# The Cyclical Advancement of Drastic Technologies

# Work-in-Progress

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Drastic technological changes are cyclical because basic R&D is carried on only at times when entrepreneurial profits for incremental technologies of the prevailing technological paradigm falls below the profits promised by the next technological paradigm. The model is essentially an endogenous technological change framework. Varieties, input to the final good production, are composite goods. Each composite good is produced by a set of intermediaries, outgrowths of basic R&D and applied R&D. The basic intermediate, product of basic R&D, is modeled as in Romer (1990). Complementary intermediates, the outgrowths of applied R&D, do show the property of falling profits. The falling character of profits implies that basic R&D becomes more yielding than applied R&D at certain points in time. Research people switches back and forth between the applied and basic research sectors, creating cycles in the advancement of drastic technologies and economic activity.

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## 1. Introduction

It has been first debated by Kondratieff (1926) that capitalism has long waves, regular fluctuations in economic life with a wavelength of 45-60 years. Schumpeter (1939) proposed that the cause of long run cycles might be *discontinuities* in the process of drastic technical innovation and hence major innovations may cluster in certain periods. Historical evidence indeed indicates that long run technological progress is hardly a smooth process (Olsson, 2001; Gordon, 2000; Mokyr 1990; van Duijn, 1983; Mensch, 1979).

Given the significant effect of technological change on economic growth and on standards of living (Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992), a better understanding of the reasons behind the cyclical evolution of technology may be useful in the Pareto sense. In particular, smoothing out that cyclical advancement may mean an improvement in the long run performance of an economy, and, for that reason, there may be room for policy-makers.

Why does drastic technological change *tends to* proceed in a cyclical fashion? This paper conjectures that the main factor behind observing that drastic technological changes appear in clusters is eventually exhausting profit opportunities in incremental technologies. We develop a simple model where researchers of R&D sector exploit profit opportunities of the prevailing technological paradigm by making incremental, non-drastic innovations. As profit opportunities become exhausted, it becomes more yielding to invest in a new technological paradigm at a certain point. Researchers then switch to work on the next drastic innovation (technological paradigm). Incremental innovation resumes within the new paradigm and it lasts as long as the existing profit opportunities are higher than investing in the next technological paradigm. In this way, drastic technological change and hence economic development proceeds in long waves.

Surprisingly, the clustered appearance of drastic technologies has not received sufficient attention from growth theory. Recently, David (1990) and especially Bresnahan and Trajtenberg (1995) have made the term general-purpose technology

(GPT) popular to the growth theory. The main aim of this literature is to emphasize the difference between drastic technologies and incremental technological changes in terms of their growth implications. Currently, the focus seems to be on whether an economy experiences a slowdown at the onset of a new technological change due to reallocation of resources from the old to the new sectors or not (see several chapters in Helpman, 1998). Hence, the focus is on temporary cycles that may be created by new technological paradigms at the onset of their introduction to the economy. Our paper states the conditions for a slowdown at the onset of a new technological paradigm in addition to explaining Kondratieff waves.

Our model is essentially an extension of Romer (1990). A snap shot of the model is as follows: We assume that there are two types of R&D-sector, basic and applied, in the model. The former advances drastic technological changes and the latter introduces incremental technologies. Each drastic technology is owned by a monopolist. Nevertheless, that monopolist is not the only beneficiary of the new technological paradigm. With a new GPT, production of a set of complementary intermediaries becomes possible (via applied R&D). Entrepreneurs are eager to develop these complementary products/ services because there are positive profit opportunities. In other words, a new GPT opens new opportunities in the market for complementary ideas that have not been available before. This characteristic of our model is based on the well-known assumption that ideas are public goods, at least partly. Clearly, each complementary intermediate needs to be invented, which is done by the applied R&D sector. We presume that both R&D sectors use the same input, namely research people, and thus they compete for scarce resources. In our set up, a patent-holder of GPT must indeed support the invention/production of complementary intermediaries because the basic produce of GPT (i.e., basic intermediate) and the complementary intermediaries complement each other in producing the new composite good, which is an input for the final good production. Hence, the new composite good does not appear in the 'statistics' unless the basic and all complementary intermediates are invented and produced.

A good example to the exercise that we advanced here is perhaps the computer (ICT). If the microprocessor is a GPT, then all other hardware and software can be classified as complementary inputs. Clearly, a microprocessor is useless unless hardware and software complement it. If ICT is a drastic technological change, then all

complementary products ranging from infrastructure-related materials and investments to re-organization of production processes may be considered as complementary goods.

The organization of the paper is as follows. The next section introduces the model in its basic form, and solves the model at long run equilibrium. This section indicates that exhausting profits in 'applied technology' can indeed be the source of clustered drastic technological changes. An interesting finding of this section is that not the level of (skilled) labor but the growth rate of it enhances endogenous growth. The last section concludes the paper.

## 2. The Model

Let us suppose that the final good Y production technology is represented by

$$Y = L^{1-\beta} \sum_{i=1}^{B} z_{i}^{\beta} \qquad \qquad 0 < \beta < 1$$
 (1)

where L represents unskilled labor used in the production of GDP,  $z_i$  is a *composite* good,  $1-\beta$  is the partial output elasticity of unskilled labor, and *i* is the index of technological paradigm (GPT). The higher the *i*, the more recent the GPT that a composite good (or any other variable) is associated. Many endogenous technological change models, following Romer (1990), presume that a vector of single inputs, which is scaled up constantly due to technological change, produces the final output (and thus generates endogenous growth). In this paper, diverging from Romer (1990), we assume that a vector of composite goods is input to the final good production. As usual, we assume that the final-good sector is a perfectly competitive market.

We suppose that the composite good production technology is represented by the following Cobb-Douglas function:

$$z_{i} = \prod_{j=0}^{n_{i}} (x_{ij})^{\alpha_{ij}} \qquad \forall i = 1, 2, \dots, B; \ \forall j = 0, 1, \dots, n_{i}; \ \sum_{j=0}^{n_{i}} \alpha_{ij} = 1, \ \alpha_{ij} > 0 \qquad (2)$$

In equation (2),  $n_i$  is a positive integer indicating the number of intermediaries that the i<sup>th</sup> composite good is made up of (at time *t*). Note that  $n_i$  represents the critical mass in this formulation.<sup>1</sup>  $x_{ij}$  is the j<sup>th</sup> intermediary used in the production of the i<sup>th</sup> composite input, and  $\alpha_{ij}$  indicates the *relative share* of j<sup>th</sup> input in the total product of composite good  $z_i$ .<sup>2</sup> Evidently,  $\alpha_{ij}$  can be equivalent to or different from  $\alpha_{ij}$  for  $\forall i, i' \in B$ . Furthermore, without loss of generality, we assume that  $\alpha_{ij} \neq \alpha_{ij'}$ ,  $\forall j, j' \in n_i$ . This assumption implies that none of any pair of  $(\alpha_{ij}, \alpha_{ij'})$  are alike.

