

Real Income Growth and Optimal Credit Use

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Borrowing may be optimal if real income is expected to increase. If income growth is uncertain, optimal credit use is not obvious. A two period model of consumption for determining optimal credit use is presented. The impact of real income growth is analyzed with numerical analysis. The results may be useful for financial counselors and educators, as well as for insight into empirical patterns of credit use. The income growth rate expected by the household plays a crucial role in determining optimal credit use for current consumption.

I. INTRODUCTION

Economic investment theory models developed by Fisher (1930) and Hirshleifer (1970) suggest consumers may increase market opportunities and their utility through judicious selection of debts and assets (Herendeen 1975). If a consumer is uncertain about future income, a small sustained growth (decrease) in real income or a substantial "expected" one-time increase (decline) might lead to borrowing (or saving) to smooth consumption over life cycle. Young consumers, especially students, and other families with temporarily low income might find borrowing rational. Clearly, the use of consumer credit makes it possible for families and individuals to have the immediate consumption of goods and services and thus raise their level of living and satisfaction. However, the dramatic growth of consumer installment debt and the holding and use of credit cards from the past two decades (Eastwood 1985; Canner 1988), has led financial planners and educators to express alarm regarding whether consumers are becoming debt-ridden and overextended. The purpose of this paper is to describe a model for determining optimal credit use decisions for consumers with uncertain future income. The vehicle of analysis is the

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familiar two-period model of consumption. Analysis is confined to credit for current consumption.

II. THE LITERATURE

There has been extensive discussion in the literature of optimal saving (borrowing) and consumption behavior under uncertainty either in the context of infinite time horizon or in two-period or multiperiod intertemporal models (e.g., Leland (1968); Levhari & Srinivasan (1969); Sandmo (1970); Mirman (1971); Dreze & Modigliani (1972); Sibley (1975); Hey (1979); Salyer (1988)). In general, the authors analyze one or two variables at a time, assuming a value for each of the other parameters. For example, in two-period models the effects of income and interest rate uncertainty on borrowing (or saving) decisions are analyzed, given an assumption of a certain lifetime. Infinite horizon or finite horizon models explore effects of the discount factor (lifetime uncertainty) on borrowing (saving) behavior while assuming absence of income and interest rate uncertainty.

In the discussion of income uncertainty and saving behavior, it is assumed that the consumer's beliefs about the value of future income can be summarized in a subjective probability density function. On the basis of the probability density function, the consumer maximizes expected utility of consumption. Leland (1968) uses a two-period model of consumption to demonstrate the effect of uncertainty on saving and concludes that with an additive utility function and the assumption of decreasing absolute risk aversion, the precautionary demand for saving is a positive function of uncertainty. Sandmo (1970) discusses the effects of increased riskiness of future income on present consumption in a two-period model and proves that increased uncertainty about future income decreases consumption (increases saving). Sibley (1975) extends the two-period result of the effects on optimal savings of increased risk in the future income to the multiperiod analysis discussed by Leland (1968). Sibley (1975) suggests that increased wage uncertainty increases saving. For the case of a constant elasticity utility function, Levhari and Srinivasan (1969) show that optimal savings can increase with increasing uncertainty. However, those authors mainly emphasize the effects of the subjective probability density function as a projection of uncertain future income on saving (or borrowing) behavior. No study has been done relating levels of risk aversion, interest rates, and income growth rates to optimal borrowing in a model incorporating uncertainty.

Kinsey and Lane (1978) point out that when consumption is accompanied by the use of consumer credit, utility maximization may be viewed in a lifetime sense, thus a life cycle approach to the allocation of income, consumption, and saving (borrowing) is appropriate. Additionally, by appropriate interpretation, two-period models can describe completely the individual's resource allocation problem during their lifetime, if the focus is on consumption in that period and total consumption in all future periods (Hey, 1979). With additional assumptions on certain risk properties of utility functions, a two-period model with uncertainty for determining

optimal credit use facing consumers is presented and illustrated with numerical analysis. Implications for a life cycle model are then discussed.

Factors affecting optimal credit use include the expected growth rate of real income, the variance of future income, the consumer's utility function (e.g., the parameter of risk aversion), the real interest rate, and the consumer's personal discount rate.

