M. Dynamic Economics

The questions in this section are old, so the dates they contain are all in the past. To get their correct answers, imagine that you were answering the questions when the questions were first written. If I ask any of these questions on an exam, I will make the dates current.


   (a) Suppose the interest rate which banks pay is \( r = 0 \). If Santa Claus gives you a choice of Bond A or Bond B for Christmas, which will you pick?

   (b) What choice will you make if \( r \) is infinite?

   (c) What choice will you make if \( r = 2 \) (that is, an interest rate of 200%)? Is this surprising, given the answers to (a) and (b)? What choice will you make for \( r = 1 \) and \( r = 3 \)?

2. Suppose you decide to go to law school after graduating from the University of Utah. Law school takes 3 years. You have three housing options.

   **Option A:** You could live in dormitories for the 3 years. The whole year’s dormitory rent is due at the beginning of the school year. So if you take this option, you will pay
   - $800 on 9/1/89,
   - $800 on 9/1/90, and
   - $800 on 9/1/91.

   **Option B:** You could buy a condominium with cash, live there for the 3 years, and then sell the condominium. (We suppose you have enough cash to buy a condominium.) In this case:
   - On 9/1/89 you will pay $51,000; and
   - on 9/1/92 you will get $64,000 for your condominium.

   **Option C:** You could take out a mortgage and buy a condominium, which you will sell after 3 years. Then you will:
   - Pay a $1000 mortgage payment on 9/1/89;
   - pay a $1000 mortgage payment on 9/1/90;
   - pay a $1000 mortgage payment on 9/1/91; and
   - get $600 (your “equity” plus appreciation) on 9/1/92, when you sell the condominium.
(a) Is it better to rent (Option A), to buy with cash (Option B), or to buy with a mortgage (Option C), if the interest rate is 0%?

(b) Which of the three options is better if the interest rate is 10%? (If you do not have a calculator with you, you may assume that the interest rate is 100% instead of 10% to make the numbers easier.)

3. Suppose that on January 1, 1988, you decide you need a motorcycle for the next two years. You visit a store which sells motorcycles, and there you discover that you can get the motorcycle you want in two ways.

The first way: You can lease (rent) the motorcycle from the store. This means that you will have to pay $1800 on 1/1/88, $1800 on 1/1/89, and that on 1/1/90 you must give the motorcycle back to the store.

The second way: You can buy the motorcycle for $3000 on 1/1/88. Then, on 1/1/90, when you do not need the motorcycle any more, you will be able to sell it for $1000.

(a) If the interest rate is 0%, what should you do?

(b) If the interest rate is 100%, what should you do?

(c) Suppose the interest rate is 100%. Also suppose that if you buy the motorcycle, instead of paying $3000 on 1/1/88, if you wish you may pay $2000 on 1/1/88 and $2000 on 1/1/89. Now what should you do: lease, buy with a payment of $3000, or buy with two payments of $2000 each?

4. Suppose you would like to subscribe for two years to Newsweek magazine. Usually, to do this you would have to pay $40 now, for a one-year subscription, and $40 in one year from now, for another one-year subscription. However, you receive a letter informing you that if you buy a two-year subscription right now, the price is only $75.

If the interest rate you can get from a bank is very high, what should you do: pay $75 now, or $40 now and $40 one year from now? What if the interest rate were zero? At what interest rate will you be indifferent between the two offers?

5. (a) What is today's price of an IOU promising to pay $12 two years from now if the interest rate will be 100% for this year and 200% for next year?
(b) What would the price of this IOU be if you tried to sell it one year from now?

6. Find the price (which is the present value) of an IOU promising to pay $3 on May 4, 1992 if the interest rate on alternative investments is: 100% between today (May 4, 1990) and May 4, 1991; and is 200% between May 4, 1991 and May 4, 1992. Explain the role of the “no regrets” criterion.

7. In the beginning of Chapter 18, Nicholson discusses the “rate of return.” He first defines it as in Figure 1; this is Robert Solow’s idea of the rate of return as being the “reward” in the future for a consumption “sacrifice” in the present. Next Nicholson explains how the rate of return is determined by using Figure 2, “the market for future goods.” Tell me all you know about criticisms of these ideas.