We make a set of assumptions. First, recall that we assumed a new technological paradigm led to 'emergence' of a *set* of intermediate goods,  $x_{ij}$ , rather than a single intermediate. For the matter of simplicity, we label *one* of these intermediate sectors as the *core* intermediary. Clearly, the core intermediary is the main outgrowth of the new paradigm. We argue that the GPT idea fuels also a set of intermediary sectors that we coin them as *complementary intermediaries*.<sup>3</sup> In accordance with this assumption, equilibrium in this work mean the state that the final good production is made by a (growing) set of technological paradigms that are fully developed.

Second, we assume that one firm holds patent right of the GPT and therefore incurs fully the patent costs. This is the core intermediary. Complementary firms, on the other hand, do not pay any patent fee to the GPT idea. That is, the new paradigm (the idea) is a *public good* for them. Nevertheless, this does not mean that these complementary firms do not involve in any patent race. Though the GPT-idea is freely available, these firms need to develop a complementary product/ service. This certainly requires incurring some costs, and we collect these costs under the name of applied R&D costs.

<sup>&</sup>lt;sup>1</sup> We will show that  $n_i$  is determined endogenously within the model.

<sup>&</sup>lt;sup>2</sup> It may also be called *cost share* and *budget share*.

<sup>&</sup>lt;sup>3</sup> Complementary intermediaries refer to "innovational complementarity" in the GPT literature.

Third, we assume that only the outgrowth of the core intermediary accumulates in the economy. The complementary intermediaries are considered as services/ 'perishable' goods. We indicate the core sector by assigning j = 0 to that sector, and, similarly, we use  $j = 1,...,n_i$  to designate complementary intermediaries. Unless otherwise stated, 0 and *j* designate the two types of intermediaries of the model from now on.

Fourth, we assume that all economic agents have perfect foresight. This assumption is crucial because determination of equilibrium in the model requires forward-looking behavior.

Finally, we assume that the basic R&D sector focuses on fundamental research and the applied R&D on inventing complementary intermediaries for the *most recent* technological paradigm in the economy. Details of the model are as follows:

### The Final Good Sector

A representative firm's profits are

$$\Pi_{Y} = L^{1-\beta} \sum_{i} z_{i}^{\beta} - \sum_{i} p_{i} z_{i} - wL$$
(3)

where composite output price is normalized to one, L is unskilled labor used in the production of final good,  $p_i$  is the user cost (price) of the composite input, and w is the rental price of unskilled labor. First order conditions with respect to  $z_i$  and L are

$$p_i = \beta L^{1-\beta} z_i^{\beta-1} \tag{4}$$

$$w = (1 - \beta)L^{-\beta}\sum_{i} z_{i}^{\beta}$$
(5)

Neither equation (5) is standard unskilled labor demand function nor is equation (4) an (inverse) input demand function. At least, not yet in their explicit form. In order to find out the explicit labor demand function and input demand function for intermediaries  $x_{i0}$  and  $x_{ij}$ , we must first associate the first order conditions of the final good market to the composite good production technology.

The Cobb-Douglas composite good technology for the latest GPT activates after the complementary intermediates of it has been fully developed (cf., the critical mass assumption in Helpman and Trajtenberg (1998)). One way to link the final good sector to intermediary markets is to use cost minimization. Let us suppose that the intermediary-good prices are denoted by  $(q_{i0}, q_{i1}, \dots, q_{in_i})$ , in which the first price is associated with the core sector,  $x_{i0}$ , and others are associated by the complementary sector,  $(x_{i1}, \dots, x_{in_i})$ . Then, total costs corresponding to the composite good *i* is  $C_i = \sum_{i=0}^{n_i} q_{ij} x_{ij}$ , and the cost-minimizing Lagrangean is

$$\Gamma_i = \sum_{j=0}^{n_i} q_{ij} x_{ij} + \lambda_i \left\{ \overline{z}_i - \prod_{j=0}^{n_i} (x_{ij})^{\alpha_{ij}} \right\}$$
(6)

Cost minimization gives

$$q_{ij} \cdot x_{ij} = \lambda_i \alpha_{ij} z_i \qquad \qquad \forall j \qquad (7)$$

Summation of equation (7) over j gives

$$\sum_{j} q_{ij} x_{ij} = C_i = \lambda_i z_i \sum_{j} \alpha_{ij} \Rightarrow$$

$$C_i = \lambda_i z_i$$
(8)

Equation (8) says that the cost of producing the composite intermediate  $z_i$  is shadow price of composite input times quantity. Throughout the study,  $\lambda_i$  will work as shadow price and unit price of composite input *i*.

Substituting the optimum condition for the j<sup>th</sup> intermediate of the i<sup>th</sup> GPT,  $x_{ij}$ , from equation (7) into equation (2) gives

$$\lambda_i = \prod_{j=0}^{n_i} \left( \frac{q_{ij}}{\alpha_{ij}} \right)^{\alpha_{ij}}$$
(9)

Equation (9) shows that the shadow price of the i<sup>th</sup> composite input,  $\lambda_i$ , is weighted geometric average of input prices (weighted by cost shares). Substituting back the value of  $\lambda_i$  from equation (9) into equation (7) gives

$$x_{ij} = \frac{\alpha_{ij} \cdot z_i}{q_{ij}} \prod_{j=0}^{n_i} \left(\frac{q_{ij}}{\alpha_{ij}}\right)^{\alpha_{ij}} \qquad \forall i, j$$
(10)

Yet,  $x_{ij}$  is not in its ultimate (explicit) form because the composite input  $z_i$  is function of  $x_{ij}$ ,  $\forall j$ , and therefore the right hand side of equation (10) is function of  $x_{ij}$ . Consequently, the final step is to get rid of  $z_i$  on the right hand side of equation (10). Naturally, the derived demand relationship between  $z_i$  and  $x_{ij}$  comes from the first order profit maximization condition of the final-good producer. Thus, when equation (4) is used, equation (10) takes the form of

$$x_{ij} = \left(\frac{\alpha_{ij}}{q_{ij}}\right) \beta^{\sigma} L \left[\prod_{j=0}^{n_i} \left(\frac{q_{ij}}{\alpha_{ij}}\right)^{\alpha_{ij}}\right]^{1-\sigma} \qquad \forall i,j$$
(11)

where  $\sigma = 1/(1 - \beta)$ . Note that we can express equation (11) also as

$$x_{ij} = \left(\frac{\alpha_{ij}}{q_{ij}}\right) \beta^{\sigma} L \left[\prod_{j=0}^{n_i} \left(\frac{\alpha_{ij}}{q_{ij}}\right)^{\alpha_{ij}(\sigma-1)}\right] \qquad \forall j$$
(12)

This is the inverse input-demand function. As there are two types of intermediary goods for any GPT, we must consider them apart.

