Endogenous Life Expectancy in a Simple Model of Growth

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Abstract

In an overlapping generations economy reproductive agents mature safely through two periods of life and face an endogenous probability of surviving for a third period. Given this probability, which depends on aggregate outcomes, each agent maximises her expected lifetime utility by choosing consumption and savings. The dynamic general equilibrium of the economy is characterised by multiple development regimes associated with different levels of economic activity and different rates of life expectancy. Transition between these regimes may or may not occur depending on parameter values and initial conditions.

1 Introduction

Over the past century, most (if not all) countries of the world have experienced dramatic increases in the longevity of their citizens. In the US, for example, life expectancy at birth has risen by almost 50 percent since 1920: in that era the average lifetime of a person was only 54 years; by 1950 this figure had increased to 68 years, by 1965 to 70 years, and by 1980 to 74 years. Nowadays, the average US citizen can expect to live upto 80 years of age, which is twice as long as the average US citizen born in 1850 (e.g., Fogel 1994; Lichtenberg 1998). There are numerous other examples of this striking trend, both for developed and developing economies (e.g., Fogel 1997; Livi-Bacci 1997; Preston 1980; Pritchett and Summers 1996; United Nations 1991; World Bank 1993). Thus it has been reported that life expectancy among developing countries increased by as much as 50 percent during the period

1950-1990, while average lifetimes in nearly all countries were extended by 9 years or more between 1960 and 1990. Other evidence reveals a remarkable decline in the total number of countries (with more than 1,000,000 inhabitants) where life expectancy is less than 50 years: this number was 70 in 1960, 43 in 1975 and now stands at around 18.[1]

In spite of the above, there exists relatively little theoretical work on the causes and consequences of increasing longevity. This is in contrast to the extensive formal treatment of fertility choice within the context of fully-specified dynamic general equilibrium models of demographic transition (e.g., Barro and Becker 1989; Becker *et al.* 1990; Pavilos 1995; Raut and Srinivasan 1994).[2] Yet changes in life expectancy are just as much a part of this transition as changes in child-bearing, and may be just as important in determining economic outcomes and policy prescriptions, as well as other trends in demographic behaviour. This is already exemplified by the few analyses that do model longevity explicitly (e.g., de la Croix and Licandro 1999; Ehrlich and Lui 1991; Hu 1995; Kalemli-Ozcan *et al.* 2000; Zhang and Zhang 2001; Zhang *et al.* 2001). Together, these analyses demonstrate how an increase in life expectancy can increase savings, increase investment in education and decrease fertility, leading to an overall improvement in the growth rate of output.[3] The implied positive correlation between life expectancy and economic development is another stylised fact revealed by the data (e.g., Barro and Sala-i-Martin 1995; Bloom *et al.* 2001; Knowles and Owen 1995, 1997; Preston 1978; Pritchett and Summers 1996). For example, cross-section evidence suggests that, in 1996, average life expectancy in the poorest countries was 50 years of age, while average life expectancy in the richest countries was 76 years of age.[4]

[1] Different ways of looking at the data give the same impression (e.g., Cutler and Sheiner 1998). For example, it has been estimated that a US citizen born in 1960 had a 71 percent chance of surviving to age 65, while the same person born in 1990 had a 90 percent chance of reaching that age. Similarly, it has also been estimated that half of the US population aged 85 or over in 1990 would not have been alive if mortality rates had been the same as those in 1960.

[2] For authoritative surveys of the demographic transition literature, see Ehrlich and Lui (1997) and Kirk (1996). Both of these indicate an urgent need for integrating mortality into the theory of demographic transition. Since the surveys were written, this need has remained largely unfulfilled.

[3] The main policy issue addressed in this literature is the sustainability of unfunded social security programmes in the presence of an ageing population (e.g., Hu 1995; Pecchino and Utendorf 1999; Weil 1997; Zhang *et al.* 2001).

[4] A similarly strong positive correlation between income and life expectancy is revealed in time series studies, as well as investigations conducted at the micro level. As regards the latter, it has been estimated that, in 1980, a US citizen aged 25 years with a family income of $5,000 could have expected to live 10 years less than another citizen of the same

While yielding important insights, all of the above analyses are incomplete in one major respect: they do not allow for the endogeneity of life expectancy, itself. On the contrary, life expectancy is treated as being purely exogenous and independent of any events that may occur in the economy, whether at the individual or aggregate levels, and whether by accident or design. Clearly, this is not true and there are many ways in which life expectancy may change with changes in individual circumstances, government policies and various other aspects of the socio-economic environment. The presumption, of course, is that economic development is conducive to longer lifetimes as technological progress, increased education and rising per capita incomes manifest themselves in the forms of higher levels of nutrition, better standards of sanitation, greater provision of health care, improved awareness of health risks, advances in medical knowledge and so on and so forth. Consequently, the relationship between longevity and development is to be seen as being fundamentally two-way causal with effects running in both directions. These effects can be significant: according to some estimates, as much as half of the increase in life expectancy among developing countries can be attributed to income gains, while each extra year of life expectancy would raise annual output by as much as 4 percent in some countries (e.g., Bloom *et al.* 2001; Preston 1980).

Greater life expectancy is likely to be associated with all-round improvements in health and it is often claimed that the historical gains in life expectancy have been due mainly to increases in income and advances in health technology (e.g., Easterlin 1996; Fogel 1994). There is also a good deal of evidence that testifies to a strong positive correlation between income and various measures of health, with the poor having a significantly worse health status than the rich (e.g., Bidani and Ravallion 1997; Gupta *et al.* 2001). As above, it is to be presumed that causality runs in both directions such that improvements in health and improvements in living standards are mutually dependent events.[5]

The purpose of the present paper is to provide a simple illustration of how an economy might evolve when the longevity of its citizens both influences and is influenced by the process of economic development. We do this by endogenising life expectancy in the basic overlapping generations model of

age with a family income of $50,000 (e.g., Deaton and Paxson 1999).

[5]Improvements in health can have important economic effects beyond those engendered by greater life expectancy. This follows from the fact that health, like education, is a form of human capital and so is likely to be related to labour market outcomes. Thus lower morbidity and better functionality can raise the productivity and wages of individuals. This is another area in which there is relatively little theoretical research. For a review of the existing (mainly empirical) literature, see Strauss and Thomas (1998).

capital accumulation and growth. In general, changes in life expectancy reflect changes in mortality at different stages in the life-cycle, especially early childhood and later adulthood. As in other analyses, we focus on the latter, partly because of the need to exercise some discretion, and partly because of the fact that most gains in longevity now tend to occur through improvements in survival at older ages, rather than reductions in deaths during infancy (e.g., Kannisto *et al.* 1994; Lee and Tuljarpurkar 1997).[6]

Agents in our model live for two periods with certainty and face a probability of surviving for a third period. An exogenous increase in this probability leads agents to increase their savings during middle-age (when they work) in order to finance consumption during old-age (when they are retired). The increase in savings is converted into an increase in capital accumulation and growth. Thus the model predicts a positive correlation between longevity, savings and investment, as reported in many empirical studies (e.g., Barro and Sala-i-Martin 1995; Doshi 1994). We endogenise life expectancy by allowing the survival probability to depend on the level of development of the economy, itself. As well as motivating this in general terms, we provide a specific justification for it based on the provision of public health care. Our analysis demonstrates how endogenising longevity can radically alter the implications of even the simplest of growth models. As development now takes place, there is an increase in life expectancy which feeds back onto the growth process. This produces mutliple development regimes such that limiting outcomes depend critically on parameter values and initial conditions. Under some circumstances, the economy evolves smoothly from a low development regime, in which life expectancy is also low, to a high development regime, in which life expectancy is also high. Under other circumstances, there is no such transition and the economy is destined to remain in the regime where it started.