8. (a) The chapter in your textbook concerning the determination of the interest rate is entitled “The Pricing of Capital.” Why is this misleading?

(b) At the beginning of this chapter, Nicholson presents Robert Solow’s ideas on the rate of return. Explain why these ideas contradict some of Keynes’s opinions. Also explain why Solow’s ideas may not work for society as a whole, even though they might work for particular members of society.

(c) Tell me everything you know about Eugen von Böhm-Bawerk and his theory of “roundaboutness.”

9. In one or two sentences, briefly explain the answers to the following questions. (You have not seen these quotes before.)

(a) What is the name of the theory of interest which the author of the following quotation believes in? “[Suppose] 100 rice ripen into 110 rice in one year’s time...[then] no steady state (real!, ‘own’!) rate of interest can obtain other than 10% per year.”

(b) Whose theory of interest does the author of the following quotation believe in? “...by sacrificing ‘h’ units of consumption in the present, society can earn an extra consumption of ‘k’ units next period and suffer no ill effects thereafter. In such a case I would of course define the own-period rate of return on investments as (k/h) - 1.”
(c) How many interest rates does the author of the following passage think there are? "...capital theory should be liberated from the concept of the rate of interest, meaning by that one rate...It is natural to call \( \frac{P_{x,t}}{P_{x,t+1}} - 1 \) the own-rate of interest for good \( x \)."

(d) How would the author of the following quotation complete the sentence, 'Interest is the price of _____?' The quotation is: "...interest does not adhere to any class of concrete goods...Thus we cannot move away from the money basis of interest...Interest is only the consequence of [entrepreneurs who use] a special method of carrying out new combinations [that is, innovations]."
In 18.1a society withdraws some current consumption ($s$) in order to gorge itself (with $x$ extra consumption) in the next period. The one-period rate or return would be measured by $x/s - 1$. The society in 18.1b takes a more long-term view and uses $s$ to increase its consumption perpetually by $y$. The perpetual rate of return would be given by $y/s$.

The terms at which present goods trade for future goods ($P_f$) through capital accumulation is set by demand and supply conditions. It is likely that $P_f^*$ will be less than one. We define the rate of return ($r$) by the formula $\frac{1}{1+r} = P_f^*$. Since $P_f^* < 1$, $r$ will be positive.

Question 7's Fig. 18.1 & 18.2
1. a. \[ PV_A = 2 + 1 + 9 = 12 \]
   \[ PV_B = 1 + 7 + 1 = 9 \] \( \text{choose } A \text{ at } r = 0 \)

b. \[ PV_A = 2 \] \( \text{choose } A \text{ at } r = \infty \)

b. \[ PV_B = 1 \]

b. \[ PV_A = 2 + \frac{1}{(1+2)} + \frac{9}{(1+2)^2} = 2 + \frac{1}{3} + \frac{9}{9} = 2 + \frac{1}{3} = 3 \frac{2}{3} \] \( \text{choose } B \text{ at } r = 2 \)

\[ \text{at } r = 1: \left\{ \begin{array}{l}
PV_A = 2 + \frac{1}{(1+1)} + \frac{9}{(1+1)^2} = 2 + \frac{1}{2} + \frac{9}{4} = 4 \frac{3}{4} \\
PV_B = 1 + \frac{7}{(1+1)} + \frac{1}{(1+1)^2} = 1 + \frac{7}{2} + \frac{1}{4} = 4 \frac{3}{4}
\end{array} \right. \]

Choose either A or B if \( r = 1 \)

\[ \text{at } r = 3: \left\{ \begin{array}{l}
PV_A = 2 + \frac{1}{(1+3)} + \frac{9}{(1+3)^2} = 2 + \frac{1}{4} + \frac{9}{16} = 2 \frac{13}{16} \\
PV_B = 1 + \frac{7}{(1+3)} + \frac{1}{(1+3)^2} = 1 + \frac{7}{4} + \frac{1}{16} = 2 \frac{13}{16}
\end{array} \right. \]