III. A TWO-PERIOD MODEL OF CONSUMPTION

To begin, consider the following model: assume that consumers attempt to maximize the expected value of utility (T) for the two periods. They will make their borrowing (or saving) decision in conjunction with their known first period income. The second period consumption will, of course, be a random variable, dependent on the actual value of second period income which is assumed to be affected by income growth rate (or decrease rate) and the probability that income growth occurs, and also dependent on the interest rate of borrowing (or saving). It is assumed that there are two states of the world in the second period—real income either increases or stays constant. (The analysis could allow for other scenarios, but the discussion is limited to this scenario because it is the most plausible scenario for borrowing to be rational). C_2 and C_{2a} represent consumption in these states. Finally, consumers are assumed to repay the loan in full in the second period. Mathematically, the problem can be formulated as:

$$T = U(C_1) + \frac{P * U(C_2) + (1 - P) * U(C_{2a})}{(1 + \rho)} \quad (1)$$

$$\text{The constraints are: } C_1 = I - S \quad (2)$$

$$C_2 = (1 + g) * I + (1 + r) * S \quad (3)$$

$$C_{2a} = I + (1 + r) * S \quad (4)$$

Variables:

- T = Total two period utility
- I = Year 1 income
- Year 2 income = $(1 + g) * I$ (if income increases in that year), otherwise,
- Year 2 income = Year 1 income
- C_1 = Consumption in Year 1
- S = The amount of savings in Year 1 (negative value means borrowing.)

- C_2 = Consumption in Year 2 if real income in Year 2 increases
 C_{2a} = Consumption in Year 2 if real income in Year 2 does not increase
 g = Growth rate in real income
 r = Real interest rate (Note that r may be higher for $S < 0$, i.e., borrowing, than for $S > 0$)
 P = Probability that real income increases
 ρ = Personal discount factor

A consumer may discount utility from future consumption because of the possibility of not being alive then, or because of other possible changes in capacity to derive utility from consumption. Discounting because of the risk of death should be small for a young adult, although some younger consumers may discount future consumption because of "impatience" or limited ability to imagine utility as a middle aged or elderly consumer.

It is possible to find the level of C_1 and C_2 (and therefore savings or dissavings) which produce the highest feasible level of utility by using calculus. Only general results can be derived from equation (1), unless restrictions are placed on the utility function. Most studies of intertemporal consumption have used a constant elasticity utility function (Hurd, 1989) which is additively time separable:

$$U = \frac{C^{1-x}}{1-x} \quad (5)$$

The elasticity of marginal utility with respect to consumption is $-x$. The elasticity of intertemporal substitution in consumption is equal to $1/x$. When this type of utility function is used for analysis of risk, the parameter x is relative risk aversion. C is consumption per time period.

Estimates of Relative Risk Aversion

Grossman and Shiller (1981) have given x an interpretation as "... a measure of the concavity of the utility function or the disutility of consumption fluctuations" (Grossman & Shiller, 1981, p. 224). The higher the value of x , the more risk averse is the consumer, and the more rapidly marginal utility decreases as consumption or wealth increases. The analysis of economic behavior under uncertainty uses relative risk aversion extensively. For intertemporal consumption, empirical estimates of x range from just under two (Skinner, 1985) to 15 (Hall, 1988). Other estimates were between these two values.

The wide range of estimates of the utility function parameter, x , relative risk aversion, may be due to several causes, including:

1. Many households face liquidity constraints (they have zero or low levels of assets, and therefore cannot dissave without borrowing at a much higher interest rate than can be obtained from safe investments).
2. The data sets used for empirical analyses did not contain appropriate variables. For instance, the theoretical measure of savings used in the literature is the amount not consumed. The measure used in many studies is the change in net worth.

However, although it is often assumed that a consumer cannot identify a utility function explicitly, it may be possible to construct hypothetical examples that allow one to intuitively identify a unique utility function parameter. It is possible to create a scenario to obtain insight into the similar parameter for the intertemporal utility function. To obtain some insight into plausible values of relative risk aversion, x , consider the following *hypothetical* situation: You are 20 years old, and know with *certainty* that you will live to be 100 in good health. Everything about your personal situation will remain the same for the next 80 years. You want to spend all of your wealth by the day of your death. Your non-asset income will be \$20,000 per year in real (constant dollar) terms. You can obtain exactly 6% per year after inflation and taxes on investments. Table 1 shows optimal consumption paths for different values of x , assuming $\rho = 0$.

