Financial Intermediaries as Firms and the Business Cycle*

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Abstract

This paper studies the propagation of shocks via financial intermediaries over the business cycle. The leverage of financial intermediaries is shown to be an important determinant of credit conditions. Importantly, in this model the relevance of financial intermediation does not stem from legal restrictions such as reserve or capital requirements, rather, agency costs due to asymmetric information are present for intermediaries as well as for other firms. The model is therefore applicable to all forms of financial intermediation, not only to banks.

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1 Introduction

The financial markets' role in business cycle propagation has been a hot topic of recent research due to the inability of most microfounded general equilibrium models to generate the long lived responses to shocks found in VAR studies. Recently, Chari, Kehoe, and McGrattan (2000) have argued that in an otherwise standard monetary RBC model price stickiness should be unreasonably high for the model to match the observed impulse-responses. Models that feature financial market imperfections address this issue by having endogenous propagation mechanisms that are collectively called *credit channels*. Broadly, these come in two flavors: The *balance sheet channel* emphasizes the importance of firms' net worth in determining the terms of their borrowing. Since net worth is procyclical, during a downswing of the business cycle firms face higher opportunity costs of external finance, which induces less investment and adds to the longevity of the recession.

As opposed to the balance sheet channel, which depends on the financial health of borrowers, the *bank lending channel* stems from the funds available to their lenders, banks. To the extent that intermediated credit is important for producers and cannot be substituted for by other means, and to the extent that banks cannot replace lost deposits with other liabilities, a monetary policy tightening that decreases reserves, and therefore deposits (due to reserve requirements) of banks will lead to reduced intermediated credit to producers and a lower level of economic activity. This effect is in addition to the liquidity effect of monetary policy changes.

Kashyap and Stein (1994), Kashyap, Stein, and Wilcox (1993), Kishan and Opelia (2000), and Ashcraft (2001) provide evidence for the existence of a bank lending channel. Of these, KSW is especially important; this paper shows that at the beginning of a downturn of the business cycle, when bank lending decreases, firms—that are able to do so—try to compensate for the loss of funding by issuing direct debt. That is, the decrease in bank lending is not only demand driven. Nilsen (2002) provides further evidence in this direction by showing that at times of tight credit firms increase their dependence on trade credit, i.e., they look for alternative means of financing. Gertler and Gilchrist (1994) show that firms of different sizes are affected differently from lack of intermediated financing. Larger firms turn to direct borrowing and decrease their production less, whereas small firms that do not have access to commercial paper markets have to decrease their activity more.

The multitude of empirical papers on the subject is not matched by theoretical studies of the bank lending channel. Repullo and Suarez (2000) and Van den Heuvel (2000), are among the few papers that have theoretical models with lending channel properties. To my knowledge, all theoretical studies of the lending channel stem from legal aspects of banking. That is, these models assert that the intermediaries are banks and then use legal restrictions on banks, such as regulation Q type interest rate ceilings (Repullo and Suarez) or capital adequacy requirements (van den Heuvel) to generate a role for internal funds of financial intermediaries in determining the amount and terms of intermediated credit. In an otherwise similar paper Zeng (2002) depends on banks' issuance of both insured and uninsured deposits to argue that monetary policy has real effects even when prices are flexible.

The standard bank lending channel argument has been criticized by Romer and Romer (1990) on the grounds that banks that lose deposits can replace these by issuing non-reservable liabilities such as certificates of deposit. Models that put limits on interest rates or capital ratios are immune to this criticism by assumption. However, assuming these features are not necessary. As Kashyap and Stein (1994) note, there will be bank lending channel as long as banks face an upward sloping supply of non-reservable funds.

This paper differs from the literature in its treatment of financial intermediaries. I model financial intermediaries (FIs) as firms that sell intermediation services. Despite producing a different 'good' than other firms, the financial intermediaries themselves are, after all, firms. A large body of literature exists on the financial frictions firms face and the resulting financial accelerator. I argue that financial intermediaries, as firms, should be subject to financial market frictions of the kind 'other firms' are subject to. Therefore, the intermediaries in my model do not have reserve requirements (or deposits in the conventional sense, for that matter) or capital adequacy requirements, but the size of their balance sheets are still constrained by their leverage.

This way of modeling the credit channel has several appealing properties. The most important one is that the friction that leads to bank lending channel is market based and therefore is applicable at all times to all financial institutions, not just to contemporary banks. This model, for example, can be used to study the effect of loss of banking capital during the great depression, a time when there were no capital requirements. It is also a nice feature that it explains the role of non-bank intermediaries, such as the consumer lending arms of automobile manufacturers in the US, as much as it explains the role of deposit banks over the business cycle.

The crux of the model is the argument that, at the margin, financial intermediaries, be them banks or not, fund themselves via non-reservable, non-insured securities and face an upward sloping supply curve of these. The fact that not all liabilities of FIs are insured is important. Obviously, if the expected recovery rate is unity following a default, there will be no agency problems as lenders will be paid back in all states of the world. The model presented in this paper does not take into account that some of the liabilities of intermediaries are covered by deposit insurance, therefore in the model agency problems apply to all liabilities of FIs. Taking partial insurance of liabilities into account would not change the central conclusions of the paper.

The rest of the paper is structured as follows: Section 2 below introduces the different types of agents and lays out the assumptions underlying the model. Sections 3 and 4 deal with the optimal contracts between the FIs and producers, and households and FIs, respectively. Section 5 shows that the net worth of financial intermediaries matter for business cycle dynamics in addition to the net worth of producers. Section 6 concludes.

2 Basics of the Model

There are two main types of agents in the economy, households and entrepreneurs. Households are risk-averse, entrepreneurs are risk-neutral. Entrepreneurs come in two types: financial intermediaries, denoted by F, and producers, denoted by P. Every period, producers buy capital from capital producers and hire labor to produce a wholesale good. At the end of the period capital is re-sold to capital producers and the wholesale good is either consumed or added to next period's capital stock.

It is assumed that households cannot observe the outcome of producers' projects at any cost. Financial intermediaries, on the other hand, can observe the outcome of a project undertaken by a producer at a cost, and can themselves be 'audited' at the same cost. In the economy, there are many financial intermediaries and many producers per intermediary. More specifically, there are a continuum of FIs distributed over the unit interval, and each intermediary has a region that it caters to exclusively, populated by a continuum of producers, again distributed over the unit interval. The law of large numbers applying at both the level of the individual intermediary and over the intermediaries comes handy when aggregating over the whole economy.

All entrepreneurs have a probability $1-\zeta$ of death at the end of the period. The deaths happen independently within intermediaries and within the producers in each region. Entrepreneurs who die consume their equity before they leave the scene and new intermediaries and producers replace the dead ones. The deaths are to ensure that neither financial intermediaries nor producers accumulate enough net worth to finance their operations without borrowing. The independence assumption guarantees that the densities of financial intermediaries and producers per FI do not change over time.

Given the informational asymmetries, contracts in this economy cannot be made contingent on the realization of the stochastic shocks, and therefore, as we will see in sections 3 and 4, the contracting parties will sign incentive-compatible, optimal contracts that minimize the probability of costly auditing. The auditing cost is a constant fraction μ of the actual outcome of the project. That is, it costs more to audit big projects than to audit small projects.

