Sequential Auctions with Endogenously Determined Reserve Prices

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Abstract

I model an auction game in which two identical licenses for participating in an oligopolistic market are sold in a sequential auction. There is no incumbent. The first auction is a standard first-price, sealed-bid type with an exogenously set reserve price, while the second has the sold price of the first unit as the reserve price. This auction rule mimics the license auction for the Turkish Global Mobile Telecommunications in 2000. For some cases this auction setup generates the same revenue as selling the monopoly right with the second-price, sealed-bid auction; for the others it creates less revenue .

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1 Introduction:

Auction theory has attracted enormous attention in the last few years. Auctions may have been part of human history, but with the aid of recent technological innovations they have been increasingly practiced. This situation has paved the way both for the ordinary person to participate in or establish an auction through the internet, also for theoreticians to analyze what happens in auctions and how they come about. Governments use auctions in the sale of treasury bills, mineral rights such as oil fields, pollution permits, state-owned firms' privatization, as well as cellular phone and cable-television licenses.[1]

There are various objectives for governments in selling those type of licenses. Some governments are worried about the competition and/or price of the service/product produced after the auction. They pay attention to the consumer side aiming to enhance the total welfare of the society. Others, however, may simply focus on extracting maximum revenue from the auction.

This paper focuses on license issues. There are many ways to allocate licenses. In "beauty contests," firms apply for a license and the government chooses one of them according to some criteria. The government is not likely to gain as much revenue as with other types of auctions. Auctions can also sell more than one license either sequentially or simultaneously, waiting until bidders stop their bids for any one license, the prize going to the highest bidder for any particular license. Although the main argument is to have a competitive market, revenues are also important for governments, sometimes more important in the short-run.

In this paper, I consider a regulatory agency or the government, the seller, which sells two licenses — for example Global System for Mobile Communica-

[1] The Israeli Cable Television License auction is a good example. For an analysis of this license auction, see Neil Gandal (1995)

tions, GSM, licenses for cellular phones — in a sequential auction format. The initial auction is for the first license with the first-price, sealed-bid rule, i.e. the firm that bids the maximum amount receives the first license by paying its bid. The seller then sets this price as a reserve price in the second auction for the second license.[2] The winner of the first license cannot participate in the second auction. Any firm satisfying predetermined criteria can bid. The 2000 Turkish Global Mobile Telecommunication license auction was done under this particular setup. To simplify, the paper first focuses on the two-bidder case. Then the model is analyzed for the general n-participant case with some parameter restrictions. The competition among firms that have a license is modeled by a reduced-form industry profit function, viz. the profit of a firm depends on the number of firms in the industry, which makes the values of licenses endogenous. The analysis concentrates on the revenue generated by the auction and compares it with other types of auctions.

The results show that this auction setup is not a superior design for a large set of values of the modelling parameters. For some parameter values, it produces the same revenue as the second-price, sealed-bid auction for a monopoly right, and for some others it produces less revenue. However, for some other modelling parameter values, this auction design gives more revenue than the revenue of selling one license.[3]

[2] For a real world case, see appendix A.2

[3] I compare the revenue of the design in this paper with selling one license because there is a general tendency to think that this design tends to create monopoly. For example Klemperer (2001) says "*..., Turkey last year auctioned two telecom licenses sequentially, with an additional twist that set the reserve price for the second license equal to the selling price of the first. One firm then bid far more for the first license than it could possibly be worth if the firm had to compete with a rival holding the second license. But the firm had rightly figured that no rival would be willing to bid that high for the second license, which therefore remained unsold, leaving the firm without a rival operating the second license!*" Hence it is

The paper is organized as follows: Section 2 gives a brief literature review. Section 3 describes the model. Section 4 solves the model assuming asymmetric information at the firm level and describes the results first for the two-firm auction as a benchmark and then for the n-firm case, deriving the bidding behaviors of participants and the revenue function for the seller. Section 4 also shows that for some values of the model's parameters, the solution to the model changes dramatically - both the behavior of the firms and the seller's revenue function change. Section 5 gives a comparison with other types of auctions. Section 6 concludes the paper. Proofs of the propositions are given in the Appendix which also describes the Turkish 2000 GSM license auction.

2 Literature Review

von Der Fehr (1994) considers a standard sequential model in which two indivisible units of a good are sold by means of two consecutive English auctions. He finds an equilibrium in which the two units are sold at the same price equal to the third-highest valuation.

Pitchik and Schotter (1988) present an experimental study of bidding behavior in sequential auctions in which there are budget constraints. The budget constraints affect the behavior of bidders who attempt to exploit the constraints of others.

Jehiel and Moldovanu(2000a) analyze the interplay between license auctions

natural to compare this design with selling one license, the monopoly right, to see which setup is more benefitial for the seller's point of view. Naturally one can ask why this set up is not compared to selling two licenses sequentially without any reserve price and without any link between the sequence of the auctions. Since one participant's valuation depends on others' types, the players' strategies and therefore the equilibrium, are not obvious and are left for future reasearch.

and market structure in a model with several incumbents and several potential entrants. The authors also study how the auction format affects the incentives for explicit or tacit collusion among incumbents. The number of incumbents and licenses play important roles in their modelling. If the number of incumbents is greater than the number of licenses, the auctioning of more licenses need not result in greater competitiveness. They also analyze an auction format in which the number of licenses is endogenously determined at the auction.

Jehiel and Moldovanu(2000b) examine the positive or negative externalities created by the auctioned object, a patent. The authors study an auction whose outcome influences the future interactions among agents, where the type of the agent, which is private information at the time of the auction, determines the impact of interaction. They derive equilibrium bidding strategies for second-price, sealed-bid auctions. In their model, they assume that if one obtains the auctioned object, its marginal cost decreases whereas others' marginal costs remain the same. They assume Cournot interactions in their analyses.

Paul Klemperer discusses several issues related to auctions in his paper "What Really Matters in Auction Design" (2001). He pays attention to radio spectrum auctions done throughout the world, electricity market, TV franchises, and football TV-rights. His paper is an excellent source of information in regards to the main questions in auctions of these rights. Also what is auctioned, where the auction took place, and under which rules they were conducted. He also mentions the 2000 Turkish mobile phone license auction as a failure due to inadequate thinking about the rules of the auction.

A similar rule in the auction setup is being used at the Spanish treasury bond auctions, Mazon and Nunez (1999). It analyzes the Spanish bond auctions both theoretically and empirically. The general rule is a mixture of uniform price and discriminatory price auction setups. The feature that is relevant to my analysis is that in the Spanish bond auctions at the second round, if it takes place, each market maker can participate in the auction and have to submit their bids at prices higher or equal to the price prevailing in the first round auction.[4]

3 The Model

Consider a market in which only firms with licenses can participate. The government has two identical licenses to sell. The market is inactive prior to the sale of the licenses. There are two risk-nuetral potential entrants (firms) that bid for these licenses. This paper analyzes an auction game in which the licenses are sold in two consecutive auctions. Each buyer can purchase no more than one of the auctioned licenses. The risk-nuetral seller uses a first-price, sealed-bid auction design with an additional rule that "bids in the second auction for the second license shall begin from the winning price of the first auction." In technical terms, the price of the first auction is taken as the reserve price at the second auction. If nobody beats the reserve price in the second auction, the second license remains unsold. If both firms bid the same amount, the usual tie rule applies, i.e. each will receive the license with probability 1/2.

After auctions, the licensed firm will produce services over one period. Each firm is able to produce any amount of service it wants, and there is no regulation on the price.