## The Core Sector--Preliminaries

Let us first consider the core sector, indexed by 0. The derived demand function of the core sector,  $x_{i0}$ , by using equation (12), is get as

$$x_{i0} = \left(\frac{\alpha_{i0}}{q_{i0}}\right)^{1+(\sigma-1)\alpha_{i0}} \cdot \beta^{\sigma} \cdot L \cdot \left[\prod_{j=1}^{n_i} \left(\frac{\alpha_{ij}}{q_{ij}}\right)^{\alpha_{ij}(\sigma-1)}\right]$$
(13)

Thus, we defined the input demand function for the very first sector of the i<sup>th</sup> GPT. In equation (13),  $1+(\sigma-1)\alpha_{i0} > 0$  due to the fact that  $\sigma > 1$ , and thus, own price elasticity of demand is negative (there is an inverse relationship between the input demand and its own price). Surely, prices of service goods do not have any (cross) price effect on the core intermediary.

We shall continue to handle the core sector's profit maximization problem in the standard way. We assume that this sector behaves à la Romer (1990). That is, there is a monopolist holding patent rights of the core product associated with a GPT. At cost of

some repetition, we would like to show the derivations. The profit equation of any intermediate firm in the complementary sector is

$$\pi_{i0} = q_{i0} \cdot x_{i0} - r\eta x_{i0} \tag{14}$$

Equation (14) tells us that each unit of production of  $x_{i0}$  uses  $\eta x_{i0}$  units of resources. Profit maximization leads to the well-known Amoroso-Robinson condition that

$$q_{i0} = r\eta \frac{\varepsilon_{i0}}{1 + \varepsilon_{i0}} \tag{15}$$

where  $\varepsilon_{i0} = -[1 + (\sigma - 1)\alpha_{i0}] < 0$  is the own price elasticity of input  $x_{i0}$ . Thus, equation (15) becomes

$$q_{i0} = r\eta \left[1 + \frac{1}{(\sigma - 1)\alpha_{i0}}\right] = r\eta \left[1 + \frac{1 - \beta}{\beta \alpha_{i0}}\right] = r\eta \phi_{i0}$$
(16)

where  $\phi_{i0} > 1$ , and the monopolist follows mark-up over unit cost pricing.

Let us analyze the relationship between  $\phi_{i0}$  and  $\alpha_{i0}$ , which, we will soon show, is also applicable to the complementary intermediaries. There is an inverse parabolic relationship between  $\phi_{i0}$  and  $\alpha_{i0}$ , which can be illustrated as:

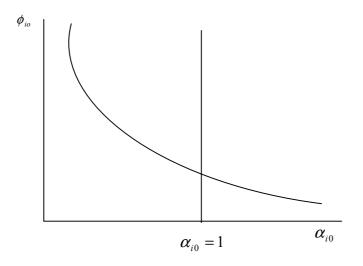


Figure 1 The inverse relationship between the  $\phi_{i0}$  and the  $\alpha_{i0}$ 

The importance of this relationship is that it is *monotonic* and *one-to-one*.<sup>4</sup> For that reason, we may switch back and forth between  $\phi_{ij}$  and  $\alpha_{ij}$  at all times. Note that markup is negatively associated with price elasticity of  $x_{ij}$  and the latter is positively associated with the relative shares. Hence, the higher the relative share of the input the lower the markup over marginal cost.

Note that it is *not* matter of substituting the core sector input price (cf. equation (16)) into the respective demand (c.f. equation (13)) in order to solve the short run equilibrium demand for  $x_{i0}$  because we need to determine input prices for services sectors before proceeding further. The next step does this.

### The Complementary Sector

The complementary sector works as follows. When a GPT appears in the market, the idea but the patent is a public good. An entrepreneur who believes that she can reap

<sup>&</sup>lt;sup>4</sup> Recall that  $\alpha_{ij}$  are assumed dissimilar to each other.

some profits by introducing a complementary good to the main technology bids a price for the blueprint of the complementary good, and the applied R&D sector invents it.<sup>5</sup>

We assume that the main input of complementary sector is skilled labor, H. Since there is perfect factor mobility across services sectors within each GPT and across GPT sectors, there is a single factor price,  $w_h$ , which is given to any producer of this kind. We assume that one unit of skilled labor produces one unit of service-good:

$$x_{ij} = h_{ij}$$
  $j = 1, ..., n_i$  (17)

where  $h_{ij}$  is the amount of *skilled labor* used in the production of  $x_{ij}$ . The profit equation of any firm in the services sector is

$$\pi_{ij} = q_{ij} \cdot x_{ij} - w_h x_{ij} \qquad j = 1, \dots, n_i$$
(18)

The market equilibrium process leads to

$$q_{ij} = w_h \phi_{ij}$$
  $j = 1, ..., n_i$  (19)

where  $\phi_{ij} = 1 + \frac{1}{(\sigma - 1)\alpha_{ij}}$ ,  $j = 1, 2, ..., n_i$ , sharing the same inverse parabolic relationship shown in figure 1.

Using (19) in (12) gives

<sup>&</sup>lt;sup>5</sup> Clearly, this sequence is only for matter of presentation and it may happen in the other way around. That is, the applied R&D may invent a new complementary product and may auction it in the market.

$$x_{ij} = \left(\frac{\alpha_{ij}}{q_{ij}}\right) \beta^{\sigma} \cdot L\left(\frac{\alpha_{i0}}{q_{i0}}\right)^{\alpha_{i0}(\sigma-1)} \prod_{j=1}^{n_i} \left(\frac{\alpha_{ij}}{q_{ij}}\right)^{\alpha_{ij}(\sigma-1)} \Rightarrow$$

$$x_{ij} = \left(\frac{\alpha_{ij}}{\phi_{ij}}\right) \cdot \beta^{\sigma} \cdot L \cdot \left(\frac{\alpha_{i0}}{q_{i0}}\right)^{\alpha_{i0}\beta\sigma} (w_h)^{-[1+(1-\alpha_{i0})\beta\sigma]} \left[\prod_{j=1}^{n_i} \left(\frac{\alpha_{ij}}{\phi_{ij}}\right)^{\alpha_{ij}\beta\sigma}\right]$$
(20)

Evidently, it is not possible to proceed further algebraically with this 'pure' form of equations. We have to make an assumption in order to make the model tractable. From now on, we will assume that the output share of the core sector,  $\alpha_{i0}$ , is same across the GPTs, that is,  $\alpha_{i0} = \alpha_{i'0} \quad \forall i, i' \in B$ . From the viewpoint of our analysis, this assumption does not change results at all. Actually, its only limit is on that the aggregate contribution of services to composite good production is fixed. Note that the number of complementary intermediaries and the share of each have nothing to do with this assumption.