We are aware of only two other analyses that attend explicitly to the joint determination of longevity and growth. In Blackburn and Cipriani (1998)

[6]We do not mean to trivialise the considerable reductions in infant and child mortality that have played such a vital role in increasing life expectancy. At least in industrialised countries, however, mortality rates of the young-age population are now very low (infant mortality is less than 1 percent) and any further reductions are likely to be small by historical standards. The general trend over recent decades has been a deceleration in the rate of mortality decline at young ages, but an acceleration in the rate of mortality decline at adult ages. According to some estimates, a US citizen who reached age 65 in 1960 (1990) faced a 26 (38) percent chance of surviving to age 85, while the share of the US population over age 65 (85) will nearly double (triple) by the year 2050 (e.g., Cutler and Sheiner 1998). In Indonesia - a country with one of the largest populations over age 65 - the number of elderly expected to be alive in 2025 is 4 times greater than the number in 1990 (e.g., Adlakha and Rudolph 1994).

a dynastic overlapping generations model is used to study the interactions between growth, fertility and infant mortality. In Blackburn and Cipriani (2002) an analysis of human capital accumulation, child-bearing and adult mortality is conducted within a framework of probalistic survival. From a broader perspective, the present paper may be seen as continuing the tradition of the economic demography literature in the progressive treatment of key demographic variables - in our case, longevity - as being endogenous, rather than exogenous, to the process of economic development. Naturally, our analysis is not meant to provide a complete account of this process, but rather is intended to draw attention to the role of life expectancy and to illustrate this formally in a simple and intuitive way within the context of a standard benchmark model. The paper may also be viewed as a contribution to the wider literatures on poverty traps, threshold externalities and the demographic transition of economies over the very long-run (e.g., Galor and Weil 1999, 2000; Kremer 1993; Jones 1999; Tamura 1999).

The remainder of the paper is organised as follows. In Section 2 we present a description of the model. In Section 3 we solve the model and analyse its implications. Concluding remarks are contained in Section 4.

2 The Model

Time is discrete and indexed by $t = 0, ..., \infty$. The economy is populated by reproductive agents who have finite but uncertain lifetimes, and who belong to overlapping generations connected by altruism. Each agent has one parent and one child, and each generation is a measure of unit mass to begin with.[7] Agents mature safely through two periods of life and face a probability of surviving for a third period. A young agent is economically inactive, being simply raised and cared for by her parent. A middle-aged agent is a child-bearer and a worker, earning income which is allocated between consumption, savings and bequests. An old agent is retired, consuming all the proceeds from her savings. Production of output is undertaken by firms, of which there is also a continuum of unit mass. Each firm manufactures a single commodity using labour and capital supplied by agents. All markets are perfectly competitive.

[7] We abstract from fertility choice for simplicity, referring the reader to other analyses that deal with this issue (e.g. Blackburn and Cipriani 2002; Ehrlich and Lui 1991; Zhang and Zhang 2001; Zhang *et al.* 2001). Allowing for endogenous fertility would tend to strengthen our results since an increase in life expectancy would then increase savings by causing both an increase in the return on savings and a decrease in the demand for children. As usual, we also abstract from complications of marriage by assuming that an agent is able to bear children on her own.

2.1 Agents

The expected lifetime utility of an agent of generation $t-1$ is given by

$$U^{t-1} = \frac{[c_t^{t-1} + v(b_t)]^{1-\sigma} - 1}{1-\sigma} + \theta\pi_t \frac{(c_{t+1}^{t-1})^{1-\sigma} - 1}{1-\sigma}, \quad \sigma > 0, \ \theta \in (0,1] \tag{1}$$

where c_t^{t-1} denotes consumption in middle-age, c_{t+1}^{t-1} denotes consumption in old-age, b_t denotes bequests to offspring and π_t is the probability of surviving to old-age. We model altruism according to the simple 'warm-glow', or 'joy-of-giving', motive for making bequests (e.g., Andreoni 1989), as reflected in the function $v(\cdot)$ which is assumed to be strictly concave and to satisfy the usual Inada conditions. Bequests are made by parents during middle-age, being invested in the capital market and becoming available to children when they, themselves, reach maturity. Our particular specification of midde-age felicity implies that the marginal rate of substitution between consumption and bequests is independent of the level of consumption. As we shall see, this leads to the convenient result that bequests are constant across generations.[8] The parameter σ is important for determining how middle-age consumption and savings respond to changes in the rate of return on savings. Under the presumption that substitution effects dominate income effects, it is normal to impose restrictions on the value of this parameter, though this is not essential for our analysis. The possibility of dying before reaching old-age is reflected in the fact that the discount factor applied to old-age consumption is $\theta\pi_t$ which depends on the probability of surviving into retirement. This probability, which is also the fraction of agents in each generation who survive to old-age, is assumed to be the same for all agents of the same generation and is discussed further below.

A middle-aged agent earns income by supplying one unit of labour to firms in return for a wage, w_t. In addition, the agent is entitled to her inheritence which is equal to the wealth bequeathed by her parent when she was young, plus the interest earned on this bequest: that is, $(1+r_t)b_{t-1}$, where r_t is the rate of interest. Given these resources, the agent consumes, saves and makes bequests to her own offspring. Denoting savings by s_t, the budget constraint for a middle-aged agent is

$$c_t^{t-1} + s_t + b_t = w_t + (1+r_t)b_{t-1}. \tag{2}$$

[8] We use the bequest motive solely as a simple device for allowing non-degeneracy in a low development equilibrium. Nevertheless, our results are equally valid for the case in which this equilibrium is reached at zero (rather than positive) levels of production and consumption.

If an agent survives to old-age, then she no longer works but finances her consumption entirely from savings. As in other models of uncertain lifetimes, we need to deal with the subtle issue of how to treat the retirement savings that are left by those agents who do not survive to old-age. As far as the present analysis is concerned, it makes no essential difference as to whether one assumes that these savings are merely wasted (e.g., Ehrlich and Lui 1991), or that they are distributed among the surviving population of savers through actuarially fair annuity markets (e.g., Hu 1995; Zhang and Zhang 2001; Zhang *et al.* 2001). For no particular reason, we follow the latter approach which implies that an old agent's return on her savings is equal to the market interest rate divided by the average survival rate of the population: that is, $\frac{1+r_{t+1}}{\pi_t}$.[9] Accordingly, the budget constraint of an old agent is

$$c_{t+1}^{t-1} = \left(\frac{1 + r_{t+1}}{\pi_t} \right) s_t. \tag{3}$$

The linchpin of our analysis is the endogenous determination of the survival probability, π_t. It is this feature that accounts for our main results and which distinguishes our analysis from most of the existing literature. In general, one may think of life expectancy as being determined by factors that are both internal and external to an individual's decisions. Examples of the former are personal expenditures on food, hygiene, exercise and medical care, while examples of the latter include parental influence and family background, environmental conditions and social infrastructure, and public expenditures on education and health. To many observers, most changes in life expectancy are due to changes in the external inputs to individual health, and the positive correlation between longevity and income is a reflection of the fact that income acts more as a proxy for these inputs, rather than as a key variable, itself, in determining survival. For example, there is considerable evidence that the education and health levels of parents, correlated positively with family income, have a significant influence on the life expectancy of offspring (e.g., Bishai 1996; Mirowski and Ross 1998; Sandiford *et al.* 1995).[10] A similarly large body of evidence suggests that it is

[9] If retirement savings were not annuitised, or if survival to old-age was certain, then the return on savings would be $1 + r_{t+1}$. A third approach to the issue (one that we do not consider) is to view the savings of the deceased as being left to the next generation in the form of unintended bequests from parents to children (e.g., Abel 1985).

