# Optimal Width of the Implicit Exchange Rate Band, and the Central Bank's Credibility

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**ABSTRACT**

In February 2001 currency peg regime collapsed in Turkey and the Central Bank switched to floating exchange rate regime. In an open and highly dollarized economy such as Turkey exchange rate volatility is not desirable because of its detrimental effects on inflation and output. At the beginning of 2002 monetary authorities looking for an effective way to limit exchange rate volatility have announced that foreign exchange interventions will be kept at a minimum and the Central Bank will only intervene in excessive fluctuations.

In this paper, the Central Bank's policy decision to intervene foreign exchange markets in only excessive fluctuations will be modelled as a policy rule of an implicit (S,s) form. If the exchange rate hits *s* or *S*, the Central Bank intervenes and realigns the exchange rate. In order to determine the optimal width of the implicit exchange rate band we use the methodology of statistical ruin problem for random walks. Our results suggest two points: the Central Bank adopts such a policy due to low level of credibility, and credibility of such a policy depends upon the level of foreign reserves, the nature of the disturbances and weights put on the foreign exchange fluctuation vis-à-vis inflation and output volatility.

## 1. Introduction

The experience with the flexible exchange rates has taught us that the monetary autonomy under the flexible exchange rate is not very different from fixed exchange rate regime. Otherwise we should not be very concerned about the volatility of exchange rates, since they may be corrected by appropriate monetary responses. However this is not the case because of the international monetary interdependence and high degree of capital mobility. The exchange rate volatility is not desirable for many countries because of their high degree of openness.

One may argue that observed excessive exchange rate volatility, and divergent and high inflation rates in some countries may be the result of monetary authorities' illusion about this mistaken monetary autonomy in a world of high capital mobility. As a result of these concerns economists as well as politicians began to look for an effective way to limit exchange rate volatility and accompanying high inflation rates.

In order to control high inflation and exchange rate volatility pegged exchange rate regime is proposed and used by many countries. Turkey had also adopted a disinflation program based on a crawling peg regime in December 1999 but the regime has collapsed when it came under speculative attacks first in November 2000 and later in February 2001. In February 2001 crawling peg regime collapsed and the Central Bank switched to floating exchange rate regime. In the followings months Turkish Lira has depreciated considerably and also exhibited large degree of volatility.

In an open and highly dollarized economy such as Turkey exchange rate volatility is not desirable because of its detrimental effects on inflation and output. In spite of output cost of high exchange rate volatility, at the beginning of 2002 the central bank announced that foreign exchange interventions will be kept at a minimum and the Central Bank will only intervene in excessive fluctuations. But what is meant by excessive fluctuations left undefined.

In the following, a model is proposed to understand the rational behind the Central Bank's policy choice.

## 2. The Model

The model consists of the following equations.

$$y_t = \eta(e_t - p_t) + u_t, \qquad \eta > 0 \tag{1}$$

$$m_t - q_t = y_t \tag{2}$$

$$q_t = \lambda p_t + (1 - \lambda)e_t, \qquad 0 < \lambda < 1 \tag{3}$$

All variables are measured as deviations from their long-run equilibrium values. The first equation describes the goods market where $y$ is the log of output, $e$ is the exchange rate measured as the price of foreign currency in terms of the domestic currency, $p$ is the log of the price level of non-traded goods, $u$ is the demand disturbance.

The second equation describes the money market. $m$ is the log of the money supply, which is assumed to be controlled directly by the monetary authority. $q$ is the log of the general price level.

The third equation gives us the general price level as the weighted-sum of the price levels of traded and non-traded goods. Equation (4) defines the inflation rate.