Recall that we assumed input-resource used in the complementary sector was sectorspecific and therefore its use was limited to this sector. Under the assumption that supply is given, it is obvious that the price of input  $w_h$  will also be sector specific. Hence, we can easily calculate the equilibrium value of the raw input (skilled labor) from the full employment condition.

Let us suppose that we are at real-time s. We assume that the model economy is in the state that the B<sup>th</sup> GPT has just appeared *in the statistics* as the latest technology. Then, the demand-supply equilibrium of resource input in the complementary sector is

$$H = \sum_{i=1}^{B} \sum_{j=1}^{n_i} h_{ij} = \sum_{i=1}^{B} \sum_{j=1}^{n_i} x_{ij}$$
(21)

Using (20) in (21) gives

$$H = \beta^{\sigma} \cdot L \cdot \left(\frac{\alpha_0}{q_0}\right)^{\alpha_0 \beta \sigma} \cdot w_h^{-[1+(1-\alpha_0)\beta\sigma]} \cdot \sum_i \left\{ \prod_{j=1}^{n_i} \left(\frac{\alpha_{ij}}{\phi_{ij}}\right)^{\alpha_{ij}\beta\sigma} \sum_j \left(\frac{\alpha_{ij}}{\phi_{ij}}\right)^{\beta} \right\}$$
(22)

Then, for given H, L, and r, the short run equilibrium wage rate for skilled labor is found as

$$w_{h} = \beta^{\sigma\chi} \cdot L^{\chi} \cdot H^{-\chi} \cdot \left(\frac{\alpha_{0}}{q_{0}}\right)^{\alpha_{0}\beta\sigma\chi} \cdot \left[\sum_{i} \left\{\prod_{j=1}^{n_{i}} \left(\frac{\alpha_{ij}}{\phi_{ij}}\right)^{\alpha_{ij}\beta\sigma} \sum_{j} \left(\frac{\alpha_{ij}}{\phi_{ij}}\right)^{\chi}\right\}\right]^{\chi}$$
(23)

where  $\chi = \frac{1-\beta}{1-\alpha_0\beta}$ . Note that  $0 < \chi < 1$  due to the fact that  $\alpha_0\beta < \beta$ . Thus, we determined the *short run equilibrium value* of rental rate of skilled labor.

Before examining the meaning of equation (23), we need to examine the very last component on the right hand side of the equation (23) because this component is a crucial component of our equilibrium analysis. First, note that it grows with new additions to GPT. Second, we know that  $\sum_{j} \alpha_{ij} = (1 - \alpha_0)$ , by definition. Then, given the fact that  $(\alpha_{ij} / \phi_{ij}) < \alpha_{ij}$ ,  $\sum_{j} (\alpha_{ij} / \phi_{ij}) < (1 - \alpha_0)$ . If this is the case, then we can deduce that  $\prod_{j=1}^{n} \left(\frac{\alpha_{ij}}{\phi_{ij}}\right)^{\alpha_{ij}\beta\sigma} \sum_{j} \left(\frac{\alpha_{ij}}{\phi_{ij}}\right) < \prod_{j=1}^{n_i} \left(\frac{\alpha_{ij}}{\phi_{ij}}\right)^{\alpha_{ij}\beta\sigma}$ . We will refer to this result later in the text. Third, it is not possible to make any conclusive statement about the contribution of the number of varieties within a GPT to the term. More specifically, we are not sufficiently equipped to say that, for example, more varieties increase the value of the term, and so forth. The ambiguity arises because  $\sum_{j} \left(\frac{\alpha_{ij}}{\phi_{ij}}\right)^{\alpha_{ij}\beta_{ij}}$  grows and

 $\prod_{j=1}^{n_i} \left(\frac{\alpha_{ij}}{\phi_{ij}}\right)^{\alpha_{ij}\beta\sigma}$  decreases as  $n_i$  grows. Our experiment with hypothetical parameter

values suggests that an increase in  $n_i$  lowers the value of the term. Fourth, if we define

$$G_2 = \sum_{i} \left\{ \prod_{j=1}^{n_i} \left( \frac{\alpha_{ij}}{\phi_{ij}} \right)^{\alpha_{ij}\beta\sigma} \sum_{j} \left( \frac{\alpha_{ij}}{\phi_{ij}} \right) \right\}, \text{ and if we assume that } G_2 > 1, \text{ which must hold}$$

intuitively for a sufficiently large B, we can note that  $H/G_2$  can be interpreted as the effective supply of skilled labor in the sense that  $G_2$  adjusts H downwards.

Several observations concerning equation (23) are in order. First, wages increase as the stock of GPTs rises for given L, H, and r. This is a 'normal' result in the sense that, as new GPTs are introduced, more intermediaries use the same (given) resource. Second, an increase in H or a decrease in L will lower wages. An (exogenous) increase in the supply of skilled labor will certainly has a direct impact on its own price. The latter is result of a rather indirect mechanism. A decrease in L lowers the 'demand for composite inputs' due to lower final good production. A decrease in demand for composite goods undercut the demand for complementary inputs. Consequently, wages for skilled labor decreases.

The short run equilibrium price of a complementary product  $q_{ij}$  mimics skilled labor wage (cf. equation (19) and (23)). One interesting characteristic of complementarygoods prices is that they are asymmetric along varieties within a GPT as much as along GPTs because  $q_{ij}$  are function of input-share parameters. More particularly,  $q_{ij} < q_{ij'}$  $\forall j, j' \in B$  because, by definition,  $\alpha_{ij} > \alpha_{ij'} \Rightarrow \phi_{ij} < \phi_{ij'}$ .<sup>6</sup> One implication of this result is that, contrary to the growth literature, we are able to asymmetrize the intermediary prices.<sup>7</sup> Actually, asymmetric behavior is the main characteristic of the model.

The short run equilibrium value of each service-input demanded by composite input sector at time s can be calculated by using equations (20) and (23):

$$x_{ij} = \left(\frac{\alpha_{ij}}{\phi_{ij}}\right) \cdot \frac{H}{G_2} \cdot \prod_{j=1}^{n_i} \left(\frac{\alpha_{ij}}{\phi_{ij}}\right)^{\alpha_{ij}\beta\sigma}$$
(24)

<sup>&</sup>lt;sup>6</sup> This is very much in accordance with the finding that a complementary firm sells more the higher the input-share parameter.

<sup>&</sup>lt;sup>7</sup> This has rarely been done in the literature. See van Zon and Yetkiner (2002).