[10] Specific instances of this are revealed by numerous case studies which indicate that better educated parents tend to have children who are less likely to take up smoking, less likely to become overweight, less likely to be sexually promsicuous and so on and so forth (e.g., Cooksey *et al.* 1996; Greenlund *et al.* 1996; Kandel and Wu 1995).

public (rather than private) spending on health care, correlated positively with aggregate income, which is the major determinant of health status and longevity among all members of society, whether rich or poor (e.g., Anand and Ravallion 1993; Gupta *et al.* 2001).[11] Historically, improvements in life expectancy can be allied to fundamental changes in the socio-economic environment, such as the establishment of public order, the introduction of revolutionary medicines and the development of an infratsructure in transport and commerce, which reduced fatalities from violence, famines, malnutrition, epidemics and contagious diseases (e.g., Lichtenberg 1998; McKeown *et al.* 1972; Schofield *et al.* 1992). We return to some of these issues later on in the paper. For the moment, we note that, if life expectancy is determined primarily by factors that are external to individuals, then it will be rational for an individual to treat her probability of survival as essentially given and beyond her own control. This is the approach that we follow in our analysis and, for this reason, we find it convenient to postpone further discussion of π_t until a more appropriate juncture and to turn our immediate attention to the choices that do confront agents in our model.

The decision problem for our representative agent of generation $t-1$ is to choose $c_t^{t-1} \geq 0$, $c_{t+1}^{t-1} \geq 0$, $s_t \geq 0$ and $b_t \geq 0$ so as to maximise (1) subject to (2) and (3). The first-order conditions for this problem yield interior solutions for all variables and may be summarised as

$$v'(b_t) = 1, \tag{4}$$

$$\frac{1}{[c_t^{t-1} + v(b_t)]^\sigma} = \frac{\theta(1 + r_{t+1})}{(c_{t+1}^{t-1})^\sigma}. \tag{5}$$

The condition in (4) implies that $b_t = b$ for all t, which confirms our earlier assertion that the optimal size of bequest is the same for every agent of every generation. The condition in (5) equates the current marginal loss with the future marginal gain of an additional unit of savings. Together with (2) and (3), these conditions may be used to establish the following optimal decision rule for savings:

$$s_t = \frac{R_{t+1}\pi_t}{1 + R_{t+1}\pi_t}[B_t + w_t], \tag{6}$$

where $R_{t+1} = [\theta(1 + r_{t+1})^{1-\sigma}]^{\frac{1}{\sigma}}$ and $B_t = r_t b + v(b)$. As indicated earlier, the effect on savings of a change in the (future) rate of interest is generally am-

[11] In relative terms, it is the poor who appear to benefit the most. For example, some estimates suggests that, for the same percentage increase in public health spending, twice as many deaths are prevented among the poor than the non-poor (e.g., Bidani and Ravallion 1997; Gupta *et al.* 2001).

biguous and depends, in part, on the value of σ.[12] By contrast, an increase in the wage has an unambiguously positive effect on savings, as does an increase in the probability of survival. The latter result is of particular interest to us and is explained by the fact that the more that an agent expects to survive to old-age, the more that she will save during middle-age in order to finance retirement consumption.

2.2 Firms

The representative firm combines l_t units of labour with k_t units of capital to produce y_t units of output according to

$$y_t = Ak_t^{\alpha}(K_t l_t)^{1-\alpha}, \quad A > 0,\ \alpha \in (0,1) \tag{7}$$

where K_t denotes the aggregate stock of capital. As in many types of endogenous growth model, we use this variable to capture externality effects in the production of output by treating it as a proxy for the stock of disembodied technological knowledge which each firm acquires freely through serendipitous learning-by-doing and which each firm takes rationally as given and beyond its own control.(e.g., Romer 1987).

A firm hires labour and capital from agents at the given wage rate w_t and the given rental rate r_t, respectively. Profit maximisation implies

$$w_t = (1-\alpha)Ak_t^{\alpha}K_t^{1-\alpha}l_t^{-\alpha}, \tag{8}$$

$$r_t = \alpha Ak_t^{\alpha-1}K_t^{1-\alpha}l_t^{1-\alpha}. \tag{9}$$

3 General Equilibrium

The solution of the model is a dynamic, competitive general equilibrium which describes aggregate economic activity based on the optimal decision rules that solve agents' and firms' maximisation problems. The equilibrium is computed by combining the relationships obtained so far with the relevant market clearing conditions in the economy.

Clearing of the labour market implies $l_t = 1$. Given this, together with the fact that $k_t = K_t$ in equilibrium, the conditions in (8) and (9) may be reduced to $w_t = (1-\alpha)Ak_t$ and $r_t = \alpha A$. Clearing of the capital market requires $k_{t+1} = s_t + b_t$, where s_t is given in (6) and $b_t = b$ in accordance with

[12] In the absence of any non-labour income (i.e., bequests), we have the usual implications that an increase in r_{t+1} has a positive, negative or zero effect on s_t according to whether $\sigma < 1$, $\sigma > 1$ or $\sigma = 1$. As we have mentioned already, and for reasons that will become clear shortly, our main results are the same in all of these cases.

(4). Together with the foregoing results, these expressions may be used to establish the following dynamic equation for capital:

$$k_{t+1} = \frac{R\pi_t}{1+R\pi_t}[B + (1-\alpha)Ak_t] + b, \tag{10}$$

This equation shows that, *ceteris paribus*, an increase in life expectancy has a positive effect on capital accumulation. The reason for this follows from our previous result that a higher probability of survival leads to a higher level of savings. Precisely how this probability, itself, is determined is a matter to which we now turn with the view to providing a complete characterisation of the equilibrium path of development of the economy.

3.1 Exogenous Life Expectancy

If the probability of survival is constant, $\pi_t = \pi$ for all t, then (10) describes a simple linear transition path along which the economy converges towards a unique steady state of either zero or positive long-run growth, depending on whether $\frac{R\pi(1-\alpha)A}{1+R\pi} \in (0,1)$ or $\frac{R\pi(1-\alpha)A}{1+R\pi} > 1$. In the case of the former, an increase in π raises the steady state level of capital, as given by $k^* = \frac{R\pi B+(1+R\pi)b}{1+R\pi-R\pi(1-\alpha)A}$. In the case of the latter, an increase in π raises the steady state growth rate of capital, as determined by $\lim_{t\to\infty}\left(\frac{k_{t+1}}{k_t}\right) = \frac{R\pi(1-\alpha)A}{1+R\pi}$. Either way, the model predicts that exogenous improvements in life expectancy lead to improvements in the prospective fortunes of the economy. This is consistent with the results obtained in other models and accords with the empirical observation of a positive correlation between life expectancy and economic development.