Based on the hypothetical example, a value of $x = 1$ (which corresponds to a natural logarithm utility function) would seem extremely miserly, as you would spend only \$4,323 of your \$20,000 income at age 20 in order to enjoy \$457,382 of consumption the last year of your life. A value of $x = 6$ might be representative of the typical American consumer, as the consumer would spend \$16,929 out of \$20,000 income at age 20, and could spend \$36,817 at age 100. It seems likely that most Americans would have a value of x between four and eight.

Kimball's (1988) hypothetical example for relative risk aversion, may imply a value between four and eight (Hanna 1988). The utility function $U(w)$, and the expected utility $EU(w)$ are specified as follows,

TABLE 1.
Optimal Consumption by X, Hypothetical Example

Age	$x=1$	$x=2$	$x=3$	$x=4$	$x=5$	$x=6$	$x=20$
20	4,323	11,104	13,942	15,421	16,323	16,929	19,073
30	7,742	14,859	16,931	17,840	18,341	18,656	19,637
40	13,865	19,885	20,560	20,637	20,608	20,558	20,217
50	24,831	26,611	24,968	23,874	23,155	22,655	20,815
60	44,468	35,611	30,320	27,618	26,017	24,966	21,431
70	78,635	47,656	36,820	31,948	29,233	27,512	22,064
80	141,614	63,774	44,713	36,959	32,846	30,318	22,716
90	255,400	85,344	54,299	42,754	36,905	33,410	23,388
100	457,382	114,210	65,939	49,459	41,467	36,817	24,079

TABLE 2.
Hypothetical Example of Relative Risk Aversion

Relative Risk Aversion	Lowest Value of <i>I</i>
0	0
1	25,000
2	33,333
3	37,796
4	40,548
6	43,665
10	46,299
20	48,209

$$U(w) = \frac{W^{1-x}}{1-x} \quad (6)$$

$$EU(w) = \sum P_i U(w_i) \quad (7)$$

where x = relative risk aversion level
 w = total wealth

A modified version of Kimball's (1988) example developed by Hanna (1988), could explain the concept of relative risk aversion in the context used.

Assume that you have one year to live, and may choose an investment to provide you with your consumption for the next year. Once you choose, it will be impossible for you to obtain income from any other source. You have no assets of any kind. You may choose one of two plans: A or B. Plan A pays you a tax free real income of \$50,000 per year, while plan B involves a gamble. If you choose plan B, the government in effect flips a coin, and there is a 50% chance of having a real income of \$100,000 tax-free, and a 50% chance of some lower income I . At what level of I would you be indifferent between Plan B and Plan A. Table 2 shows how your answer corresponds to your level of relative risk aversion.

In the context of the expected utility model, relative risk aversion relates to the extra utility of increased consumption if the gamble pays off compared to the lost utility because of decreased utility if you lose the gamble. For instance, if you have a relative risk aversion level of four, you value the gain of utility from increasing your consumption from \$50,000 to \$100,000 the same as the loss of utility from decreasing your income from \$50,000 to \$40,548 (Hanna, 1988). Intuitively, a level of at least four would seem reasonable for most people.

By combining intertemporal consumption analysis with risk aversion, we can obtain the optimal amount of saving in terms of Year 1 income, interest rate, income growth rate, and probability of that income increases. To give some intuitive insight into optimal credit a model with perfect certainty will be examined first.

IV. OPTIMAL CREDIT WITH PERFECT CERTAINTY

If a consumer is certain that real income will increase with a growth rate g , and the consumer faces a real interest rate r , we can derive by calculus the optimal pattern of consumption (expressed as the growth rate of real consumption), as shown in equation (8). For plausible values of the real interest rate r , the personal discount rate, ρ , and relative risk aversion x , the optimal growth rate in consumption approximately equals the interest rate minus the personal discount rate, divided by relative risk aversion. Note that the optimal growth rate in real consumption does not depend upon income patterns, although income patterns over time may influence the interest rate faced by a household, and also the feasibility of particular consumption patterns.