Three characteristics of equation (24) are in order. First, equilibrium values of intermediaries are dissimilar within a GPT and along GPTs. The first component on the right hand side of the equilibrium is the source of asymmetry across complementary services. Second, the equilibrium value declines with the  $\alpha_{ii}$ 's. It is straightforward to

see this result by checking  $\frac{\partial(\alpha_{ij} / \phi_{ij})}{\partial \alpha_{ij}}$ , which is positive. In other words, the later the intermediate appears in the market, the less its equilibrium value. Third, for given H, B is associated negatively with  $x_{ij}$  and H is associated positively with  $x_{ij}$  for given B. We can see this result by noticing that  $G_2$  is inversely related with  $x_{ij}$ .

The profits of the j<sup>th</sup> firm in the i<sup>th</sup> GPT (in the complementary intermediaries) is

$$\boldsymbol{\pi}_{ii} = (\phi_{ii} - 1) \cdot \boldsymbol{w}_h \cdot \boldsymbol{x}_{ii} \,. \tag{25}$$

Substituting the respective values of  $w_h$  and  $x_{ij}$  from (23) and (24) in (25) gives us

$$\pi_{ij} = \left(\frac{1}{\sigma - 1}\right) \left(\frac{1}{\phi_{ij}}\right) \cdot L^{\chi} \cdot \beta^{\sigma\chi} \cdot \left(\frac{H}{G_2}\right)^{1 - \chi} \cdot \left(\frac{\alpha_0}{q_{i0}}\right)^{\alpha_0 \beta \sigma\chi} \cdot G_1$$
(26)

where  $G_1 = \prod_{j=1}^{n_i} \left(\frac{\alpha_{ij}}{\phi_{ij}}\right)^{\alpha_{ij}\beta\sigma}$ . Note that  $G_1$  is source of asymmetry *across GPTs*. The most obvious characteristic of profits in equation (26) is its falling nature. Profits are descending function of composite-good's production share indicators. We assumed before that the  $\alpha_{ij}$  decline monotonically in the order of appearance in the market. Thus, *the later the intermediate appears, the less the profit it earns*. Another aspect of profits is its asymmetric nature within a GPT and across GPTs.

Finally, we need to calculate  $x_{i0}$ . From equation (13),  $x_{i0}$  is

$$x_{i0} = \beta^{\sigma} \cdot L \cdot \left(\frac{\alpha_0}{q_0}\right)^{(1+\alpha_0\beta\sigma)} \cdot w_h^{-(1-\alpha_0)\beta\sigma} G_1$$
(27)

Substituting the respective value of equilibrium wage from equation (23) gives us<sup>8</sup>

$$x_{i0} = \beta^{\sigma\chi} \cdot L^{\chi} \cdot \left(\frac{\alpha_0}{q_0}\right)^{\sigma\chi} \cdot \left(\frac{H}{G_2}\right)^{1-\chi} \cdot G_1$$
(28)

This is the short run equilibrium of  $x_{i0}$  at time *s*. Due to  $G_1$ ,  $x_{i0}$ 's are asymmetric across the GPTs. Note that  $x_{i0}$  implies the following equilibrium profit for the i<sup>th</sup> GPT:

$$\pi_{i0} = r \eta \cdot (\phi_{i0} - 1) \cdot \beta^{\sigma_{\chi}} \cdot L^{\chi} \cdot \left(\frac{\alpha_0}{q_0}\right)^{\sigma_{\chi}} \cdot \left(\frac{H}{G_2}\right)^{1-\chi} \cdot G_1$$
(29)

As in case of  $x_{i0}$ ,  $\pi_{i0}$  are dissimilar across the core sectors (i.e., along the GPTs).

As we now have all information concerning the composite input, we can proceed to find the short run equilibrium values of 'aggregate variables'. Using (24) and (28) in equation (2) gives us<sup>9</sup>

$$z_{i} = (x_{i0})^{\alpha_{0}} \prod_{j=1}^{n_{i}} (x_{ij})^{\alpha_{ij}} \Longrightarrow$$

$$z_{i} = \beta^{\alpha_{0}\sigma\chi} \cdot L^{\alpha_{0}\chi} \cdot \left(\frac{\alpha_{0}}{q_{0}}\right)^{\alpha_{0}\sigma\chi} \cdot \left(\frac{H}{G_{2}}\right)^{(1-\alpha_{0})\sigma\chi} \cdot \prod_{j=1}^{n_{i}} \left(\frac{\alpha_{ij}}{\phi_{ij}}\right)^{\alpha_{ij}\sigma}$$
(30)

<sup>&</sup>lt;sup>8</sup> It is helpful to see that  $1 - (1 - \alpha_0)\beta\sigma\chi = \chi$ .

Next, we can show that final output Y is

$$Y = \beta^{\alpha_0 \beta \sigma \chi} \cdot L^{\chi} \cdot \left(\frac{\alpha_0}{q_0}\right)^{\alpha_0 \beta \sigma \chi} \cdot \left(\frac{H}{G_2}\right)^{1-\chi} \cdot \sum_{i=1}^B G_i$$
(31)

Equation (31) has many interesting properties. One of the important characteristics of equation (31) is  $P = \left(\sum_{i} G_{1}\right) / G_{2}^{1-\chi}$ , which works like a 'productivity parameter' at a given time. Clearly, it contains more information than the usual ones. We know from our previous discussion that  $\prod_{j=1}^{n_{i}} \left(\frac{\alpha_{ij}}{\phi_{ij}}\right)^{\alpha_{ij}\beta\sigma} \sum_{j} \left(\frac{\alpha_{ij}}{\phi_{ij}}\right) < \prod_{j=1}^{n_{i}} \left(\frac{\alpha_{ij}}{\phi_{ij}}\right)^{\alpha_{ij}\beta\sigma}$ . Thus,  $P > 1.^{10}$ 

Furthermore, it is intuitive to expect that P increases as new GPTs are introduced into the economy. Clearly, P looks like the love of variety variable A in Romer 1990. Second, contrary to Romer (1990), our model shows that the growth rate of labor rather than the levels of labor has a determining impact on the growth rate of output, besides technology variables P. Third, the very existence of a number of complementary varieties within each technology creates the difference between the Romer model and ours. Unfortunately, it is difficult to make any conclusive statement on the impact of complementarities on the growth performance of final good. Third, contrary to several models in the literature, the growth impact of each GPT is not identical in the sense that output does not grow linearly in proportion to the stock of knowledge. Hence, *the cycles are not only along the levels but also at growth rates*.

We can calculate aggregate capital and check if the ratio of the two is constant, fitting to stylized facts. Aggregate capital is obtained by summing  $x_{i0}$ 's along the GPTs:

<sup>&</sup>lt;sup>9</sup> Note that  $1 - \alpha_0 \chi = (1 - \alpha_0) \sigma \chi$ .