Each firm may be either high cost or low cost, denoted by c_H and c_L respectively, where $c_H > c_L$. At the end of the auctions, if only one license is sold, the winning firm will be a monopoly and its profit is the monopoly profit of its type, H and L respectively. However, if both licenses are sold then there will be duopoly and the profit for the I type firm is IJ, where J is the type of

[4] This price is WAP, the weighted average price of accepted bids. For more information about the Spanish Treasury bond auctions look at Mazon & Nunez (1999) and Alvarez (2001).

the other firm in the duopoly.[5] A firm that does not receive a license, the loser, receives zero profits.

The probability a firm is high-cost type p, and the complementary probability that is low-cost type $1-p$ are common knowledge; firms' types are private information.

4 The Solution

In this paper, I assume that a firm's cost is its private information, therefore, no one knows exactly what type the other firms are. Meaning, neither the seller nor the firm's rivals know other firms' cost types. The other firms and the seller know only that a particular firm is a high-cost type with probability p and a low-cost type with probability $(1-p)$. Altough there may be equilibrium in asymmetric setting, this paper uses symmetric strategy setting. Since there is no Nash equilibrium in symmetric pure-strategies, I am looking for a symmetric equilibrium in mixed strategies .

The equilibrium strategies depend on the gross profit orders of the variuous outcomes. There are three cases to consider. In the first, LL and LH are less than H, which is always less than L, i.e. $LL < LH < H < L$, the second $H < LL < LH < L$, and the third $LL < H < LH < L$.[6] In the first case, a type-L firm can deter the entry of a type-H firm by bidding just a bit more than H, and this bid also deters the entry of a type-L firm at the second auction, because at the second auction it cannot beat the reserve price, which in this scenario, is at least H. In the latter two cases, however, a type-H firm will not bid H, since doing so pays negative net profit, and it decreases its bid to

[5] Throughout the paper, I stands for the monopoly profit of type-I, IJ stands for the duopoly profit of type-I when the other firm is type-J.

[6] As HL is less than other profits, I dropped it from these orderings.

HL. A type-L firm, knowing this, may bid just above HL. But, if the other firm is of type L, it can come in and get the second license, which is possible with some probability. Type-L may eliminate this possibility for some modelling parameter values by increasing its bids above LL; therefore, it is not feasible for any bidder to receive the second license at the second auction. However, it may give the same expected profit to the type-L if it lowers its bid below LL (keeping it above HL). Though it lowers the possibility of being a monopoly, in this strategy, the payment is also low. On the other hand, for some other modelling parameter values, it may not be possible to deter the entry of a second firm to the market by receiving the second license. All these will be clearer throughout the sections 4.1.1, 4.1.2 and 4.1.3. Briefly, there are three cases: The first one is $LL < LH < H < L$, the second is $H < LL < LH < L$, and the third is $LL < H < LH < L$. The strategies of players and the revenue for the seller all depend on which case we are in as stated earlier. Now find the solution for each case.

4.1 Solution for Two-Firm Auction:

4.1.1 The first case $LL < LH < H$

Since the monopoly profit of an L-firm is greater than the monopoly profit of an H-firm, an L-firm can deter the entry of firm-H at the second auction simply by bidding slightly higher than H in the first auction. Since an H-firm knows this and since its rival may be higher cost too, firm-H bids its monopoly profit, H, to make its expected profit maximum which is zero. It makes zero expected net profit if it receives the first license - it pays H, and receives H.- Therefore, for firm-H, bidding its monopoly profit is a weakly dominating strategy.

Now, let us find the strategy of firm-L. Since I am looking for the solution in symmetric setting, firm-L's bid, B, should satisfy

$$B \geq pH + (1-p)\,B, \tag{1}$$

which implies

$$B \geq H. \tag{2}$$

The maximum profit of firm-L is L. Therefore the bid of firm-L cannot be outside the interval determined by the monopoly profit of the high-cost type and its own monopoly profit, i.e.,

$$B \in [H, L]\,. \tag{3}$$

Expected profit of type-L, when its bid is x, is

$$[p + (1-p)\,F(x)]\,(L-x)\,, \tag{4}$$

where $F(x)$ is the cumulative distribution function for bidding x. Since there are no point masses in the equilibrium density, which is shown by Appendix A.1, the cumulative distribution function is continuous on the interval specified below. If $f(x)$ is the density corresponding to $F(x)$, then $f(x) = F^{'}(x)$ almost everywhere.

Since bidding L gives zero profit for sure, type-L wants to bid less than L in order to make positive expected profits, implying that there is a point in the specified interval above which type-L does not want to bid. Denote this point as $\overline{b}$. i.e., we have $B \in \left[H, \overline{b}\right]$.

Since bidding H and $\overline{b}$ should give the same profit for firm-L as they are two points of the interval on which firm-L mixes its strategies, we have

$$p\,(L-H) = \left(L - \overline{b}\right), \tag{5}$$

which implies

$$\overline{b} = (1-p)\,L + pH. \tag{6}$$

Bidding $x \in \left(H, \overline{b}\right)$ and bidding $\overline{b}$ should give the same expected profit for some cumulative distribution function $F(x)$ and probability density function $f(x)$, i.e.,

$$[p + (1-p)\,F(x)]\,(L - x) = L - \overline{b}. \tag{7}$$

Inserting $\overline{b}$ from (6) into (7) gives

$$[p + (1-p)\,F(x)]\,(L - x) = p\,(L - H)\,. \tag{8}$$

Solving (8) for $F(x)$ gives

$$F\,(x) = \left(\frac{(L-H)}{(L-x)} - 1\right)\frac{p}{1-p}, \tag{9}$$

and the density of $F(x)$ is

$$f(x) = \begin{cases} \frac{(L-H)p}{(L-x)^2(1-p)} & x \in (H, \overline{b}] \\ 0 & otherwise. \end{cases} \tag{10}$$

This result states that firm-H bids its monopoly profit, and firm-L randomizes its bid x in the interval $(H, \overline{b}]$ according to the probability density function $f(x)$ in the first auction.

The results for this case are summarized in the following proposition.

Proposition 1 *In the auction with asymmetric information and with $LL < LH < H$, firm-L randomizes its bid x in the interval $(H, \overline{b}]$ according to the* *probability density function $f(x)$ given by (10) and firm-H bids H. Only the first license is sold.*

Proof. *See Appendix A.2.* ■

By playing such a strategy, every player maximizes expected profit. Firm-L bids above the monopoly profit of firm-H to deter its entry into the market and to eliminate the possibility of its receiving a license either in the first auction or in the second auction. Also, since $LL < H$, firm-L's bid x can disincline type-L rival to bid at the second auction, because in order to receive the license at the second auction, a type-L is obligated to pay at least x, which is greater than LL, the post auction profit.

4.1.2 The second case, $H < LL < LH$

In the previous case, in which $LH < H$, type-H bids H, which is its equilibrium strategy. However now, as $H < LH$, bidding H does not prevent the possibility of receiving the first license if the rival is type-L. Type-L rival may allow type-H to receive the first license, and type-L still can buy the second license. This creates a huge loss for type-H firm − type-H bids H and can make only HL. Therefore, type-H lowers its bids to the level HL. If a type-L firm bids more than HL, then that amount is enough to eliminate the entry of a type-H firm

in the second auction. However, if the second firm is type-L, it can surpass the price of the first auction if the price is lower than the duopoly profit LL, which is greater than HL. Therefore, in this case there is a threat of entry at the second auction if the price in the first auction is less than the duopoly profit LL.