Producer i in region j who has net worth N_{t+1}^{ji} at the end of period t, will borrow B_{t+1}^{ji} given the price of capital Q_t, and her desired capital stock K_{t+1}^{ji}:

$$B_{t+1}^{ji} = Q_t K_{t+1}^{ji} - N_{t+1}^{ji}. \qquad (1)$$

The return to producer ji will have three components, an idiosyncratic shock, ι_{t+1}^{ji}, a region specific shock, κ_{t+1}^{j}, and an average aggregate return to capital, R_{t+1}^{k}. R_{t+1}^{k} includes the aggregate productivity shock, ρ_{t+1}, and the resale value of capital. For the partial equilibrium problem, consider Q as constant. ι_{t+1}^{ji} and κ_{t+1}^{j} are *i.i.d.* shocks, uncorrelated across producers and regions.

Both ι_{t+1}^{ji} and κ_{t+1}^{j} are log-normal variables with mean one.[1] Then, their

[1] Any increasing transformation of a Gaussian variable that admits a positive domain would satisfy a regularity condition that will be required in Sections 3 and 4. Log-normal is chosen because it is easy to work with. See Appendix A on the properties of the log-normal.

multiplication, $\phi^{ji}_{t+1} \equiv \iota^{ji}_{t+1}\kappa^{j}_{t+1}$, is also a log-normal variable with its mean equal to unity and the total return to producer ji is $\phi^{ji}_{t+1}R^{k}_{t+1}Q_tK^{ji}_{t+1}$. ϕ^{ji}_{t+1} is the only stochastic component in this expression.

This setup is close to the one used in Bernanke, Gertler, and Gilchrist (1999), but BGG do not have a role for financial intermediaries other than diversifying risk. That is, their producers face an asymmetric information problem when contracting with financial intermediaries, but financial intermediaries and households contract frictionlessly. The model presented in this paper treats all entrepreneurs similarly, therefore FIs, as well as producers face financial market frictions in securing external funding.

Defining $N^{Pj}_{t+1} = \int_0^1 N^{ji}_{t+1}di$ as the total net worth and B^{Pj}_{t+1} as the total borrowing of producers in region j, intermediary j has total lending $B^{Pj}_{t+1} = \int_0^1 B^{ji}_{t+1}di$ and therefore has to borrow

$$B^{Fj}_{t+1} = B^{Pj}_{t+1} - N^{Fj}_{t+1}. \tag{2}$$

The total value of capital stock in region j will be the sum of producers' net worth and their borrowing from the financial intermediary, and the total borrowing of the producers will be equal to the financial intermediary's net worth supplanted by its borrowing from households:

$$Q_tK^{j}_{t+1} = N^{Pj}_{t+1} + B^{Pj}_{t+1} = N^{Pj}_{t+1} + N^{Fj}_{t+1} + B^{Fj}_{t+1}.$$

Having aggregate uncertainty when one of the parties is risk-averse complicates matters. I assume that households demand the parties they lend to take on the aggregate risk. Therefore, financial intermediaries as a whole guarantee the households the riskless rate of return on their lending to the FIs. The implications of the arrangement will be clear in section 4.

It should be intuitive that since aggregate risk is insured by the intermediaries, as the idiosyncratic producer risk is diversified by the FIs and the region specific risk is diversified at the FI level, there is no uncertainty in terms of the total return to the representative household.

Most of the non-standard assumptions made above are to facilitate aggregation and to make sure that the risk averse households do not actually have to worry about risk. As a final twist, to ease aggregation, I assume that total net worth of the financial intermediaries be re-distributed so as to make $N_{t+1}^{Fj}/N_{t+1}^{Pj}$ equal across regions. Ex-ante, this is a mean preserving spread that the risk-neutral financial intermediaries will not object to. Note that we only require the FI net worth to aggregate (within the region) producer net worth to be equal across regions, but not across time. Indeed, this ratio will change over time as the aggregate FI to P net worth ratio changes.

3 Properties of the Optimal Contract between Intermediaries and Producers

The financial intermediary and the producer contract in an asymmetric information environment. Only the borrower (the producer) costlessly observes the outcome of her project. The lender (financial intermediary) must incur a cost proportional to the outcome to learn the true state, $\mu\phi_{t+1}^{ji}R_{t+1}^{k}Q_tK_{t+1}^{ji}$. This set-up is the one studied first by Townsend (1979) and later by Gale and Hellwig (1985). They show that in such a costly-state-verification framework, the optimal contract is a 'standard debt contract', where the borrower pays a fixed rate if she can and becomes bankrupt by defaulting on her payment if she cannot, in which case the lender monitors and retains the remainder of the proceeds. Equating the default, monitoring, and bankruptcy states maximizes the costs of defaulting for the borrower and therefore minimizes the states that involve costly monitoring.

Denote the contractual payment rate for the loan as R^{ji}_{t+1}. There exists a value of the combined shock, $\overline{\phi}^{ji}_{t+1}$, such that at this outcome of the project or better, the borrower is able to fulfill her contractual obligations

$$\overline{\phi}^{ji}_{t+1} E_t\{R^k_{t+1}\} Q_t K^{ji}_{t+1} = R^{ji}_{t+1} B^{ji}_{t+1}. \tag{3}$$

where $E_t\{R^k_{t+1}\}$ is the expected value of R^k_{t+1} at the end of period t. In the model presented here, aggregate returns to capital are public knowledge, therefore producers and intermediaries write contracts where the contractual payment is a function of the aggregate state of the world. It is important to note that financial intermediaries are not assuming all of the aggregate risk, in good (bad) states of the world the returns to both financial intermediaries and producers go up (down).

$\overline{\phi}^{ji}_{t+1}$ and R^{ji}_{t+1} should satisfy

$$\begin{aligned} & [1 - F(\overline{\phi}^{ji}_{t+1})] R^{ji}_{t+1} B^{ji}_{t+1} + (1-\mu) \int_0^{\overline{\phi}^{ji}_{t+1}} \phi^{ji}_{t+1} E_t\{R^k_{t+1}\} Q_t K^{ji}_{t+1} dF(\phi) \\ = \; & R^{Fj}_{t+1} B^{ji}_{t+1} \end{aligned} \tag{4}$$

In the above expression, $F(\cdot)$, is the *c.d.f.* of ϕ. The first term on the left hand side of equation (4) is then the expected full contractual payment weighted by the probability of the borrower being able to fulfill her obligation. The second term is expected amount recovered in cases of default. The sum of these two as a fraction of $E_t\{R^k_{t+1}\} Q_t K^{ji}_{t+1}$, call it $\Phi(\overline{\phi}^{ji}_{t+1})$, is the expected share of proceeds going to the lender. The net expected return to the lender should be enough to cover the opportunity cost of funds, R^{Fj}_{t+1}, of the financial intermediary.

The cost of funds of the intermediary depends on the terms of the contract between the FI and the households. In the absence of agency problems between intermediaries and households, the FI would borrow at the riskless rate, R_{t+1}, as there is no aggregate risk for the households, and require the riskless rate of return on his net worth as the FI is risk neutral. In this case, R^{Fj}_{t+1} would be equal to R_{t+1}.[2] We will see that having a costly-state-verification framework for the contracts between households and FIs will push R^{Fj}_{t+1} up, and will thus increase the external finance costs of producers.