Based on a regression of U.S. household expenditures on age and age squared, it can be estimated that real expenditures of households tend to increase with age until age 49 (Bae, 1993, p. 137). Consumption levels of individual households tend to be relatively stable over a lifetime, although there are several plausible theoretical explanations for this stable pattern (Yunker, 1992). Households with substantial amounts of financial assets face real, after tax interest rates of 0% to 6% on safe, liquid assets. Households who would have to borrow might face real interest rates between 3% (home equity loan) and 20% (finance company.) The fact that the elderly are most likely of any age group to have declining levels of real consumption implies (at least within the life cycle framework) that the personal discount rate depends on the risk of death. There does not seem to be strong evidence of extra "impatience" by young consumers. If the personal discount rate is related to the risk of death, it is unlikely to be an important factor in credit decisions. It will be shown below that credit use can be explained by an observable factor, growth rates of real income, so there are advantages to dispensing with the unobservable factor of the personal discount rate. In any case, as the approximation in equation (8) shows, the effect of a higher personal discount rate can be approximated by using a lower real interest rate.

$$\frac{(C_2 - C_1)}{C_1} = \left(\frac{1+r}{1+\rho} \right)^{\frac{1}{x}} - 1 \approx \frac{r-\rho}{x} \quad (8)$$

Equation (9) gives optimal savings as a proportion of Year 1 income. The consumer's relative risk aversion is x . For particular values of r and g , the greater the relative risk aversion, the more the consumer should borrow. This seemingly paradoxical result is due to the fact that the two period model with certainty involves no risk, but only intertemporal allocation. If the consumer faces a higher interest rate for borrowing than for saving, there may be some growth rates for which neither borrowing nor saving is optimal. If the ratio is negative, borrowing is optimal.

$$\frac{S}{I} = \frac{\left(\frac{1+r}{1+\rho}\right)^{\frac{1}{x}} - (1+g)}{\left(\frac{1+r}{1+\rho}\right)^{\frac{1}{x}} + \left(\frac{1+r}{1+\rho}\right)} \quad (9)$$

The natural log utility function ($U = \ln[C]$) has been used frequently, although the example from Table 1 implies extremely miserly behavior. It is simple to analyze. Substituting the value of $x = 1$ in equation (9), then for reasonable values of r , equation (10) is a good approximation.

$$\frac{S}{I} = \frac{(r-\rho) - g}{2(1+r)} \quad (10)$$

Note that if $r-\rho$ is less than g , saving is negative, so that some borrowing is optimal. For a given value of r , as g increases, the optimal amount to borrow will increase. For instance, if $g = 20\%$, $\rho = 0\%$ and $r = 10\%$ then $S/I = -4.5\%$. If this year's income is \$10,000, the optimal amount to borrow is \$450.

For other values of x , Equation (9) is somewhat complicated. However, an intuitive sense of the patterns can be obtained by using approximations, resulting in equation (11). If the real interest rate (r) minus the personal discount rate (ρ), divided by the relative risk aversion, is less than the expected growth rate in real income (g), borrowing is optimal. For instance, if r is 12%, ρ is 0%, and x is 6, then borrowing is optimal if g is greater than 2%. Clearly, one does not need to rely on the assumption of a high personal discount rate to explain borrowing for current consumption. An analysis of households in the Survey of Consumer Finances interviewed in 1983 and re-interviewed in 1986 (Chang and Lindamood, 1993) shows that, based on two year household income in 1982/1983 and two year household income in 1984/1985, the median growth rate in real income for households under the age of 35 was 20%. The annualized median growth rate for young households was almost 10%. For the households under 35, 25% had an annualized growth rate of 22% or higher. Consumers who were certain of a high growth rate might borrow even if they faced rather high interest rates.

$$\text{Borrow if } \frac{r-\rho}{x} < g \quad (11)$$

V. OPTIMAL CREDIT WITH UNCERTAINTY

There is no simple, closed analytical solution for optimal savings or credit with uncertainty. Therefore, simulations were used to find optimal savings/credit.

Simulation Results: Factors Affecting Optimal Credit Use

Equations (1) through (4) were used with simulations to find the value of s that maximized expected lifetime utility for particular values of the parameters. In this section, we shall discuss and illustrate effects of real income growth on optimal savings/credit use, for two levels of relative risk aversion and three probability levels. In order to focus on scenarios with borrowing, it was assumed that the consumer faced either constant real income or a real income growth rate g with a probability p .

