First Draft

Measurement of Potential Output for Turkey: Unobserved Components Model

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Abstract

This paper specifies a basic univariate and a bivariate unobserved components models to estimate potential output using information from observable aggregates and presents results for the Turkish economy. The most important motivation behind measuring output gap for the Turkish economy is the absence of a certain measure for the output gap that can be used for both near-term and long-term inflation forecasts especially within the "inflation-targeting framework". The first specification used in the paper, i.e. univariate approach, decomposes actual output into potential output that follows a random walk with a time-varying potential growth rate and a stationary output gap. The univariate specifications commonly ignore some economic content, which might be relevant for the measurement of output gap. In this respect, the univariate model is extended by utilizing the relationship between inflation and the output gap, namely the Phillips curve. Whereas both models give similar output gap estimates, signal extraction statistics suggest that incorporating the supply side to the system reduces the parameter uncertainty and the total standard error and improves the gap estimate.

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1. Introduction

Potential output is the level of production that can be achieved with the existing level of factors of production without putting pressure on inflation. Therefore level of output above potential will often be seen as a source of inflationary pressures and a signal for the monetary authorities. The deviation of actual output from the potential, namely output gap, is a key variable for monetary policy issues in order to understand the historical development of the inflation and assess the extent of current inflationary pressures. Unfortunately, there is a lot of uncertainty related with the measurement issue, as potential output so the output gap cannot be observed directly. In general, methods to estimate these unobserved variables may be classified as univariate (non-structural), structural and multivariate non-structural (or mixed) approaches.

Univariate approaches perhaps are the most popular ones since they require less information due to their solely reliance on the actual data. Consequently, they are merely statistical approaches and suffer from the shortcoming that they disregard other information, such as, inflation, unemployment, and capacity utilization. On the other hand, structural approaches, like production function, use economic theory to estimate potential output. However, data requirements for Turkish economy impose tight constraints on the application of these kinds of approaches. The mixed approach combines the time series techniques with the economic theory and can be viewed as an extension to the univariate ones. These kinds of extensions have been applied by Laxton and Tetlow (1992) for Canada, Kuttner (1994) for the U.S., Gerlach and Smets (1999) for EMU-area and Benes and N'diaye (2001) for Czech economy. These authors utilized the relation between inflation and the real output (in some cases aggregate demand relationship is also included) in order to obtain a more meaningful measure of potential output.

This paper describes a basic univariate and a bivariate[1] unobserved components models to estimate potential output using information from observable aggregates and presents results for the Turkish economy. The most important motivation behind measuring output gap for the Turkish economy is the absence of a certain measure for the output gap that becomes important for both the near-term and long-term inflation forecasts especially within the "inflation-targeting framework" given that one of the key issues for the inflation-targeting framework is the estimation of potential output.

[1] It is called bivariate since output and inflation equations together form a bivariate unobserved components model.

The paper is organized as follows. The following section briefly discusses the fundamentals of the univariate model. The third section extends the univariate model by adding information about inflation and tries to describe output-inflation relation. Empirical results for Turkish economy are presented in section 4 and the final part gives some concluding remarks.

2. Basic Univariate Unobserved Components Model

The unobserved components model is a method to estimate the unobserved variables such as potential output, trend growth rate and output gap using the information from observed variables. Once the model is specified in the state space form and given the initial values for the unobserved state vector, the unobserved variables can be estimated by a recursive algorithm known as Kalman filter.

Kalman filter uses the initial values for the unobserved state vector in order to predict the unobserved variables and then updates the guesses based on the prediction errors. When all the observations have been processed, the smoothing equations give the best estimators of the unobserved variables based on all the information[2].

The simplest way of measuring potential output is the univariate methods, in which only the real output data is used. Early specification of the trend component of the output is a linear trend, which is based on a strong assumption that the supply side of the economy is deterministic and economic fluctuations usually depend on the changes in the demand-side. Later, Nelson and Plosser (1982) suggest that the nonstationarity in economic activity should be removed by first- differencing, which means the trend component is a random walk with drift rather than a straight line. It was followed up by Watson's (1986) specification, which characterized potential output and output gap as random walk with drift and AR(2) respectively. However in all these settings, economy's trend growth rate is assumed to be constant and there is no reason for these components to be constant over time, especially when an economy is experiencing considerable structural change. Hence Clark (1987) and Kuttner (1999) constructed a variable growth rate model given the decline of the U.S. productivity growth in the 1970s, reduction of labor force growth in the 1980s and the apparent increase in the trend growth in the mid-1990s.

