An Endogenous Growth Model with Quality Ladders and Consumers' Heterogeneity

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Abstract

This paper develops an endogenous growth model with quality ladders where consumers heterogeneity is assumed and is modelled through non homothetic preferences. We show that in such a model, unlike mainstream quality ladders models, the steady state equilibrium is characterised by a duopoly were the state of the art technology and the one immediately below it are both able to survive and thrive, under given conditions for the income distribution. In the words of Schumpeter, this model delivers only partial creative destruction. Furthermore, we show that under duopoly, an increase in the degree of income inequality, raises the intensity of research activities and the growth rate of the economy.

JEL classification: L10, O15, O31, O41

1. Introduction

The endogenous growth literature can by now be defined as abundant.
More than a decade has passed since the seminal works of Romer, in which for the first time those who engaged in innovative activities with the goal of fostering technological change, were formally modelled (informally, Schumpeter had talked about this long before), as enjoying a degree of market power (Romer, 1990). Thereby, technological change and growth could be determined endogenously as the result of rational decisions taken by economic agents in pursuit of profit incentives.
In the years that followed there were other important contributions, the two most seminal probably being Grossman and Helpman (1991) and Aghion and Howitt (1992), henceforth G.H. and A.H. respectively. These studies split the field of endogenous growth into two branches, with one preoccupying itself with introducing technological change through growth in the number of goods produced (horizontal differentiation), and another which preferred to introduce technological change through quality improvements in production (vertical differentiation, also known as quality ladders). Even the latter stream soon became quite abundant, as the initial studies by G.H. And A.H. were followed by many others that tried to use the conceptual framework provided by the quality ladder models to broaden their scope to the study of open economies, trade, foreign direct investment, developed versus developing countries and so on.
However, the mainstream models with quality ladders all predicted total obsolescence, or, in the parlance first introduced by Schumpeter, total creative destruction. Put simply, these models predicted that an innovation in the quality goods sector would force lower quality goods out of the market by

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bringing their sales down to zero. As a result, in these models, the quality goods sector featured a monopoly of the good that embodied the highest quality on the ladder.
However, in the real world there are many examples of markets for quality goods with duopolistic or oligopolistic market structure. Examples include video and radio cassettes vs. digital video disks and compact disks, various generations of computer processors (Pentium II, III, IV, Celeron, etc.) and so on. Perhaps the most telling example is a comparison of the car market in developed vs. developing countries. The former is typically characterized by a monopoly of the latest models, while in the latter one can often see several generations of cars being produced alongside each other and all making a profit. A possible explanation for these phenomena may lie in the different way income is distributed across markets or countries. The main stream models by G.H. and A.H. cannot account for income distribution differences.
In A.H., the consumption good is produced by inputting intermediate goods according to the Cobb Douglas production function: $Y = Ax^{\alpha}, \alpha < 1$. This is a homothetic specification which implies that the rate of substitution between inputs does not change with income.
In G.H., household utility is of the form $\ln D_t = \ln \Sigma_m q_m x_m$, thus products along the quality ladder are perfect substitutes and there is positive demand only for the product that carries the lowest price per unit of quality, which is the highest product on the quality ladder. Again, that occurs regardless of the level of income.
In order to introduce income distribution differences into the quality ladder framework, we employ the conceptual apparatus produced by that particular branch of industrial organization known as vertical quality differentiation literature. Among the most important contributions to this literature, we cite Gabszewicz and Thisse (1979), and Shaked and Sutton (1982). These studies model preferences as follows: $u(I, q_i) = q_i(I - p_i)$, where I is income, q_i is a quality index, and p_i is the price of a good of quality I. Thus consumers problem is not how much to buy of some good (as in A.H. and G.H.) But whether to buy one unit of the good, and which quality to buy.
Consumer preferences such as those just described above, have been employed in the context of a quality ladder model in two other papers, Li (1998) and Zweimuller and Brunner (1998). In the quality ladder literature these are the studies that come closest to the model developed here, but there are some important differences too that differentiate them from the present study.
In Li (1998), income inequality is introduced through assuming that labor income has a uniform distribution with mean preserving spread. This assumption simplifies somewhat the analysis, but it does not allow for a comparison of countries that enjoy very different mean incomes. Hence, it does not allow for a comparison of the developed world vs. the developing world.
In Zweimuller and Brunner (1998), labor income is assumed to be the same across individuals, while income inequality is introduced through heterogeneity in “other wealth”, which is endogenous, and whose source is the stake that each individual owns in the firms that produce the quality goods. Here too a simplifying assumption is made, so that the other wealth is not uniformly distributed, as in Li (1998) and in previous vertical quality differentiation literature, but consumers are divided into two categories, the rich and the poor, according to a discrete distribution.
While we accept that a uniform distribution may fail to give an accurate distribution of income in the real world, we nevertheless feel that it is worthwhile to maintain this assumption, in order not to lose the rich framework provided by the vertical quality differentiation literature.

Therefore, in this study, income inequality is introduced through other wealth which originates from having a stake in the firms that produce the quality goods, like in Zweimuller and Brunner. This other wealth is assumed to be uniformly distributed, but without the restrictive mean preserving spread assumption.

The rest of the paper is organized as follows: section 2 lays down the model, introduces income heterogeneity and describes the features of the market structures of monopoly and duopoly.

Sections 3 to 5 introduce the remaining main building blocks of this model, namely the research and development sector (in short R&D), the labor markets, and how the growth rate of the economy is being computed. Sections 6 and 7 perform steady state equilibrium analysis in monopoly and duopoly respectively. Section 8 carries out some further and interesting comparative statics, while section 9 comprises the conclusion and some directions for future research. The appendix has all the remaining mathematical details.

2 The Model

The economy is populated by L individuals, whose life-span is infinite. The representative individual consumes two types of goods each period. The first type is a good that is subject to quality innovation over time. Each individual consumes at most one unit per period of these goods. The quality good is denoted as q_{it} where t indexes time, and $i=1,2$ indexes quality, in ascending order. An innovation raises the quality of good q_{it} by a constant factor γ. Therefore $q_1 = \gamma$ denotes the product that sits second from top on the quality ladder, and $q_2 = \gamma q_1 = \gamma^2$ denotes the good that occupies the highest position on the ladder. The second type of good is a homogeneous product, that can be thought of as a composite commodity that comprises all other purchases made beyond the quality goods sector. Let this homogeneous good be denoted by h_t (again t indexes time).