In the cases of certainty (i.e., probability of income increase equals one), the greater the relative risk aversion, the less the consumer will save, or the more the consumer will borrow. The relative risk aversion is related to how much more utility the consumer will lose due to low consumption in Year 1 than they will gain from higher consumption in Year 2. For any given real income increase, the consumer will borrow more in order to smooth out consumption as much as is justified by the utility function and the real interest rate on loans. When uncertainty is added to the total period utility function (i.e., probability of income increases between zero and one), the borrowing-relative risk aversion relationship observed for certainty does not always hold. The simulations were based on the following assumptions:

1. The real interest rate on loans = 14.095% (e.g., nominal rate of 19.8% with 5% inflation.)
2. The real, after tax interest rate on savings = 1% (e.g., nominal interest rate of 8.4%, subject to 28% tax rate and 5% inflation.)
3. Expected utility from all possible borrowing levels (at 14.095%) is compared to expected utility from all possible saving levels (at 1%) and optimal saving/borrowing is that which produces highest expected utility.
4. The personal discount rate, ρ , is zero.

Figure 1 shows the relationship between the optimal ratio of amount saved in Year 1 to Year 1 income and the rate of increase of real income, for three probabilities (50%, 95% and 100%) that income will increase, each for two levels of relative risk aversion (one and six). Table 3 shows similar information, except expressed as the amount *borrowed* as a percent of Year 1 income. There are threshold levels of real income growth for any borrowing to be optimal (Table 4). For real income growth levels below these threshold levels, there is a range of growth levels for which neither borrowing nor saving is optimal. At very low levels of growth, a small amount of saving is optimal. For instance, for relative risk aversion of 1.0, if the consumer is certain that real income will remain constant, the optimal amount to save out of Year 1 income is 0.50% of income. If the consumer thinks there is a 50% chance that real income will increase by 1%, optimal saving will be 0.25% of income for relative risk aversion of 1.0 but 0.0% for relative risk aversion level of 6.0.

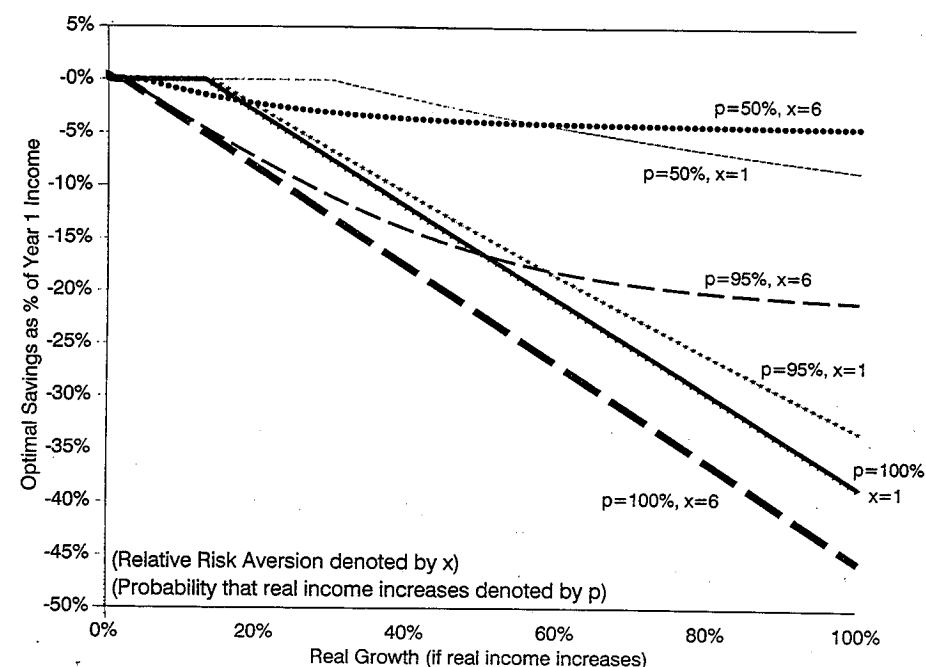


Figure 1. Optimal Savings (borrowing) as % of Income, by Growth Rate, Probably and Risk Aversion