3.2 Endogenous Life Expectancy

As argued earlier, it is more natural to think of life expectancy as being endogenous, rather than exogenous, to the process of development. Both conceptually and empirically, there are good reasons for supposing that changes in longevity are not only a cause, but also a consequence, of changes in prosperity. Allowing for such a possibility is the main innovation of our analysis. As we shall see, the implications of the model can change dramatically with the introduction of this additional, new dimension.

3.2.1 A General Characterisation

For reasons given earlier, we treat life expectancy as being determined primarily by factors that are largely external to individuals and which correlate positively with the level of development. A specific example of this is provided later when we consider public policy. For now, we choose not to be so precise, but rather seek to establish some basic principles and key implications from a broader, more inclusive perspective. To this end, we take the most immediate approach towards endogenising life expectancy by making the fairly general assumption that the probability of survival is an increasing, but bounded, function of the (aggregate) stock of capital: that is, $\pi_t = \pi(k_t)$, where $\pi'(\cdot) > 0$, $\pi(0) = \underline{\pi} > 0$ and $\lim_{k\to\infty} \pi(\cdot) = \overline{\pi} \leq 1$. Essentially, $\pi(\cdot)$ may be thought of as a reduced form of some other underlying relationships through which life expectancy is linked to economic activity.[13] Given this, then (10) is now understood to define a transition function, $F(\cdot)$, such that

$$k_{t+1} = F(k_t) \equiv \frac{R\pi(k_t)}{1 + R\pi(k_t)}[B + (1-\alpha)Ak_t] + b, \tag{11}$$

where $F'(\cdot) > 0$ and $F''(\cdot) \gtreqless 0$. A steady state equilibrium with zero growth corresponds to a fixed point of this mapping, $k^* = F(k^*)$. Such a point is stable if $\lim_{k\to k^*-} F(\cdot) > k^*$ and $\lim_{k\to k^*+} F(\cdot) < k^*$, but unstable if $\lim_{k\to k^*-} F(\cdot) < k^*$ and $\lim_{k\to k^*+} F(\cdot) > k^*$. In the event of the latter, there is the possibility of a non-stationary long-run equilibrium in which growth occurs at a positive, constant rate.

The key implication of endogenising life expectancy is the existence of multiple development regimes which may lead to multiple steady state equilibria such that the limiting outcomes of the economy are non-ergodic but depend crucially on initial conditions. The clearest illustration of this is provided by the case in which $\pi(\cdot)$ takes the form of a simple step function, such as

$$\pi(k_t) = \begin{cases} \underline{\pi} & \text{if } k_t < k^c, \\ \overline{\pi} & \text{if } k_t \geq k^c, \end{cases} \tag{12}$$

for some critical level of capital, $k^c > 0$. The transition function may then be written as

$$F(k_t) = \begin{cases} \underline{f}(k_t) \equiv \frac{R\underline{\pi}}{1+R\underline{\pi}}[B + (1-\alpha)Ak_t] + b & \text{if } k_t < k^c, \\ \overline{f}(k_t) \equiv \frac{R\overline{\pi}}{1+R\overline{\pi}}[B + (1-\alpha)Ak_t] + b & \text{if } k_t \geq k^c, \end{cases} \tag{13}$$

where $\underline{f}(0) < \overline{f}(0)$ and $\underline{f}'(\cdot) < \overline{f}'(\cdot)$. Based on (13), we are led to distinguish between two types of development regime: the first - a low development

[13] This is illustrated in our subsequent example of public policy. Since $y_t = Ak_t$ in equilibrium, it makes no difference as to whether one specifies $\pi(\cdot)$ in terms of capital or output.

regime - is characterised by low levels of capital and life expectancy (i.e., $k_t < k^c$ and $\pi_t = \underline{\pi}$); the second - a high development regime - is characterised by high levels of capital and life expectancy (i.e., $k_t \geq k^c$ and $\pi_t = \overline{\pi}$). For illustrative purposes, we assume that $\underline{f}'(\cdot) \in (0,1)$, implying the existence of a stable stationary equilibrium at $k_L^* = \underline{f}(k_L^*)$. If $\overline{f}'(\cdot) \in (0,1)$ as well, then there is another such (but higher) equilibrium at $k_H^* = \overline{f}(k_H^*)$, while if $\overline{f}'(\cdot) > 1$, there is a non-stationary equilibrium that entails perpetual growth.[14] Precisely which of these equilibria the economy converges to depends essentially on the initial stock of capital, k_0, and the relationship between k^c and k_L^*. This is illustrated in Figure 1.

Suppose that $k_0 < k^c < k_L^*$. In this case the economy starts off in a situation where agents have a relatively small probability of survival, $\underline{\pi}$, and development takes place along the low capital accumulation path, $\underline{f}(\cdot)$. At some point in time, k_t reaches k^c and the survival probability increases to $\overline{\pi}$. This propels the economy onto the high capital accumulation path, $\overline{f}(\cdot)$, by causing it to jump from $\underline{f}(k^c)$ to $\overline{f}(k^c)$, after which it then either converges to the high steady state equilibrium, k_H^*, or grows perpetually at a constant positive rate. This chain of events describes a process of transition from the low development regime to the high development regime. But there is nothing in the model to guarantee such an outcome. To be sure, suppose that $k_0 < k_L^* < k^c$. In this case the economy is destined for the low steady state equilibrium, k_L^*, being locked forever on the low capital accumulation path, $\underline{f}(\cdot)$, without any improvement in life expectancy. To the extent that the high steady state equilibrium, k_H^*, or the positive growth equilibrium would be attained if $k_0 > k^c$, the model now presents a situation in which limiting outcomes depend fundamentally on initial conditions.

The above results are preserved under more general specifications of $\pi(\cdot)$ for which changes in life expectancy occur smoothly, rather than discontinuously. Naturally, the transition function, $F(\cdot)$, is also continuous in these circumstances, the implications of which may be conveyed broadly as follows. Given the restrictions on $\pi(\cdot)$, then there must exist a $\widehat{k} \geq 0$ such that $\pi''(\cdot) < 0$ for all $k > \widehat{k}$, with $\lim_{k\to\infty}\pi'(\cdot) = \lim_{k\to\infty}\pi''(\cdot) = 0$. Given that $\lim_{k\to\infty} k\pi'(\cdot) = 0$ as well, then it may be verified that $\lim_{k\to\infty} F'(\cdot) = \frac{R\overline{\pi}(1-\alpha)A}{1+R\overline{\pi}}$ and $\lim_{k\to\infty} F''(\cdot) = 0$. Thus, as in the case of a step function, long-run growth is either zero or positive according to whether $\frac{R\overline{\pi}(1-\alpha)A}{1+R\overline{\pi}} \in (0,1)$ or $\frac{R\overline{\pi}(1-\alpha)A}{1+R\overline{\pi}} > 1$. A fixed point of the transition map-

[14] The precise expressions for these terms are obtained from (13) as $\underline{f}'(\cdot) = \frac{R\underline{\pi}(1-\alpha)A}{1+R\underline{\pi}}$, $\overline{f}'(\cdot) = \frac{R\overline{\pi}(1-\alpha)A}{1+R\overline{\pi}}$, $k_L^* = \frac{R\underline{\pi}B+(1+R\underline{\pi})b}{1+R\underline{\pi}-R\underline{\pi}(1-\alpha)A}$ and $k_H^* = \frac{R\overline{\pi}B+(1+R\overline{\pi})b}{1+R\overline{\pi}-R\overline{\pi}(1-\alpha)A}$. A necessary condition for positive long-run growth is that $(1-\alpha)A > 1$.