The log-normal distribution of ϕ implies that the left hand side of equation (4) has a unique interior maximum. Intuitively, as the cutoff value, $\overline{\phi}^{ji}_{t+1}$ increases, expected payoff increases since the contractual interest rate is higher (notice the positive relationship between $\overline{\phi}^{ji}_{t+1}$ and R^{ji}_{t+1} in equation (3)). On the other hand, the probability of fulfilling the contract obligations decreases as $\overline{\phi}^{ji}_{t+1}$ increases, and after the maximum this latter effect dominates, making expected returns to lender decrease as the cutoff gets larger. Technically, having a unique interior maximum depends on the properties of $F(\cdot)$. Specifically, the requirement is that the hazard function of ϕ, $h(\phi) = f(\phi)/[1 - F(\phi)]$, should satisfy $\partial \phi h(\phi)/\partial \phi > 0$, which is indeed satisfied by all monotone increasing transformations of Gaussian variables, and by the log-normal in particular.[3]

Clearly, $\overline{\phi}^{ji}_{t+1}$ greater than the maximum cannot be an equilibrium as the lender can have higher expected returns by decreasing the cutoff. Thus, if R^{Fj}_{t+1} is so high that no $\overline{\phi}^{ji}_{t+1}$ smaller than the maximum can satisfy equation (4) the borrower will be rationed. I rule out rationing equilibrium by assumption and consider only interior values of $\overline{\phi}^{ji}_{t+1}$.

[2] This is the model studied by Bernanke, Gertler, and Gilchrist (1999). If there are no agency problems involved when intermediaries are borrowing, having a regional shock does not matter and one could think of ϕ as a single shock and ignore its components.

[3] See Appendix B on the details of why this condition is required for a unique interior maximum.

Let the expected *gross* share of proceeds going to the lender, $[1 - F(\overline{\phi}^{ji}_{t+1})] \overline{\phi}^{ji}_{t+1} + \int_0^{\overline{\phi}^{ji}_{t+1}} \phi^{ji}_{t+1} dF(\phi)$, be denoted by $\Upsilon(\overline{\phi}^{ji}_{t+1})$. Then the producer has an expected share $\Lambda(\overline{\phi}^{ji}_{t+1}) = 1 - \Upsilon(\overline{\phi}^{ji}_{t+1})$,

$$\Lambda(\overline{\phi}^{ji}_{t+1}) = \int_{\overline{\phi}^{ji}_{t+1}}^{\infty} \phi^{ji}_{t+1} dF(\phi) - [1 - F(\overline{\phi}^{ji}_{t+1})]\overline{\phi}^{ji}_{t+1}. \tag{5}$$

Note that the expected net shares of proceeds going to the lender and the borrower do not add up due to the monitoring costs[4]

$$\Phi(\overline{\phi}^{ji}_{t+1}) + \Lambda(\overline{\phi}^{ji}_{t+1}) = 1 - \mu \int_0^{\overline{\phi}^{ji}_{t+1}} \phi^{ji}_{t+1} dF(\phi). \tag{6}$$

Now, the maximization problem of the producer can be stated concisely as

$$\begin{aligned} &\max_{\overline{\phi}^{ji}_{t+1}, Q_t K^{ji}_{t+1}} \Lambda(\overline{\phi}^{ji}_{t+1}) E_t\{R^k_{t+1}\} Q_t K^{ji}_{t+1} \\ &s.t.\ \Phi(\overline{\phi}^{ji}_{t+1}) E_t\{R^k_{t+1}\} Q_t K^{ji}_{t+1} = R^{Fj}_{t+1}(Q_t K^{ji}_{t+1} - N^{ji}_{t+1}). \end{aligned} \tag{7}$$

The solution of the maximization problem is presented in Appendix B. It is shown that $E_t\{R^k_{t+1}\}/R^{Fj}_{t+1}$, the expected return to capital discounted by marginal cost of financing, is an increasing function of the cutoff:

$$\frac{E_t\{R^k_{t+1}\}}{R^{Fj}_{t+1}} = \varsigma^P(\overline{\phi}^{ji}_{t+1}), \quad \varsigma^{P\prime}(\cdot) > 0. \tag{8}$$

Of course, in a frictionless economy, investment in capital would be undertaken to the point where marginal return to capital is equated to the marginal cost of funds, $E_t\{R^k_{t+1}\}$ would be equal to R^{Fj}_{t+1}, and R^{Fj}_{t+1} would be equal to

[4] Substituting from equation (4) shows that $\Lambda(\overline{\phi}^{ji}_{t+1}) R^k_{t+1} Q_t K^{ji}_{t+1} = [1 - \mu \int_0^{\overline{\phi}^{ji}_{t+1}} \phi^{ji}_{t+1} dF(\phi)] E_t\{R^k_{t+1}\} Q_t K^{ji}_{t+1} - R^{Fj}_{t+1} B^{Pi}_{t+1}$. The borrower internalizes the monitoring costs.

R_{t+1}. We see that the agency problems introduce a wedge between these returns. The solution of the maximization problem also yields the leverage of the producer as an increasing function of the cutoff:

$$\frac{Q_t K_{t+1}^{ji}}{N_{t+1}^{ji}} = \vartheta^P(\overline{\phi}_{t+1}^{ji}),\ \vartheta^{P\prime}(\cdot) > 0. \tag{9}$$

Inverting the function in equation (8) and combining it with equation (9) gives us a relationship between the leverage of the producer and the return to capital discounted by its expected marginal cost

$$\frac{Q_t K_{t+1}^{ji}}{N_{t+1}^{ji}} = \Omega^P \left(\frac{E_t\{R_{t+1}^k\}}{R_{t+1}^{Fj}} \right), \quad \Omega^{P\prime}(\cdot) > 0. \tag{10}$$

Equation (10) relates the capital choice of a given producer to financial conditions in her region and her net worth. This relationship shows that the choice of capital of a producer is a function of the condition of her balance sheet. More specifically, $Q_t K_{t+1}^{ji}$ is proportional to net worth, where this proportion is determined by $E_t\{R_{t+1}^k\}/R_{t+1}^{Fj}$. This is the channel linking the producer's capital expenditure, net worth and financial conditions in the region together.