[2] See Harvey (1990) for the technical details.

The general form of the system that encompasses a wide range of possibilities[3] can be written as follows:

$$x_t = x_t^* + z_t \tag{1}$$
$$x_t^* = x_{t-1}^* + \mu_t + \eta_t \tag{2}$$
$$\mu_t = (1-\rho)\mu_0 + \rho\mu_{t-1} + \varepsilon_t \tag{3}$$
$$\phi(L)z_t = \xi_t \tag{4}$$

where x_t is the log of seasonally adjusted real gdp, x_t^* is the potential output, μ_t is trend growth rate and z_t is the output gap. η_t, ε_t and ξ_t are independent normal white-noise processes with standard deviations, σ_η, σ_ε and σ_ξ respectively and $\phi(L)$ is the finite autoregressive polynomial with the lag operator L. Here, η is the shock to the level of potential output and related to capacity effects and investment, ε is the shocks to the trend's growth rate and related to technology, changing trends in factor inputs and finally, ξ is the gap shock.

In the system, potential output follows a random walk with drift and trend growth rate can be shaped with respect to different values of ρ. For example, potential growth rate can be assumed to follow a random walk ($\rho = 1$) in which the output is I(2) or a serially correlated potential growth ($0 \langle \rho \langle 1$) with nonzero σ_ε.

The dynamics of potential output and the output gap depend on the nature of the shocks, in other words relative importance of the supply and demand shocks. This relative importance, which determines the smoothness of the trend component, can be denoted by a parameter λ, which is the ratio of the variance of the cycle to the variance of trend fluctuations. A small λ implies that shocks to the economy are mainly supply shocks where potential output moves nearly with the data and hence a small output gap is expected. In the opposite case, choosing a higher value of λ, a larger weight on smoothness in the trend, means shocks to the economy are principally shocks to aggregate demand. All these point out the effective selection of this smoothness parameter. Unfortunately, methodology itself cannot provide this information. Therefore, it should be either selected a priori, in which the judgment of knowledgeable expert is essential or estimated along with the other parameters.

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[3] For example, as a special case, famous Hodrick-Prescott filter can be achieved by imposing the restrictions of no level shock ($\sigma_\eta = 0$), random walk trend growth rate with nonzero σ_ε ($\rho = 1$) and serially uncorrelated output gap ($z_t \sim WN(0, \sigma_\xi^2)$). Smoothing parameter of 1600 for quarterly data is equivalent to $\sigma_\varepsilon = 0.025\sigma_\xi$.

3. Bivariate Unobserved Components Model

In general, univariate methods lack important economic content. Therefore, in this section potential output and the output gap are determined conditional on the information in the system (1)-(4), together with the ability of the gap measure to explain inflation as in Kuttner (1994). The supply side of the economy is illustrated by a Phillips curve relationship. In order to specify this aggregate supply relationship, general to specific modeling approach is used and the final empirical model is as follows:

$$\pi_t = \alpha_1 \pi_{t-1} + \alpha_2 \pi_{t-2} + \beta \pi_t^{pub} + \gamma z_{t-1} + \psi \pi_t^m + v_t \qquad (5)$$

In equation (5), variable π_t is the measure of inflation and defined as the quarterly logarithmic difference of CPI. In the equation, the influence of excess demand is captured through the output gap whereas the other terms, the public manufacturing price inflation (π_t^{pub}) and the import price[4] inflation (π_t^m), are the exogenous factors effecting headline inflation. Finally, v_t represents shocks to the inflation. Coefficients on the right hand-side of the inflation equation sum up to one in order to satisfy the natural-rate hypothesis (NRH).

Note that import price effect is immediate (there is no delay) and output gap enters through the first lag. Two lags of quarterly inflation represent the effects of things like contractual lags or other cost adjustment that lead to stickiness in prices. In addition to that backward looking behavior dominates the price expectations of the agents due to highly inflationary environment. However, with the implementation of the IT regime, as the credibility of the policies increases, the portion of the agents who tend to form their expectations in a forward-looking manner is expected to increase since the inflation target itself is a nominal anchor for monetary policy and inflation expectations.

