Labour values, prices of production and the missing equalization of profit rates: Evidence from the German economy

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Abstract

During the last years several empirical studies found out that deviations from labour values to market prices are quite small. However, most of these articles do not offer a detailed reason for this result. In this paper two theoretical justifications of the labour theory of value are brought together with some data concerning labour values, prices of production and market prices on the base of German IO tables from 2000 and 2004. In addition, the statistical characteristics of profit rates are analyzed. Both of the theoretical arguments are much in line with the empirical observations because there is only a slight transformation tendency and at the same time profit rates and capital intensity are negatively correlated. Moreover, during the period under observation the German economy seems to be in a state of statistical equilibrium.

Key words: Labour values, Prices of production, Transformation problem, Profit rates
JEL Classification: B51, D46, D57

1 Introduction

In non-mainstream economic theory there are usually two ways of explaining market prices: Firstly, labour theory of value which states that prices are driven by vertically integrated labour time (labour values). This approach, originally used by Karl Marx in “Capital, Volume 1”, evoked the famous transformation problem because a profit rate equilibrium is only possible in case of a uniform capital intensity or zero profits. Secondly, the discussion about labour values has let to the development of neoricardian prices of production based on the work of Pierro Sraffa and his followers. These authors believe the transformation debate reached its well deserved end because their model provides prices generating an equilibrium profit rate. Hence, it is typically viewed as state of the art and even prominent marxian authors stated that labour values “play no

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role whatsoever in the discussion of exchange and price" (Roemer, 1981, p. 200). Different points of view arguing that the transformation problem is likely to be negligible failed to prevail.

On the other hand, there is a growing body of empirical studies claiming that deviations from values to prices are quite small (Shaikh, 1984; Petrović, 1987; Ochoa, 1989; Cockshott and Cottrell, 1997, 1998, 2003; Tsoulfidis and Maniatis, 2002; Zachariah, 2006; Tsoulfidis and Mariolis, 2007; Tsoulfidis, 2008). The authors found correlation coefficients and coefficients of determination R^2 to be considerably larger than 0.9. Therefore, labour values might be as good in explaining market prices as neoricardian prices of production are. Although these results are rarely linked to theoretical debates, they are a serious challenge for traditional approaches of classical economics. Unsurprisingly, fundamental critique has taken place to doubt these outcomes (Kliman, 2002, 2005; Díaz and Osuna, 2005–06, 2007, 2009).

The aim of this paper is to connect theoretical and empirical arguments for the validity of the labour theory of value. To realise this intention it is useful to start Section 2 with a brief sketch of marxian and neoricardian economics followed by the theoretical arguments given by Farjoun and Machover (1983) and Shaikh (1984). Both of these approaches offer a solution for the transformation problem but have not reached a perceptible influence on classical economics until today. Section 3 serves to outline the data and to explain the estimation procedures. After that the empirical results are put forward in Section 4. Finally, Section 5 gives a summary and presents some conclusions.

2 Theoretical framework

2.1 The law of value

Consider an economy with n sectors and a uniform period of production.[1] Each sector is producing a single output. The economy is described by a linear, constant-returns-to-scale technology $\{\boldsymbol{A}, \boldsymbol{l}\}$, where $\boldsymbol{A} = (a_{ij})$ is an indecomposable, productive $(n \times n)$-Matrix of input coefficients and $\boldsymbol{l}$ is the $(1 \times n)$-vector of direct labour inputs.[2] Labour value $\lambda_i, i = 1, \ldots, n$, is the sum of direct and indirect labour inputs needed to produce commodity i with respect to $\{\boldsymbol{A}, \boldsymbol{l}\}$. Therefore, the $(1 \times n)$-vector of labour values $\boldsymbol{\lambda}$ is obtained by the following equation:

$$\boldsymbol{\lambda} = \boldsymbol{\lambda A} + \boldsymbol{l}. \tag{1}$$

Since we have assumed $\boldsymbol{A}$ to be indecomposable and productive we may rewrite (1) as

$$\boldsymbol{\lambda} = \boldsymbol{l}\,(\boldsymbol{I} - \boldsymbol{A})^{-1}. \tag{2}$$

For a moment we are adopting that the whole net product of the economy is paid to workers because there are no capitalists. The corresponding wage rate is called w^* and the resulting $(1 \times n)$-vector of prices is denoted by $\boldsymbol{p}^*$. Net product is given by the $(n \times 1)$-vector $\boldsymbol{y}$. In this case prices are solely determined by labour values. We get

$$\boldsymbol{p}^* = \boldsymbol{p}^*\boldsymbol{A} + w^*\boldsymbol{l}. \tag{3}$$

[1] For the usual framework of marxian economics see for instance Pasinetti (1977), Roemer (1981) or Mohun (2004).