The right hand side of equation (10) does not depend on producer specific characteristics, $E_t\{R_{t+1}^k\}/R_{t+1}^{Fj}$ is the same for all producers in the region. Therefore this equation can be aggregated over producers to yield a relationship between aggregate capital expenditures, aggregate net worth, and the region's financial conditions

$$Q_t K_{t+1}^{ji} = \Omega^P \left(\frac{E_t\{R_{t+1}^k\}}{R_{t+1}^{Fj}} \right) N_{t+1}^j, \quad \Omega^{P\prime}(\cdot) > 0. \tag{11}$$

We have to know more about the characteristics of R_{t+1}^{Fj} to be able to aggregate over regions, and for that we need to know the terms of the contract between the intermediaries and households. Before turning to this issue, we establish that all producers in a given region have the same cutoff value of ϕ_{t+1}. Combining equation (4) with equation (11) we have

$$\begin{aligned}
&\{[1-F(\overline{\phi}^{ji}_{t+1})]\overline{\phi}^{ji}_{t+1}+(1-\mu)\int_0^{\overline{\phi}^{ji}_{t+1}}\phi^{ji}_{t+1}dF(\phi)\}E_t\{R^k_{t+1}\}\Omega^P\left(\frac{E_t\{R^k_{t+1}\}}{R^{Fj}_{t+1}}\right) \\
=\;& R^{Fj}_{t+1}[\Omega^P\left(\frac{E_t\{R^k_{t+1}\}}{R^{Fj}_{t+1}}\right)-1] \qquad (12)
\end{aligned}$$

The right hand side of this equation is the same for all producers while the left hand side is increasing in $\overline{\phi}^{ji}_{t+1}$ therefore there is a unique value of $\overline{\phi}^{ji}_{t+1}$, call it $\overline{\phi}^{j}_{t+1}$, that satisfies the equality for all producers. From equation (3) it can be seen that this implies a constant contractual rate for all producers. The intuition here is that the contractual rate and the cutoff value are functions of leverage ratio and as all producers have the same leverage ratio they all face the same contractual rate. Now, we are ready to study the contract between households and intermediaries.

4 Properties of the Optimal Contract between Households and Intermediaries

The contract between households and intermediaries is a lot like the contract between FIs and producers. Given the contract between FIs and producers, intermediary j expects to receive a fraction $\Phi(\overline{\phi}^{j}_{t+1})$ of the total proceeds of investment in region j. For the FI, the idiosyncratic shocks of producers are irrelevant as they wash out when the producers are aggregated (the intermediary perfectly diversifies idiosyncratic producer shocks) thus, at the level of the financial intermediaries, the uncertainty comes from the regional shock and the realization of aggregate returns to capital. Given any realization of the regional shock, κ^j_{t+1}, the share of total return to capital accruing to the FI is

$$\begin{aligned}\Phi(\overline{\phi}^j_{t+1} \mid \kappa^j_{t+1}) &= \kappa^j_{t+1}\{[1 - F^\iota\left(\frac{\overline{\phi}^j_{t+1}}{\kappa^j_{t+1}}\right)]\left(\frac{\overline{\phi}^j_{t+1}}{\kappa^j_{t+1}}\right) \\ &\quad +(1-\mu)\int\limits_0^{\overline{\phi}^j_{t+1}/\kappa^j_{t+1}} \frac{\phi^{ji}_{t+1}}{\kappa^j_{t+1}} dF^\iota(\phi/\kappa)\} \\ &= \kappa^j_{t+1}\Phi^\iota\left(\frac{\overline{\phi}^j_{t+1}}{\kappa^j_{t+1}}\right), \qquad (13)\end{aligned}$$

where the superscript ι denotes that the relevant distribution is that of ι. Since $\overline{\phi}^j_{t+1} = \overline{\kappa_{t+1}\iota_{t+1}}^{ji}$, for any realization of κ^j_{t+1}, producers whose idiosyncratic shocks are smaller than $\overline{\phi}^{ji}_{t+1}/\kappa^j_{t+1}$ will default and the remainder of the producers will fulfill their contractual obligations, as formalized by equation (13). Then, for a given κ^j_{t+1} the total return to the financial intermediary is

$$\kappa^j_{t+1}\Phi^\iota\left(\frac{\overline{\phi}^j_{t+1}}{\kappa^j_{t+1}}\right) R^k_{t+1} Q_t K^j_{t+1} \qquad (14)$$

The regional shock dependent term in equation (14), $\kappa^j_{t+1}\Phi^\iota(\overline{\phi}^j_{t+1}/\kappa^j_{t+1})$, has mean $\Phi(\overline{\phi}^j_{t+1})$.[5] It is easier and more intuitive to work with the transformed variable,

$$z^j_{t+1} \equiv \frac{\kappa^j_{t+1}\Phi^\iota\left(\frac{\overline{\phi}^j_{t+1}}{\kappa^j_{t+1}}\right)}{\Phi(\overline{\phi}^j_{t+1})}, \qquad (15)$$

which has mean one. The return to the intermediary can now be written as:

$$z^j_{t+1}\Phi(\overline{\phi}^j_{t+1}) R^k_{t+1} Q_t K^j_{t+1} = z^j_{t+1}\widetilde{r} R^{Fj}_{t+1} B^{Pj}_{t+1}$$

[5] Since $\kappa^j_{t+1}\Phi^\iota(\overline{\phi}^j_{t+1}/\kappa^j_{t+1})$ is the conditional expectation, taking expectations over the conditioning variable, κ^j_{t+1}, gives the unconditional mean, $\Phi(\overline{\phi}^j_{t+1})$.

where $\widetilde{r} = R^k_{t+1}/E_t\{R^k_{t+1}\}$ and the equality follows from the aggregate from of equation (4). Now, the return to the financial intermediary has the form of the return to a producer: an observable term multiplied by a mean one random variable that is observable only to the intermediary. κ^j_{t+1} affects z^j_{t+1} in two ways: As the regional productivity gets larger, the pie gets larger (the first term in the numerator of equation (15)), but the intermediary receives a smaller share of the pie, as suggested by the second term in the numerator. It is, therefore, important to verify that z^j_{t+1} is increasing in κ^j_{t+1}. Noting that $\Phi(\overline{\phi}^j_{t+1})$ is a number, not a random variable,

$$\frac{\partial z^j_{t+1}}{\partial \kappa^j_{t+1}} = \frac{1}{\Phi(\overline{\phi}^j_{t+1})}(1-\mu) \int_0^{\overline{\phi}^j_{t+1}/\kappa^j_{t+1}} \frac{\phi^{ji}_{t+1}}{\kappa^j_{t+1}} dF^{\iota}(\phi/\kappa) + \mu \left(\frac{\phi^{ji}_{t+1}}{\kappa^j_{t+1}}\right)^2 f^{\iota}\left(\frac{\phi^{ji}_{t+1}}{\kappa^j_{t+1}}\right) > 0,$$

where $f^{\iota}(\cdot)$ is the $p.d.f$ of ι. Thus, z is a monotonically increasing transformation of κ, which is itself a monotonically increasing transformation of a Gaussian variable. Since the hazard rate for the Gaussian distribution is positive and increasing, $zh(z)$ can be shown to be increasing in z.

Now, the problem in hand is in the same form the problem solved in section 3 above. Given the total lending (to producers), B^{Pj}_{t+1}, and borrowing (from households), B^{Fj}_{t+1}, of intermediary j and the contractual payment rate, R^j_{t+1}, there exists a $\overline{z}^j_{t+1}$ such that[6]

$$\overline{z}^j_{t+1} \widetilde{r} R^{Fj}_{t+1} B^{Pj}_{t+1} = R^j_{t+1} B^{Fj}_{t+1}. \tag{16}$$

The relevant interest rate is the riskless rate because the financial intermediaries insure the households against aggregate risk, so only region specific risk is left, which is diversifiable. Equation (16) shows that the contractual payment

[6] More properly, there exists a schedule of $\overline{z}^j_{t+1}$ contingent on the realization of the aggregate shock.

rate expected from the intermediary is increasing in $\overline{z}^j_{t+1}$.

