zero.

6. Hedging to Protect Against Correlation

This section shows how to eliminate correlation over time between: (i) the payouts for the annuity the consumer will purchase; and (ii) the consumer's affordable annuity income if his or her assets were invested in one of the six bond funds listed in Section 3, mixed with cash. Section 2 suggested "zero correlation" is likely to be a relevant goal for the long run, so since this paper analyzes the short run this discussion will be brief.

Forty-eight ways of achieving zero correlation were calculated, one for each of the eight age/gender categories and six bond funds that could be chosen as the hedging vehicle. Only results using long corporate bonds are reported because every one of the other bond funds required a larger bond-to-cash ratio for every one of the eight age/gender categories, and many of those required ratios were greater than 100%. Long corporate bonds achieved zero correlation when mixed with cash using the following corporate bond percentages: M60: 67%; F60: 74%; M65: 62%; F65: 65%; M70: 54%; F70: 59%; M75: 52%; F75: 51%. For a 70-year-old man, optimal hedging against correlation instead of against volatility changes Figure 2 into Figure 3. Figure 3 shows that although the hedging asset mixes in this section perfectly achieve the Correlation Goal, they result in wide monthly swings in annuity

income, and are no better than the simple payouts at meeting the Volatility Goal.

Conclusion

For a consumer close to an annuity purchase who does not want to be exposed to interest-rate risk, a rough rule of thumb would be to hold about one-fifth of funds destined to buy the annuity as long-term corporate bonds, and hold the rest in cash: a defensive strategy that protects from interest rate swings by "locking in," as much as possible, currently-available annuity income. If a consumer is less interested in "locking in" annuity income and more interested in reducing its correlation with interest rates, the proportion of assets in long corporates should increase, up to half or more of the asset mix.

Holding such asset mixes would reduce the incentive for a prospective purchaser in a low-interest-rate environment to delay his or her annuity purchase, and in a high-interest-rate environment it would reduce the incentive to annuitize prematurely. Following this strategy is so simple that many retail investors may already be doing something close to it inadvertently, just by holding part of their money earmarked for an annuity purchase as bonds. Conversely, if a consumer does not want to hedge because he thinks interest rates will soon rise, he should keep bond duration below those of the hedging portfolios described in this paper, otherwise the rise in interest rates will bring him no net benefit; and if he thinks interest rates will soon fall, his bond duration should be greater than those of the above-described portfolios.

Among the limitations of this study are that it restricted the consumer to holding only two assets; it used only one of many potential mathematical descriptions of how consumers feel about fluctuations in affordable annuity income; it depended on data from a particular historical period which will differ from the future (although *prudent* annuity company portfolios will probably be managed in the future not unlike they have been in the past); and it ignored, because of lack of data on inflation-indexed annuities, the possibility of unexpectedly high inflation, which would have adverse effects on long bonds and on annuitants' long-run real income.

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