Now, think about raising the bid above the duopoly profit. This eliminates the threat available for the second auction for firm-L, however, the cost of this strategy is paying more, which makes expected profit less. However, letting the second license be sold and lowering its own bid, a type-L firm can make the same expected profit. Here, there are two forces working in the opposite directions: bidding high increases the probability of receiving the first license and can make sure the sale of only one license giving profit of $L - (high\ bid)$, whereas bidding low decreases the probability of selling only one license and if two licenses are sold, profits fall to $LL - (low\ bid)$. Note that profits in both cases can be the same; multiplication of the difference between a high payment and a high return, L ,with a higher probability of receiving the first license can be equal to the multiplication of the difference between a low payment and a low return, LL, with a lower probability of receiving the first license. So a natural guess is that the symmetric equilibrium strategy has two separate supports, $(HL, \overline{\overline{b}}]$ and $(LL, \overline{b}]$, where $\overline{\overline{b}}$ and $\overline{b}$ are some upper bounds of these supports. Indeed, it can be shown like in the proof of proposition 1 that any symmetric Bayesian Nash Equilibrium takes this form and is unique.

$Figure 1$: Bidding regions on the profit line.

HL	$\overline{\overline{b}}$	LL	$\overline{b}$	L
(———————	]———————	(———————	]———————	
$region 1$		$region 2$		

Now there are two subcases: In the first one the strategies give some weight to both supports, $(HL, \overline{\overline{b}}]$ and $(LL, \overline{b}]$; in the second playing on $(LL, \overline{b}]$ is not feasible due to the model's parameter values, i.e. players bid only on $(HL - \overline{\overline{b}}]$, possibly for a different $\overline{\overline{b}}$. We can calculate the symmetric Bayesian Nash Equilibrium.

Subcase i: strategies on two separate regions

Let there be probability distribution function $F(x)$ on the profit line such that

$$F(x) = \begin{cases} 0 & x \leq HL \\ F_1(x) & x \in (HL, \overline{\overline{b}}] \\ F_1\left(\overline{\overline{b}}\right) + F_2(x) & x \in (LL, \overline{b}] \\ 1 & x > \overline{b} \end{cases}$$

where $F_1(x)$ and $F_2(x)$ are parts of the distribution function $F(x)$ in *regions* 1 and 2 respectively. Appendix A.1 shows that these probability like distribution functions are continuous. Also, let $F_1\left(\overline{\overline{b}}\right) = q$. As discussed earlier, a type-H firm bids HL.

Since the expected profits of bidding $\overline{\overline{b}}$ and LL are the same

$$p\left(L - \overline{\overline{b}}\right) + (1-p)F_1\left(\overline{\overline{b}}\right)\left(LL - \overline{\overline{b}}\right) = p\,(L - LL) + (1-p)F_2\,(LL)\,(L - LL)\,,$$

which gives

$$\overline{\overline{b}} = LL - \frac{(1-p)q}{(1-p)q + p}\,(L - LL)\,. \tag{11}$$

Similarly equating $p\,(L - LL) + (1-p)F_2\,(LL)\,(L - LL)$ to $L - \overline{b}$ gives

$$\overline{b} = (1-t)L + tLL, \tag{12}$$

where $t = p + (1-p)q$.

The condition $\overline{\overline{b}} \geq HL$ is always satisfied.[7]

In order to find $F_1(x)$, equate the expected profit of bidding any x in $(HL, \overline{\overline{b}}]$ to the expected profit of bidding $\overline{\overline{b}}$. This yields

$$F_1(x) = \frac{t(L - LL) - p(L - x)}{(1-p)(LL - x)} \qquad for\ any\ x\ \epsilon\ (HL, \overline{\overline{b}}]. \tag{13}$$

[7] $\overline{\overline{b}} \geq HL$ since $\left(1 - \frac{L-LL}{L-HL}\right)(LL - HL) \geq 0$ which is always true since $HL < LL$ and $HL < L$.

In order to find $F_2(x)$, equate the expected profit of bidding x in $(LL, \overline{b}]$ to the expected profit of bidding $\overline{b}$, and then solve it for $F_2(x)$. This gives

$$F_2(x) = \left[\frac{t(L-LL)}{(L-x)} - p\right]\frac{1}{1-p} - q \qquad for\ x \in (LL, \overline{b}]. \tag{14}$$

Since $F_1(HL) = 0$,

$$q = \frac{p}{1-p}\frac{(LL-HL)}{(L-LL)}. \tag{15}$$

However, for some values of p, q can be greater than 1, which is not possible since q is a portion of a cumulative distribution function and can be at most one. The values of p that make $q \leq 1$ are

$$p \leq \frac{L-LL}{L-HL}. \tag{16}$$

Therefore, when $p \leq \frac{L-LL}{L-HL}$, we obtain the distribution function of the following form

$$F(x) = \begin{cases} 0 & x \leq HL \\ \frac{p}{1-p}\frac{x-HL}{LL-x} & for\ \ x \in (HL, \overline{\overline{b}}] \\ \frac{p}{1-p}\left[\frac{x-HL}{L-x}\right] & for\ \ x \in (LL, \overline{b}] \\ 1 & x > \overline{b} \end{cases}$$

whose density is

$$f(x) = \begin{cases} \frac{p}{1-p}\frac{LL-HL}{(LL-x)^2} & for\ \ x \in (HL, \overline{\overline{b}}] \\ \frac{p}{1-p}\frac{L-HL}{(L-x)^2} & for\ \ x \in (LL, \overline{b}] \\ 0 & otherwise, \end{cases} \tag{17}$$

where

$$\overline{\overline{b}} = \frac{LL^2 + HL(L-2LL)}{L-HL},$$

$$\overline{b} = (1-p)L + pHL.$$

The results are summarized in the following proposition:

Proposition 2 $\left(p \leq \frac{L-LL}{L-HL} \right)$*In an auction with two firms under asymmetric information, firm-L randomizes its bid x in the intervals* $\left(HL, \overline{\overline{b}}\right]$ *and* $\left(LL, \overline{b}\right]$ *according to the probability density function* $f(x)$ *given by (17) and firm-H bids* HL*. Under these bidding strategies, one or two licenses can be sold depending on the realization of bids at the first auction. If, in the first auction, at least one bid is in*$\left(LL, \overline{b}\right]$ *or both firms are type-H, then only one license is sold, whereas if both bids are in* $\left(H, \overline{\overline{b}}\right]$, *then two licenses are sold at a price equal to the maximum bid of the first auction.*

Proof. *See Appendix A.3.* ■

Subcase ii: the upper region vanishes $\left(p > \frac{L-LL}{L-HL}\right)$

For values of p greater than the right-hand side of (16), q, given by (15), is greater than one. But this is not possible, because q is just a portion of the cumulative distribution function $F(x)$. Hence if p is greater than the critical value, q should be taken as 1. Therefore,

$$q = \begin{cases} \frac{p}{1-p} \frac{(LL-HL)}{(L-LL)} & if \quad p \leq \frac{L-LL}{L-HL} \\ 1 & otherwise \end{cases} . \tag{18}$$