We can now proceed to solving for the contract between the financial intermediary and the representative household. The maximization problem is

$$\max_{\overline{z}^j_{t+1},\ B^{Pj}_{t+1}} E_t\{\Lambda(\overline{z}^j_{t+1})\widetilde{r}R^{Fj}_{t+1}B^{Pj}_{t+1}\}$$
$$s.t.\ E_t\{\Phi(\overline{z}^j_{t+1})\widetilde{r}R^{Fj}_{t+1}B^{Pj}_{t+1}\} = R_{t+1}(B^{Pj}_{t+1} - N^{Fj}_{t+1}). \tag{17}$$

Details of the solution of this problem are presented in Appendix C. The algebra is slightly more involved than the solution of the contract between intermediaries and producers due to the presence of aggregate uncertainty, but the central conclusion is the same. There is a positive a relationship between the leverage of the intermediary and the expected discounted return to lending (the external finance premium)

$$\frac{B^{Pj}_{t+1}}{N^{Fj}_{t+1}} = \Omega^F\left(\frac{E_t\{\widetilde{r}R^{Fj}_{t+1}\}}{R_{t+1}}\right), \quad \Omega^{F\prime}(\cdot) > 0. \tag{18}$$

The equation above relates the total lending of the financial intermediary to the intermediary's net worth. This relationship is analogous to the relationship that was established between producers' choice of capital and their net worths. This resemblance is because of the way intermediaries are modeled. Since financial intermediaries are not different from producers fundamentally, their level of economic activity is an increasing function of their net worths, just like the producers' level of economic activity are a function of their net worths. Both producers and financial intermediaries are firms, and this makes them intrinsically alike.

One problem with equation (18) is that the right hand side involves a FI specific variable, R^{Fj}_{t+1}, and therefore we cannot easily integrate over the financial intermediaries. The assumption of constant intermediary to aggregate producer net worths across regions takes care of this problem by equalizing R^{Fj}_{t+1} in all regions. To see this, write equation (10) as

$$\frac{B^{Pj}_{t+1}}{N^{Pj}_{t+1}} = \Omega^P \left(\frac{R^k_{t+1}}{E_t\{\widetilde{r} R^{Fj}_{t+1}\}} \right) - 1, \tag{19}$$

and divide equation (18) by equation (19) to find

$$\frac{N^{Pj}_{t+1}}{N^{Fj}_{t+1}} = \frac{\Omega^F \left(\frac{E_t\{\widetilde{r} R^{Fj}_{t+1}\}}{R_{t+1}} \right)}{\Omega^P \left(\frac{R^k_{t+1}}{E_t\{\widetilde{r} R^{Fj}_{t+1}\}} \right) - 1}. \tag{20}$$

The numerator of the expression on the right hand side of equation (20) is increasing in R^{Fj}_{t+1}, and the expression in the denominator is decreasing in R^{Fj}_{t+1}. Thus, the right hand side as a whole is increasing in R^{Fj}_{t+1}, however, the left hand side is constant across regions by assumption, and therefore the unique value of R^{Fj}_{t+1} that satisfies equation (20) is constant across regions. This establishes that in equilibrium, there is a single required rate of return on financial intermediaries' lending in the economy which is independent of any given intermediary's net worth. The net worth only determines the scale of lending.

Having a constant R^{Fj}_{t+1} across regions allows us to aggregate equation (18) over financial intermediaries to find an aggregate relationship between financial intermediary net worth and total lending to producers. This also allows us to aggregate equation (11), which is already aggregated over producers in a region, across regions. Thus, total spending on capital in the economy can be expressed as an increasing function of producers' net worth. Also, equation (12) points out that if R^{Fj}_{t+1} is the same for all regions, $\overline{\phi}^j$ is also the same.

An argument similar to the one made at the end of section 3 and using the fact that R^{Fj}_{t+1} is the same across regions shows that $\overline{z}^j_{t+1} = \overline{z}_{t+1}$ is common for all financial intermediaries (but still contingent on $\widetilde{r}$), and therefore, ex-post a measure $\overline{z}_{t+1}$ of them fail to make their contractual payments and default. Having a common $\overline{\phi}_{t+1}$ for all producers and a common $\overline{z}_{t+1}$ for all intermediaries makes analyzing the dynamics possible.

Before turning to dynamics, observe from equation (20) that R^F_{t+1} is increasing in $N^{Pj}_{t+1}/N^{Fj}_{t+1}$. This is important because the lending channel depends on intermediaries' charging higher interest rates (higher R^F_{t+1}) during contractions and lower rates during expansions. This will happen only if $N^{Pj}_{t+1}/N^{Fj}_{t+1}$ is countercyclical.

5 Dynamics

An immediate problem with the setup presented above is that in each period some producers default and lose all their net worth, also new producers that replace dead ones arrive with no net worth. Since capital spending is a multiple of net worth, as established by equation (10), producers without any net worth cannot undertake any projects, and thus their net worth never increases. Over time, as producers die and go bankrupt, total net worth of producers falls to zero. This is not an interesting steady state to analyze. To allow producers who have zero net worth at the beginning of a period to accumulate some wealth and get started, I follow Carlstrom and Fuerst (1997) and BGG in letting producers inelastically supply one unit of labor as workers.

The aggregate production function has the standard Cobb-Douglas form

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \tag{21}$$

where labor is an aggregate of household and producer labor, L^h_t and L^P_t respectively:

$$L_t = \left(L_t^h\right)^{\Theta} \left(L_t^P\right)^{1-\Theta}. \tag{22}$$

Think of Θ as being very close to one so that the income producers generate from selling their labor is very small and the changes in it are of second order when compared to changes in producers' equity. The total net worth of producers is the sum of the value of equity of the surviving producers and the labor income

$$N_{t+1}^P = \zeta[R_t^k Q_{t-1} K_t - \Upsilon(\overline{\phi}_t) R_t^k Q_{t-1} K_t] + (1-\Theta)(1-\alpha) Y_t. \tag{23}$$

The first term in equation (23) is the equity value of surviving producers. The gross payments (payments inclusive of the monitoring costs) to intermediaries are deducted from the total return to producers' projects. The second term in the equation is the wage income of producers. In competitive labor markets, producers are paid their marginal product as real wages.

We do not need a mechanism to re-start financial intermediaries whose net worth falls to zero because wealth re-distribution that equalizes intermediary to producer net worth ratios across regions takes care of this. Aggregate financial intermediary net worth evolve according to

$$\begin{aligned} N_{t+1}^F &= \zeta[\widetilde{r} R_t^F (Q_{t-1} K_t - N_t^P) - R_t (Q_{t-1} K_t - N_t^P - N_t^F) \\ &\quad -\mu \textstyle\int_0^{\overline{z}_t} z_t \widetilde{r} R_t^F (Q_{t-1} K_t - N_t^P) dF(z)]. \end{aligned} \tag{24}$$

As the intermediaries have no labor income, their net worth evolves only with their equity value which is determined by the return to their lending to producers net of the payments they make to households, internalizing the monitoring costs.