In any period t, the utility achieved by the representative individual is given by:

$$u_t = q_{it} h_t$$

We take advantage of a utility functions' property by which these are only defined up to a monotonic transformation, to rewrite the above utility in logs as follows:

$$\ln u_t = \ln q_{it} + \ln h_t$$

The intertemporal utility maximization problem for the representative individual is:

$$Max \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t \ln u_t$$

where ρ is the rate of time preference.

We assume that consumers are endowed with assets k, earn a wage w from supplying one unit of labor, and make expenditures C on the two types of goods defined above. Over time, in any given period, assets next period must be equal to the sum of the assets, augmented at the current interest rate, and the wage income earned from working this period, minus consumption expenditure this period, according to the following budget constraint:

$$k_{t+1} = (1 + r_t) k_t + w_t - C_t$$

Since consumers purchase one unit of the quality good and spend the rest on the homogeneous good, demand for the latter is given by:

$$h_t = C_t - p_{it}$$

where p_{it} is price in period t of one unit of good of quality $i = 1,2$.

Having stated what the representative individual's intertemporal problem is, we will restrict our attention to the steady state, utility-maximizing, consumption pattern of such an individual[2], which is:

$$C = w + \rho k.$$

2.1 Introducing Income Heterogeneity.

In order to introduce income heterogeneity, we shall assume that w, wage income, is the same for everybody who supplies one unit of labour and exogenously given, whereas assets k, which we will henceforth refer to as "other wealth" (other than wage income), is uniformly distributed on a support $[k_l, k_u]$.

We further define: $\frac{v}{L} = \frac{k_u+k_l}{2}$ as the mean other wealth (per capita), a measure of the position of the distribution concerning other wealth, and $\frac{x}{L} = \frac{k_u-k_l}{2}$ as a measure of the spread of the distribution.

Given the linear relationship between other wealth and steady state consumption, the latter is also uniformly distributed on support $[C_l, C_u]$.

These assumptions imply that individuals have same preferences and wage income, but differ in their other wealth and, as a result, in their levels of consumption. The consumption pattern of individual $i \in [l, ..., u]$ is thus given by $C_i = w + \rho k_i$, and the mean consumption pattern, as a case of special interest, is given by: $C_m = w + \rho \frac{v}{L}$.

2.2 Bertrand Duopoly Game.

Each individual on the wealth scale makes a decision regarding the purchase of the quality good, namely whether to buy it, and which quality to buy. This decision depends on the income of the individual in question and, again because of the linear relationship between income and consumption, on the level of consumption that this individual can afford.

Let C_{eq} denote the consumption level of the individual who is indifferent between purchasing one unit of good 1 at price p_1 and one unit of good 2 at price p_2.

For such an individual, utility derived from these two consumption patterns must be the same:

$$\ln u(q_1) = \ln u(q_2) \Leftrightarrow \ln q_1 + \ln(C_{eq} - p_1) = \ln q_2 + \ln(C_{eq} - p_2)$$

Solving for C_{eq} yields:

$$C_{eq} = \frac{q_2 p_2 - q_1 p_1}{q_2 - q_1} = \frac{\gamma p_2 - p_1}{\gamma - 1}.$$

The individual with consumption pattern C_{eq} divides the population into two groups. There are individuals $C_i \in [C_l, C_{eq}]$ who buy good 1, and individuals $C_i \in [C_{eq}, C_u]$ who buy good 2.[3]

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[2] In order to solve for the steady state, we write down the Bellman equation for this problem: $V(k_{t-1}) = \max_{C_t, k_t}\left[u_t(C_t) + \frac{1}{1+\rho} V(k_t)\right]$, subject to the intertemporal budget constraint: $C_t = -k_{t+1} + (1+r_t)k_t + w_t$. We recall that $u_t(C_t) = \ln(C_t - p_{it}) + \ln q_{it}$. Substituting from the budget constraint into the Bellman equation, we can rewrite the latter as follows: $V(k_{t-1}) = \max_{k_t}\left[u_t(k_t) + \frac{1}{1+\rho} V(k_t)\right]$. The Euler equation is: $u_t' = \frac{1}{1+\rho} u_{t+1}' f'(k_t)$. Here: $\frac{1+r_t}{C_t - p_{it}} = \frac{1}{1+\rho} \frac{1+r_{t+1}}{C_{t+1} - p_{it+1}} (1+r_t) \Leftrightarrow \frac{C_{t+1} - p_{it+1}}{C_t - p_{it}} = \frac{1+r_{t+1}}{1+\rho}$. In steady state we set $C_{t+1} = C_t = C$ and $p_{it+1} = p_{it} = p_i$ to derive: $1 = \frac{1+r_{t+1}}{1+\rho} \Rightarrow r_t = r = \rho$. Steady state can be arrived at by setting $k_t = k, C_t = C, w_t = w, r_t = \rho$ in the budget constraint, to get: $k = (1+\rho)k + w - C \Leftrightarrow C = w + \rho k$

[3] Underlying this statement, there is an assumption that the market is "covered", that is, everybody buys the quality good, although some people prefer good 1 and some others buy good 2. Another equilibrium

Therefore, demand for goods 1 and 2 is respectively:

$$D_1 = LF(C_{eq})$$
$$D_2 = L(1 - F(C_{eq}))$$

Where $F(\cdot)$ is the cumulative distribution function (cdf) of consumption levels, which has density $f(\cdot)$. Under duopoly, firms compete for customers by choosing prices (Bertrand competition).
Their objective is to maximize the following profits:

$$\pi_1 = D_1(p_1 - wc) = LF(C_{eq})(p_1 - wc)$$
$$\pi_2 = D_2(p_2 - wc) = L(1 - F(C_{eq}))(p_2 - wc)$$

where π_i is profit accruing to firm producing good of quality $i = 1, 2$ and selling it at price p_i and wc is a cost per unit produced (assumed to be the same for both qualities, and equal to wage income multiplied by a labor coefficient c<1).
We can use first order conditions for profit maximization ($\frac{\partial \pi_i}{\partial p_i} = 0$) to derive the following two equilibrium conditions:

$$1 - 2F(C_{eq}) = f(C_{eq})(C_{eq} - wc)$$
$$f(C_{eq})\left(B_{eq} - \frac{wc(\gamma+1)}{\gamma-1}\right) = 1$$

where we define $B_{eq} = \frac{\gamma p_2 + p_1}{\gamma - 1}$.
Further manipulations yield the equilibrium prices:

$$p_1^e = (\gamma - 1)\frac{F(C_{eq})}{f(C_{eq})} + wc$$

$$p_2^e = \frac{\gamma - 1}{\gamma}\frac{1 - F(C_{eq})}{f(C_{eq})} + wc$$

And equilibrium profits:

$$\pi_1^e = (\gamma - 1)\frac{L[F(C_{eq})]^2}{f(C_{eq})}$$

$$\pi_2^e = \frac{\gamma - 1}{\gamma}\frac{L[1 - F(C_{eq})]^2}{f(C_{eq})}$$

This formulation of equilibrium relationships offers the advantage of being valid for any distribution of consumption patterns, not just the uniform case.
This consideration enables us to state the following lemma for any distribution of consumption levels:
Lemma: In a duopolistic market where everybody buys one of the two quality goods, the following is always true:

$$F(C_{eq}) \le \tfrac{1}{2} \Leftrightarrow C_{eq} \le C_m$$

Proof: Recall that one of the two equilibrium conditions is:

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scenario, which is ruled out here, is when some consumers prefer not to buy the quality good at all. In this instance, we say that the market is not covered. It is not difficult to show that, in order for the market to be covered, the following condition must hold: $C_{eq} \le 2C_l - \frac{1}{\gamma-1}wc$ The following is an interpretation of this condition: The equilibrium household, which is indifferent from buying good 1 at price $p_1$and good 2 at price p_2, must have consumption that is less than twice the consumption of the poorest household minus a weighted marginal cost, where the weight $\frac{1}{\gamma-1}$is a measure of the quality differential between good 1 and good 2.

$$1-2F(C_{eq})=f(C_{eq})(C_{eq}-wc)$$

Notice that the RHS of this condition is ≥ 0 by definition, so too must be the LHS.
But this implies: $F(C_{eq})\leq\frac{1}{2}$ (q.e.d.).
From this point onwards, we shall restrict ourselves to the uniform distribution case. For later use, we rewrite equilibrium profits as a function of mean other wealth v and spread x, as follows:

$$\pi_1^e(v)=(\gamma-1)\rho\frac{\left[x-\frac{v}{3}-\frac{wL(1-c)}{3\rho}\right]^2}{2x}$$
$$\pi_2^e(v)=\frac{\gamma-1}{\gamma}\rho\frac{\left[x+\frac{v}{3}+\frac{wL(1-c)}{3\rho}\right]^2}{2x}.$$

2.3 Monopoly
Here we want to describe the conditions that in equilibrium yield a monopoly, that is that market structure, where the quality good sector is characterized by everybody buying the state-of-the-art. Moreover, just as we did when discussing duopoly, we want to determine the monopoly equilibrium price and profit that accrues to the firm which produces the state-of-the-art.
We start by writing the demand schedule facing the firm selling good 2:

$$D_2=L\int_{C_{eq}}^{C_u}f(C)dC=\frac{L}{C_u-C_L}(C_u-C_{eq})=\frac{L}{C_u-C_l}\left(C_u-\frac{\gamma}{\gamma-1}p_2+\frac{1}{\gamma-1}p_1\right)$$

If both firm producing good 1 and firm producing good 2 were to adopt marginal cost pricing (so that $p_1=wc$ and $p_2=wc$), output would be: $\frac{L}{C_u-C_l}(C_u-wc)$. A monopolist facing the above mentioned demand schedule, which is linear in p_2, would therefore choose output that is half of that chosen when both firms adopt marginal cost pricing: $\frac{L}{C_u-C_l}(C_u-wc)/2$. Because every individual buys at most one unit of the quality good, monopoly occurs whenever the size of the population is not greater than the number of units produced by a profit-maximizing monopolist, or in symbols:

$$L\leq\frac{L}{C_u-C_l}(C_u-wc)/2\Leftrightarrow C_u\leq 2C_l-wc$$

We can rewrite the above condition in terms of mean other wealth v and spread x, as follows:

$$x\leq\frac{1}{3}\left(v+\frac{wL(1-c)}{\rho}\right).$$

The economic interpretation of this condition is that for monopoly to obtain in equilibrium, the spread in the distribution of other wealth, cannot exceed one third of the expression in brackets. The latter is the sum of other wealth v and the discounted flow (discounted at the rate of time preference ρ) of the total amount of salaries wL earned in sectors other than manufacturing of the quality good (this exclusion is obtained by multiplying wL by the coefficient $1-c$). Salaries in the manufacturing of quality goods sector are excluded because they also represent a cost for households/entrepreneurs. The condition for monopoly can therefore be restated in words as: “the spread in other wealth must be at most equal to one third of the total wealth in the system”[4]. Under these circumstances the market for the quality good is a monopoly. The profit accruing to the monopolist is: $\pi_m=L(p_m-wc)$. In order to maximize this profit, the monopolist will set the maximum price compatible with a monopolist market structure. This amounts to setting the highest price such that $C_{eq}\leq C_l$. Recall that under monopoly, the firm producing the inferior good is assumed to set a price equal to marginal cost, $p_1=wc$ which yields no profit so that

[4] Obviously, the condition for duopoly is the complement of this and reads “duopoly obtains as soon as the spread in other wealth is larger than one third of the total wealth in the system”.

the firm does not start production in the first place. As a result, the above inequality may be rewritten as: $\frac{\gamma p_m - wc}{\gamma - 1} \leq C_l$. To get the price that maximizes monopoly profits, notice that the above constraint is binding and solve for p_m to obtain:

$$p_m^e = \frac{\gamma-1}{\gamma} C_l + \frac{wc}{\gamma}.$$

Putting this price back into the expression for profit, yields equilibrium profit under monopoly:

$$\pi_m^e = L\frac{\gamma-1}{\gamma}(C_l - wc).$$

Both monopoly equilibrium price and profit depend on C_l only. Thus these relationships are valid for any distribution of consumption patterns. Nevertheless, in order to maintain a parallel with the duopoly case, we shall restrict our analysis to the uniform distribution case.

For later use, we rewrite monopoly price and profit as function of mean other wealth v and spread x:

$$p_m^e = \frac{\gamma-1}{\gamma}\left[w + \rho\left(\frac{v}{L} - \frac{x}{L}\right)\right] + \frac{wc}{\gamma}$$

$$\pi_m^e = \frac{\gamma-1}{\gamma}[\rho(v-x) - wL(1-c)]$$

3. The R&D Sector

In order to close the model, and before passing to the description of steady states with associated equilibrium analyses, we need to introduce two more elements.