Then the story changes dramatically. Now, there is no *region* 2. Players only play in *region* 1 with a new definition of upper bound of *region* 1, $\overline{\overline{b}}_1$. This indicates type-L gives all the weight to *region* 1. Since p is high, a firm-L is unlikely to have a type-L rival in the auction. Trying to deter the entry of type-L rival no longer has the priority, since trying to do so cannot give as much expected profit as squeezing the bids to *region* 1. Again by a similar method used to find $\overline{\overline{b}}$ previously, the upper bound of the support for the case with $q = 1$, $\overline{\overline{b}}_1$, becomes

$$\bar{\bar{b}}_1 = (1-p)LL + pHL, \tag{19}$$

so that the distribution function becomes

$$G(x) = \frac{p}{1-p}\frac{(x-HL)}{(LL-x)} \qquad for\ x \in (HL, \bar{\bar{b}}_1]. \tag{20}$$

As a result, the distribution function when $p > \frac{L-LL}{L-HL}$ is

$$G(x) = \begin{cases} 0 & x \leq HL \\ \frac{p}{1-p}\frac{x-HL}{LL-x} & for\ \ x \in (HL, \bar{\bar{b}}_1] \\ 1 & x > \bar{\bar{b}}_1, \end{cases}$$

whose density function is

$$g(x) = \begin{cases} \frac{p}{1-p}\frac{LL-HL}{(LL-x)^2} & for\ x \in (HL, \bar{\bar{b}}_1] \\ 0 & otherwise. \end{cases} \tag{21}$$

Therefore

Proposition 3 $\left(p > \frac{L-LL}{L-HL}\right)$*In an auction with two firms under asymmetric information, firm-L randomizes its bid* x *in the interval* $\left(HL, \bar{\bar{b}}_1\right]$ *according to the probability density function* $g(x)$ *given by (21), and firm-H bids* HL. *Under this bidding strategy, one or two licenses may be sold depending on the types of the firms. If both firms are type-H or only one is type-L, then only one license is sold. However, if both firms are type-L, then two licenses are sold at a price equal to the maximum bid of the first auction.*

Proof. *Similar argument as in the proof of Proposition 2. See Appendix A.3.*

■

4.1.3 The third case, $LL < H < LH$

Again, as in the second case, since type-H makes negative net profit when it bids H, it lowers its bid below H by a similar argument made in the previous case. Type-H bids HL in this case as well. The functional form of type-L's bid is exactly the same with the bid functions driven in the previous case.[8]

Proposition 4 $\left(p \leq \frac{L-LL}{L-HL} \right)$*In an auction with two firms under asymmetric information, firm-L randomizes its bid x in the intervals $\left(HL, \overline{\overline{b}}\right]$ and $\left(LL, \overline{b}\right]$, according to the probability density function $f(x)$ given by (17), and firm-H bids HL. Under these bidding strategies, one or two licenses can be sold depending on the realization of bids at the first auction. If, in the first auction, at least one bid is in$\left(LL, \overline{b}\right]$ or both firms are type-H, then only one license can be sold, whereas if both bids are in $\left(H, \overline{\overline{b}}\right]$ then two licenses are sold at a price that is equal to the maximum bid of the first auction.*

Proof. *Similar argument as in the proof of Proposition 2. See Appendix A.3.*

■

Proposition 5 $\left(p > \frac{L-LL}{L-HL} \right)$*In an auction with two firms under asymmetric information, firm-L randomizes its bid x in the interval $\left(HL, \overline{\overline{b}}_1\right]$, according to the probability density function $g(x)$ given by (21), and firm-H bids HL. Under this bidding strategy, one or two licenses can be sold depending on the types of the firms. If both firms are type-H, or only one is type-L, then only one license is sold. However, if both firms are type-L, then two licenses are sold at a price equal to the maximum bid of the first auction.*

[8] Recall that only the functional form is the same. Qualitatively, the solution is different than the solution of the second case due to the change in the ordering of profits.

Proof. *Similar argument as in the proof of Proposition 2. See Appendix A.3.*

■

The following theorem summarizes the results from these propositions:

Theorem 6 ***(2-firm Equilibrium Strategies)*** *In the auction game described above, there exists a unique symmetric equilibrium strategy in the following form:*[9]

When $\mathbf{LL} < \mathbf{LH} < \mathbf{H}$, *firm-L randomizes its bid* x *in the interval* $(H, \overline{b}]$ *according to the probability density function*

$$f(x) = \begin{cases} \frac{(L-H)p}{(L-x)(1-p)} & for \quad x \in \left(H, \overline{b}\right] \\ 0 & otherwise, \end{cases}$$

where

$$\overline{b} = (1-p)\,L + pH.$$

If $\mathbf{H} < \mathbf{LL} < \mathbf{LH}$, *or* $\mathbf{LL} < \mathbf{H} < \mathbf{LH}$, *we have two subcases:*

Subcase i $\left(\text{when } p \leq \frac{L-LL}{L-HL}\right)$*: Firm-H bids* HL *and firm-L randomizes its bid* x *in the intervals* $\left(HL, \overline{\overline{b}}\right]$ *and* $(LL, \overline{b}]$ *according to the probability density function*

$$f(x) = \begin{cases} \frac{p}{1-p}\frac{LL-HL}{(LL-x)^2} & for\ x \in (HL, \overline{\overline{b}}] \\ \frac{p}{1-p}\frac{L-HL}{(L-x)^2} & for\ x \in (LL, \overline{b}] \\ 0 & otherwise, \end{cases}$$

where

$$\overline{\overline{b}} = \frac{LL^2 + HL(L - 2LL)}{L - HL},$$

and

$$\overline{b} = (1-p)L + pHL.$$

[9] i.e. This strategy is unique among the symmetric strategies. There may be equilibria with asymmetric strategies.

Subcase ii $\left(p > \frac{L-LL}{L-HL}\right)$ *:Firm-H bids HL and firm-L randomizes its bid x in the interval $\left(HL, \bar{\bar{b}}_1\right]$ according to the probability density function*

$$g(x) = \begin{cases} \frac{p}{1-p}\frac{LL-HL}{(LL-x)^2} & for\ x \in (HL, \bar{\bar{b}}_1] \\ 0 & otherwise, \end{cases}$$

where

$$\bar{\bar{b}}_1 = (1-p)LL + pHL.$$

4.2 Solution for the n-Firm Auction:

4.2.1 Solution for n firms when $LL < LH < H$

If there are n firms participating in the auction, then using the same logic, the respective equations become

$$\bar{b}_n = \left(1 - p^{n-1}\right) L + p^{n-1} H, \tag{22}$$

where $\bar{b}_n$ is the upper limit of bidding for firm L with n participants in the first auction.

$$F_n(x) = \left(\left(\frac{L-H}{L-x}\right)^{\frac{1}{n-1}} - 1\right)\frac{p}{1-p} \tag{23}$$

is the cumulative distribution function of bidding, and

$$f_n(x) = \begin{cases} \frac{p}{(n-1)(1-p)}\left(\frac{L-H}{(L-x)^n}\right)^{\frac{1}{n-1}} & x \in \left(H, \bar{b}_n\right] \\ 0 & otherwise \end{cases} \tag{24}$$

is the density function. Firm H bids its monopoly profit and firm L randomizes its bid x over the interval $\left(H, \bar{b}_n\right]$ according to the probability density function $f_n(x)$ given by (24).