ping satisfies $k^* = F(k^*)$ which may be re-written as $G(k^*) = H(k^*)$, where $G(\cdot) = k^*$ and $H(\cdot) = R\pi(k^*)\{B + b + [(1-\alpha)A - 1]k^*\} + b$. Sufficient conditions for a unique, stable equilibrium are that $\pi(\cdot)$ is strictly concave (i.e., $\widehat{k} = 0$) and $(1-\alpha)A \leq 1$ (i.e., zero long-run growth). Under such circumstances, $H(\cdot)$ crosses $G(\cdot)$ only once and does so from above. If either or both of these conditions are not satisfied, however, then there may be multiple steady state equilibria which alternate between stability and instability. For example, if $\pi''(\cdot) > 0$ for all $k < \widehat{k}$ (where $\widehat{k} > 0$), then there is the possibility of an equilibrium triple, $\{k_L^*, k^c, k_H^*\}$, while if $(1-\alpha)A > 1$ as well, then there is the possibility of just an equilibrium pair, $\{k_L^*, k^c\}$. We illustrate these outcomes in Figure 2. As development now takes place, there is a gradual improvement in life expectancy which feeds back on to savings and capital accumulation. But at what level of development the economy ends up depends critically on what level of development the economy starts off at: if $k_0 < k^c$, then the limiting outcome is the low steady state equilibrium, k_L^*, associated with low life expectancy; if $k_0 > k^c$, then the limiting outcome is either the high steady state equilibrium, k_H^*, or perpetual growth, associated with high life expectancy. Only when the transition path lies everywhere above the 45^0 line will a poor economy evolve into a rich economy: otherwise, poverty or prosperity to begin with implies poverty or prosperity in the future.

The existence of mutliple equlilbria means that countries with essentially the same structural features, but different initial conditions, may face very different prospects in terms of their economic development and demographic transition. Of course, these prospects may change with changes in circumstances, whether by accident or design. Thus, for a given threshold level of development, k^c, exogenous shifts in the stock of capital may cause a switch in development regime by pushing the economy either above or below that threshold. Likewise, for a given stock of capital, changes in the values of structural parameters (e.g., shifts in production and health technologies) may produce a similar turn of events by altering the transition function, $F(\cdot)$, and the threshold, itself. In both cases a switch in regime is more likely to occur the closer is an economy to k^c to begin with. Accordingly, should circumstances change for the better, then it is those countries at the upper end of the distribution below k^c that are most likely to benefit, while those countries in the lower tail are left behind. In addition, should different countries not share the same structural characteristics, then there would be a distribution of development paths that may hold little prospects for cross-country convergence. These observations suggest that the divisions between poor and rich countries are unlikely to vanish quickly or easily, if at all.

3.2.2 Public Policy

As mentioned earlier, there is a large body of evidence which indicates that state-provided health care, which accounts for a significant fraction of the public purse in most countries, is a major determinant of health status, in general, and life expectancy, in particular (e.g., Anand and Ravallion 1993; Bidani and Ravallion 1997; Gupta *et al.* 2001). This holds for all classes of citizen, though it is especially true for the poor who tend to benefit more than the rich from an expansion in public health programmes, the impact of which on the poor's health status tends to be much greater than the impact of an increase in private spending on health care. Both directly and indirectly, the evidence suggests that the positive correlation between income and life expectancy is due, in large part, to the fact that wealthier nations are more able to fund a better provision of essential health-improving public services, such as sanitation, medical care, epidemiological protection, environmental safeguards and education. In other words, it is not income growth *per se* that matters for longevity, but rather the extent to which higher incomes are used to support public health and welfare programmes, the benefits of which are distributed among the whole population. In what follows we present a simple formalisation of this idea, from which we obtain a probability of survival function and a transition path for capital that are identical in all essential respects to those that formed the basis of our previous analysis.

The model is now extended to include a government which undertakes various types of public expenditure that support and improve the health, well-being and life expectancy of its citizens. For simplicity, we consolidate these expenditures into a single term, x_t, and assume that they are financed each period by constant proportional taxes on the labour incomes of agents. Denoting the tax rate by $\tau \in (0,1)$, the government's balanced budget condition is $x_t = \tau w_t$, while the budget constraint facing a middle-aged agent reads $c_t^{t-1} + s_t + b_t = (1-\tau)w_t + (1+r_t)b_{t-1}$. Given the latter (together with (1) and (3)), each agent chooses an optimal level of savings equal to $s_t = \frac{R_{t+1}\pi_t}{1+R_{t+1}\pi_t}[B_t + (1-\tau)w_t]$. The probability of survival is specified initially as $\pi_t = p(x_t)$, where $p'(\cdot) > 0$, $p(0) = \underline{\pi} > 0$ and $\lim_{x\to\infty} p(\cdot) = \overline{\pi} \leq 1$. As before, (8) and (9) yield $w_t = (1-\alpha)Ak_t$ and $r_t = \alpha A$ in equilibrium. Thus we may write $x_t = \tau(1-\alpha)Ak_t$, implying $\pi_t = p(\tau(1-\alpha)Ak_t) = \pi(k_t)$, where $\pi'(\cdot) > 0$, $\pi(0) = \underline{\pi} > 0$ and $\lim_{x\to\infty} \pi(\cdot) = \overline{\pi} \leq 1$. It follows that the equilibrium path of capital accumulation satisfies

$$k_{t+1} = F(k_t) \equiv \frac{R\pi(k_t)}{1+R\pi(k_t)}[B + (1-\tau)(1-\alpha)Ak_t] + b. \tag{14}$$

Evidently, all of our previous results are preserved in this modified version of the model, where the relationship between longevity and development is derived from a more explicit and specific set of microfoundations relating to public policy. An additional parameter is the tax rate, τ, changes in which have ambiguous effects on capital accumulation because of ambiguous effects on savings: on the one hand, an increase in τ means that agents have less disposable income which causes a fall in savings; on the other hand, an increase in τ implies that agents have a higher life expectancy which induces an increase in savings as a means of financing higher expected retirement consumption.

Rather than repeating fully our previous analysis, we study the implications of (14) using numerical simulations of a calibrated version of the model. As well as confirming our qualitative results, these simulations may be used to obtain an idea of the quantitative orders of magnitude involved. For illustrative purpose, we confine our attention to the following parameterisations of the functions $v(\cdot)$ and $p(\cdot)$:[15]

$$v(b_t) = \Gamma b_t^{\gamma}, \quad \Gamma > 0, \gamma \in (0,1) \tag{15}$$

$$p(x_t) = \frac{\underline{\pi} + \overline{\pi}\Phi x_t^{\phi}}{1 + \Phi x_t^{\phi}}, \quad \Phi, \phi > 0. \tag{16}$$