We believe that the change in the public manufacturing prices affects the CPI inflation by the following channels: increasing public manufacturing prices increases the cost of private sector in which the publicly produced inputs are intensively used and the past experience shows that this is perceived as a signal for an increase in CPI inflation through the expectations.

It is worth to note that there is no guarantee that this reduced-form equation specifies a true output-inflation trade-off. Given that the data come predominantly from previous regimes, where inflation was closely tied to exchange rate movements and inertia, it is to be expected that it will be hard to identify the effect of the output gap, and there is a risk that econometric

[4] It is adjusted for the changes in the exchange rate.

estimates will understate the value with the new IT regime. In addition, with the floating exchange rate regime, it is believed that effect of the output gap will increase, and the output gap will become a more pronounced variable. In fact, it may be worth to say that latest released data may be considered as an early signal.

It is believed that the joint estimation of the univariate and the aggregate supply relationship will provide more meaningful measure of output gap *to the extent that inflation is related to the level of the output gap*. Kuttner (1994) says that

> "Estimating the model amounts to choosing the unknown parameters to yield the z_t most consistent with observed inflation, subject to the smoothness restrictions implicitly stochastic trend specification for GDP. In this way, the bivariate potential output model adds an element of economic content that is absent from the univarite detrending methods"

In other words, by using mixed approach the strengths of the statistical models are integrated with the structural ones.

Equations (1)-(5) can be written in state-space form[5] and estimates of the parameters of the model and the unobserved state variables can be obtained by maximizing the likelihood function using the Kalman filter.

4. Empirical Results

Measurement of the potential output and output gap are shown to be sensitive to the model specification (consequently various assumptions related to initial state vector), estimation period and the method of estimation. Before going through the results, it is important to underscore that the short estimation period and the assumption of the constant economic structure in the estimation period limit the accuracy of our estimates.

In this paper, actual output is defined as the logarithmic seasonally adjusted gross domestic product at 1987 constant prices. Sample period covers between 1987:Q1 and 2002:Q4[6]. The issue of seasonality is handled with the commonly used program named TRAMO/SEATS (see Gomez and Maravall, 1998).

We believe that the sustainable steady-state real growth rate for our economy is 4.5 percent on annual basis[7], which corresponds to 1.125 percent per quarter. It is appropriate to assume the

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[5] See Appendix for the state-space representations.

[6] Note that forecasted values are added for the last three quarter in order to increase the end point precision and data description can be found in Appendix.

[7] In general, it is thought that the potential output corresponds to the ideal equilibrium position for all the output variables; this particular position corresponds to the so-called "steady state". Here, this steady-state value is set by judgment in the light of the historical values.

potential growth rate as a time varying process in order to capture the slow down and the increases in the trend growth rate. In most of the early specifications, the trend growth rate is assumed to follow a random walk, so the potential and the observed output are I(2). However, we consider that it is more proper to use a mean-reverting model where trend growth rate converges slowly to the steady- state rate of 4.5 percent. Therefore, as in the Czech model (Benes and N'diaye, 2001), parameter ρ is set to 0.9[8] and a serial correlation is assumed in the trend growth rate model. Furthermore the output gap is modeled as an AR(1) process[9].

By using the initial guess of the state vector and its covariance matrix and then applying the Kalman Filter, maximum likelihood estimates of the parameters in the models are reported in Table 1.

At a first glance, focusing on the bivariate model estimates, model is consistent with the natural-rate hypothesis in other words; it is super-neutral[10]. Coefficient of the lagged inflation terms sum to 0.57, which indicates high level of inflation persistence. Here, gap units are defined as the percentage of the potential output. Therefore, the estimated coefficient of output gap indicates that a one-percentage point increase of output over potential raises the inflation by 0.16 percent in the next quarter. Also bear in mind that output gap follows an AR(1) process and the estimated coefficient is 0.82. For this reason, the future inflation rates will also be influenced due to this persistence.

Table 1: Maximum Likelihood Estimates of Model (1)-(5) (Quarterly Data : 1987:Q1-2002:Q4)

	ϕ_1	σ_ξ	α_1	α_2	β	γ	ψ	σ_v
Univariate Model	0.867 (4.91)	0.027 (16.26)						
Bivariate Model	0.824 (5.81)	0.026 (15.75)	0.321 (7.09)	0.253 (4.67)	0.256 (4.66)	0.163 (2.19)	0.161 (3.26)	0.022 (9.08)

Note: t-statistics in parenthesis.