Choose either A or B if \( r = 3 \)

\[ \text{This is a diagram showing which choices are best for which interest rates. It's different from examples we saw before because one choice is only good for intermediate levels of } r \]
<table>
<thead>
<tr>
<th>Date</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>9/1/89</td>
<td>-800</td>
<td>-51,000</td>
<td>-1000</td>
</tr>
<tr>
<td>9/1/90</td>
<td>-800</td>
<td></td>
<td>-1000</td>
</tr>
<tr>
<td>9/1/91</td>
<td>-800</td>
<td></td>
<td>-1000</td>
</tr>
<tr>
<td>9/1/92</td>
<td></td>
<td>+64,000</td>
<td>+600</td>
</tr>
</tbody>
</table>

\[
a. \quad PV_A = -800 + \frac{-800}{1+0} + \frac{-800}{(1+0)^2} = -2400
\]

\[
PV_B = -51,000 + \frac{64,000}{(1+0)^3} = +13,000 \quad \text{Best Choice}
\]

\[
PV_C = -1000 + \frac{-1000}{1+0} + \frac{-1000}{(1+0)^2} + \frac{600}{(1+0)^3} = -2400
\]

\[
b. \quad PV_A = -800 + \frac{-800}{1.1} + \frac{-800}{(1.1)^2} = -2188.4 \quad \text{Best Choice}
\]

\[
PV_B = -51,000 + \frac{64,000}{(1.1)^3} = -2915.9
\]

\[
PV_C = -1000 + \frac{-1000}{1.1} + \frac{-1000}{(1.1)^2} + \frac{600}{(1.1)^3} = -2284.7
\]

For 100% interest rate:
\[
PV_A = -800 \left[ 1 + \frac{1}{2} + \frac{1}{4} \right] = -1400 \quad \text{Best Choice}
\]

\[
PV_B = -51,000 + \frac{64,000}{8} = -51,000 + 8000 = -43,000
\]

\[
PV_C = -1000 \left[ 1 + \frac{1}{2} + \frac{1}{4} \right] + \frac{600}{8} = -1675
\]
<table>
<thead>
<tr>
<th>(part)</th>
<th>V1/88</th>
<th>V1/89</th>
<th>V1/90</th>
</tr>
</thead>
<tbody>
<tr>
<td>First way</td>
<td>-1800</td>
<td>-1800</td>
<td></td>
</tr>
<tr>
<td>Second way</td>
<td>-3000</td>
<td></td>
<td>+1000</td>
</tr>
<tr>
<td>Third way</td>
<td>-2000</td>
<td>-2000</td>
<td>+1000</td>
</tr>
</tbody>
</table>

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4 pts  First way: \( PV = -1800 - \frac{1800}{1+1} = -1800 - 900 = -2700 \)  

4 pts  Second way: \( PV = -3000 + \frac{1000}{(1+1)^2} = -3000 + \frac{1000}{4} = -2750 \)  

3 pts  The first way is better.

4 pts  Third way: \( PV = -2000 - \frac{2000}{1+1} + \frac{1000}{(1+1)^2} = -2000 - 1000 + \frac{1000}{4} \)  

\[ = -2750, \text{ so retry is still better.} \]
4) The $40 now, $40 one year from now option has a present discounted cost of \( 40 + \frac{40}{1+r} \). The "$75 now" option has a present discounted cost of $75. If \( r \) is very large then \( \text{PDC}_{40,40} \approx 40 + 0 = 40 \), which is less than $75 — so in this case you should choose the "$40, $40" option. If \( r = 0 \) then \( \text{PDC}_{40,40} = 40 + 40 = 80 > 75 \), so you should pay $75 now.

\[ \text{PDC}_{40,40} = \text{PDC}_{75} \text{ when } 40 + \frac{40}{1+r} = 75 \]

\[ \frac{40}{1+r} = 35 \]

\[ \frac{40}{35} = 1+r \quad \Rightarrow \quad r = \frac{1}{7} \approx 14.3\% \]

5) a)

<table>
<thead>
<tr>
<th></th>
<th>Today</th>
<th>1 Year from Today</th>
<th>2 Years from Today</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy the IOU</td>
<td>Pay $12</td>
<td>-</td>
<td>Receive $12</td>
</tr>
<tr>
<td>Don't Buy the IOU</td>
<td>Put $p in a</td>
<td>Bank balance ( p(1+r) )</td>
<td>Bank balance ( \frac{p(1+r)}{1+r} )</td>
</tr>
<tr>
<td></td>
<td>bank</td>
<td></td>
<td>at start of the second year</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1+r) with ( r = 200% )</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1+r) with ( r = 100% )</td>
<td></td>
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over \(
\)
Imposing the "No Replots" criterion gives

\[
12 = \left[ p \left(1 + \frac{1}{2}\right) \right] \left(1 + \frac{1}{2}\right) = 6p \Rightarrow p = 2.
\]

b) Let \( \hat{p} \) be the price one year from now.