The specification of $p(\cdot)$ satisfies the relevant restrictions, while being fairly general and flexible. The quantity $\widehat{x} \equiv \left[\frac{\phi-1}{\Phi(1+\phi)}\right]^{\frac{1}{\phi}}$ defines a turning point such that $p''(\cdot) > 0$ for $x_t < \widehat{x}$ and $p''(\cdot) < 0$ for $x_t > \widehat{x}$. If $\phi \in (0,1)$, then $\widehat{x} < 0$, implying that $p(\cdot)$ is strictly concave for all x_t. More generally, the parameters Φ and ϕ determine jointly both the value of $\widehat{x}$ and the speed at which $p(\cdot)$ traverses the interval $\{\underline{\pi}, \overline{\pi}\}$. *Ceteris paribus*, an increase (decrease) in the value of Φ (ϕ) reduces the value of $\widehat{x}$ and raises the speed of transition (the limiting case of which is when $p(\cdot)$ changes value from $\underline{\pi}$ to $\overline{\pi}$ instantaneously, which corresponds to the case of a step function). Naturally, these properties are also reflected in the function $\pi(\cdot) = \frac{\underline{\pi}+\overline{\pi}\Phi[\tau(1-\alpha)A]^{\phi}k_t^{\phi}}{1+\Phi[\tau(1-\alpha)A]^{\phi}k_t^{\phi}}$, for which $\pi''(\cdot) > 0$ if $k_t < \widehat{k}$ and $\pi''(\cdot) < 0$ if $k_t > \widehat{k}$, where $\widehat{k} \equiv \frac{\widehat{x}}{\tau(1-\alpha)A}$. We focus on the case in which the high development regime is characterised by transitional dynamics towards a balanced, endogenous growth path. Treating each period as 25 years, our baseline set of parameter values is $\{\sigma = 1.00, \theta = 0.62, \gamma = 0.50, \Gamma = 0.50, A = 7.80, \alpha = 0.30, \underline{\pi} = 0.15, \overline{\pi} = 0.95, \phi = 0.40, \Phi = 0.01, \tau = 0.20\}$. Along the balanced growth path, these parameter values imply an annual discount factor of 0.98, an annual per capita income

[15] Our results remain broadly unchanged under other parameterisations which share the same basic properties.

growth rate of 2 percent and a life expectancy of 74 years. The low steady state equilibrium occurs at $k_L^* = 0.14$, where life expectancy is 54 years, while the threshold level of development occurs at $k^c = 2.49$, where life expectancy is 61 years. Given these outcomes, an economy which is close to k_L^* would require an extremely large (18-fold) increase in its capital stock to take it just beyond k^c into the high development regime, implying a substantial (13 percent) leap in life expectancy.

Different configurations of parameter values are associated with different transition paths, different steady states and different threshold levels of capital. Parameters of particular interest are those relating to production and health technologies. In Table 1 we summarise the results of our simulations when we vary, in turn, the values of the shift parameters, A and $\underline{\pi}$. An increase in either of these has the effect of pushing up the transition path, $F(\cdot)$, such that the low steady state equilibrium, h_L^*, is raised, while the threshold level, h^c, is lowered. Increasing the value of A ($\underline{\pi}$) has a negligible (positive) effect on life expectancy at h_L^*, but a positive (zero) effect on long-run growth beyond h^c. For example, if A were to rise from 8.90 to 10.90, then an economy in the high development regime would experience an increase in its long-run annual growth rate from 2.48 to 3.32 percent, while if $\underline{\pi}$ were to rise from 0.20 to 0.30, then an economy in the low development regime would experience an increase in the steady state life expectancy of its citizens from 55 to 58 years. For $A > 15.90$ ($\underline{\pi} > 0.40$), the transition path lies everywhere above the 45^0 line and the multiplicity of development regimes vanishes. Our numerical analysis may also be used to study how these different scenarios might translate into different populations of poor and rich countries. As a simple illustration of this, consider an initial situation in which the world stock of capital is uniformly distributed over a continuum of (otherwise identical) economies, a unit mass of which is located within the interval $\{0, k^c\}$ at our benchmark parameter configuration. An increase in the value of either A or $\underline{\pi}$ would have the effect of reducing this mass by causing the interval to shrink. Thus, using the same examples as above, if $A = 8.90$ (10.90), or $\underline{\pi} = 0.20$ (0.30), there would be 19 (44) percent, or 5 (18) percent, fewer countries in the low development regime compared to the benchmark case.

4 Concluding Remarks

Changes in life expectancy are an integral part of the process of demographic transition. To many observers, such changes are both a significant cause and a fundamental consequence of rising prosperity. On the one hand, greater chances of survival encourage investment in both physical and human capital by raising the expected returns on savings and education. On the other hand, higher levels of income foster lower rates of mortality by allowing more resources to be spent on essential life-preserving services, as well as by engendering the adoption of healthy lifestyles among individuals. As yet, there are very few analyses that model explicitly this two-way interaction between life expectancy and economic development. The purpose of the present paper has been to take a step forwards towards filling this gap.

Our analysis indicates how endogenising life expectancy can radically alter the implications of even the simplest of growth models. Depending on parameter values, there may be multiple equilibria associated with threshold effects which imply that the limiting outcomes in the economy are determined by historical, or initial, conditions. An economy that is poor to begin with may be destined to remain poor unless there are major changes in circumstances which allow the threshold to be breached or which eliminate the threshold altogether. Our explanation for this is distinct from other accounts of poverty traps, being derived from a different perspective that focuses on the co-evolution of individual life-cycles and economic progress. This perspective yields further insights into the perplexing issue of why initial inequalities between countries might persist, if not remain indefintely. The transition from a low development regime (with low life expectancy) to a high development regime (with high life expectancy) may not just be difficult to achieve, but may simply lie beyond the capability of a country.

While the model that we have used is deliberately stylised, the results from our numerical simulations are both instructive and revealing. Small changes in production and health technologies, which shift the transition function, can nudge an economy from just below to just above the threshold level, endowing it with the prospect of long-run prosperity, rather than long-run poverty. For countries at the lower end of the distribution, however, such changes (in particular, exogenous increases in the probability of survival) may succeed in prolonging life while having little effect on long-term economic development: citizens of poor countries may well live longer as a result of advances in health technologies, but they may nevertheless remain poor. The latter implication accords with the observation that, over the past half century, life expectancy rates have converged significantly across countries without there having been a similar cross-country convergence in per capita income levels (e.g., Easterlin 1996). This is probably due to intrinsic differences in the characteristics of health and production technologies. Whatever the reason, it is certainly true that most parts of the world have experienced a mortality revolution which is continuing to this day. Incorporating this into models of development and demography would appear to be a rewarding, if not essential, avenue of research to pursue further.

References

[1] Abel, A., 1985. Precautionary savings and accidental bequests. *American Economic Review*, 75, 777-791.

[2] Adlakha, A. and D. Rudolph, 1994. Ageing trends: Indonesia. *Journal of Cross-cultural Gerentology*, 9, 99-108.

[3] Anand, S. and M. Ravallion, 1993. Human development in poor countries: on the role of private incomes and public services. *Journal of Economic Perspectives*, 7, 133-150.

[4] Andreoni, J., 1989. Giving with impure altruism: applications to charity and Ricardian Equivalence. *Journal of Political Economy*, 97, 1447-1458.

[5] Barro, R.J. and X. Sala-i-Martin, 1995. *Economic Growth*. McGraw-Hill, New York.

[6] Bidani, B. and M. Ravallion, 1997. Decomposing social indicators using distributional indicators. *Journal of Econometrics*, 77, 125-139.

[7] Bishai, D., 1996. Quality time: how parents' schooling affects child health through its interaction with child-care time in Bangladesh. *Health Economics*, 5, 383-407.

[8] Blackburn, K. and G.P. Cipriani, 1998. Endogenous fertility, mortality and growth. *Journal of Population Economics*, 11, 517-534.