The corresponding proposition for the n-firm auction becomes

Proposition 7 *In the auctions with asymmetric information with n participating firms, firm-L randomizes its bid x in the interval $\left(H, \overline{b}_n\right]$, according to the probability density function $f_n(x)$ given by the equation (24), and firm-H bids H. Only the first license is sold.*

Proof. *See Appendix A.4.* ■

4.2.2 Solution for n-firm when $H < LL < LH$ or $LL < H < LH$

A straightforward extension of the two-firm results gives the following propositions for the n-firm auctions for these cases:

Proposition 8 $\left(p \leq \left(\frac{L-LL}{L-HL}\right)^{\frac{1}{n-1}}\right)$ *In an auction with n firms under asymmetric information, firm-L randomizes its bid x in the intervals $\left(HL, \overline{\overline{b}}_n\right]$ and $\left(LL, \overline{b}_n\right]$ according to the probability density function*

$$f_n(x) = \begin{cases} \frac{p}{1-p}\frac{1}{n-1}\left(\frac{LL-HL}{(LL-x)^n}\right)^{\frac{1}{n-1}} & for\ x \in (HL, \overline{\overline{b}}_n] \\ \frac{p}{1-p}\frac{1}{n-1}\left(\frac{L-HL}{(L-x)^n}\right)^{\frac{1}{n-1}} & for\ x \in \left(LL, \overline{b}_n\right] \\ 0 & otherwise, \end{cases}$$

where

$$\overline{\overline{b}}_n = \frac{LL^2 + HL(L-2LL)}{L-HL},$$

and

$$\overline{b}_n = (1-p^{n-1})L + p^{n-1}HL,$$

and firm-H bids HL. Under these bidding strategies, one or two licenses can be sold depending on the realization of bids at the first auction. If in the first auction any one of bids is in$\left(LL, \overline{b}_n\right]$, if all the firms are type-H, or if there is only one type-L, then only one license can be sold, whereas if there is more than one type-L and all the bids are in $\left(HL, \overline{\overline{b}}_n\right]$, then two licenses are sold at a price *that is equal to the maximum bid of the first auction. So there is a possibility of selling both licenses.*

Proof: *Similar argument as in the proof of Proposition 2.*

Proposition 9 $\left(p > \left(\frac{L-LL}{L-HL}\right)^{\frac{1}{n-1}} \right)$ *In an auction with n firms under asymmetric information, firm L randomizes its bid x in the interval* $\left(HL, \overline{\overline{b}}_{n,1}\right]$, *according to the probability density function*

$$g_n(x) = \begin{cases} \frac{p}{1-p}\frac{1}{n-1}\left(\frac{LL-HL}{(LL-x)^n}\right)^{\frac{1}{n-1}} & for\ x \in (HL, \overline{\overline{b}}_{n,1}] \\ 0 & otherwise, \end{cases}$$

where

$$\overline{\overline{b}}_{n,1} = LL - p^{n-1}(LL - HL),$$

and firm H-bids HL, where $\overline{\overline{b}}_{n,1}$ *denotes the upper bound of the bidding interval. Under this bidding strategy, one or two licenses can be sold depending on the types of the firms. If all the firms are type-H or only one is type-L, then only one license is sold. Otherwise two licenses are sold at a price equal to the maximum bid of the first auction.*

Proof: *Similar argument as in the proof of Proposition 2.*

4.3 The Seller's Revenue

4.3.1 When $LL < LH < H$

Expected revenue of the seller for the two-firm auction is given by

$$R_2 = p^2 H + p^2 \left[\left(\frac{L-H}{L-\overline{b}} \right)^2 (2\overline{b} - L) - 2H + L \right], \tag{25}$$

and for the n-firm auction, it is

$$R_n = p^n H + p^n \left[\left(\frac{L-H}{L-\overline{b}_n} \right)^{\frac{n}{n-1}} \left(n\overline{b}_n - (n-1)L \right) - nH + (n-1)L \right]. \quad (26)$$

4.3.2 When $H < LL < LH$ or $LL < H < LH$:

With the symmetric strategies as specified in Theorem 6 when there are two firms, the revenue of the seller for $p \leq \frac{L-LL}{L-HL}$ is given by

$$\begin{aligned} E(Revenue) &= p^2 HL + 2p(1-p)\left[\int_{HL}^{\overline{\overline{b}}} xf(x)dx + \int_{LL}^{\overline{b}} xf(x)dx \right] \\ &+ (1-p)^2 \left[\begin{array}{c} 4q^2 \int_{HL}^{\overline{\overline{b}}} xF(x)f(x)dx + 2q(1-q)\int_{LL}^{\overline{b}} xf(x)dx \\ +2(1-q)^2 \int_{LL}^{\overline{b}} xF(x)f(x)dx \end{array} \right]. \end{aligned} \quad (27)$$

Also, for the values of $p > \frac{L-LL}{L-HL}$, the expected revenue is given by

$$\begin{aligned} E(Revenue) &= p^2 HL + 2p(1-p)\left[\int_{HL}^{\overline{\overline{b}}_1} xg(x)dx \right] \\ &+ 4(1-p)^2 \int_{HL}^{\overline{\overline{b}}_1} xG(x)g(x)dx. \end{aligned} \quad (28)$$

The expected revenue functions can be calculated analytically, but are very long, and are not easy to interpret it from the analytical formula. Therefore, the expected revenue functions are calculated for all possible values of p and various values of HL, LL, LH, H and L . Figure 1 and 2 give the expected revenue functions for four different value combinations of those parameters - in the figures the curve named as 2-licenses. As seen from the graphs, for some values of parameters as the probability of type-H increases, the revenue first decreases then increases to some extent, reaches a local maximum and then decreases. At first glance, it seems revenue should decrease as p increases. However, because of the rule about the reserve price of the second auction, there is another effect pushing up the revenue for some p values. Although the bids are becoming smaller as p increases, the probability of selling two licenses also increases. Therefore, the probability of receiving twice the maximum bid of the first auction becomes larger. The result is that the revenue increases for these values of p. If one looks at Figure 3 and 4, which gives the expected number of licenses sold for the same parameter values I used to calculate the revenues in Figure 1 and 2, the local maximum point of the revenue function achieved at the peak of graphs in Figure 3 and 4. The amount of increase at the revenue function depends on the expected number of licenses sold, for a larger expected number of licenses sold, there is a larger increase at the revenue. In the light of these facts, we have the following theorem for the revenue of the seller.

Theorem 10 ***(Revenue)*** *In the auction game described above, under the unique symmetric equilibrium strategies given in Theorem 6, the seller has the following expected revenue functions:*

If $LL < LH < H$, only the first license is sold. The seller's expected revenue is

$$R = p^2 H + p^2 \left[\left(\frac{L-H}{L-\overline{\overline{b}}_2} \right)^2 (2\overline{b}_2 - L) - 2H + L \right].$$