In the cases of certainty, there is a virtually linear increase in the optimal amount to borrow as g increases. With $x = 6.0$, some borrowing is optimal even if the growth rate is only 5%. With $x = 1.0$, the growth rate must equal 20% for borrowing to be optimal. If the consumer is 95% sure that real income will increase, the pattern is almost the same as certainty if $x = 1.0$, but the pattern is very different if $x = 6.0$, with optimal borrowing at $g = 100\%$ for $p = 95\%$, less than half the amount

TABLE 3.
Optimal Amount to Borrow, as Percent of Year 1 Income, by Growth Rate(g),
Relative Risk Aversion (x) and Probability that Income Increases

Growth Rate	$x=1$			$x=6$		
	$p=50\%$	$p=95\%$	$p=100\%$	$p=50\%$	$p=95\%$	$p=100\%$
10%	0.00%	0.00%	0.00%	0.98%	3.37%	3.69%
20%	0.00%	2.57%	3.10%	2.25%	7.42%	8.34%
30%	0.02%	6.66%	7.52%	3.05%	11.05%	12.99%
40%	1.60%	10.69%	11.95%	3.54%	14.08%	17.64%
50%	3.03%	14.65%	16.37%	3.85%	16.42%	22.29%
60%	4.34%	18.55%	20.80%	4.05%	18.10%	26.94%
70%	5.52%	22.36%	25.22%	4.17%	19.26%	31.59%
80%	6.61%	26.01%	29.65%	4.25%	20.04%	36.24%
90%	7.59%	29.68%	34.07%	4.31%	20.55%	40.89%
100%	8.49%	33.17%	38.50%	4.35%	20.90%	45.54%

TABLE 4.
Minimum Growth Rate Needed for Borrowing to
be Optimal, by Relative Risk Aversion(x) and
Probability that Income Increases

Probability	$x=1$	$x=6$
50%	30%	5%
95%	14%	3%
100%	14%	3%

for $p = 100\%$. If there is a 50% chance that real income will increase, borrowing is not optimal for $x = 1.0$ unless growth of 35% or more is expected; while with $x = 6.0$, some borrowing is optimal even for growth of 5%, but the amount is limited.

A consumer expecting a high probability of a substantial increase in real income may rationally borrow a large amount of money for current consumption. The importance of a correct assessment of the probability of an income increase may be seen in Figure 1. For any particular level of relative risk aversion, optimal borrowing is substantially greater for higher probabilities. Note, however, that regret is not assumed to enter the utility function. If a consumer with an income of \$40,000 has a relative risk aversion of six and a probability of 95% that real income will increase by 50%, then there is a 5% probability of having the discomfort of repaying the \$5,860 borrowed out of an unchanged income of \$40,000. This will result in a drop in real consumption from \$45,860 in Year 1 to \$33,314 in Year 2, or a drop of 27.4%.

VI. EXTENSIONS OF THE MODEL

Durable Goods

The analysis presented assumes that all spending is for current consumption, which may be realistic for a consumer who rents a home and leases automobiles. Use of credit for some types of durable goods, such as automobiles and kitchen/laundry appliances, may be rational even if real income is not expected to increase. However, for decisions about how expensive the durable good should be beyond minimum standards (e.g., reliable transportation), the analysis presented in this article may give some insights into how such choices should be made.

Extensions to More Than Two Periods

If the analysis is extended to three periods, but the assumption is made that the real income level during the third year is whatever the real income level is during the second year, then optimal credit for the first year is higher than the corresponding level shown in this article for the two period model. Allowing for changes between

Year 2 and Year 3 introduces a much higher degree of complexity, and has not been addressed by the authors in this article.

Allowing for Decreases in Real Income

If there is a possibility that real income will decrease, optimal saving may be positive. For instance, if real income will either remain constant or decrease, and both states of the world are equally likely, the consumer should save some money from Year 1 income in order to prevent too much of a decrease in Year 2 consumption. If, however, the probability that real income decreases is small, optimal saving may be low.

Extensions to More Than Two States of the World

If there are more than two states of the world, analysis of optimal behavior is complex. For consumers with a very small chance of a large decrease in real income, and approximately equal chances of constant real income or a substantial increase in real income, the optimal saving (credit) pattern will be approximately the same as the patterns presented in this article for the two states of the world model. For some consumers, a small possibility of a substantial decrease in income could be dealt with through help from relatives and the social safety net. Consumers could also implicitly assume that decreases in real income could be dealt with by default.