[9] Blackburn, K. and G.P. Cipriani, 2002. A model of longevity, fertility and growth. *Journal of Economic Dynamics and Control*, 26, 187-204.

[10] Bloom, D.E., D. Canning and J. Sevilla, 2001. Health, human capital and economic growth. Working Paper WG1:8, Commission on Macroeconomics and Health.

[11] Cooksey, E.C., R.R. Rindfuss and D.K. Guilkey, 1996. The initiation of adolescent sexual and contraceptive behaviour during changing times. *Journal of Health and Social Behaviour*, 37, 59-74.

[12] Cutler, D.M. and L. Sheiner, 1998. Demographics and medical care spending: standard and non-standard effects. Working Paper 6866, National Bureau of Economic Research.

[13] Deaton, A. and C. Paxson, 1999. Mortality, education, income and inequality among American cohorts. Working Paper 7140, National Bureau of Economic Research.

[14] De la Croix, D. and O. Licandro, 1999. Life expectancy and economic growth. *Economics Letters*, 65, 255-263.

[15] Doshi, K., 1994. Determinants of the savings rate - an international comparison. *Contemporary Economic Policy*, 12, 37-45.

[16] Easterlin, R.A., 1996. *Growth Triumphant*. University of Michigan Press, Ann Arbor.

[17] Ehrilich, I., and F.T. Lui, 1991. Intergenrational trade, longevity and economic growth. *Journal of Political Economy*, 99, 1029-1060.

[18] Ehrlich, I. and F.T. Lui, 1997. The problem of population and growth: a review of the literature from Malthus to contemporary models of endogenous population and endogenous growth. *Journal of Economic Dynamics and Control*, 21, 205-242.

[19] Fogel, R.W., 1994. Economic growth, population theory and physiology: the bearing of long-term processes on the making of economic policy. *American Economic Review*, 84, 369-395.

[20] Fogel, R.W., 1997. New findings on secular trends in nutrition and mortality: some implications for population theory. In M. Rosenzweig and O. Stark (eds.), *Handbook of Population and Family Economics*, Elsevier, Amserdam.

[21] Galor, O. and D.N. Weil, 1999. From Malthusian stagnation to modern growth. *American Economic Review*, 89, 150-154.

[22] Galor, O. and D.N. Weil, 2000. Population, technology and growth: from Malthusian stagnation to the demographic transition and beyond. *American Economic Review*, 90, 806-828.

[23] Greenlund, K.J., K. Lui, A.R. Dyer, C.I. Kiefe, G.L. Burke and C. Yunis, 1996. Body mass index in young adults: association with parental body size and education in CARDIA study. *American Journal of Public Health*, 86, 480-485.

[24] Gupta, S., M. Verhoeven and E. Tiongson, 2001. Public spending on health care and the poor. Working Paper 01/127, International Monetary Fund.

[25] Hu, S., 1995. Demographics, productivity growth and the macroeconomic equilibrium. *Economic Inquiry*, 22, 592-610.

[26] Kalemli-Ozcan, S., H. Ryder and D. Weil, 2000. Mortality decline, human capital investment and economic growth. *Journal of Development Economics*, 62, 1-23.

[27] Kandel, D.B. and P. Wu, 1995. The contribution of mothers and fathers to the intergenerational transmission of cigarette smoking in adolescence. *Journal of Research on Adolescence*, 5, 225-252.

[28] Kannisto, V., J. Lauritsen, A.R. Thatcher and J. Vaupel, 1994. Reductions in mortality at advanced ages: several decades of evidence from 27 countries. *Population and Development Review, 20, 793-810.*

[29] Kirk, D., 1996. Demographic transition theory. *Population Studies*, 50, 361-387.

[30] Knowles, S. and P.D. Owen, 1995. Health capital and cross-country variations in per capita income in the Mankiw-Romer-Weil model. *Economics Letters*, 48, 99-106.

[31] Knowles, S., and P.D. Owen, 1997. Education and health in an effective-labour empirical growth model. *Economic Record*, 73, 314-328.

[32] Lee, R.D. and S. Tuljapurkar, 1997. Death and taxes: how longer life will affect social security. *Demography*, 34, 67-82.

[33] Lichtenberg, F.R., 1998. Pharmaceutical innovation, mortality reduction and economic growth. Working Paper 6569, National Bureau of Economic Research.

[34] Livi-Bacci, M., 1997. *A Concise History of World Population.* Blackwell, Malden, MA.

[35] McKeown, T., R.C. Bower and G. Record, 1972. An interpretation of the modern rise of population in Europe. *Population Studies*, 26, 45-82.

[36] Mirowsky, J. and C.E. Ross, 1998. Education, personal control, lifestyle and health - a human capital hypothesis. *Research on Ageing*, 20, 415-449.

[37] Murray, C.J.L. and C.H. Chen, 1992. Understanding morbidity change. *Population and Development Review*, 18, 481-503.

[38] Pecchenino, R.A. and K.R. Utendorf, 1999. Social security, social welfare and the ageing population. *Journal of Population Economics*, 6, 353-362.

[39] Preston, S.H., 1978. The changing relation between mortality and economic development. *Population Studies*, 29, 231-248.

[40] Preston, S.H., 1980. Mortality decline in less developed countries. In R. Easterlin (ed.), *Population and Economic Change in Developing Countries*, Chicago University Press, Chicago.

[41] Pritchett, L. and L. Summers, 1996. Wealthier is healthier. *Journal of Human Resources*, 31, 844-868.

[42] Sandiford, P., J. Cassel, M. Monetnegro and G. Sanchez, 1995. The impact of women literacy on child health and its interaction with access to health services. *Population Studies*, 49, 5-17.

[43] Schofield, R., D. Reher and A. Bideau, 1991. *The Decline of Mortality in Europe*. Oxford University Press, Oxford.

[44] Strauss, J. and D. Thomas, 1998. Health, nutrition and economic development. *Journal of Economic Literature*, 36, 766-817.

[45] United Nations, 1991. *Population Bulletin of the United Nations 30*. United Nations, New York.