<sup>&</sup>lt;sup>10</sup> Recall that  $\chi < 1$ . Thus,  $G_2^{1-\chi} < G_2$ .

$$K = \eta \cdot \sum_{i} x_{i0} \tag{32}$$

By using (28) in (32), we get

$$K = \eta \cdot \beta^{\sigma_{\chi}} \cdot L^{\chi} \cdot \left(\frac{\alpha_0}{q_0}\right)^{\sigma_{\chi}} \cdot \left(\frac{H}{G_2}\right)^{1-\chi} \cdot \sum_{i=1}^B G_i$$
(33)

at times. Note that

$$\frac{K}{Y} = \left(\frac{\alpha_0}{q_0}\right) \cdot \beta = \frac{\alpha_0 \beta}{r \phi_0},\tag{34}$$

which is constant in the equilibrium if r is constant.

## The Basic Research Sector

We assume in this model that research sectors are isolated from the other sectors of the economy in terms of the type of input used. More specifically, we conjecture that basic and applied research sectors use only research people in order to handle their research activities and to produce blueprints of GPTs and complementary inputs. In that respect, we are diverting from the general tradition that sets up a link between the research sector and the final goods sector (or the intermediate sector). Technically speaking, our excuse for this assumption is the fact that the very existence of the two types of research sectors is sufficient to determine the equilibrium in this market.

Before elaborating the basic research sector, let us make clear that the model distinguishes between real time s and GPT time t. The latter represents the time intervals (on real time) that GPTs arrive. We conjecture that blueprints accumulate according to the following difference function:

$$B_{t+1} - B_t = \delta R_B B_t \tag{35}$$

where  $\delta$  represents the productivity of the blueprint generation process, and  $R_B$  is the amount of research people used in generating blueprints of GPTs. The way we defined the GPT generation mechanism is a simple difference equation (with only homogenous part) and its solution is

$$B_t = (1 + \delta R_B)^t \tag{36}$$

Clearly, the mechanism generates (discrete) perpetual growth. That is, the stock of blueprints for GPTs shows the following behavior:

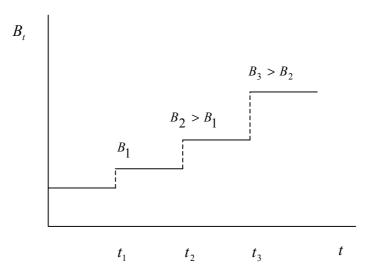


Figure 2 The growth of GPT blueprints

We can interpret figure 2 in two ways. First, the stock of GPTs increase at increasing rates at equal time distances. Second, the time between two GPTs is shortening. This result is due to the public good character of ideas itself (cf. Romer (1990)).

As usual, whenever the basic R&D sector undertakes research, the proceeds of blueprints are paid as wages. Suppose that  $B_{t-1}$  has been already invented (thus given). The profits for the next basic R&D activity at time t would be

$$\pi_{B,0}^{R} = P_{B}(1 + \delta R_{B})B_{t-1} - w_{B,0}^{R}R_{B}$$
(37)

The profits of the basic R&D are the price  $P_B$  of designs invented at time t times the number of GPTs added at the basic research period. Profit maximization gives

$$w_{B0}^{R} = P_{B} \delta B_{t-1}, \qquad (38)$$

a condition that must be satisfied had research staff is ever employed in the basic R&D. Note that the stock of  $B_{t-1}$  is taken as given. Next, let us describe the applied sector.

## The Applied R&D Sector

The dynamics of the applied R&D sector is slightly different than the basic sector due to the observation that blueprints of varieties are not public good to each other. The applied R&D accumulation function (for the latest GPT) is supposed to be

$$n_{s+1} - n_s = \xi B_{t-1} R_A \tag{39}$$

where  $\xi$  represents the productivity of the product-development process,  $B_{t-1}$  denotes the stock of technology paradigms that complementary services of the *newest GPT* enjoys freely, and  $R_A$  is the amount of research people employed in generating blueprints of complementary goods. Note that applied R&D is possible only for the most recent GPT, and therefore, there is no need to index *n*'s by their associated GPT. The way we defined the complementary-good generation mechanism is a simple difference equation (with homogenous and particular parts) and its solution is

$$n_s = s \cdot \xi \cdot B_{t-1} \cdot R_A + n_0 \tag{40}$$

Given the fact that  $n_0$  is zero at the time that the GPT has arrived, the solution becomes

$$n_s = s \cdot \xi \cdot B_{t-1} \cdot R_A \tag{41}$$

According to equation (41), the number of complementary varieties grows as a linear positive function of the amount of research labor used. Clearly, the complementary intermediary supply will continue to grow as long as research labor is a positive amount.

The profits of the j<sup>th</sup> design will be

$$\pi_{A,j}^{R} = P_{A,j}\xi B_{t-1}R_{A} - w_{A,j}^{R}R_{A}$$
(42)

In (42),  $P_{A,j}$  is the price of the j<sup>th</sup> design. Profit maximization gives

$$w_{A,j}^{R} = P_{A,j} \xi B_{t-1}.$$
(43)

This is the wage rate that the applied R&D sector must pay in order to undertake research in the sector.

We have already shown in equation (26) that profits in the complementary sector are falling down as new intermediaries are introduced into the market. Hence, due the fact that price of a blueprint is present discounted value of profits of the respecting

intermediary producer, wages received by the research people (employed in the applied R&D) are not identical across the complementary goods. More specifically,  $P_{A,j}$  is a declining function of  $\alpha_{ij}$ . We next show how the falling nature of blueprint prices creates the cyclical use of research staff.

## The Cyclical Use of R&D-labor

Before showing the switching conditions between the two alternative uses of research labor, let us look at the sequence of events we used in the paper. Recall that we denote real time by s and GPT time by t in the model. The following figure illustrates the sequence of the R&D efforts:

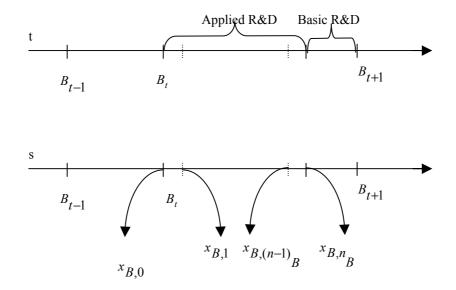


Figure 3. The Timing of Basic R&D and the Applied R&D

In the upper part of figure 3, t denotes the time that the GPT stock realizes level  $B_t$  and (t+1) represents the time that the GPT stock becomes  $B_{t+1}$ . Evidently, these points also refer to some s on the real-time scale. According to the figure,  $\Delta t$  is made up of two intervals: the applied-research interval and the basic-research interval.