Extensions to a Life Cycle Model

The two period model can be extended to a life cycle model if certainty is assumed. For probabilities greater than 98% that real income will increase, there may not be substantial differences in optimal credit use, if it can be assumed that real income will either increase or remain constant after the first year. For many households, the simplifying assumption that income will either increase or remain constant is very unrealistic. However, if there is a small probability that there will be a substantial drop in real income, a consumer who has taken on credit for current consumption has the option of default or some form of bankruptcy. These options, as well as the opportunity to repay credit early, can be viewed as put options which have implicit value for the borrower. To model the costs of bankruptcy is beyond the scope of this paper.

Taking Default into Account

The model used in this article ignores the possibility of default and/or bankruptcy. The results described are based on the assumption that the consumer must repay the loan in full. There are obvious costs of default and various forms of bankruptcy. If these costs were low, even more consumers would become overextended. It is difficult to specify the monetary value of the costs of default, etc., but given the increasing number of bankruptcies, it is plausible that a priori, borrowing

even if bankruptcy is possible is rational for some consumers. It is also plausible that many consumers may underestimate the true costs of bankruptcy, and therefore take too much risk with credit use. Most consumers who use credit face at least a small risk of default. The analysis presented in this article provides a starting point to development of a realistic evaluation of rational credit use.

VII. SUMMARY AND CONCLUSION

A two-period model of consumption is developed to analyze optimal credit use decisions, based on the probability that future real income will increase, for different levels of relative risk aversion. With the assumptions that the utility function be additive between periods, and reflect constant relative risk aversion, effects of parameters on optimal borrowing and saving decisions and the interacting relationships are discussed and demonstrated using numerical simulation technique and graphs. We have shown that the optimal amount of credit use increases with increasing income growth rate and with increasing probability of real income growth. There are threshold levels of real growth rates for borrowing to be optimal, at relatively low levels for certainty, but at high levels for lower probability levels that growth will take place. The optimism of the middle of the 1980s and the longest peacetime expansion (see discussion in Chang and Lindamood, 1993) may have led to the increase in consumer credit and may have contributed to the increase in personal bankruptcy rates. If young consumers in the future expect stagnant real income levels and have high levels of uncertainty about their prospects, the amount of consumer credit will continue to decrease. Only optimistic consumers will borrow for current consumption.

The simplest possible model of optimal credit use dealing with uncertainty has been developed. Clearly it would be desirable to extend the model to the borrowing and saving decisions over more than two time periods. More complicated models such as multiperiod or a life span analysis of optimal saving and borrowing decisions, however, may require complex computer programming techniques. Further analyses using empirical data may be needed for comparisons between theoretical and empirical results.

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The Individual's Tax-Exempt Bond Portfolio Decision Under Income Uncertainty

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In this article, an individual's tax-exempt bond portfolio decision is investigated. A model capturing the relationship between income uncertainty and optimal portfolio choice is defined when an individual decision-maker has the opportunity to hold higher yielding private-activity bonds. The findings in this article show that in most cases risk-averse individuals will maximize the expected utility of after-tax income by holding a large proportion of private-activity bonds in their portfolio even under income uncertainty and the risk of a minimum tax liability. Those individuals who would benefit from holding private-activity bonds in a tax-exempt portfolio are identified and the magnitude of the benefit is quantified.

I. INTRODUCTION

The Tax Reform Act of 1986 created two general classes of municipal investments, essential-purpose and nonessential-purpose or private-activity bonds. Essential-purpose bonds yield interest income that is exempt from any form of federal income tax. However, interest income on private-activity bonds issued after August 7, 1986 is classified by the Internal Revenue Service as preference income and is exempt from federal income tax only if an individual is not subject to the alternative minimum tax.¹ The alternative minimum tax (or minimum tax) is a flat tax rate applied to minimum taxable income which is the total of regular taxable income plus preference income and adjustments. An individual is subject to the minimum tax if the minimum tax liability exceeds the regular tax liability.² The probability of an individual being subject to minimum tax increased with 1990 tax reform when the minimum tax rate was increased from 21% to 24%.

The separation of municipal bonds into two distinct classes alters individual decision-making regarding the tax-exempt portfolio because of the relatively high yields and the uncertain tax treatment of private-activity bonds. In this article, the

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