Before turning to the effects of shock to aggregate productivity, it is useful to make some observations about the steady state capital stocks under different assumptions about the nature of the agency problems. When there are no asymmetric information problems in the economy, expected marginal returns to capital and bank lending are equated to the riskless interest rate, $E_{t-1}\{R_t^k\} = R_t^F = R_t$. This is the first-best situation.

When there are agency problems between producers and financial intermediaries, but intermediaries are able to borrow frictionlessly from households, $E_{t-1}\{R_t^k\}$ is greater than R_t^F which is equal to R_t. That $R_t^F = R_t$ in a frictionless contracting environment is immediate. To see that $E_{t-1}\{R_t^k\} > R_t^F$ note that $\Omega^P(1) = 1$, so to the extent producers are leveraged ($QK > N^P$), $E_{t-1}\{R_t^k\}$ must be greater than R_t^F, which implies that capital stock is lower than first-best.

If the intermediaries also face agency problems when they are leveraged, we have $E_{t-1}\{R_t^k\} > R_t^F > R_t$. The first inequality was already established, the second inequality follows from a similar argument made for the leverage of financial intermediaries. $\Omega^F(1) = 1$, so if the intermediaries are leveraged (if they are borrowing from households), R_t^F must exceed the safe interest rate. In this case, the wedge between $E_t\{R_{t+1}^k\}$ and R_t is even wider and the steady state capital stock is even lower.

Equations (10), (18), (23), and (24) allow us to study the partial equilibrium dynamics of this system. Consider a one-time, small negative aggregate productivity shock, ρ, that lowers R_t^k below $E_{t-1}\{R_t^k\}$ for a single period.

First consider the first best contracting environment where there are no frictions in the financial markets. In this case, the investment decisions of producers are independent of their net worths, and a temporary shock at period t has no effects on the quantity of capital bought in period $t+1$.

Now consider the case where financial intermediaries face no agency problems while contracting with households, but there are agency problems between intermediaries and producers. This is the case when the intermediaries can be costlessly monitored, $R_t^F = R_t$. Differentiating equation (23) with respect to the ex-post aggregate return to capital, we have

$$\frac{\partial N_{t+1}^P}{\partial R_t^k} = \zeta\{[1-\Upsilon(\overline{\phi}_t)]R_t^k Q_{t-1}K_t\} > 0. \tag{25}$$

Thus, as expected, a negative aggregate productivity shock reduces the end of period net worth of producers. But from equation (10), a smaller net worth will lead to a smaller capital stock in the subsequent period even though aggregate productivity has returned to its normal level:

$$\frac{\partial Q_t K_{t+1}}{\partial R_t^k} = \Omega^P\left(\frac{E_t\{R_{t+1}^k\}}{R_{t+1}^F}\right)\frac{\partial N_{t+1}^P}{\partial R_t^k} > 0. \tag{26}$$

This is the balance sheet channel in action. A negative shock that decreases net worth in one period is propagated due to the agency problem between producers and financial intermediaries. Notice that what happens to the intermediaries is not important as they are not constrained by their balance sheets in making their lending decisions when they don't face agency problems.

Finally, consider the case where all entrepreneurs are treated symmetrically, and intermediaries as well as producers face agency problems. In this case we start from a situation where $E_{t-1}\{R_t^k\} > R_t^F > R_t$.

The effect of temporarily lower aggregate return to capital on choice of capital in the subsequent period is:

$$\begin{aligned}\frac{\partial Q_t K_{t+1}}{\partial R_t^k} \quad = \quad & \Omega^P\left(\frac{E_t\{R_{t+1}^k\}}{R_{t+1}^{Fj}}\right)\frac{\partial N_{t+1}^P}{\partial R_t^k} \\ & +\Omega^{P\prime}\left(\frac{E_t\{R_{t+1}^k\}}{R_{t+1}^{Fj}}\right)\frac{-E_t\{R_{t+1}^k\}}{\left(R_{t+1}^{Fj}\right)^2}\frac{\partial R_{t+1}^F}{\partial R_t^k}N_{t+1}^P.\end{aligned} \tag{27}$$

The first term on the right hand side is the balance sheet channel found in equation (25). To establish that the agency problems between financial intermediaries and households add to the propagation of the one-time productivity shock, we need to show that the second term is positive. $\Omega^{P\prime}(E_t\{R^k_{t+1}\}/R^F_{t+1}) > 0$ from equation (18), so the requirement is reduced to showing $\partial R^F_{t+1}/\partial R^k_t < 0$.

From the discussion at the end of the previous section, showing $\partial R^F_{t+1}/\partial R^k_t < 0$ amounts to showing $N^{Pj}_{t+1}/N^{Fj}_{t+1}$ becomes larger when ex-post returns to capital are lower than expected.[7] This is easier accomplished by calculating the elasticities of N^P_{t+1} and N^F_{t+1} with respect to an unanticipated decrease in aggregate productivity. N^P_{t+1}/N^F_{t+1} will be larger if the elasticity of intermediaries' net worth is larger than the elasticity of producers' net worth.

Since fluctuations in the labor income of producers is negligible in comparison to the fluctuations in the value of their equity, we compare the elasticities of the equities. The equity value of all producers at the end of period t (before deaths, but this does not matter) is:

$$V^{Pe}_t = [1 - \Upsilon(\overline{\phi}_t)]R^k_t Q_{t-1} K_t,$$

and the elasticity is

$$\frac{\partial V^{Pe}_t / E_{t-1}\{V^{Pe}_t\}}{\partial R^k_t / E_{t-1}\{R^k_t\}} = \frac{[1 - \Upsilon(\overline{\phi}_t)] Q_{t-1} K_t E_{t-1}\{R^k_t\}}{E_{t-1}\{V^{Pe}_t\}} = 1, \tag{28}$$

where the second equality can be verified by taking the expectation of V^{Pe}_t. The intuition behind this result comes from the design of the contract between producers and intermediaries. Since the contract specifies the *share* of total returns to capital each party receives, a one percent increase in the return to capital increases the absolute amount the producer gets by one percent, hence

[7] See equation (20).

the elasticity is unity.

The ex-post equity value of intermediaries can be written as:

$$V_t^{Fe} = [(1 - \Upsilon(\overline{z}_t)]\Phi(\overline{\phi}_t)R_t^k Q_{t-1}K_t.$$

They receive a fraction $\Phi(\overline{\phi}_t)$ of the returns to capital and retain $1-\Upsilon(z)$ of it after repaying the households and covering the monitoring costs. The elasticity of V_t^{Fe} with respect to the ex-post return to capital is then:

$$\begin{aligned}\frac{\partial V_t^{Fe}/E_{t-1}\{V_t^{Fe}\}}{\partial R_t^k/E_{t-1}\{R_t^k\}} \quad &= \quad \frac{(1-\Upsilon(\overline{z}_t)]\Phi(\overline{\phi}_t)Q_{t-1}K_t E_{t-1}\{R_t^k\}}{E_{t-1}\{V_t^{Fe}\}} \\ &\quad -\frac{E_{t-1}\{R_t^k\}}{E_{t-1}\{V_t^{Fe}\}}\Upsilon'(\overline{z}_{t+1})\frac{\partial \overline{z}_t}{\partial R_t^k}\Phi(\overline{\phi}_t)R_t^k Q_{t-1}K_t. \quad (29)\end{aligned}$$

The first term is the direct effect of changes in the aggregate productivity on intermediary equity. Like the producers, this elasticity is unity. The second term captures the changes in financial intermediaries equity value stemming from the change in the (ex-post) value of the cutoff due to higher productivity. $\Upsilon'(\overline{z}_{t+1})$ is positive and (as shown in Appendix C) $\partial\overline{z}_{t+1}/\partial R_t^k$ is negative. When aggregate productivity is high, a lower fraction of financial intermediaries default, and the non-defaulting intermediaries pay a lower contractual rate because the shortfall of defaulting FIs they have to cover is lower.