[2] Every matrix, vector and scalar used in this paper is real and nonnegative.

where prices can be normalized such that

$$w^* = \boldsymbol{p}^* \boldsymbol{y} = 1. \tag{4}$$

Applying (4) to (3) and recalling (1) immediately shows that

$$\boldsymbol{p}^* = \boldsymbol{\lambda}. \tag{5}$$

But in reality, a certain fraction of the net product goes to capitalists simply because they are commanding the means of production. From a classical perspective, workers receive a subsistence wage basket instead of w^*, that is a $(n \times 1)$-vector of commodities $\boldsymbol{b}$. Therefore,

$$w = \boldsymbol{p}_v \boldsymbol{b} = \gamma w^* \, , \, \gamma < 1, \tag{6}$$

where $\boldsymbol{p}_v$ denotes "value prices". Profits are now allowed to be positive. By rearranging (3) we obtain

$$\boldsymbol{p}_v = \boldsymbol{p}_v \boldsymbol{A} + w\boldsymbol{l} + \frac{1-\gamma}{\gamma} w\boldsymbol{l}. \tag{7}$$

Equation (7) shows that prices are made up of three components: material costs $\boldsymbol{p}_v \boldsymbol{A}$, labour costs $w\boldsymbol{l}$ and profits $\frac{1-\gamma}{\gamma} w\boldsymbol{l}$. Defining the surplus rate $e := \frac{1-\gamma}{\gamma}$ we get the profit equation

$$\boldsymbol{\pi}_v = ew\boldsymbol{l}. \tag{8}$$

Thus, profits are based on "exploited" labour. Prices are account to

$$\boldsymbol{p}_v = w\boldsymbol{l} \, (\boldsymbol{I} - \boldsymbol{A})^{-1} \, (1 + e) = w\boldsymbol{\lambda}(1 + e). \tag{9}$$

Here, $w\boldsymbol{\lambda}$ can be interpreted as "monetary labour values". Deviations from prices to monetary labour values are caused by the level of $w = \gamma w^*$, which depends on the workers share of nominal net product γ. The greater this share is, the lower the deviations are. In other words, if all profits are zero, (9) is equivalent to (5). Note that w and e are globally defined because $\boldsymbol{b}$ is the same for all workers. However, exchange ratios are not affected by this consideration because calculating relative prices and recalling (9) always yields:

$$\frac{(p_v)_i}{(p_v)_j} = \frac{\lambda_i}{\lambda_j} \, , \, i \neq j \text{ and } i, j = 1, \ldots, n. \tag{10}$$

Equation (10) now gives us the exact meaning of the famous phrase "law of value": Labour values, that is direct and indirect labour time socially necessary to produce a commodity, are regulating the exchange of commodities. There are

$$\tau = \binom{n}{2} = \frac{n(n-1)}{2} \tag{11}$$

relative prices. The same applies to relative labour values. For notational convenience, we will call them $\boldsymbol{\rho}_v$ and $\boldsymbol{\upsilon}$, respectively. Now (10) becomes

$$\boldsymbol{\rho}_v = \boldsymbol{\upsilon}. \tag{12}$$

But one problem remains. The derivation of (9) is based on (8) which means that profits are proportional to direct labour. Since this is equivalent to profit rates negatively connected to capital intensity we are dealing with the simple labour theory of value from "Capital, Volume I and II" (Marx, 2001, 1972). The phrase "simple" means besides any considerations of the transformation problem.

The question whether differences in sectoral capital intensity are disrupting the law of value leads to the next section.

2.2 Prices of production

Because of the last statement, most authors refuse, among other things, profit determination by (8) preferring

$$\boldsymbol{\pi}_n = r\,\boldsymbol{p}_n\boldsymbol{A} \tag{13}$$

instead.[3] The scalar r indicates the uniform profit rate which is the equilibrium criterion in this approach. Profits are distributed in proportion to the price of the means of production. We symbolize this fact by the subscript "n" since $\boldsymbol{p}_n$ is the $(1 \times n)$-vector of neoricardian prices of production. Because of the proportionality between $\boldsymbol{\pi}_n$ and $\boldsymbol{p}_n\boldsymbol{A}$ it can be said that, in some sense, profits are now based on "engaging" capital. It follows that

$$\boldsymbol{p}_n = (1+r)\boldsymbol{p}_n\boldsymbol{A} + w\boldsymbol{l}. \tag{14}$$