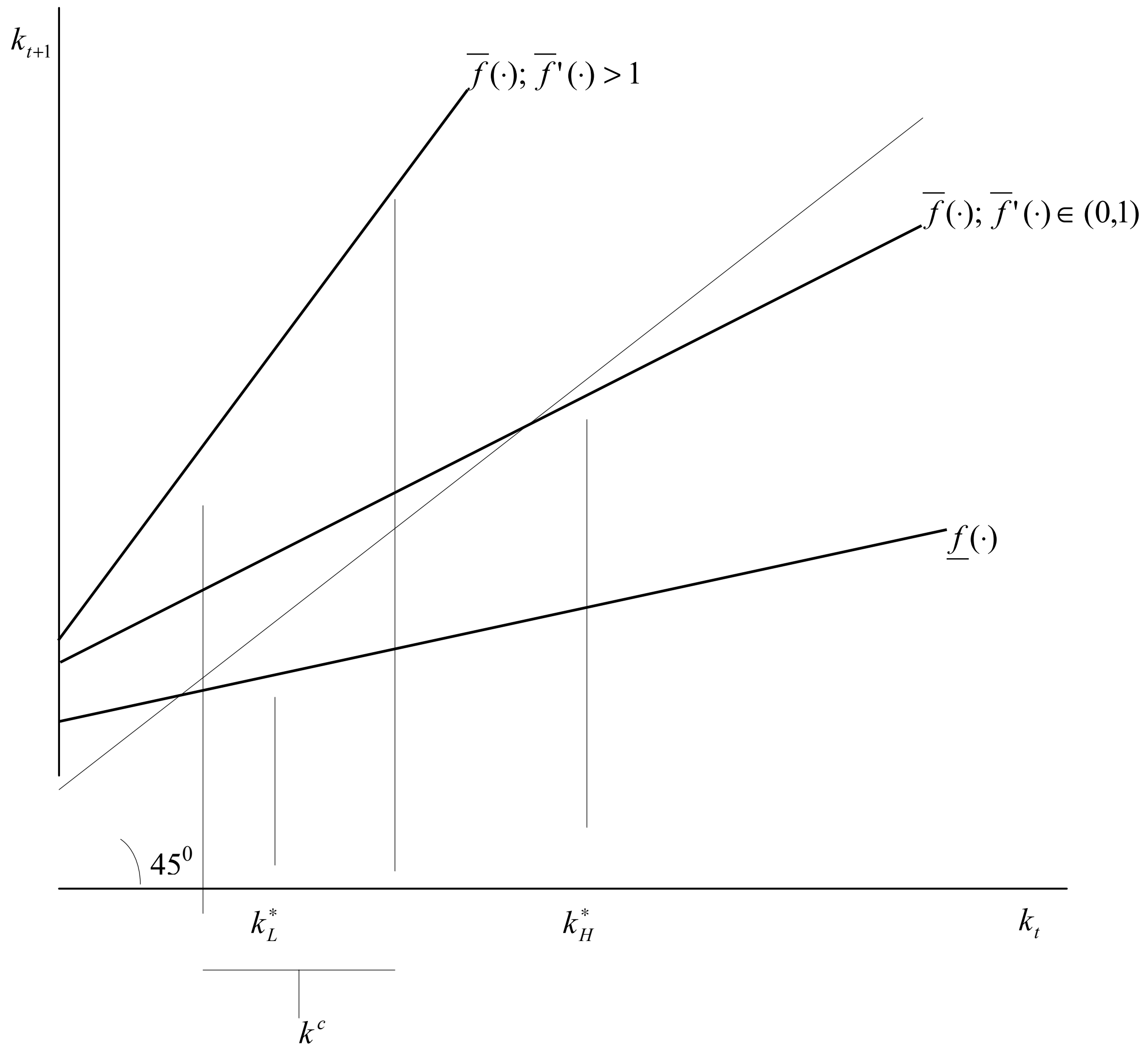
[46] Weil, D.N., 1997. The economics of population ageing. In M. Rosenzweig and O. Stark (eds.), *Handbook of Population and Family Economics*, Elsevier, Amserdam.

[47] World Bank, 1993. *World Development Report*. Oxford University Press, New York.

[48] Zhang, J. and J. Zhang, 2001. Longevity and economic growth in a dynastic family model with an annuity market. *Economics Letters*, 72, 269-277.

[49] Zhang, J., J. Zhang and R. Lee, 2001. Mortality decline and long-run economic growth. *Journal of Public Economics*, 80, 485-507.

Figure 1



k_{t+1}
$\overline{f}(\cdot); \overline{f}'(\cdot) > 1$
$\overline{f}(\cdot); \overline{f}'(\cdot) \in (0,1)$
$\underline{f}(\cdot)$
45^0
k_L^*
k_H^*
k_t
k^c

Figure 2

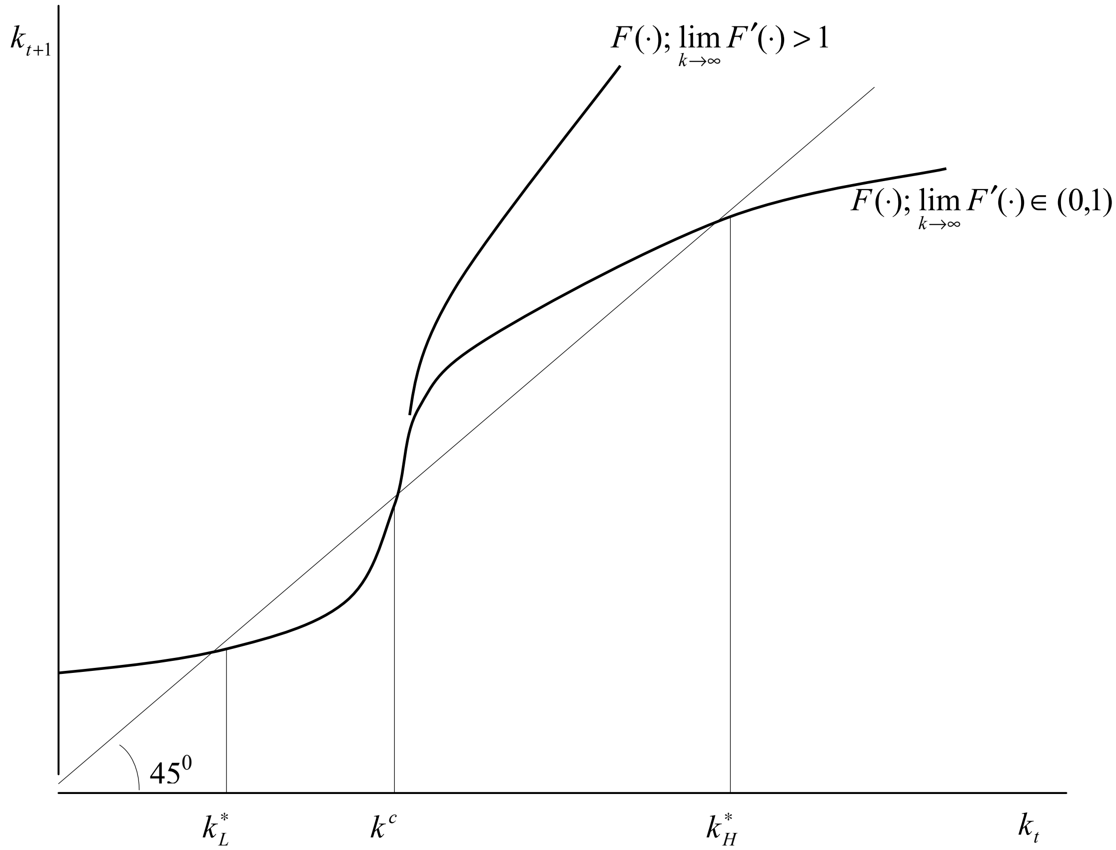

k_{t+1}
$F(\cdot);\ \lim_{k\to\infty} F'(\cdot) > 1$
$F(\cdot);\ \lim_{k\to\infty} F'(\cdot) \in (0,1)$
45^0
k_L^*
k^c
k_H^*
k_t

Table 1

A	($\underline{\pi}$)	k_L^*	k^c	Life expectancy at k_L^*	Balanced annual growth rate
7.90	(0.15)	0.14	2.49	54	2.00
8.90	(0.20)	0.15 (0.18)	2.01 (2.37)	54 (55)	2.48 (2.00)
9.90	(0.25)	0.17 (0.24)	1.66 (2.23)	54 (56)	2.92 (2.00)
10.90	(0.30)	0.19 (0.34)	1.39 (2.05)	54 (58)	3.32 (2.00)
11.90	(0.35)	0.21 (0.53)	1.18 (1.80)	54 (59)	3.68 (2.00)