If we solve the model for $e$ and $p$ we get

$$\mathrm{p} = m_t - \left[\frac{1-\lambda+\eta}{\eta}\right] y_t + \left[\frac{1-\lambda}{\eta}\right] u_t \tag{4}$$

$$e_t = m_t + \left[\frac{\lambda-\eta}{\eta}\right] y_t - \left[\frac{\lambda}{\eta}\right] u_t \tag{5}$$

## 3. Policy Choices

At the beginning of each period prices are given and monetary authority faces a stabilisation problem due to the presence of random shocks. Since we have only three equations but four endogenous variables, the first decision that monetary authority has to take is to decide the relevant loss function. For example if the loss function is given by

$$L_1 = \tfrac{1}{2}[p^2], \qquad 0 < \theta < 1 \tag{6}$$

this indicates that the monetary authority only cares on price level fluctuations On the other hand if the loss function is given by

$$L_2 = \tfrac{1}{2}[q^2] \tag{7}$$

this it indicates that the monetary authority also cares for exchange rate fluctuations.

Note that under certainty the choice of the loss function is trivial but under uncertainty the economy's structure and the nature of shocks becomes of importance and the decision about the relevant loss function becomes a decision problem. Consider the first case.

If the loss function is given by $L_1$, then the monetary authority's problem is to minimise the expected value of (6) subject to (4). The optimal money supply in this case can be found as

$$m_t^* = \left[\frac{1-\lambda+\eta}{\eta}\right] y_t ,$$

and the expected welfare loss is

$$E\left[L_1^*\right] = \left[\frac{1-\lambda}{\eta}\right]\sigma^2(u)$$

where $\sigma^2(u)$ is the variance of $u$.

In the second case monetary authority minimises (7) subject to (4) and (5). In this case the optimal money supply is given by

$$m_t^* = y_t,$$

and the expected welfare loss will be zero.

### 4. Cost of Foreign Exchange Market Interventions

Suppose that the Central Bank decides to control the value of its currency. One adopted strategy is to allow the nominal exchange rate to float within some boundaries. For example if the nominal exchange rate rises above some upper limit the Central Bank intervenes to pull back exchange rate within the accepted boundaries. Similar situation is valid for a decline that falls below a lower bound.

In the case of exchange rate interventions, we may distinguish three types of cost items, which the Central Bank should consider.

*1. The credibility cost of exchange rate interventions:* This cost item is difficult to measure because it is difficult to put a quantitative value of credibility cost of foreign exchange market interventions. But frequent interventions indicate policy mistakes, and raise doubts about the sustainability of the policy. We will express this cost as $\gamma E[f]$, where $E[f]$ is the expected number of interventions, and $\gamma$ measures the unit credibility cost of intervention.

*2. The cost of reserve holdings:* In order to intervene to the exchange rates, the central bank should have sufficient reserves at their disposal; otherwise the intervention will not be credible. The cost of reserves can be written as $rR$, where $R$ is the total amount of reserves and $r$ is the interest rate. We are assuming that the central bank can borrow from foreign central banks or international financial institutions without any constraint. A high $r$ indicates low central bank credibility.

*3. Opportunity cost of pegging:* Since the central bank can always choose to follow a policy with respect to $L_1$, that is let the exchange rate float freely, the opportunity cost of exchange rate interventions in terms of policy choice can be written as

$$E\left\lfloor L_2^* \right\rfloor - E\left\lfloor L_1^* \right\rfloor = -\left[\frac{1-\lambda}{\eta}\right]\sigma^2(u)$$

Therefore we can write the necessary condition of intervention as

$$E\left[L_2^*\right] - E\left[L_1^*\right] < 0,$$

which is always satisfied. In words, the Central Bank chooses to intervene as long as the expected welfare loss under exchange rate intervention is less than the welfare cost under floating. But this condition is not sufficient. Furthermore, the gain from intervention should also consider the political and financial costs of foreign exchange market interventions. Therefore the net welfare gain from intervention can be expressed as

$$G = \left[\frac{1-\lambda}{\eta}\right]\sigma^2(u) - \gamma f - rR$$

**5. Optimal Exchange Rate Band**

Monetary authority's objective is to determine a policy to maximise the expected gain from intervention. Then the problem becomes to

$$\max \mathrm{E}[G] = \left[\frac{1-\lambda}{\eta}\right]\sigma^2(u) - \gamma E[f] - rE[R]. \tag{8}$$