Unlike the procedure in expression (6), neoricardian theorists do not fix w by assuming a wage basket. Instead, there are two income parameters w and r usually treating the latter as being exogenous. Expressing prices by an arbitrary $(n \times 1)$-commodity vector $\boldsymbol{d}$ we get

$$\boldsymbol{p}_n = w\boldsymbol{l}\,(\boldsymbol{I} - (1+r)\boldsymbol{A})^{-1} \tag{15}$$

with $0 < r < r^*$ and

$$\boldsymbol{p}_n\boldsymbol{d} = 1, \tag{16}$$

where r^* refers to the profit rate in case of zero wages. Comparing (9) to (15) we can see that in general the law of value is not fulfilled. According to the aforementioned framework there are only two exceptions for the labour theory of value to hold: Either profits are zero or there is a uniform capital intensity across all sectors (Kurz and Salvadori, 1997, pp. 110–113, 120). Both conditions are not compatible to real capitalist economics. In this view, therefore, the labour theory of value is a rather strange special case of neoricardian price theory. Hence, in reality there should be significant deviations from prices to values.

2.3 Decomposing prices

On the other hand, decomposing an arbitrary price system into profits and wages shows that these deviations are likely to be quite small (Shaikh, 1984, pp. 64–68). To reproduce the argument let

[3] See Pasinetti (1977) or Kurz and Salvadori (1997) for details.

us go back to equation (7). There we have seen that prices are simply the sum of corresponding wage bill, profit and material costs. Now we use this statement without any assumption about profit determination such as (8) or (13):

$$\boldsymbol{p} = \boldsymbol{p}\boldsymbol{A} + w\boldsymbol{l} + \boldsymbol{\pi}. \tag{17}$$

Solving (17) for $\boldsymbol{p}$ provides

$$\boldsymbol{p} = w\boldsymbol{l}\,(\boldsymbol{I} - \boldsymbol{A})^{-1} + \boldsymbol{\pi}\,(\boldsymbol{I} - \boldsymbol{A})^{-1} = \boldsymbol{\delta} + \boldsymbol{\theta}, \tag{18}$$

where

$$\boldsymbol{\delta} := w\boldsymbol{l}\,(\boldsymbol{I} - \boldsymbol{A})^{-1}, \boldsymbol{\theta} := \boldsymbol{\pi}\,(\boldsymbol{I} - \boldsymbol{A})^{-1}. \tag{19}$$

Thus, any arbitrary price is made up by two components: Vertically integrated labour costs, that is monetary labour values and vertically integrated profits. After rearranging (18) and some algebraic manipulation we get

$$\boldsymbol{p} = \boldsymbol{\delta}\left(\boldsymbol{I} + w^{-1}\boldsymbol{\Lambda}^{-1}\boldsymbol{\Theta}\right), \tag{20}$$

with

$$\boldsymbol{\Lambda} := \mathrm{diag}(\lambda_1, \ldots, \lambda_n), \boldsymbol{\Theta} := \mathrm{diag}(\theta_1, \ldots, \theta_n). \tag{21}$$

Here, $w^{-1}\boldsymbol{\Lambda}^{-1}\boldsymbol{\Theta}$ is the $(n \times n)$-diagonal matrix of vertically integrated surplus rates. Its i-th element is a convex combination of profit-wage ratios that enters sector i via direct or indirect means of production (Shaikh, 1984, p. 68). Again, we should not be interested in absolute prices but in relative prices:

$$\frac{p_i}{p_j} = \frac{\delta_i\left(1 + \dfrac{\theta_i}{\delta_i}\right)}{\delta_j\left(1 + \dfrac{\theta_j}{\delta_j}\right)} = \frac{\lambda_i\left(1 + \dfrac{\theta_i}{\delta_i}\right)}{\lambda_j\left(1 + \dfrac{\theta_j}{\delta_j}\right)}, \; i \neq j \text{ and } i, j = 1, \ldots, n. \tag{22}$$

To facilitate the analysis now define

$$\boldsymbol{Z} := \mathrm{diag}(z_1, \ldots, z_\tau), \text{ where } z_k := \frac{1 + \dfrac{\theta_i}{\delta_i}}{1 + \dfrac{\theta_j}{\delta_j}}, \; k = 1, \ldots, \tau. \tag{23}$$

Comparing (10) to (22) we can see that (12) becomes

$$\boldsymbol{\rho} = \boldsymbol{\upsilon}\boldsymbol{Z}. \tag{24}$$

Now it is very important to recognize that all elements of $\boldsymbol{Z}$ are likely to be rather small because they depend on the degree of which different convex combinations of direct profit-wage ratios differ from each other. As a consequence, even large variations in sectoral profit-wage rates are reduced to small ones in the corresponding integrated ratios. Therefore, (24) is a *modified law of value* with $\boldsymbol{Z}$ containing some kind of probably negligible disturbance factors. If there is any transformation problem, it is most likely moderate. But this is an empirical question.