Next step is to analyze the gap estimates. Figure 1 points out that estimate of the output gap using only the output data is similar to one estimated by using the information regarding the inflation. It is important to remind that the aggregate supply relationship is able to provide more meaningful measures to the extent that inflation is related to the level of the output gap.

8 It means that, in the absence of shocks, output growth would converge to the within 1 percent of the steady-state rate in just about 10 years.

9 Initially, following Watson (1986), an AR(2) process for the output gap is tested. However, estimation results indicate that the second term is insignificant.

10 Neither the level of the money supply nor the rate of growth of the money supply influences the steady-state real equilibrium in other words there is no long-run trade-off between output and inflation or the level of prices.

Figure 1 displays that both univariate and bivariate estimates of the output gap reach the lowest level at 1994Q2 and indicate an increasing growth until 1998Q4. Moreover, actual output is below the potential since the first quarter of 2001 and according to bivariate specification, the current output (2002:Q1) is 5.8 percent below the potential where as the percentage is 6.2 for the univariate one.

The output gap is estimated in two different ways depending on what information is used. The filtered estimate at time t is one-sided and it uses information up to time t ($z_{t|t}$). According to Kuttner (1999), the one sided estimate corresponds approximately to a real time estimate[11]. On the other hand, a smoothed value is two sided and uses information from the whole sample, up to time T ($z_{t|T}$ where $0 \leq t \leq T$) i.e. it uses the future information to compute the current gap. Figure 2 shows one versus two-sided gap estimates according to the bivariate approach. It is worth to note that the filtered and smoothed values are identical for the last observation of the sample.

Because gap estimates are not precise, it is important to report the estimates with a measure of uncertainty. The unobserved components method has the advantage that it allows the construction of confidence bands for the potential output and output gap. The estimates of the error covariance matrix of the state variables can be obtained by Kalman recursion. Figure 3 and 4 illustrate the two-sided univariate and bivariate output gap estimates with ± 1.645 standard error bands, which corresponds to a 90 percent confidence. First of all, due to the larger standard error of univariate gap estimate, there is more uncertainty around univariate gap estimate as compared to bivariate one. In general, error bands indicate that 1993 and 1997-1998 are the expansions, whereas 1989, 1994 and 2001-2002 are the recessions[12] and also it is obvious from the figures that the bivariate gap estimates are more significant.

Another advantage of the unobserved component models is to provide a measure of the uncertainty related with the unknown parameters. Total variance of the state vector can be decomposed into filter variance (variance related with the signal extraction) and the parameter variance (parameter uncertainty)[13]. The parameter uncertainties are computed by Monte Carlo simulation with 300 draws and the results are presented in Table 2. In the table, the average

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[11] As noted by Kuttner (1999), "The correspondence is not exact; however, as the parameters are estimated over the entire sample and final, revised data are used".

[12] The significance of the gap estimates truly depends on the statistical criteria used; explicitly other assumptions may designate different dates.

[13] See Kuttner (1994) for further information.

size of the error assignable (imputable) to filter and parameter uncertainty appears for one and two sided estimates of the univariate and bivariate models.

Table 2: Signal Extraction Statistics

	Univariate Model	Bivariate Model
One Sided		
Filter variance	($1.54*10^{-3}$)	($8.2*10^{-4}$)
Parameter uncertainity	($1*10^{-3}$)	($3.8*10^{-4}$)
Total standard error	($4.9*10^{-2}$)	($3.4*10^{-2}$)
Two Sided		
Filter variance	($1.1*10^{-3}$)	($5.1*10^{-4}$)
Parameter uncertainity	($1.2*10^{-3}$)	($3.4*10^{-4}$)
Total standard error	($4.6*10^{-2}$)	($2.9*10^{-2}$)