Then, the second term in equation (29) is negative and therefore:

$$\frac{\partial V_t^{Fe}/E_{t-1}\{V_t^{Fe}\}}{\partial R_t^k/E_{t-1}\{R_t^k\}} > 1. \qquad (30)$$

We have shown that the net worth of financial intermediaries respond more to a given shock than the net worth of producers. This is because the financial intermediaries carry more of the aggregate risk due to their contract with households. The countercyclicality of N_{t+1}^P/N_{t+1}^F implies that R_t^F is also countercyclical ($\partial R^F_{t+1}/\partial R^k_t < 0$) and therefore the rate producers borrow during recessions is higher.

Now we can go back to equation (27). By the arguments made above, the second term in this equation is positive, and so the response of capital spending in the second period when financial intermediaries as well as producers face agency problems is greater than the response when only producers suffer from asymmetric information. This is due to the lending channel generated by the contract between intermediaries and households.

The lending channel that results from the agency problems of intermediaries is fundamentally different from the standard interpretation of bank lending channel. It does not depend on draining reserves or making bank capital requirements bind, rather, it is the result of normal operations of financial intermediaries as firms. In this sense the lending channel generated in this paper is a *financial intermediary balance sheet* channel, rather than a *bank lending channel.*

This analysis shows that a given one time (negative) shock will be propagated in the economy in two ways. The producers will have to pay higher spreads on their borrowing from intermediaries because they are less credit worthy due to eroded net worth. The intermediaries, however, also suffer from the same problem and they will have to promise higher returns to households to be able to borrow. The higher borrowing costs of intermediaries will be passed on to the producers, exacerbating the financing problem and further propagating the initial shock.

Any shocks to the net worths of intermediaries will have direct effects on the economy. FIs with low net worth cannot borrow from households and cannot provide intermediation services. The resulting disintermediation chokes the whole economy as producers face high financing costs.

This is why after financial crises countries pay special attention to recapitalizing the banking sector before the producers. Although financial intermediaries are like other firms, they form a bottleneck in the economy because credit flows depend on their financial well being.

6 Conclusions and Directions for Future Work

The notable result of the is paper is that, without making very fancy assumptions, only by treating financial intermediaries as risky firms being subject to agency problems, a lending channel that propagates a given shock can be shown to exist. This is important because such a mechanism is independent of the legislation governing financial intermediation and therefore is applicable to all kinds of intermediaries, not only to banks, and is–at least–as relevant in the 1930s economy as it is today.

The model presented in the paper does not capture all of the general equilibrium dynamics. Studying the effects of this two layered balance sheet mechanism in a in a general equilibrium setting would be valuable and is not difficult to do. The supply of credit is modeled here, the demand side of the economy and a government sector can be added to the model to complete the general equilibrium setting.

A second, and perhaps more challenging extension is to endogenize the financing mix of producers. In this paper the producers must borrow from intermediaries if they are going to borrow at all, but it is possible to allow them to directly borrow from households and let them choose between direct and intermediated external financing. This will require imposing some more structure on the distribution of producers' net worths. It would be interesting see if the model can replicate the empirical findings of Kashyap, Stein, and Wilcox (1993), and Gertler and Gilchrist (1994).

The results in the paper re-emphasize the importance of a well capitalized financial sector for the overall well being of the economy. Prudential regulation and monitoring of the financial sector by the government are also important to the extent that these activites ameliorate the information asymmetry between financial intermediaries and their lenders, and increase the capitalization of the intermediaries. An undercapitalized financial sector turns into a bottleneck in the production process.

Appendix A. The Log-normal Distribution

A random variable x is log-normally distributed if

$$x = e^y, \quad y \sim N(\mu, \sigma^2), \tag{A.1}$$

that is, if $\ln(x)$ has a Gaussian distribution. The mean of x is $\mu + 0.5\sigma^2$, therefore when $y \sim N(-0.5\sigma^2, \sigma^2)$ x has mean one. Since the sum of two Gaussian variables is Gaussian, it is easy to show that the multiplication of two log-normals $x_1 = e^{y_1}$ and $x_2 = e^{y_2}$ is also a log normal.

$$x_1 x_2 = e^{(y_1+y_2)}, \quad y_{1+}y_2 \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2) \tag{A.2}$$

where we assumed that x_1 and x_2 are independent. If both variables are mean one their multiplication is also mean one.

Appendix B. The Contract I

The appendix outlines the solution of the maximization problem presented in equation (7).[8] The first order of business is to show that $\Phi(\overline{\phi}_{t+1})$ has a unique interior maximum. Taking the derivative of $\Phi(\cdot)$, we have:

$$\Phi'(\overline{\phi}_{t+1}) = [1 - F(\overline{\phi}_{t+1})]\{1 - \mu \frac{\overline{\phi}_{t+1} f(\overline{\phi}_{t+1})}{1 - F(\overline{\phi}_{t+1})}\}.$$

The first term of this expression is always positive, the second term is unity for $\overline{\phi}_{t+1} = 0$, and is monotonically decreasing as $\partial\overline{\phi}_{t+1} h(\overline{\phi}_{t+1})/\partial\overline{\phi}_{t+1}$ is monotonically increasing. Therefore, there exists a value of the cutoff that satisfies $\Phi'(\overline{\phi}^*_{t+1}) = 0$, maximizing $\Phi(\overline{\phi}_{t+1})$. We are only interested in the values of the

[8] For a more detailed solution of a similar problem, and sufficient conditions for an interior solution, see Bernanke, Gertler, and Gilchrist (1999).

cutoff below the maximum.