We defined the blueprint production functions linearly on purpose. The research labor decides on the use of their labor by comparing the (expected) real wages offered by the two research-sectors at any time *s*. Suppose that the applied R&D sector employs the whole research staff currently. If the (expected) real wage offered by the basic R&D sector for the next technology paradigm is higher than the wage rate offered by the applied R&D for the next incremental technological innovation, the whole research people will move to the basic research sector. Otherwise, they stay in the applied R&D. Certainly, the condition satisfies at some point because the profits in the complementary sector are falling down. Then, the research labor shifts to the basic R&D sector because it becomes more rewarding to work in that sector. The reason why we introduced research production functions linearly in terms of R&D people must be clear now. The linearity brings into corner solutions in the employment of research people, which *stylizes* our model.

Given that only corner solutions on the allocation of research people between applied R&D and basic R&D is possible, we can determine the switching points between the two R&D sectors. Now, suppose that we are at time  $\tau$ , the GPT stock is at  $B_t$  and some complementary intermediaries have already been produced (research people are employed in the applied R&D sector). The switching condition is the wage rate offered to the research people by the basic R&D and the applied R&D sectors. As long as  $w_{A,j}^R > w_{B_{t+1},0}^R$  is satisfied, the research staff will remain in the applied sector. Otherwise, all research labor will shift to the basic R&D. The condition on wage rate can be further boiled down to the following condition:

$$\begin{cases} Stay \ in \ Applied \ R \& D \ if \ PV_{j} > \frac{\delta}{\xi} \cdot PV_{B+1} \\ Stay \ in \ Basic \ R \& D \ if \ PV_{j} < \frac{\delta}{\xi} \cdot PV_{B+1} \\ Any \ Combination \ if \ otherwise \end{cases}$$

$$(44)$$

It is straightforward to calculate the present value of profits (confiscated by the R&D sectors). The present value of profits of the core sector for the *next* GPT is

$$PV_{B_{t+1},0} = \sum_{s=\tau}^{\infty} (1+r)^{-(s-\tau)} \pi_{B_{t+1},0} \Longrightarrow$$

$$PV_{B_{t+1},0} = r\eta \cdot (\phi_{i0} - 1) \cdot \beta^{\sigma \chi} \cdot \left(\frac{\alpha_0}{q_0}\right)^{\sigma \chi} \sum_{s=\tau}^{\infty} (1+r)^{-(s-\tau)} \frac{(G_{1,B_{t+1}})^{\beta \sigma} L^{\chi} \cdot H^{1-\chi}}{(G_2)^{1-\chi}}$$
(45)

In Equation (45),  $\tau$  denotes a specific real time. We assume that the growth dynamics of *L* and *H* are known to the system. Similarly, the present value of the j<sup>th</sup> service-good at time  $\tau$ , where the latest GPT stock available at that time is  $B_t$ , will be

$$PV_{B_{i},j} = \frac{1}{\sigma - 1} \cdot \frac{1}{\phi_{B_{i},j}} \cdot \beta^{\sigma \chi} \cdot \left(\frac{\alpha_{0}}{q_{0}}\right)^{\alpha_{0}\beta\sigma \chi} \sum_{s=\tau}^{\infty} (1 + r)^{-(s-\tau)} \frac{\left(G_{1,B_{i}}\right)^{\beta\sigma} L^{\chi} \cdot H^{1-\chi}}{\left(G_{2}\right)^{1-\chi}}$$

$$\tag{46}$$

where  $G_{1,B_t}$  denotes  $G_1$  that starts with GPT stock  $B_t$ , and correspondingly  $G_{1,B_{t+1}}$  denotes  $G_1$  that starts with GPT stock  $B_{t+1}$ .

Comparing equations (45) and (46) according to the switching condition defined in (44) gives us the following condition:

$$\frac{1}{\sigma - 1} \frac{1}{\phi_{ij}} > \frac{\delta}{\xi} \cdot r \eta \cdot (\phi_0 - 1) \frac{(G_{1,B_{t+1}})^{\beta\sigma}}{(G_{1,B_t})^{\beta\sigma}} \frac{\sum_{s=\tau}^{\infty} (1 + r)^{-(s-\tau)} \frac{L^{\chi} H^{1-\chi}}{(G_{2,B_{t+1}})^{1-\chi}}}{\sum_{s=\tau}^{\infty} (1 + r)^{-(s-\tau)} \frac{L^{\chi} H^{1-\chi}}{(G_{2,B_t})^{1-\chi}}}$$
(47)

Clearly, *ex post*, the only decision variable is  $\alpha_{ij}$  in equation (47).<sup>11</sup> We know that  $\alpha_{ij}$ are declining and thus  $\phi_{ij}$ 's are rising. Thus, the left-hand side of equation (47) is declining in terms of  $1/\phi_{ij}$  and therefore at some point the switching condition succeeds. After that point, the research staff shifts to basic research and adds new GPTs to the stock. After the invention of  $B_{t+1}$ <sup>th</sup> stock, the research staff checks whether it is worth to move to the applied R&D sector to produce complementary products for the new cluster of GPT stock or not. If yes, they move. If not, they start to work on the  $B_{t+2}$ <sup>th</sup> stock. Hence, a cycle emerges in the employment of research people due to falling profits in the complementary sector.

### Closure of the Model

In order not to complicate the model further, we assume that consumption is determined by an exogenous saving rate proportional to income (cf., Solow (1956):

$$C_s = (1-s)Y_s \tag{48}$$

where  $C_t$  is consumption, and s is exogenous saving rate.<sup>12</sup>

#### Dynamics of the Output and the Broader Concept of Output

<sup>11</sup> *i* refers to the latest GPT in the context. <sup>12</sup> It would be possible to close the model under endogenous saving assumption. In particular, Max  $U_t = \sum_{t=0}^{\infty} \beta^t u(C_t)$  s.t.  $W_{t+1} - W_t = rW_t + w_L L + w_H H - C_t$ , where  $\beta$  is discount factor,  $U(C) = \frac{C^{1-\sigma} - 1}{1-\sigma}$  is utility, C is consumption,  $1/\sigma$  is intertemporal elasticity of consumption, and W

is the asset stock of the society. The maximization yields  $\frac{C_{t+1}}{C_t} = \beta^{\sigma} (1+r_t)^{\sigma}$  and  $K_{t+1} - K_t = Y_t - C_t$ 

given that W = K and  $rW_t + w_L L_t + w_H H_t = Y_t$ . Evidently, it is not possible to calculate explicit solution of unknowns in the model, though, then, r would be also endogenously determined.

Our model has two types of output: equilibrium output and the broader concept of output. The dynamics of the equilibrium output, Y, is quite clear. Let us first re-produce figure 3 again to illustrate its dynamics.