In this section we describe the R&D sector, while the next section is dedicated to the labor markets. Both these sectors are crucial building blocks of this model, but, because in this paper nothing novel is added to them that did not appear in the previous quality ladders literature, here we provide a concise account of them, and refer the interested reader to that literature (e.g. Grossman and Helpman, 1991) for a more detailed description.

We assume that innovations are random and arrive according to a Poisson arrival rate μ. μdt describes the probability that the next innovation occurs within dt from now, when the innovator research effort is μ. In steady state, the value of an innovation V will be such that the following arbitrage equation holds:

$$\rho V = \pi_2 - \mu V + \mu V^n$$

In turn:

$$\rho V^n = \pi_1 - \mu V^n$$

where V^n is the value of the innovation next period, after it has become second best due to the arrival on the market of a better product (i.e. after it has been pushed one step lower on the quality ladder); π_i is profit for the firm producing good of quality $i = 1, 2$, and ρ is the rate of time preference.

After substitution, we get the asset arbitrage equation:

$$V = \frac{\pi_2}{\rho+\mu} + \frac{\mu\pi_1}{(\rho+\mu)^2} \qquad \textbf{(AA)}$$

Free entry in R&D implies zero profit for the innovator, or:

$$\mu V - \mu wa = 0,$$

where μwa is the cost of doing research, whose components are the intensity of research μ, wage income w, and a labor coefficient a. We can and rewrite this free entry condition as follows:

$$\mu(V - wa) = 0$$

and conclude that positive but finite research investments can take place only when $V = wa$. In symbols:

$$V = wa, \mu > 0$$

$$V < wa, \mu = 0$$

Putting together the asset arbitrage and free entry conditions yield the following equilibrium relationship for the R&D sector:

$$wa = \frac{\pi_2}{\rho+\mu} + \frac{\mu\pi_1}{(\rho+\mu)^2} \qquad \textbf{(A)}.$$

When wealth and consumption distributions are such that, in equilibrium, the market structure is a monopoly, sales of the good of lower quality are zero and $\pi_1 = 0$. As a result, next period value is $V^n = 0$ and the R&D condition **(A)** reduces to:

$$wa = \frac{\pi_m}{\rho+\mu} \qquad \textbf{(A')}.$$

4. The Labor Markets

The final element of this model is the labor market. In this market, total demand is the sum of demand for labor in each sector.

With research intensity μ and a research sector characterized by a labor coefficient *a*, demand for labor in research is equal to *am.*

In the manufacturing sector, the demand for labor in manufacturing the quality good is cL.

Demand for labor in manufacturing the homogeneous good is $\frac{h}{w}$, and it is obtained as follows: The technology for the homogeneous good is assumed to be such that every individual contributes equally to its production. Given perfect competitive settings, we equate marginal product of labor $\frac{h}{L}$ to the going wage *w*, which, upon solving for *L*, yields the demand for labor in manufacturing the homogeneous good as stated above.

The supply of Labor is simply given by *L***.**

Equating labor demand and labor supply, yields the full employment condition:

$$a\mu + cL + \frac{h}{w} = L.$$

We recall that *h* is given by $h_t = C_t - p_{it}$ which enables us to derive the following expressions for *h*:

$$h = \int_{C_l}^{C_{eq}} \frac{L}{C_u - C_l} h(C)dC + \int_{C_{eq}}^{C_u} \frac{L}{C_u - C_l} h(C)dC$$

$$h = \int_{C_l}^{C_{eq}} \frac{L}{C_u - C_l} (C - p_1)dC + \int_{C_{eq}}^{C_u} \frac{L}{C_u - C_l} (C - p_2)dC$$

This is the expression we get for duopoly. Replace the above in the full employment condition, so that the latter can be rewritten in terms of firms' profits:

$$a\mu + \frac{LC_m}{w} - \frac{\pi_2}{w} - \frac{\pi_1}{w} = L$$

We repeat the same calculations for monopoly. Now h is given by:

$$h = \int_{C_l}^{C_u} \frac{L}{C_u - C_l} h(C)dC = \int_{C_l}^{C_u} \frac{L}{C_u - C_l} (C - p_m)dC$$

And the full employment condition can be once again rewritten in terms of firm's profits as follows:

$$a\mu + \frac{LC_m}{w} - \frac{\pi_m}{w} = L.$$

5. Growth Rate

The steady state growth rate of this economy stems from the quality upgrading process in the quality goods sector. As in the earlier quality ladders literature, we recall that innovations arrive at Poisson rate μ, and when they do arrive, the size of the jump up the quality ladder is $\ln\gamma$, to derive the following expression for the steady state growth rate:

$$g = \mu \ln\gamma$$

6. Steady State Analysis - Monopoly

In steady state equilibrium, the model is fully described by the research equation **(A)** and the full employment condition **(L)**. Under monopoly, these equations take the following form:

$$wa = \frac{\pi_m(v)}{\rho+\mu} \qquad \textbf{(A')}$$

$$w\mu a = \pi_m(v) - \rho v \qquad \textbf{(L')}$$

Where the two inverted commas are intended to differentiate these relationships from those that will be met when studying duopoly. Monopoly profit, as a function of other wealth v, has in turn been found to be:

$$\pi_m(v) = \frac{\gamma-1}{\gamma}[\rho(v-x) - wL(1-c)]$$

After plugging the profit equation into **(A')** and **(L')**, we get a system of two equations in two unknowns (v, μ) that is amenable to analysis. We state the following:

Proposition 1: If the market structure in the quality goods sector is a monopoly, there exists a unique steady state equilibrium characterized by positive wealth level v, research intensity μ and positive growth rate g. (Proof in the Appendix).

Proposition 2: An increase in the degree of inequality, as measured by an increase in the spread x, is harmful for innovative activity μ and the economy growth rate g. (Proof in the Appendix).

< Picture 1 goes here >

To sum up our monopoly analysis, we have seen that, in order to have an equilibrium with monopoly, the spread in other wealth must not exceed one third of total wealth. Hence, monopoly as an equilibrium outcome, is a function of both the level of wealth and the inequality in the distribution of wealth. The degree of inequality that is permitted before the system switches to duopoly is greater, the higher the level of wealth. Thus monopoly in the quality goods markets is more likely to be found in richer societies.