In order to solve this problem, we will use the methodology of statistical ruin problem for random walks. Let us assume that $u_t$ follows a symmetric random walk with zero origin and unit steps. Each step takes equal amount of time, which is denoted by $t$. The Central Bank adopts a policy of (S,s) form (for the optimality of (S,s) policies see Scarf (1960), Caballero and Engel (1991)). If the value of $u$ hits $s$ or $S$, the Central Bank intervenes and realigns the exchange rate. In other words, random walk $z$ has absorbing barriers at $s$ and $S$. Furthermore let the optimal band be symmetric around the long run equilibrium level of exchange rate $\hat{e}$, and let the lower bound be denoted by $s$ and upper bound by $S$, that is $s < S$. We will convert the unit step size into a monetary unit by multiplying it by $e$, i.e., $s^* = s\hat{e}$, and $S^* = S\hat{e}$. Without loss of generality we can set the long run equilibrium level of exchange rate equal to one. Then due to the symmetry around origin -$s^*$ = $S^*$, and we will solve the problem in terms of $S^*$.

1. *Derivation of* $E[f]$

Let $e_0$ be the initial exchange rate. Expected duration for the first passage time of a symmetric random walk given that probability of the exchange rate moving up or down equal can be found as (See: Feller (1968, p.348-349))

$$D(e_0) = e_0(S^* - e_0) \tag{9}$$

then for a policy horizon T, the expected number of intervention can be written as

$$E[f] = \frac{T}{e_0(S^* - e_0)\tau}$$

where $\tau$ is the duration of a single step of a random walk.

2. *Derivation of* $E[R]$**:**

Let $\overline{R}$ represents the minimum amount of reserves used to pull back the exchange rate one unit below S*. Then

$$E[R] = \frac{T\overline{R}}{e_0(S^* - e_0)\tau}$$

*3. Derivation of* $\sigma^2$(u)**:** Since each step takes $\tau$ time and expected duration is given as in (9), and furthermore unit steps can be expressed as $\hat{e}$ in currency units then $\sigma^2$(u) can be expressed as

$$\sigma^2(\text{u}) = \frac{e_0(S^* - e_0)\hat{e}^2}{\tau}$$

Therefore the problem can be expressed as

$$\max \mathrm{E[G]} = \left[\frac{1-\lambda}{\eta}\right]\frac{e_0(S^*-e_0)\hat{e}^2}{\tau} - \frac{\gamma T}{e_0(S^*-e_0)\tau} - \frac{rT\overline{R}}{e_0(S^*-e_0)\tau} \qquad (10)$$

with respect to $S^*$. The optimal $S^*$ can be found as

$$S^* = e_0 + \frac{1}{\hat{e}^2\sqrt{(\lambda-1)/\eta}}\sqrt{T\gamma + r\overline{R}}$$

It can be shown that $\frac{dS^*}{dT} > 0$, $\frac{dS^*}{d\gamma} > 0$, $\frac{dS^*}{dr} > 0$, $\frac{dS^*}{d\overline{R}} > 0$. In words, the width of the optimal exchange rate band gets larger longer the policy horizon, higher the political cost of intervention, higher the cost of borrowing and higher the average amount of intervention required to settle the market.

## 6. Conclusion

The model used in this paper is minimalist in structure and unrealistic in its stochastic specification. Furthermore the probability of speculative bubbles kept out by stochastic construction. Furthermore given the minimalist structure of the model a sequential sampling approach would bring some reality to the model, and moreover a continuous stochastic modeling based on a diffusion process would be more realistic. In spite of these shortcomings the model, I hope, put some light on the Central Bank's decision to let exchange rates float freely as long as the exchange rate fluctuations remains in an acceptable but unannounced limits, and the recent experience with the floating regime showed that the acceptable limits are indeed very large.

The currency crisis of November 2000 and February 2001 were costly for the Central Bank. They caused the Central Bank to loose reputation and credibility. An announced exchange rate band with narrow limits would suffer credibility, and difficult to defend. Therefore the policy choice of the Central Bank seems realistic given the recent crisis and the level of foreign reserves.

**References**

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