<table>
<thead>
<tr>
<th>1 Year from Today</th>
<th>2 Years from Today</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy the IOU 1 Year from Today</td>
<td>Pay ( \hat{p} )</td>
</tr>
<tr>
<td>Don't buy the IOU 1 Year from Today</td>
<td>Put ( \hat{p} ) in a bank</td>
</tr>
</tbody>
</table>

No regrets \( \Rightarrow 12 = \hat{p} \left(1 + \frac{1}{2}\right) \Rightarrow \hat{p} = 4. \)

8 pts for some kind of explanation, such as the table on the last page (though other explanations may also get full credit)
### Table 6

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Buy the IOU</strong></td>
<td>Pay $p</td>
<td></td>
<td>Receive $3</td>
</tr>
<tr>
<td><strong>Don't Buy the IOU</strong></td>
<td>Put $p in a bank</td>
<td>Bank balance is $p(1+100%) = p(1+1)$ = 2$p$</td>
<td>Bank balance is $[2p][1+200%] = [2p][1+2]$ principal = $2p \cdot 3$ = 6$p$</td>
</tr>
</tbody>
</table>

The "no regrets" criterion says that come May 4, 1992, one should not regret making either choice on May 4, 1990. Thus means that the $3 you receive on May 4, 1992 if you had bought the IOU should equal the $6$p$ you will have on May 4, 1992 if you had not bought the IOU. Therefore,

$$3 = 6p \Rightarrow p = \frac{504}{2} = 252$$

### About Fig. 18.1:

1. Vague: Which "r" is the rate of return, [18.1] or [18.2]?  
2. Hard to relate to the behavior of firms and consumers in real markets.  

### About Fig. 18.2:

This is a fictional market.
a) The interest rate is the price of credit, not the price of capital.

Also, since there are many different kinds of capital goods, how can there be only one "price" of capital?

b) Keynes said that when consumption falls in one year, this might cause a slump, in which case consumption would fall in the next year, not rise, as so few claims.

Any one individual can sacrifice $1 of consumption today for more than $1 of consumption tomorrow, by lending out his $1 to someone else. But society as a whole can't lend out $1 to anyone else, because there is no one else.

c) Question: How can a businessman promise to pay $1.05 next year if you give him $1 this year?

Bohm-Bawerk's answer: Assume that businessmen spend all their credit to buy capital goods. (This is a dubious assumption, but Bohm-Bawerk and many other authors make it.) Then the businessmen can pay more next year because investing in indirect ("roundabout") production processes is better than using direct production processes.

[Give an example of a roundabout production process]
9) a) This is an example of the fructification theory of interest, which holds that interest arises from the ability of living things to reproduce more of themselves. [The name comes from Chapter II, Book VII of Bohm-Bawerk's *Capital and Interest*, where the theory is attributed to Henry George. See also Irving Fisher, *The Theory of Interest*, Chapter III, Section 4. The quotation is from pp. 22-3 of Paul Samuelson.]

b) This is Robert Solow's idea of the rate of return as being a "consumption bonus" which "society" gets from "sacrificing "consumption today in the future. [The quotation is from p.19 of Solow's book *Capital Theory and the Rate of Return*].

c) The author believes there is one interest rate for each commodity in the economy — so there are as many interest rates as there are commodities. (5pts) (If there are more than two dates then there are even more interest rates.) This is the idea behind "own-rates of interest." [Debreu's *Theory of Value* uses own-rates, as does Keynes in the *General Theory*. This quotation is from p. 10 and p. 52 of Christopher Bliss.]

d) "Interest is the price of credit." This is because the author says that interest does not adhere to real, physical goods, but has its basis in money (or finance). None particularly, interest comes from profits of innovating entrepreneurs who needed credit. [The quotation is from Joseph Schumpeter, *The Theory of Economic Development*, p. 211]