If monopoly does obtain, then the economy grows at a positive rate because of a positive amount of research activity. But any increase in the degree of inequality, also increases the threat of entry in the market by the good of inferior quality and forces the monopolist to set lower prices. As a result, incentives for carrying research are lower and the impact on the economy growth rate is negative.

7. Steady State Analysis - Duopoly

In equilibrium, the two equations that define the model under duopoly are:

$$wa = \frac{\pi_2(v)}{\rho+\mu} + \frac{\mu\pi_1(v)}{(\rho+\mu)^2} \qquad \textbf{(A)}$$

$$\mu wa = \pi_2(v) + \pi_1(v) - \rho v \qquad \textbf{(L)}^{5}$$

Where the two expressions for profits are as follows:

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[5] Equation **(A)** is exactly as derived in the R&D section, whereas equation **(L)** is as derived from the Labor Markets section, with $C_m = w + \rho\frac{v}{L}$, and after multiplying through by w.

$$\pi_1^e(v) = (\gamma - 1)\rho \frac{\left[x - \frac{v}{3} - \frac{wL(1-c)}{3\rho}\right]^2}{2x}$$

$$\pi_2^e(v) = \frac{\gamma - 1}{\gamma}\rho \frac{\left[x + \frac{v}{3} + \frac{wL(1-c)}{3\rho}\right]^2}{2x}$$

In the above, the two endogenous variables are μ, which denotes intensity of research, and v, which measures the total value of other wealth (other than wage income w). Of great importance in our analysis will also be x, a measure of the spread of the distribution of v. The equilibrium analysis is at first carried out for fixed spread x. Later we shall let it vary and measure the impact of such variation on the endogenous pair (μ, v) in the new steady state equilibrium.

Proposition 3: If the market structure in the quality goods sector is a duopoly, there exists a unique steady state equilibrium characterized by positive wealth level v, research intensity μ and positive growth rate g.

We sketch the proof of proposition 3 here, while the entire proof can be found in the appendix.

Proof: this follows the same methodology that we used to prove proposition 1 regarding monopoly. We note that both **(A)** and **(L)** implicitly define functions $v = v_A(\mu)$ and $v = v_L(\mu)$. Our proof is in three steps: 1) prove that the function $v = v_A(\mu)$ has positive slope for all $\mu \geq 0$, $v \geq 0$.

2) prove that the function $v = v_L(\mu)$ has negative slope for all $\mu \geq 0$, $v \geq 0$. This condition holds, provided that the leap in quality brought about by the latest innovation, as measured by γ, is sufficiently small (in particular, the slope is negative for $1 < \gamma < 2$).

3) Compute $v_A(\mu)\mid_{\mu=0}$ and $v_L(\mu)\mid_{\mu=0}$ and notice that a unique equilibrium exists if and only if $v_A(\mu)\mid_{\mu=0} < v_L(\mu)\mid_{\mu=0}$. In words, this latest step consists of showing that the vertical intercept of the function defined by the **(A)** schedule occurs at a higher point than the vertical intercept of the function defined by the **(L)** schedule. This fact, together with steps 1 and 2 ensures that the two schedules meet only once in the positive quadrant of the (μ, v) plane and thereby determine a unique and positive pair (μ, v).

< Picture 2 goes here >

Further insight can be obtained by merging together the two equations **(A)** and **(L)**. Such a calculation yields the following relationship between μ and v:

$$\mu = \mu_1(v) = \frac{\pi_2(v) - aw\rho}{2aw - v} \qquad \textbf{(*)}$$

Notice that, upon dividing equation **(L)** through by aw, the latter also provides a direct relationship between μ and v:

$$\mu = \mu_2(v) = \frac{\pi_2(v) + \pi_1(v) - \rho v}{aw} \qquad \textbf{(**)}$$

Study of **(*)** and **(**)** yields another way of looking at the steady state equilibrium under duopoly.

We have already seen, earlier on in the text, that under monopoly, the value of other wealth is given by $v = wa$. Under duopoly, the remaining value of the good that is second best must be added to the value of the latest innovation, so that the value of wealth falls in the range $wa < v < 2aw$. Therefore, we want to study the behavior **(*)** and **(**)** for values of v falling into this range.

Starting with **(*)**, notice that the latter has a vertical asymptote at $v = 2aw$. For $v < 2aw$, μ approaches +∞ as v approaches $2aw$ from below, provided that $\pi_2(v) - aw\rho > 0$. The latter can be ensured for example by taking L to be sufficiently large. For $aw \leq v < 2aw$, μ is a monotonically increasing function of v, because $\pi_2(v)$ in the numerator is monotonically increasing in v, and the denominator goes down as v rises. As for **(**)**, we already know from our earlier analysis that μ decreases as v increases, provided that $1 < \gamma < 2$. The two curves will meet once if and only if :

$$\mu_1(aw) = \frac{\pi_2(aw) - aw\rho}{aw} < \mu_2(aw) = \frac{\pi_2(aw) + \pi_1(aw) - aw\rho}{aw} \Leftrightarrow \frac{\pi_1(aw)}{aw} > .0$$

Further, in order to ensure that at the point of intersection $\mu > 0$, we need that either $\pi_2(aw) - aw\rho > 0$ on the schedule **(*)**, or that $\pi_2(aw) + \pi_1(aw) - \rho v > 0$ on the schedule **(**)**.

< Picture 3 goes here >

The curves **(*)** and **(**)** are also a useful and very simple tool to study how the endogenous variable μ, and thereby the growth rate g, respond to changes in the degree of inequality x.

Proposition 4: Under duopoly, an increase in the degree of inequality x raises the intensity of research μ and thereby the growth rate g.

Proof: Since both in **(*)** and in **(**)** inequality x only enters the two profit functions, which in turn enter $\mu_1(v)$ and $\mu_2(v)$ with a positive sign, it is sufficient to show that $\frac{\partial \pi_1}{\partial x} > 0$ and $\frac{\partial \pi_2}{\partial x} > 0$.