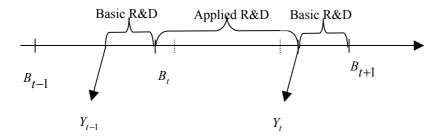


Figure 4 The sequence of output

We see that gross output realizes upward shifts at times that a new composite good initiated by a drastic change has started to be produced. Clearly, output between these equilibrium states is not constant. This brings us to the notion of *broader concept of gross output*.<sup>13</sup>

The dynamics of broader concept of gross output,  $Q_s$ , is as follows. Suppose that the model economy has realized  $Y_t$  at time s. Time s is also the time that the core intermediate of the next GPT, say  $x_{B_{i+1},0}$ , has been introduced into the economy. Hence, the economy is realizing two jumps. First, the gross output jumps from  $Y_{t-1}$  to  $Y_t$ . broader concept of gross Second, the output jumps from  $Y_t$  to  $Q_s = Y_t + x_{B_{t+1},0} \cdot (B_{t+1} - B_t)$ . Note that we multiply  $x_{B_{t+1},0}$  by  $(B_{t+1} - B_t)$  because the latter is the number of new GPTs at time t. s+1 onwards, the economy starts to produce complementary goods according to the dynamics of the applied R&D sector. Hence,  $Q_{s+1} = Y_t + x_{B_{t+1},1}(B_t - B_{t-1})(n_{s+1} - n_s)$ . That is, number of new GPTs times new complementarities added for each GPT at that time gives the number of  $x_{B,1}$  produced at that time. Is  $Q_{s+1} > Q_s$ ? This very much depends on many things like the growth rate of H and L. Since our model focuses on equilibrium states, we cannot give any clear answer to this question. If H and L had been constant, then we could have argued that the broader concept of output would have declined since new intermediaries would have also used H and L. Since these two stocks are not constant, the broader concept of output may increase or decrease. Hence, our model shows that the current debate in GPT and growth literature on whether output declines at the onset of a new GPT is inconclusive, and the answer depends on many factors. Below, we illustrate the time paths of output and broader concept of output on the assumption that the growth in Hand L outpaces the decline in these resources due to additional complementary intermediaries.

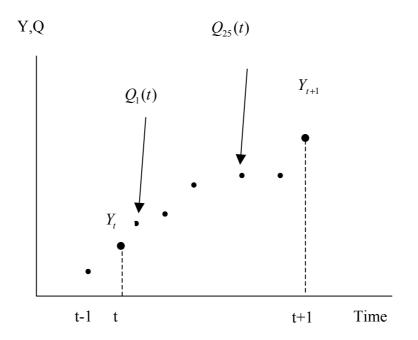


Figure 5 The Path of Output and Broader Concept of Output

### Solow's Productivity Paradox

Nobel Prize-winning economist Robert Solow has said "we see computers everywhere except in the productivity statistics". That productivity measures do not seem to show

<sup>&</sup>lt;sup>13</sup> See Chapter 5 of Barro and Sala-i-Martin (1995) for details.

any impact from new computer and information technologies has been labeled the "productivity paradox". Though the term is used to indicate a present anomaly, the paradox has been there since the Industrial Revolution in Britain.<sup>14</sup>

One important aspect of GPTs, contrary to it is being labeled radical or drastic, is that they *infiltrate* into an economy *slowly* (cf., Helpman, 1998). If GPTs infiltrate an economy slowly, than this can also be the explanation of the late arrival of (productivity) growth into statistics. It takes time to develop new sectors or to change old sectors substantially. Hence, it takes time before a GPT shows its real impact on the economy. This property of GPTs is expressed by the term *critical mass* (see, for example, Helpman and Trajtenberg (1998). Our study models that property of GPTs in an explicit way. Hence, the productivity of final-good sector increases only after a GPT has expanded fully in the economy.

#### A Digression: Wage Differentials between the Skilled and Unskilled Workers

The wage of unskilled labor from equation (5) is

$$w = \frac{\beta^{\alpha_0 \beta \sigma \chi}}{1 - \beta} \cdot L^{\chi - 1} \cdot \left(\frac{\alpha_0}{q_0}\right)^{\alpha_0 \beta \sigma \chi} \cdot \left(\frac{H}{G_2}\right)^{1 - \chi} \cdot B^{\chi} \cdot (G_1)^{\beta \sigma \chi}$$
(49)

If we look at the ratio skilled to unskilled:

$$w_h / w = (1 - \beta) \cdot \beta \cdot \left(\frac{L}{H / G_2}\right)$$
(50)

<sup>&</sup>lt;sup>14</sup> Initial studies made by economic historians on (productivity) growth during the Industrial Revolution argued that Britain experienced high productivity growth rates at that time, especially in sectors that were the primal subject of Industrial Revolution. Later studies moderated these estimates by using 'better' data and alternative measures of productivity growth (e.g., dual approach). Most of these studies confirmed the fact that, initially, the productivity growth rate was very low (close to zero!) with some acceleration in the late period of the Industrial Revolution. There are other examples to the late arrival of (productivity) growth after a 'technological breakthrough' such as the introduction of electric motors. As David (1990) has shown, the benefits of electrification only became apparent in the 1920s, more than 20 years after the widespread adoption of electric lighting and the use of electric motors.

In equation (50), the first two terms on the right-hand side are less than one. Thus, the conclusive element on the relative wage is the relative ratio of supplies of unskilled labor to the *effective* skilled labor. Note that the supply of skilled labor declines as the number of GPTs increases (recall that  $G_2$  is positive function of B). That is why effective supply of skilled labor is in the equation. If the supply of unskilled labor stock is larger than of effective skilled labor supply, than the relative wages of skilled labor is higher. Equation (50) pinpoints that the dynamics of wage differential is function of (i) relative stock dynamics, (ii) the pace of introduction of technological paradigms. Hence, it is possible that the positive wage differential between skilled and unskilled labor may endure even the growth rate of the latter is less than the former one.

# 3. Conclusion

This study showed that exhausting profits in the incremental technologies with the existing technological paradigm could be the source of long run business cycles. New technological paradigms are advanced cyclically because R&D activities focus on the existing technological paradigm as long as there remains positive profit opportunities on it. Focus returns to basic R&D whenever the profit opportunities of the next bundle of drastic technologies are higher than that of the existing paradigm. The switch of the R&D on basic and applied technologies creates long run cycles in the economy. The paper showed also that temporary falls in growth at the onset of a new technological paradigm might be because the pace of growth of inputs was not meeting the additional resource needs created by the new paradigm. An interesting finding of this study is that the growth rate of economy is shown to be function of the growth rate of inputs, rather than its levels, contrary to Romer (1990).

This paper has many possible extensions. One of them is very exciting. The very existence of long run Kondratieff cycles brings into the scene the question of "are these cycles make 'us' better off or worse off? This is an interesting question because the

model shows that agents create cycles in return to their response to market opportunities. In that sense, the market is efficient (though there are externalities that are ignored by the players). Nonetheless, we believe that it is still an open question that "are these cycles 'beneficial' to the economy or is there a room for policy-makers to smoothen these cycles?". A future study on the comparison of welfare implications of cycles and of policy responses aiming to smoothen these cycles is highly valuable.

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