If $H < LL < LH$ or $LL < H < LH$, one or two licenses can be sold depending on the realization of bids at the first auction. If, in the first auction, any one of the bids is in $(LL, \overline{b}]$, then only one license can be sold, whereas if both bids are in $\left(HL, \overline{\overline{b}}\right]$, then two licenses are sold at a price that is equal to the maximum bid of the first auction. Under these conditions and with $p \leq \frac{L-LL}{L-HL}$, the seller's expected revenue is

$$E(Revenue) = p^2 HL + 2p(1-p) \left[\int_{HL}^{\overline{\overline{b}}} x f_1(x) dx + \int_{LL}^{\overline{b}} x f_2(x) dx \right]$$

$$+(1-p)^2 \left[\begin{array}{c} 4q^2 \int_{HL}^{\overline{\overline{b}}} x F_1(x) f_1(x) dx + 2p(1-q) \int_{LL}^{\overline{b}} x f_2(x) dx \\ +2(1-q)^2 \int_{LL}^{\overline{b}} x F_2(x) f_2(x) dx \end{array} \right],$$

and with $p > \frac{L-LL}{L-HL}$, *the seller's expected revenue is*

$$E(Revenue) = p^2 HL + 2p(1-p)\left[\int_{HL}^{\bar{\bar{b}}_1} xg(x)dx\right] + 4(1-p)^2 \int_{HL}^{\bar{\bar{b}}_1} xG(x)g(x)dx,$$

where

$$\bar{\bar{b}} = LL - \frac{(1-p)q}{(1-p)q+p}\left(L - LL\right),$$
$$\bar{\bar{b}}_1 = (1-p)LL + pHL,$$

and

$$\bar{b} = (1-p)L + pHL.$$

5 Comparison with other auctions:

As seen from the results, the seller can sell only one license by using this auction format if H, the monopoly profit of type-H, is greater than LH, the duopoly profit of type-L firms when the other firm is type-H. This result occurs because there is no threat to the winner of the first auction. Once a firm has won the first license, its bid deters entry to the second auction. The winner of the first auction may be type-H or type-L. If it is type-H, then all the other firms are also type-H, and since the winner pays its monopoly profit, nobody can pay more at the second auction as they are all type-H. If the winner is type-L, then the remaining firms for the second auction can be either type-L or type-H. However, the types of the firms do not matter anymore, since any type-H cannot bid more than its monopoly profit, the winner of the first pays at least the monopoly profit of type-H, and any type-L cannot beat the bid of the winner. If it wins, it would get only duopoly profit whereas it pays more.

Now, let us compare the results with a second-price, sealed-bid auction setup to sell only one license, the monopoly right. In that type of auction, the revenue for the seller is

$$R_n = H(p^n + np^{n-1}(1-p)) + L(1 - p^n - np^{n-1}(1-p)) \tag{29}$$

If we put the definition for $\overline{b}_n$, (22), into the revenue function, (26), we get exactly the same function as in (29). This tells us that if $LH < H$ holds, then there is no difference between using this Turkish style auction and the second-price, sealed-bid auction to sell the monopoly right.

If $H < LL < LH$ or $LL < H < LH$ holds, then the story changes dramatically. This time, there is a symmetric mixed strategy equilibrium with two separate supports. The expected profits are equal throughout these supports. In these cases, the seller's revenue is given (27) and (28) under the parametric restrictions specified previously. Since analytically it is very difficult to compare (27) and (28) with (29), the same values of HL, H, LL, LH, and L from $Figure$ 1 and 2 are used to calculate the revenue of the seller if the seller sells only one license with second-price, sealed-bid auction for comparison. The revenue functions are the solid curves in Figure 1 and Figure 2. As seen from the graphs, selling the monopoly right with second-price, sealed-bid auction format always gives more revenue to the seller than the Turkish style auction setup. The following theorem summarizes the results about the revenues.

Theorem 11 *If $LL < H$, the design in this auction produces exactly the same revenue for the seller as selling the monopoly right. Otherwise, selling the monopoly right with the second-price, sealed-bid auction creates more revenue for the seller than the Turkish style auction.*

6 Conclusion:

In the cases analyzed above, one firm is planning to attempt to become a monopoly in the industry. Low-cost type firm has the resources to reach that aim under the conditions mentioned above. If the firms have the same cost, one of them still becomes a monopoly in the industry. Increased number of firms in the race forces firms to bid more aggressively. Not only the upper boundary of the bidding interval $\overline{b}$ increases, but also the participants give more weight to the upper sections of the bidding interval. In any case, the efficient firm receives the license, and the government receives more revenue as the number of firms in the auction increases as expected. Under some other parametric restrictions, the bidders' strategies change. Since the threat of resulting in a duopoly is credible, they arrange their bidding behaviors accordingly taking this threat into account and now bid from two separate regions. In addition, both licenses can be sold if the participants play the symmetric strategies specified in this paper. This auction setup creates the same revenue as selling the monopoly right with the second-price, sealed-bid auction for some profit orders; for the other profit orders it creates less revenue for the seller.

References

[1] Alvarez, Francisco (2001). "Multiple Bids in a Multiple-Unit Common Value Auction: Simulations For The Spanish Auction," Mimeo, Universidad Complutense.

[2] Bernhardt, Dan and Scoones, David (1994). "A Note on Sequential Auctions," AER, Vol.84 (3).pp.653-57.

[3] Gandal, Neil.(1997). "Sequential Auctions of Interdependent Objects: Israeli Cable Television Licenses," J. of Industrial Economics, Vol.45 (3), pp.227-44.

[4] http://www.milliyet.com.tr/2000/04/13/ekonomi/eko00.html

[5] Jehiel, Philippe and Moldavanu, Benny (2000a). "License Auctions and Market Structure," discussion paper, University of Mannheim.

[6] Jehiel, Philippe and Moldavanu, Benny (2000b). "Auctions with Downstream Interactions Among Buyers," RAND Journal of Economics, Vol. 31 (4), 2000, pp. 768-91.

[7] Jehiel, Philippe and Moldavanu, Benny (1998). "Efficient Design with Interdependent Valuations," Northwestern University Center for Mathematical Studies in Economics and Management Science, Discussion Paper: 1244. p.28.

[8] Klemperer, Paul (1999). "Auction Theory: A Guide to the Literature," J. of Economic Surveys, Vol 13 (3). pp. 227-86.

[9] Klemperer, Paul (2000). "Why Every Economist Should Learn Some Auction Theory," Invited Lecture to 8th World Congress of the Econometric Society.

[10] Klemperer, Paul (2001). "What Really Matters in Auction Design," Mimeo, Oxford University.

[11] Lambson, Val E. and Thurston, Norman (2001). "Sequential Auctions: Theory and Evidence from the Seattle Fur Exchange," mimeo, Brigham Young University.

[12] Mazon, Christina and Nunez, Soledad (1999). "On the Optimality of Treasury Bond Auctions: The Spanish Case," Mimeo, Universidad Complutense de Madrid.

[13] Pitchik, Carolyn and Schotter, Andrew (1988). "Perfect Equilibria in Budget Constrained Sequential Auctions: An Experimental Study," RAND Journal of Economics, 19 (3). pp.363-88.

[14] Varian, Hal R. (1980). "A Model of Sales." American Economic Review, Vol. 70 (4). pp. 651-59.

[15] von der Fehr, Nils-Henrik Morch (1994). "Predatory Bidding in Sequential Auctions." Oxford Economic Papers, Vol. 46 (3). pp. 345-56.