We find $\frac{\partial \pi_1}{\partial x} = (\gamma - 1)\rho \frac{\left(x - \frac{v}{3} - \frac{wL(1-c)}{3\rho}\right)\left(x + \frac{v}{3} + \frac{wL(1-c)}{3\rho}\right)}{2x^2}$ and $\frac{\partial \pi_2}{\partial x} = \frac{\gamma - 1}{\gamma}\rho \frac{\left(x + \frac{v}{3} + \frac{wL(1-c)}{3\rho}\right)\left(x - \frac{v}{3} - \frac{wL(1-c)}{3\rho}\right)}{2x^2}$. Both $\frac{\partial \pi_1}{\partial x} > 0$ and $\frac{\partial \pi_2}{\partial x} > 0$ follow from the fact that x, under duopoly, must satisfy the condition $x > \frac{v}{3} + \frac{wL(1-c)}{3\rho}$.

We have seen that under duopoly too, the equilibrium outcome features a positive amount of research activity and a growing economy. Notice, however, that as inequality in wealth increases, consumers of quality goods of two different types are driven further apart. Producers of both types are able to earn larger profits as a result of less competition due to more inequality (notice that the derivatives of the profit functions with respect to the spread x are both positive. Here, therefore, an increase in wealth inequality is beneficial to the growth of the economy.

Given that monopoly is more likely in wealthier societies while duopoly is more likely in poorer societies, and given the contrasting impact of an increase in inequality on the growth rate of the economy under the two different regimes, one might go as far as concluding that an increase in inequality is bad for wealthier economies, but is good for poorer ones.

8. Comparative statics on the number of qualities as a function of inequality, given low or high wages, and as a function of wages, given low or high inequality.

In order to study the behaviour of the level of inequality and wages that determine the switch from monopoly to duopoly (from one quality to two qualities), the relevant regime to look at is monopoly (this of course implies that duopoly would be the relevant regime if we were to study the threshold between two and three qualities).

Under monopoly, other wealth v is equal to the value of innovation. This in turn is equal to wa because of the free entry into R&D condition. Therefore we can write: $v = wa$. Notice that the equilibrium v in monopoly depends positively on wages, but does not depend on inequality x.

We recall that, in terms of inequality, the threshold level we are studying is[6]:

$$x^* \le \frac{1}{3}\left(v(w) + \frac{wL(1-c)}{\rho}\right);$$

We can see that:

$$x^*(w_L) < x^*(w_H).$$

Where w_L and w_H are low wages and high wages, respectively.

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[6] In this section, the superscript "*" indicates equilibrium values of the variable concerned, while the subscripts "L" and "H" stand for "low" and "high".

< Picture 4 goes here >

In terms of wages, the threshold level is:

$$w^* \geq \frac{\rho}{L(1-c)}(3x - v(w^*)) \Leftrightarrow w^* \geq \frac{3\rho x}{L(1-c)+a\rho}$$

Here, as x increases, there is only a direct effect on w^*, because the equilibrium v does not depend on x in monopoly. Thus we have:

$$w^*(x_L) < w^*(x_H).$$

< Picture 5 goes here >

Notice that these results fit in well with anecdotal evidence whereby qualities increase with inequality and decrease with wealth.

9. Conclusion

This paper has developed a quality ladder model characterized by non homothetic consumer preferences, in line with the vertical quality differentiation literature. Such modelling of consumer preferences in the context of a quality ladder model is the novel element of this study, and it is an attempt to explain real world phenomena like the survival of older generations of goods along with the state-of-the-art, something which was not accounted for in the mainstream quality ladder models of G.H. And A.H.

The next step in this line of research might be to introduce such preferences in open economy quality ladder models, such as that of Grossman and Helpman (1991, chp.12). Indeed, in an open economy framework, the possibility of having monopoly of the best quality in a richer North with a higher and less unequal distribution of wealth, and duopoly in a poorer South with a lower and more widespread distribution of wealth, might be obtained as an endogenous outcome of the model.

Such model would then make it possible to study equilibrium outcomes and comparative statics involving changes in the spread of the wealth distribution, much like has been done in the closed economy model developed in the present paper.

Appendix

Proof of proposition 1:

We plug monopoly profits into equations **(A')** and **(L')** so to have them in explicit form:

$$wa = \frac{\frac{\gamma-1}{\gamma}[\rho(v-x)+wL(1-c)]}{\rho+\mu} \qquad \textbf{(A')}$$

$$w\mu a = \frac{\gamma-1}{\gamma}[\rho(v-x)+wL(1-c)]-\rho v \qquad \textbf{(L')}$$

It is useful to rewrite **(A')** and **(L')** as follows:

$$A'(\mu,v)=0 \Leftrightarrow \frac{\gamma-1}{\gamma}[\rho(v-x)+wL(1-c)]-wa(\rho+\mu)=0$$

$$L'(\mu,v)=0 \Leftrightarrow \frac{\gamma-1}{\gamma}[\rho(v-x)+wL(1-c)]-\rho v-w\mu a=0$$

Proof of proposition 1 will be done in three steps:

Step 1: we shall prove that $\frac{dv}{d\mu} \mid_{A'} > 0$,

Step 2: we shall prove that $\frac{dv}{d\mu} \mid_{L'} < 0$,

Step 3: we shall write down the condition under which $v_{A'}(\mu) \mid_{\mu=0} < v_{L'}(\mu) \mid_{\mu=0}$.

In step 1: $\frac{dv}{d\mu} \mid_{A'} = -\frac{-wa}{\frac{\gamma-1}{\gamma}\rho} > 0.$

In step 2: $\frac{dv}{d\mu}\mid_{L'} = -\frac{-wa}{-\frac{1}{\gamma}\rho} < 0$.

In step 3: to compute $v_{A'}(\mu)\mid_{\mu=0}$ we set $\mu = 0$ in **(A')** and solve for v:

$\frac{\gamma-1}{\gamma}[\rho(v-x)+wL(1-c)] - wa\rho = 0 \Leftrightarrow v_{A'}(\mu)\mid_{\mu=0} = x + \frac{\gamma}{\gamma-1}wa - \frac{wL(1-c)}{\rho}$.

We compute $v_{L'}(\mu)\mid_{\mu=0}$ in the same way:

$\frac{\gamma-1}{\gamma}[\rho(v-x)+wL(1-c)] - \rho v = 0 \Leftrightarrow v_{L'}(\mu)\mid_{\mu=0} = (\gamma-1)\left[\frac{wL(1-c)}{\rho} - x\right]$.

A unique equilibrium with a positive pair (v, m) exists if and only if:

$x + \frac{\gamma}{\gamma-1}wa - \frac{wL(1-c)}{\rho} < (\gamma-1)\left[\frac{wL(1-c)}{\rho} - x\right] \Leftrightarrow x < \frac{wL(1-c)}{\rho} - \frac{1}{\gamma-1}wa$.