Going back to equation (7), the optimal contracting problem is:

$$\max_{\overline{\phi}_{t+1},\, Q_t K_{t+1}} \Lambda(\overline{\phi}_{t+1}) E_t\{R^k_{t+1}\} Q_t K_{t+1}$$
$$s.t.\ \Phi(\overline{\phi}_{t+1}) E_t\{R^k_{t+1}\} Q_t K_{t+1} = R^F_{t+1}(Q_t K_{t+1} - N^P_{t+1}).$$

Let $r^P = E_t\{R^k_{t+1}\}/R^F_{t+1}$ and $k = Q_t K_{t+1}/N^P_{t+1}$ and let λ^P be the Lagrangian multiplier associated with the constraint. Then the first order conditions, omitting the $t+1$ subscripts, are:

$$\Upsilon'(\overline{\phi}) - \lambda^P \Phi'(\overline{\phi}) = 0, \tag{B.1}$$

$$[\Lambda(\overline{\phi}) + \lambda^P \Phi(\overline{\phi})] r^P - \lambda^P = 0, \tag{B.2}$$

$$\Phi(\overline{\phi}) r^P k - k + 1 = 0. \tag{B.3}$$

Equation (B.1) defines the Lagrangian multiplier in terms of the cutoff value of the producer's combined shock:

$$\lambda^P(\overline{\phi}) = \frac{\Upsilon'(\overline{\phi})}{\Phi'(\overline{\phi})}, \quad \lambda^{P'}(\cdot) > 0.$$

The next first order condition, equation (B.2), yields a relationship between the external finance premium, r^P, and the cutoff, $\overline{\phi}$:

$$r^P(\overline{\phi}) = \frac{\lambda^P(\overline{\phi})}{\Lambda(\overline{\phi}) + \lambda^P(\overline{\phi})\Phi(\overline{\phi})}, \ r^{P'}(\cdot) > 0.$$

It will come handy to define the inverse relationship that defines the cutoff as a function of r^p:

$$\overline{\phi} = \phi(r^P), \quad \phi'(\cdot) > 0. \tag{B.4}$$

Lastly, equation (B.3) can be solved to express the leverage of the producer as a function of the cutoff value:

$$k(\overline{\phi}) = 1 + \frac{\lambda^P(\overline{\phi})\Phi(\overline{\phi})}{\Lambda(\overline{\phi})}, \quad k'(\cdot) > 0.$$

Combining the equation above with equation (B.4) we finally arrive at a function that describes total spending on capital as an increasing function of producer's net worth and financial conditions, $k(r^P) = \Omega^P(r^P)$:[9]

$$QK = \Omega^P\left(\frac{E\{R^k\}}{R^F}\right)N^P, \quad \Omega^P(\cdot) > 0. \tag{B.5}$$

Equation (B.5) (equation (10) in text) documents a key relationship that generates the balance sheet channel. A worsening in financial conditions, for example an increase in R^F, will lead to less capital spending, a lower net worth next period, and therefore a lower spending on capital next period, even if the change in R^F was temporary.

Appendix B. The Contract II

This appendix provides some details of the contract between households and financial intermediaries. This contract is a lot like the contract between financial intermediaries and producers, studied in Appendix B. Given the contract between intermediaries and producers, the FI expects to receive a fraction $\Phi(\overline{\phi})$ of the proceeds of capital investment. At the level of the intermediary, the only source of uncertainty is the regional shock as idiosyncratic producer shocks are diversified. As explained in Section 4, given any realization of κ, the return to

[9] It is easy to show that $\lim_{\overline{\phi}\to 0} \lambda(\overline{\phi}) = 1$. Thus, $\lim_{\overline{\phi}\to 0} r^P$ is also 1 or, from equation (B.4), $\lim_{r^P\to 1} \overline{\phi} = 0$. $\lim_{\overline{\phi}\to 0} k = 1$ so $\lim_{r^P\to 1} k = 1$. That is, $R^k = R^F$ only when all capital spending is financed by producers' internal funds.

the FI is

$$\kappa\Phi^{\iota}(\overline{\phi/\kappa})R^k QK.$$

It is easier to work with the transformed return

$$z\widetilde{r}R^F B^P,$$

where $\widetilde{r} \equiv R^k/E\{R^k\}$ and $z \equiv \kappa\Phi^{\iota}(\overline{\phi}/\kappa)/\Phi(\overline{\phi})$, is a monotonically increasing transformation of κ with mean one. Therefore it satisfies the regularity condition, $\partial zh(z)/\partial z > 0$, so that the expected return to lender (the household), $\Phi(\overline{z})$, has a unique global maximum. Once again, we are only interested in the values of $\overline{z}$ below the maximum.

The contracting problem between the financial intermediary and the household is:

$$\max_{\overline{z},\ B^P} E\{\Lambda(\overline{z})\widetilde{r}R^F B^P\}$$
$$s.t.\ E\{\Phi(\overline{z})\widetilde{r}R^F B^P\} = R(B^P - N^F).$$

The first order conditions of the maximization problem above are:

$$\Upsilon'(\overline{z}) - \lambda^F[\Phi'(\overline{z})] = 0, \quad \text{(C.1)}$$

$$E\{\Lambda(\overline{z})\widetilde{r}R^F + \lambda^F[\Phi(\overline{z})\widetilde{r}R^F - R]\} = 0, \quad \text{(C.2)}$$

$$\Phi(\overline{z})\widetilde{r}R^F B^P - R(B^P - N^F) = 0. \quad \text{(C.3)}$$

where λ^F is the ex-post value of the Lagrangian multiplier on the constraint that households receive the riskless rate in expectation on their lending to financial intermediaries. Equation (C.1) (which is the same as equation (B.1)) implies that this multiplier is:

$$\lambda^F(\overline{z}) = \frac{\Upsilon'(\overline{z})}{\Phi'(\overline{z})} > 0, \quad \lambda^F(\cdot) > 0. \tag{C.4}$$

Defining $r^F = R^F/R$ as the external finance premium of the intermediary and $b = B^P/N^F$ as the lending to net worth ratio, equation (C.3) can be implicitly differentiated to calculate:

$$\frac{\partial \overline{z}}{\partial r^F} = -\frac{\Phi(\overline{z})}{\Phi'(\overline{z}) r^F} < 0, \tag{C.5}$$

$$\frac{\partial \overline{z}}{\partial b} = \frac{1}{\Phi'(\overline{z}) \widetilde{r} r^F b} > 0. \tag{C.6}$$

Next, implicit differentiation of equation (C.2) provides an expression for $\partial b/\partial r^F$:

$$\frac{\partial b}{\partial r^F} = \frac{\Lambda(\overline{z}) + \lambda^F(\overline{z})\Phi(\overline{z}) - \Gamma'(\overline{z})\left(\frac{\partial \overline{z}}{\partial r^F}\right)}{\Gamma'(\overline{z})\left(\frac{\partial \overline{z}}{\partial b}\right)} \tag{C.9}$$

where $\Gamma'(\overline{z}) \equiv \lambda^{F'}(\overline{z}) - \widetilde{r} r^F [\Lambda'(\overline{z}) + \lambda^{F'}(\overline{z})\Phi(\overline{z}) + \lambda^F(\overline{z})\Phi'(\overline{z})]$. Using equation (C.3), this expression simplifies to

$$\Gamma'(\overline{z}) = \lambda^{F'}(\overline{z})[1 - \Phi(\overline{z})\widetilde{r} r^F] = \frac{\lambda^{F'}}{b} > 0.$$

Since $\partial \overline{z}/\partial r^F$ is negative and $\partial \overline{z}/\partial b$ is positive, from equation (C.9), $\Gamma'(\overline{z})$ being positive implies that

$$\frac{\partial b}{\partial r^F} > 0,$$

and therefore the lending to net worth ratio of financial intermediaries can be written as an increasing function of the external finance premium:

$$B^F = \Omega^F\left(\frac{R^F}{R}\right) N^F, \quad \Omega^{F'}(\cdot) > 0. \tag{C.8}$$

Equation (C.8) (equation (18) in text) is analogous to equation (B.5). It is the crucial link that relates lending by an intermediary to the intermediary's balance sheet.

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