A Appendix

A.1 The Distribution function F(x) is continuous

Proof. There is no point mass in the density function. If the amount x were bid with positive probability mass, i.e. if there is a jump in the graph of F(x), there would be a positive probability of a tie at x. If deviant bids slightly higher, i.e. x+ε , with the same probability with which the other bids x, it looses an amount ε however, it increases the probability of recieving the license with the amount of the jump. Thus, giving a positive probability to point x cannot be part of a symmetric equilibrium. Therefore, the distribution function of bidding has no jumps from zero to the potential maximum bid, implying it is a continuous function in this interval. Can there be a jump at the potential maximum bid? No, because bidding the potential maximum and bidding a bit less than it gives the same payoff. Hence, there is no meaning in increasing the weight of the potential maximum bid. ■

A.2 Proposition 1:

Proof: *If a firm of type H bids less than H, given the other players' bids as specified in the proposition, then the firm-H is going to loose the auction for sure and receives zero. If a type L firm bids less than H, then it is going to loose the first auction, given the other players play the proposed strategy. Therefore, firm-L does not want to lower its bid below H . What about bidding more than $\overline{b}$? Is it better than bidding any x in $\left(H, \overline{b}\right]$? If the firm-L bids $\overline{b} + \epsilon$, then it is going to receive*

$$L - \overline{b} - \epsilon. \tag{30}$$

However, if it bids any x from the specified interval, it is going to receive

$$p(L-H). \tag{31}$$

Placing $\overline{b}$ *from (6) into (30) gives* $p(L-H)-\epsilon$*, which is obviously less than the value in (31). Bidding* x *in the specified interval is better for firm-L than bidding outside of this interval.*

Lastly, let us see that bidding any x *in* $\left(H,\overline{b}\right]$ *gives the same expected profit.*

$$E\ (profit) = (L-x)\left[p+(1-p)F(x)\right]. \tag{32}$$

Placing $F(x)$ *from (9) into (32) gives*

$$E\ (profit) = p(L-H), \tag{33}$$

which is constant irrespective of the choice of x*. Therefore, any* x *in* $\left(H,\overline{b}\right]$ *gives the same expected profit. Since no firm wants to deviate, the specified bidding strategy is an equilibrium.*

A.3 Proposition 2:

Proof: *If the firm-H bids less than* HL*, given the other player bids the specified amount in the proposition, then the firm-H is going to lose the auction for sure and receives zero. If firm-L bids less than* HL*, then it is going to lose the first auction, given the other player plays the proposed strategy. Therefore, firm-L does not want to lower its bid below* HL*. What about bidding* $x \in (\overline{\overline{b}}, L]$*? Is this a good idea for firm-L? Now, bidding* $x \in (\overline{\overline{b}}, L]$ *cannot increase* $F_1(x)$ *and does not change* $F_2(x)$*, i.e. there is no change in the winning probability. However, bidding* $x \in (\overline{\overline{b}}, L]$ *decreases expected profit because the bidder is, now, going to pay more if it wins, although the expected revenue stays the same implying less* *expected net profit. To see this, let firm-L bid* $\overline{\overline{b}} + \epsilon$*. Then it receives*

$$p(L - \overline{\overline{b}} - \epsilon) + (1-p)q(LL - \overline{\overline{b}} - \epsilon).$$

However, bidding $\overline{\overline{b}}$ *gives*

$$p(L - \overline{\overline{b}}) + (1-p)q(LL - \overline{\overline{b}}),$$

which is obviously greater. So there cannot be any bid in $(\overline{\overline{b}}, L]$ *.*

Can there be any bid x *in* $(\overline{b}, L]$ *? Again by bidding* $\overline{b}$*, the bidder is going to receive the license definitely, and therefore there is no reason to increase the payment in order to receive the license, since this will decrease the expected profit of the bidder. To see this, let the firm-L bid* $\overline{b} + \epsilon$ *then it is going to receive*

$$L - \overline{b} - \epsilon = (p + (1-p)q)(L - LL). \tag{34}$$

However, if it bids any x *from the specified interval, it is going to receive*

$$\begin{aligned} &p(L-x) + (1-p)(F_2(x) + q)(L-x) \\ &= (p + (1-p)q)(L-x) + (1-p)F_2(x)(L-x), \end{aligned} \tag{35}$$

which is obviously greater than (34). Therefore, bidding in the specified interval is better than bidding outside of this interval.

Lastly, let us see that bidding any x *in* $(\overline{\overline{b}}, L]$ *and* z *in* $(LL, \overline{b}]$ *give the same expected profit. Any* x *in* $(\overline{\overline{b}}, L]$ *gives*

$$p(L-x) + (1-p)F_1(x)(LL - x) = (p + (1-p)q)(L - LL). \tag{36}$$

Also, any z *in* $(LL, \overline{b}]$ *gives*

$$p(L-x) + (1-p)(F_2(x) + q)(L-x) = (p + (1-p)q)(L - LL). \tag{37}$$

Since the terms in equations (36) and (37) are the same and independent from the choice of x or z, any bid in either of these intervals produces the same expected profit.

A.4 Proposition 3:

Proof*: The logical flow is the same as in the previous proof. This time, firm-H bids H, because any other bid give negative net profit. If firm-L bids less than H, then it is going to lose the first auction, given the other players play the proposed strategy. Therefore, firm-L does not want to lower its bid below H. What about bidding more than $\overline{b}_n$? Is it better than bidding any x in $\left(H, \overline{b}_n\right]$? If the firm-L bids $\overline{b}_n + \epsilon$, then it is going to receive*

$$L - \overline{b}_n - \epsilon. \tag{38}$$

However, if it bids any x from the specified interval, it is going to receive

$$p^{n-1}(L - H). \tag{39}$$

Placing $\overline{b}$ from (22) into (38) gives $p^{n-1}(L - H) - \epsilon$, which is obviously less than the value in (39). Bidding x in the specified interval is better for firm-L than bidding outside of this interval.

Does any x in $\left(H, \overline{b}\right]$ give the same expected profit?

$$\begin{aligned} E(profit) = (L - x) \Big[& p^{n-1} + \tbinom{n-1}{1} p^{n-2}(1-p)F_n(x) \\ & + \tbinom{n-1}{2} p^{n-3}(1-p)^2 F_n^2(x) \\ & + \ldots. \\ & + \tbinom{n-1}{n-2} p(1-p)^{n-2} F_n^{n-2}(x) + (1-p)^{n-1} F_n^{n-1}(x) \Big], \end{aligned}$$

which can be written as

$$= (L - x) \left[p + (1-p)F_n(x) \right]^{n-1}. \tag{40}$$

Placing $F_n(x)$ from (23) into (40) gives

$$= p^{n-1}(L - H), \tag{41}$$

which is constant irrespective of the choice of x. Therefore, any x in $\left(H, \overline{b}_n\right]$ gives the same expected profit. As a result, no firm wants to deviate, meaning that the specified bidding strategy is an equilibrium.

A.5 2000 Turkish Mobile Phone License Auction As An Example:

The described auction design was used by the Turkish Government in selling second generation GSM licenses on April, 2000. The government offered two licenses to the market. Five groups participated in the race. Each group was composed of by a group of domestic firms and a foreign partner. These groups were: 1) Isbankasi-Telecom Italia, 2)Dogan-Dogus-Sabanci Holding Companies and Telefonica Spain, 3) Genpa-Atlas Construction, Atlas Finance, Demirbank and Telenor Mobile Communications, Norway, 4)Fiba-Suzer -Nurol Holding Companies, Finansbank, Kentbank and Telecom France, and 5) Koctel Telecommunication Services and SBC Communications Inc.,US. Their bids are given in Table 1.

According to the bids given in the Table 1, group 1 received the first license. Other groups were invited to the second auction but did not participate, since the reserve price for the second auction was set at 2.525 billion US$, which is the price of the first unit. As a result, only one of the two licenses was sold.