Proof of Proposition 2: This proposition can be proved in two ways.

Proof 1: an increase in inequality x, as measured by: $\frac{\partial\mu}{\partial x}\mid_{A'} = -\frac{-\frac{\gamma-1}{\gamma}\rho}{-wa} < 0$, and by: $\frac{\partial\mu}{\partial x}\mid_{L'} = -\frac{-\frac{\gamma-1}{\gamma}\rho}{-wa} < 0$, moves to the left both the **(A')** and the **(L')** schedule. As a result, we have a new equilibrium with less research intensity μ and growth g (recall that $g = \mu\ln\gamma$). Furthermore, since the effect of an increase in x on both schedules is of the same magnitude, at the new equilibrium, the level of other wealth v is unchanged.

Proof 2: In a monopoly equilibrium, the value of other wealth v is determined by the value of the most recent innovation. As shown in the text, the latter is equal to the costs faced by the most recent innovator: $v = wa$. We plug this result into **(A')** and **(L')**, to get the same relationship:

$$\frac{\gamma-1}{\gamma}[\rho(wa-x)+wL(1-c)] - wa(\rho+\mu) = 0 \qquad \textbf{(A', L')}$$

We then solve for μ to get:

$$\mu = \frac{\gamma-1}{\gamma}\left[\frac{L(1-c)}{a} - \frac{\rho x}{wa}\right] - \frac{1}{\gamma}\rho$$

From the above it is obvious that μ decreases in x.

Proof of Proposition 3:

The proof is done in three steps. Schedules **(A)** and **(L)** implicitly define functions $v = v_A(\mu)$ and $v = v_L(\mu)$.

Step 1: prove that $\frac{dv}{d\mu}\mid_A > 0$. Rewrite **(A)** as follows: $G(\mu, v) = 0 \Leftrightarrow -wa + \frac{\pi_2(v)}{\rho+\mu} + \frac{\mu\pi_1(v)}{(\rho+\mu)^2} = 0$.

Then, by the implicit function theorem: $\frac{dv}{d\mu}\mid_A = -\frac{\frac{\partial G}{\partial\mu}}{\frac{\partial G}{\partial v}}$. We find that $\frac{\partial G}{\partial\mu} = -\frac{\pi_2}{(\rho+\mu)^2} - \frac{\pi_1(\rho-\mu)}{(\rho+\mu)^3}$. We set $\frac{\partial G}{\partial\mu} = 0$ and solve for μ to get $\widehat{\mu} = \rho\frac{\pi_1-\pi_2}{\pi_1+\pi_2}$. Notice that, in duopoly, $\widehat{\mu} < 0$ since $\pi_1 < \pi_2$. As $\frac{\partial G}{\partial\mu} < 0$ for $\mu > \widehat{\mu}$ and as we are only interested in non-negative values of μ, we conclude that $\frac{\partial G}{\partial\mu} < 0$ in the relevant range. Next, we compute $\frac{\partial G}{\partial v} = \frac{\frac{\partial\pi_2}{\partial v}}{\rho+\mu} + \frac{\mu\frac{\partial\pi_1}{\partial v}}{(\rho+\mu)^2} = \frac{1}{\rho+\mu}\left[\frac{\partial\pi_2}{\partial v} + \frac{\mu\frac{\partial\pi_1}{\partial v}}{\rho+\mu}\right]$ which takes the sign of $\frac{\partial\pi_2}{\partial v} + \frac{\mu\frac{\partial\pi_1}{\partial v}}{\rho+\mu}$.

We find that $\frac{\partial\pi_2}{\partial v} = \frac{\gamma-1}{\gamma}\rho\frac{x+\frac{v}{3}+\frac{wL(1-c)}{3\rho}}{3x} > 0$ and $\frac{\partial\pi_1}{\partial v} = (\gamma-1)\rho\frac{x-\frac{v}{3}-\frac{wL(1-c)}{3\rho}}{x}\left(-\frac{1}{3}\right) < 0$.

However $\frac{\partial\pi_2}{\partial v} + \frac{\partial\pi_1}{\partial v} = x + \frac{v}{3} + \frac{wL(1-c)}{3\rho} + \gamma\left(x - \frac{v}{3} - \frac{wL(1-c)}{3\rho}\right) > 0$ in duopoly, as the expression in brackets is guaranteed to be positive with such a market structure. $\frac{\partial\pi_2}{\partial v} + \frac{\mu\frac{\partial\pi_1}{\partial v}}{\rho+\mu} > 0$ follows from $\frac{\mu}{\rho+\mu} < 1$. Hence $\frac{\partial G}{\partial v} > 0$. The latter enables us to conclude that $\frac{dv}{d\mu}\mid_A = -\frac{\frac{\partial G}{\partial\mu}<0}{\frac{\partial G}{\partial v}>0} > 0$

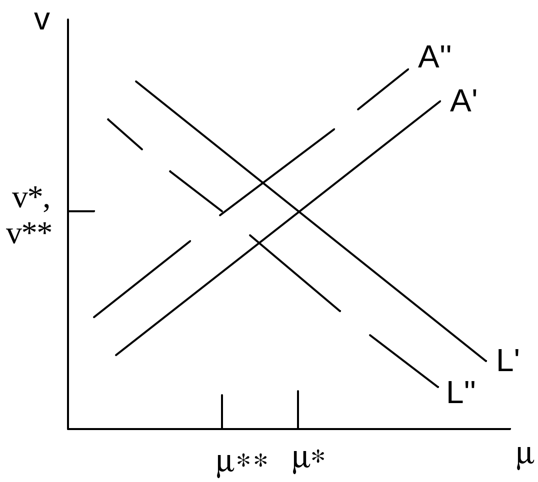
Step 2: prove that $\frac{dv}{d\mu}\mid_L < 0$. Rewrite **(L)** as $H(\mu, v) = 0 \Leftrightarrow \frac{\pi_2(v)+\pi_1(v)-\rho v}{\mu} - wa = 0$. By the implicit function theorem: $\frac{dv}{d\mu}\mid_L = -\frac{\frac{\partial H}{\partial\mu}}{\frac{\partial H}{\partial v}}$. First, we compute $\frac{\partial H}{\partial\mu} = -\frac{\pi_2(v)+\pi_1(v)-\rho v}{\mu^2}$. $\frac{\partial H}{\partial\mu} < 0$ follows from