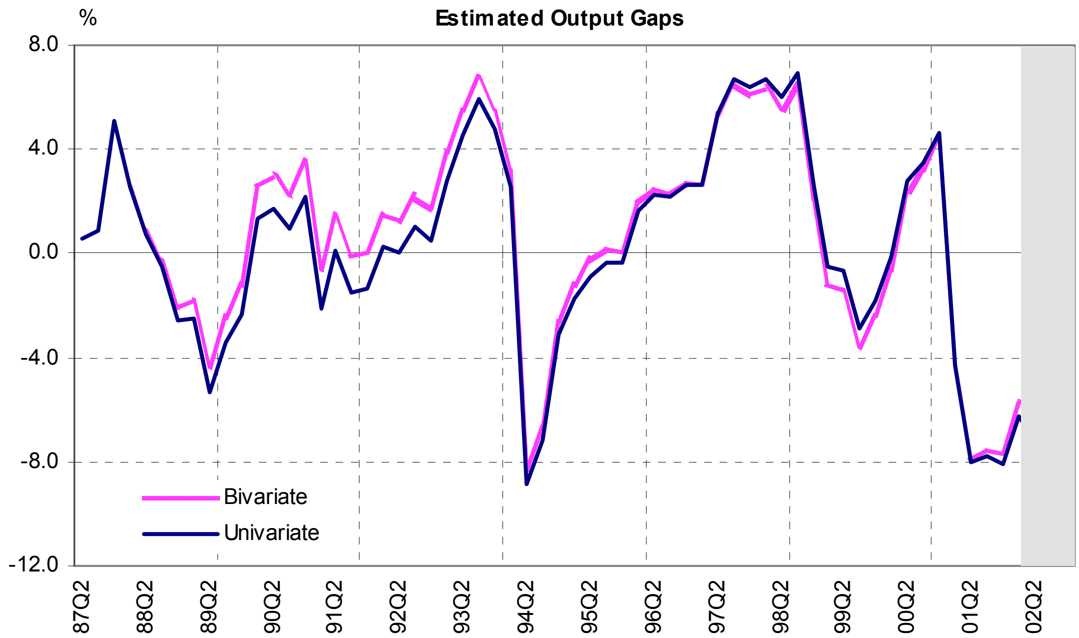
Note: The first eight quarters of the sample are excluded in these outcomes, so that the influence of the initial conditions is reduced.

For both of the univariate and bivariate models, the filter variance is larger than the parameter variance in the one sided estimate. While the filter variance in two sided estimate is 70 percent of what it was in the one sided case for univariate model, it is only 60 percent of the one sided case for the bivariate model.

The total standard error is reduced from 4.9 percent to 4.6 percent by moving to the two-sided estimate in univariate model. In bivariate model, the effect of moving to the two-sided estimate is more pronounced; the total standard error reduces from 3.4 percent to 2.9 percent. Results show the notable evidence of using information from the whole sample. This finding underlines the importance of the careful analysis of the end-point estimate since the filtered and smoothed values are the same for the last observation of the sample.

On the other hand, the comparison of univariate and bivariate models yields 1.7 percentage points less total standard error in the favor of the bivariate one. In addition, it is worth to note that among the four alternative models the smallest total standard error is in two-sided estimate of the bivariate model.

Figure 1:



Note: Shaded area refers to the forecasted values from this point onward. Both estimates are two-sided.

Figure 2:

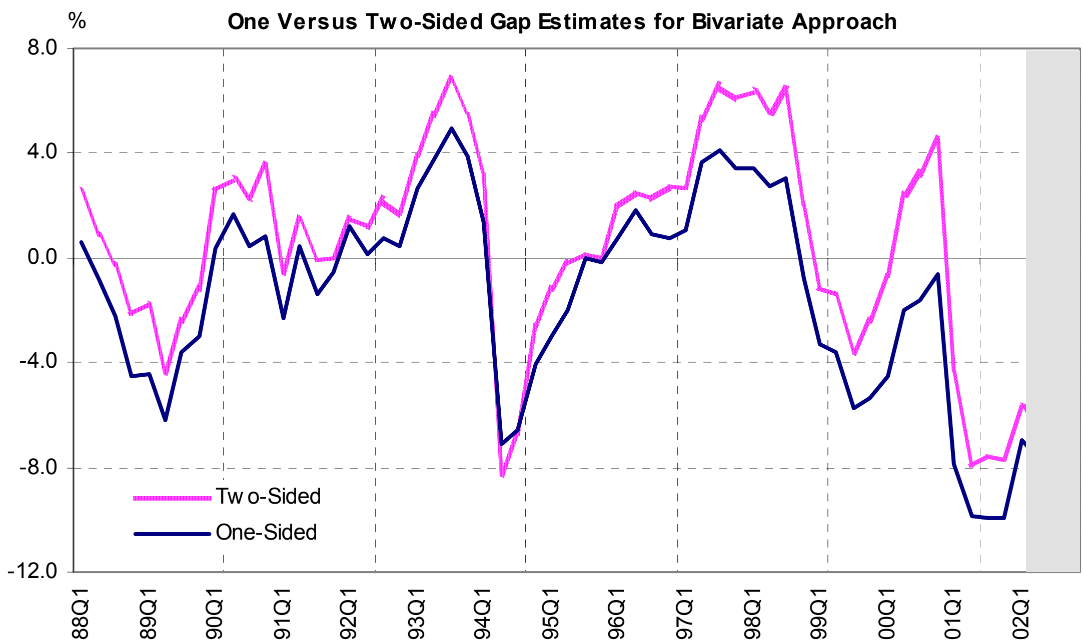


Figure 3: Basic Univariate Model Results (using only the output data)

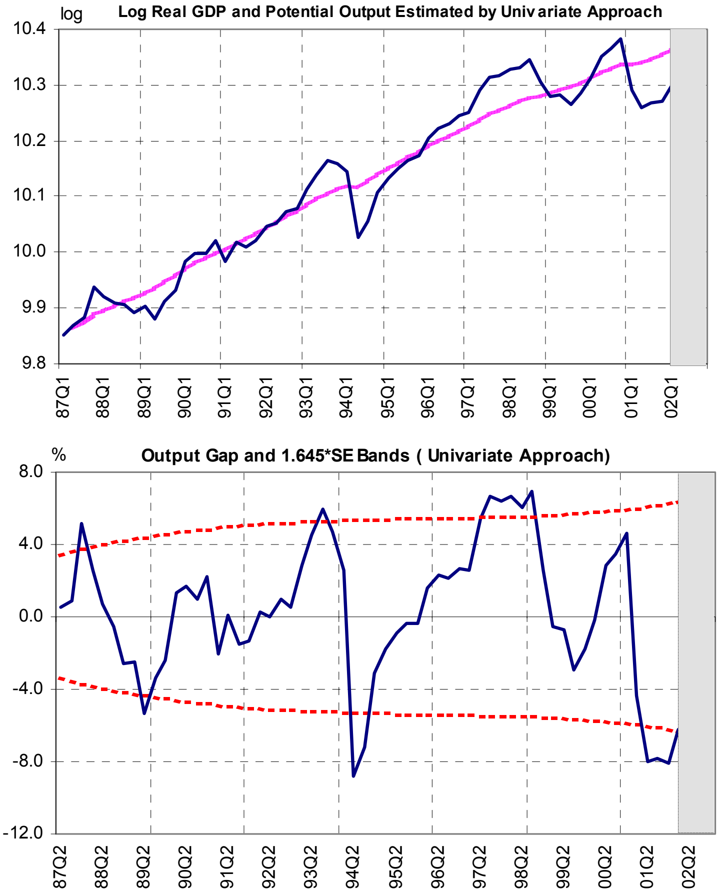
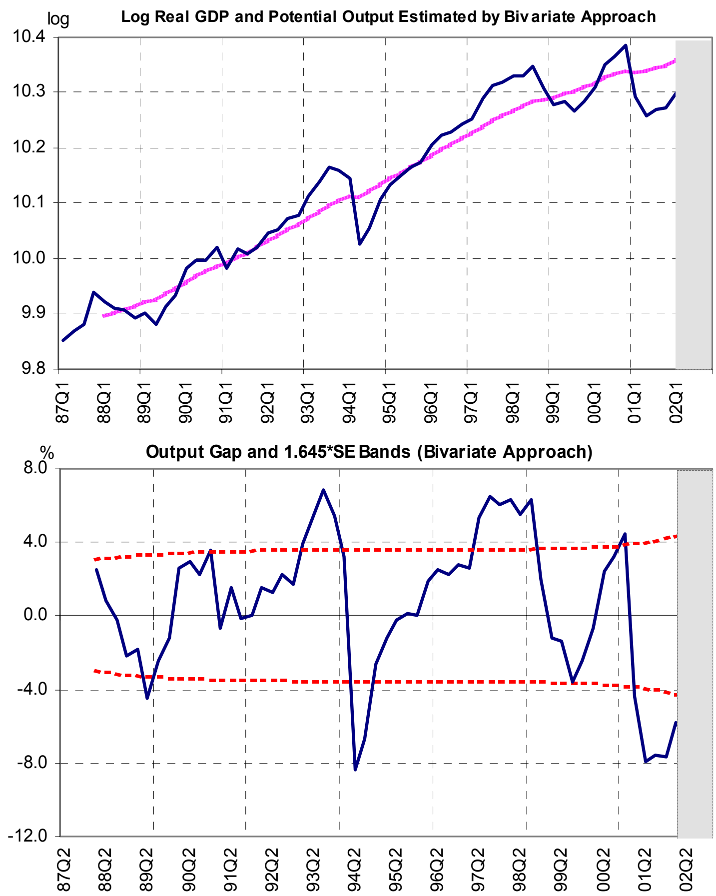