Group	Bid
1	2,525
2	1,350
3	1,224
4	1,017
5	1,207

Table 1: Bids in the second generation GSM license auction in Turkey in 2000, in million $US

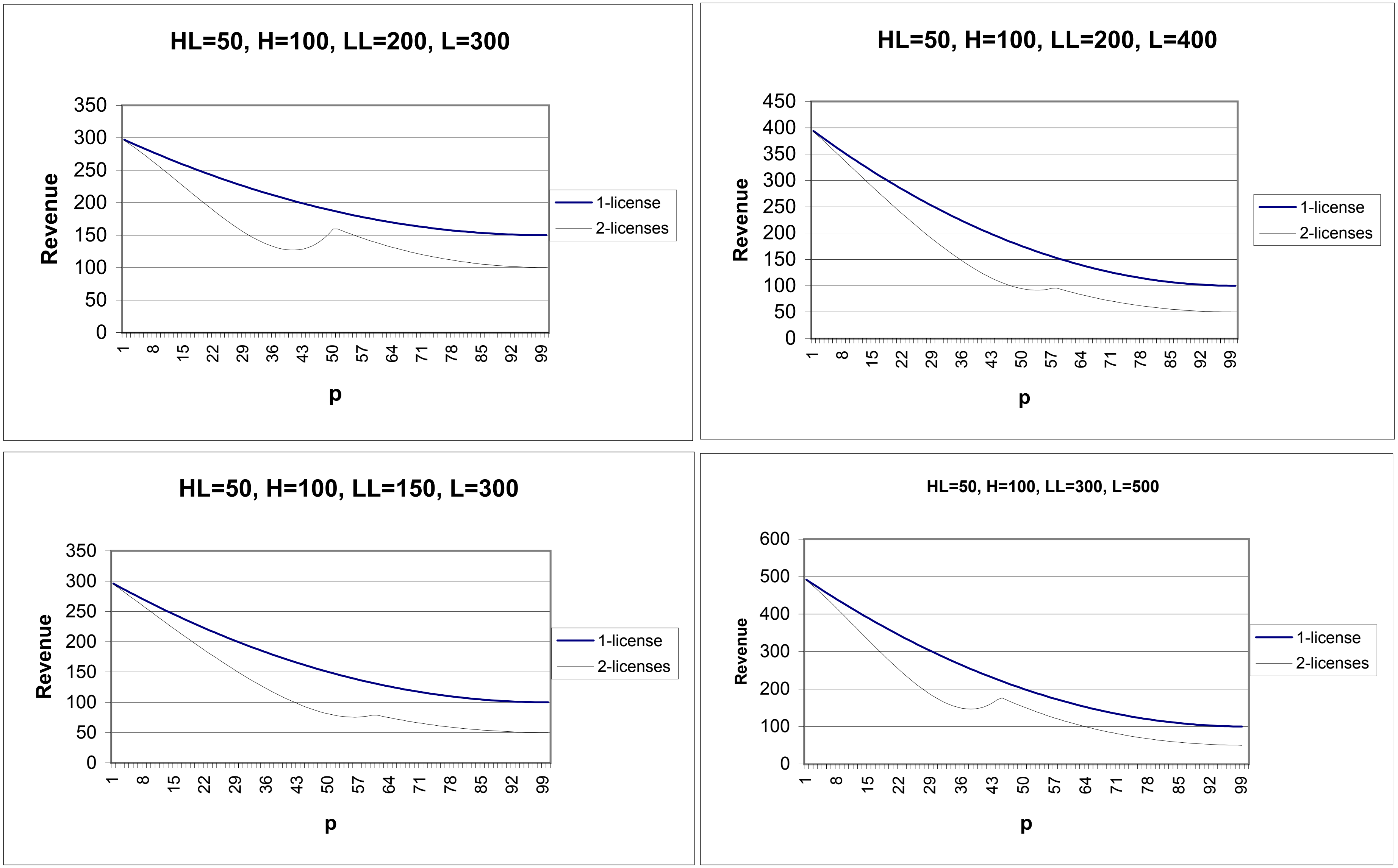

Figure 1: Comparison of the revenue generated by the model and selling the monopoly right. In all cases, selling monopoly right generates more revenue for the seller.

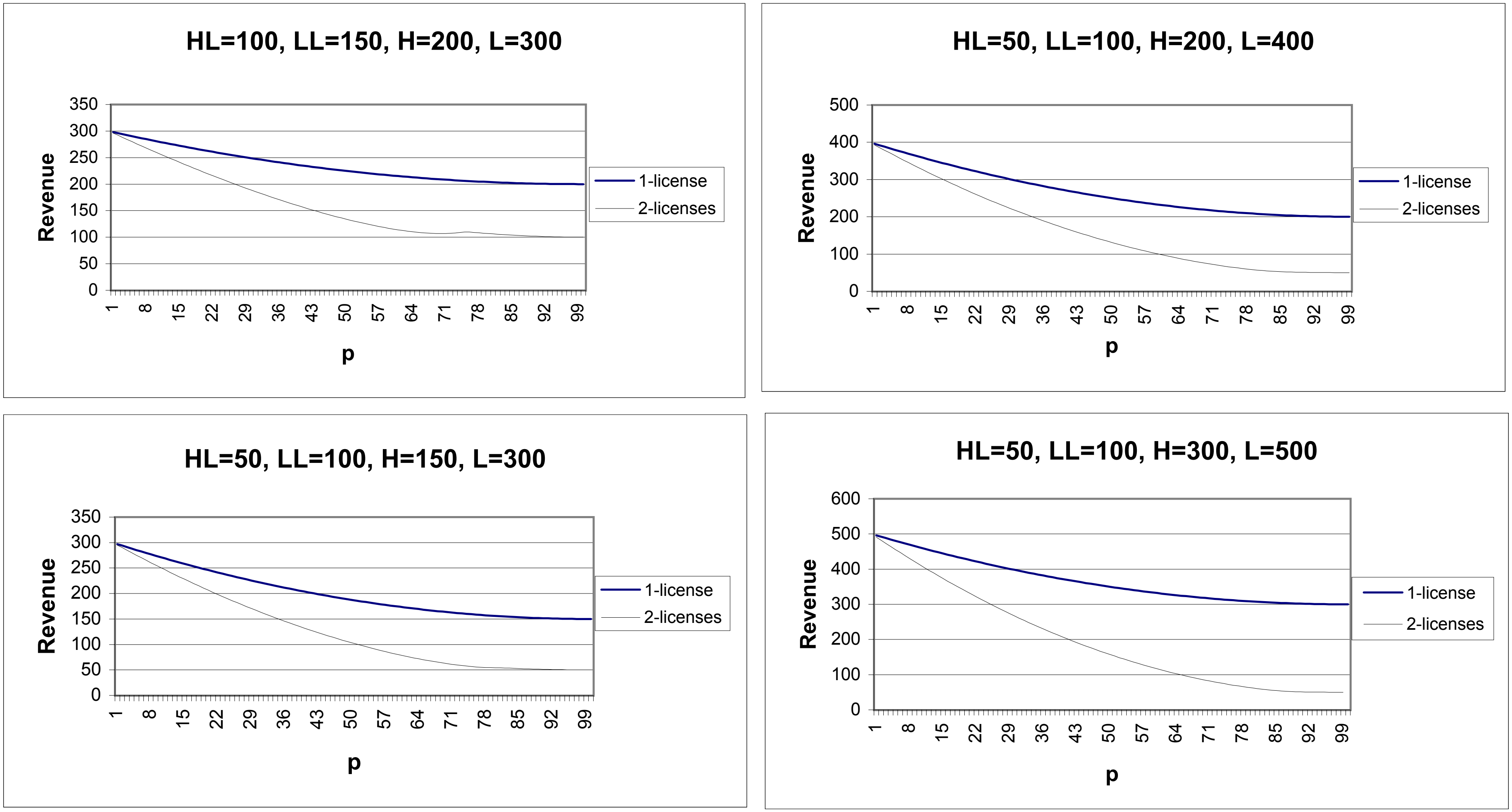

Figure 2: Comparison of the revenue generated by the model and selling the monopoly right. In all cases, selling monopoly right generates more revenue for the seller.