$\pi_2(v) + \pi_1(v) - \rho v = \mu wa > 0$(see **(L)**). Next, we compute $\frac{\partial H}{\partial v} = \frac{\frac{\partial \pi_2}{\partial v} + \frac{\partial \pi_1}{\partial v} - \rho}{\mu}$. We need to determine the sign of $\frac{\partial \pi_2}{\partial v} + \frac{\partial \pi_1}{\partial v} - \rho$. A few calculations reveal that $\frac{\partial \pi_2}{\partial v} + \frac{\partial \pi_1}{\partial v} - \rho < 0$ provided that $x > \frac{1-\gamma}{2-\gamma}\left[\frac{v}{3} + \frac{wL(1-c)}{3\rho}\right]$ Since x is always positive, this condition certainly holds if its RHS is negative. That is indeed the case if $1 < \gamma < 2$. Then we have that $\frac{\partial H}{\partial v} < 0$. Therefore, we conclude that for $1 < \gamma < 2$, $\frac{dv}{d\mu} \mid_L = -\frac{\frac{\partial H}{\partial \mu} < 0}{\frac{\partial H}{\partial v} < 0} < 0$.

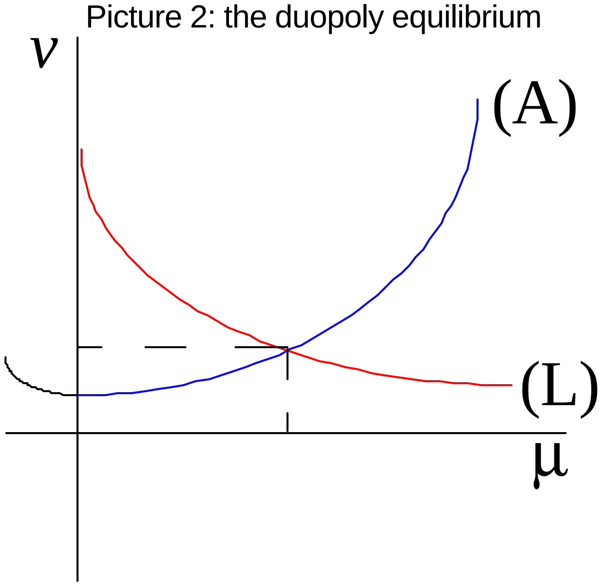
Step 3: a unique equilibrium with a positive pair (μ, v) and a positive growth rate g exists if and only if $v_A(\mu) \mid_{\mu=0} < v_L(\mu) \mid_{\mu=0}$, where $v_A(\mu) \mid_{\mu=0} = 3\left(wa\frac{\gamma}{\gamma-1}2x\right)^{\frac{1}{2}} - 3x - \frac{wL(1-c)}{\rho}$and $v_L(\mu) \mid_{\mu=0}$ is implicitly determined in **(L)** with $\mu = 0$.

Pictures

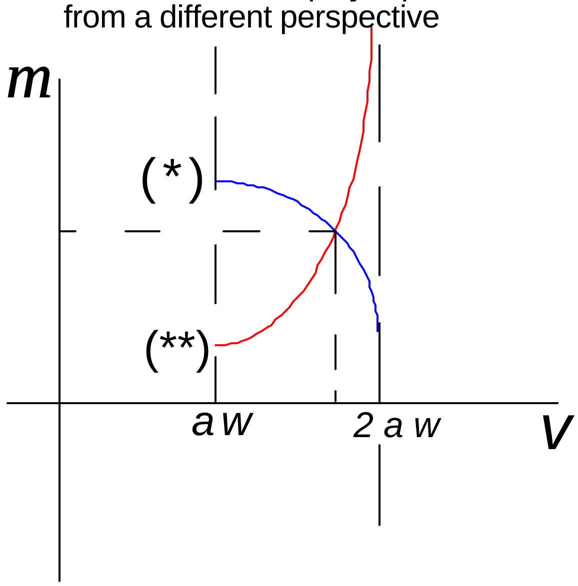
Picture 1: monopoly equilibrium
and comparative statics



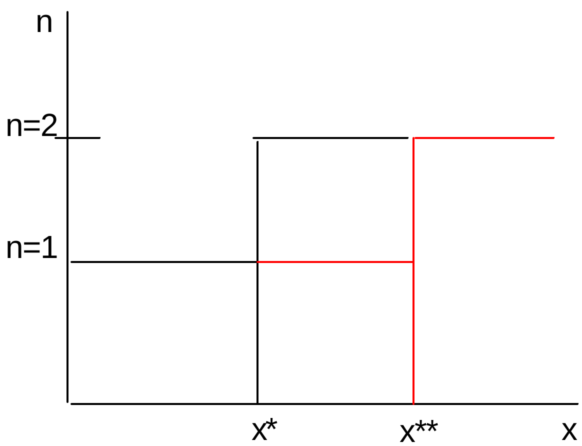
Picture 2: the duopoly equilibrium



Picture 3: the duopoly equilibrium from a different perspective

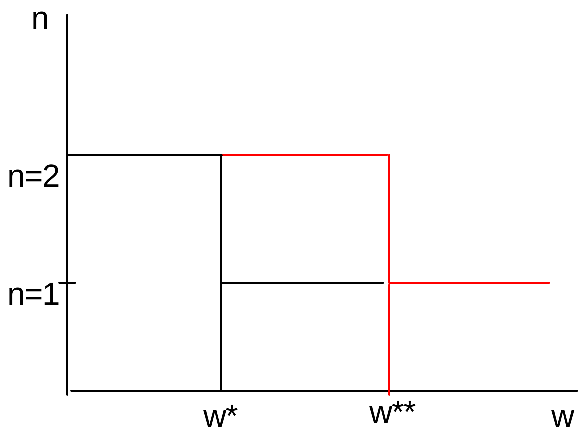


Picture 4: n = f(x/w)
(Red line indicates extension due to higher w).



$$x^* = \frac{1}{3}\left(v(w_L) + \frac{w_L L(1-c)}{\rho}\right)$$
$$x^{**} = \frac{1}{3}\left(v(w_H) + \frac{w_H L(1-c)}{\rho}\right)$$

Picture 5: n = f(w/x)
(red line indicates extension due to higher x)



$$w^*(x_L) = \frac{3\rho x_L}{L(1-c)+a\rho}$$
$$w^*(x_H) = \frac{3\rho x_H}{L(1-c)+a\rho}$$

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