Figure 4: Bivariate Model Results (adding information concerning inflation)



Note: All estimates are two-sided estimates.

5. Conclusion

The absence of a certain measure for the output gap for the Turkish economy is the main motivation of this study. Intensive usage of the output gap in the assessment of the economic activity and also its noteworthy role in the near and the long-term inflation forecasts within the inflation-targeting monetary regime are the main reasons for the development of the numerous methods containing a variety of judgment and complexity levels. Among these methods, unobserved component model is preferred to estimate the potential output or, equivalently the output gap due to its distinctive advantages.

In the basic univariate setup, we let the data speak by just using the time series properties of the actual output data. On the other hand, it is known that building models that incorporate more economic content can increase the reliability of the estimated gaps. For this reason, in the second round, the output gap is estimated conditional on the information in the system (1)-(4), together with the ability of the gap measure to explain inflation. Although the signal extraction statistics suggest that incorporating the supply side to the system reduces the parameter uncertainty and the total standard error and improves the gap estimate, the figure of univarite and bivariate estimates does not show a clear distinction. According to our view, the reason for that is the limited degree of relation between the inflation and the level of the output gap due to the dominance of the previous regimes where inflation is closely tied to exchange rate movements and inertia. However we think that the extent of the relation is subject to change with the floating exchange rate and the new IT regime.

It is important to bear in mind that one of the reasons of the estimation of the output gap is to provide information about excess capacity in the economy at a particular point in time. However, it is not an easy job to identify the absolute size of the output gap. Assumptions related with the shocks, steady-state growth rate, and the model specification used can place an important role in the decomposition and so in the determination of this absolute size given that diverse assumptions may bring about a range of possible solutions that may differ in their policy implications. All these issues emphasize the importance of a careful analysis and expert judgment.

This study can be considered as a preliminary step to measure output gap for the Turkish economy. Although both models give similar output gap estimates, signal extraction statistics suggest promising results. Along with this, latest economic developments may suggest that further extensions especially regarding the demand side may deserve attention.

Appendix:

A.1. Data Description

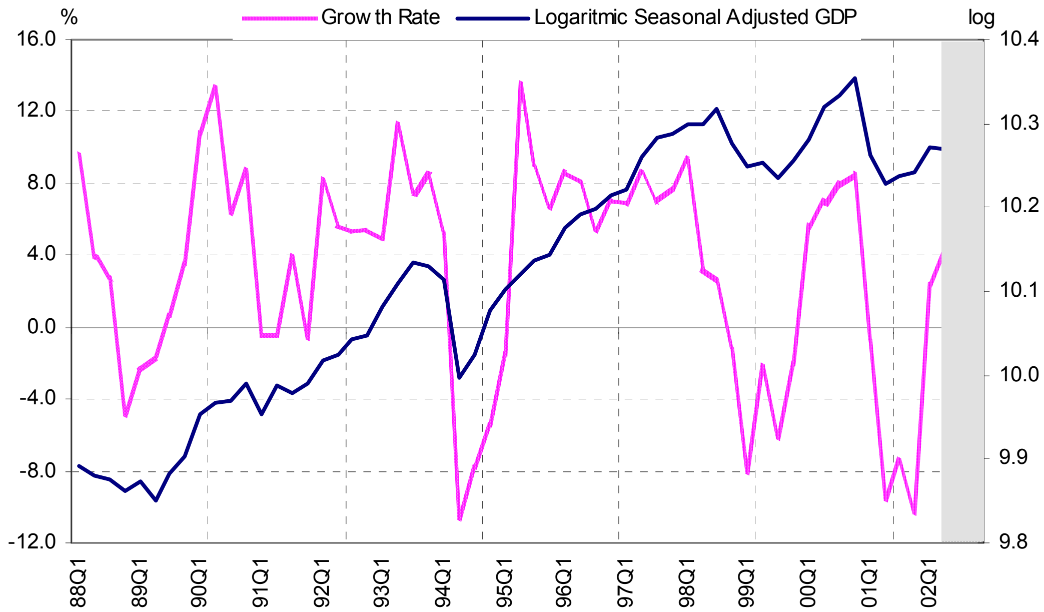
x_t : Logarithmic seasonally adjusted gross domestic product at 1987 constant prices

π_t : Quarterly log difference of consumer price index, (1994=100)

π_t^{pub} : Quarterly log difference of public manufacturing price index, (1994=100)

π_t^{m} : Quarterly log difference of import price index, adjusted for exchange rate, (1994=100)

Figure 1: GDP and Growth Rate



A.2. State Space Representations

The vector of observed variables (output and inflation) is denoted as **x**, while the vector of unobserved state variables (potential output, trend growth rate and the output gap) is denoted by **z**. Then the measurement equation where the evolution of the observed variables is described as a function of the unobserved state variables and transition equation are:

$$\begin{aligned} x_t &= Cz_t + Du_t + Hw_t \\ z_t &= Az_{t-1} + Be_t + Gw_t \end{aligned} \qquad \textbf{A.2.1}$$

where the **u** and **e** denotes vectors of normally distributed iid shocks which are assumed to be uncorrelated and **w** is the vector of exogenous variables. Equations (1)-(5) can be written as:

$$[x_t] = [1 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0]\begin{bmatrix} x_t^* \\ x_{t-1}^* \\ \mu_t \\ \mu_{t-1} \\ z_t \\ z_{t-1} \\ z_{t-2} \end{bmatrix} \qquad \textbf{A.2.2}$$

$$\begin{bmatrix} x_t^* \\ x_{t-1}^* \\ \mu_t \\ \mu_{t-1} \\ z_t \\ z_{t-1} \\ z_{t-2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_1 & \phi_2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x_{t-1}^* \\ x_{t-2}^* \\ \mu_{t-1} \\ \mu_{t-2} \\ z_{t-1} \\ z_{t-2} \\ z_{t-3} \end{bmatrix} + \begin{bmatrix} \sigma_\eta & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sigma_\varepsilon & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_\xi \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ (1-\rho)\mu_0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}[1] \qquad \textbf{A.2.3}$$

and when inflation-output relationship is included in the model, representation is:

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \gamma & 0 \end{bmatrix}\begin{bmatrix} x_t^* \\ x_{t-1}^* \\ \mu_t \\ \mu_{t-1} \\ z_t \\ z_{t-1} \\ z_{t-2} \end{bmatrix} + \begin{bmatrix} 0 \\ \sigma_v \end{bmatrix}[1] + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_1 & \alpha_2 & \beta & \psi \end{bmatrix}\begin{bmatrix} 1 \\ \pi_{t-1} \\ \pi_{t-2} \\ \pi_t^{pub} \\ \pi_t^m \end{bmatrix} \qquad \textbf{A.2.4}$$

$$\begin{bmatrix} x_t^* \\ x_{t-1}^* \\ \mu_t \\ \mu_{t-1} \\ z_t \\ z_{t-1} \\ z_{t-2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \phi_1 & \phi_2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} x_{t-1}^* \\ x_{t-2}^* \\ \mu_{t-1} \\ \mu_{t-2} \\ z_{t-1} \\ z_{t-2} \\ z_{t-3} \end{bmatrix} + \begin{bmatrix} \sigma_\eta & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \sigma_\varepsilon & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_\xi \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ (1-\rho)\mu_0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ \pi_{t-1} \\ \pi_{t-2} \\ \pi_t^{pub} \\ \pi_t^m \end{bmatrix} \qquad \textbf{A.2.5}$$

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