2.4 Probabilistic Political Economy

In the probabilistic approach developed by Farjoun and Machover (1983), all variables such as prices, labour values, profit rates etc. are random ones. In place of analysing a deterministic system with "mechanical" equilibrium properties like traditional marxian or neoricardian theorists do, they scrutinize the elements of an economic system similar to the way the behaviour of ideal gas molecules enclosed in a container is described by statistical mechanics (Farjoun and Machover, 1983, pp. 39–56). In their view, the transformation problem occurs because of using an inappropriate concept of equilibrium, namely the adoption of a uniform profit rate (Farjoun and Machover, 1983, pp. 28–38). Instead, they suppose profit rates to be described by a gamma distribution and replace the assumption of equalizing profit rates by the more sophisticated assertion that for a given country in a state of equilibrium the probability density function (PDF) of profit rates is virtually independent of time (Farjoun and Machover, 1983, pp. 64–66). This concept is even compatible with large differences between sectoral profit rates whereas traditional approaches must assume that the distribution of empirical profit rates should be quite narrow – which is almost not the case.

Remarkably, this procedure results in relationships similar to Section 2.1, that is the labour theory of value probably holds in spite of heterogeneous capital intensity. We should give a brief survey of the relevant proceeding.

First of all, declare all prices to be measured in "wage units", that is units of the average hourly wage. Then consider the sample space of all products bought and sold during a given period, assuming that there are T transactions. In the next step, pick up randomly one transaction t ($t = 1, 2, \ldots, T$) and define the *specific price* of the commodity under investigation by

$$\Omega_t := \frac{\hat{p}_t}{\hat{\lambda}_t}, t = 1, \ldots, T. \tag{25}$$

In (25) we are no longer dealing with unit prices p_i and unit labour values λ_i because t refers to a specific quantity. For that reason $\hat{p}_t$ and $\hat{\lambda}_t$ are used to indicate price and value aggregates, respectively. In terms of Adam Smith, Ω_t can be interpreted as ratio of labour commanded to labour embodied. Surely, it cannot generally be less than one because then the selling price of a commodity does not even meet its direct and indirect wage costs. Furthermore, if Ω_t would not be a random variable but degenerate at unity, we would fall back to our introductory world without capitalists in equation (3) and (4).

Now look back on (18). It says that prices are made of integrated labour costs and integrated profits. Therefore,

$$\hat{p}_t = \hat{\delta}_t + \hat{\theta}_t, t = 1, \ldots, T. \tag{26}$$

Next we are dividing equation (26) by its corresponding labour value and get

$$\Omega_t = \frac{\hat{\delta}_t}{\hat{\lambda}_t} + \frac{\hat{\theta}_t}{\hat{\lambda}_t}, \tag{27}$$

with the expected value

$$E(\Omega) = E\left(\frac{\hat{\delta}}{\hat{\lambda}}\right) + E\left(\frac{\hat{\theta}}{\hat{\lambda}}\right). \tag{28}$$

Because the first term on the right-hand side of equation (28) can be expressed as

$$E\left(\frac{\hat{\delta}}{\hat{\lambda}}\right) = \sum_{t=1}^{T} \phi_t \left(\frac{\hat{\delta}_t}{\hat{\lambda}_t}\right), \tag{29}$$

with weights

$$\phi_t = \frac{\hat{\lambda}_t}{\sum_{t=1}^{T} \hat{\lambda}_t}, \tag{30}$$

we immediately obtain

$$E\left(\frac{\hat{\delta}}{\hat{\lambda}}\right) = \frac{\sum_{t=1}^{T} \hat{\delta}_t}{\sum_{t=1}^{T} \hat{\lambda}_t} = E(w), \tag{31}$$

and similarly,

$$E\left(\frac{\hat{\theta}}{\hat{\lambda}}\right) = \frac{\sum_{t=1}^{T} \hat{\theta}_t}{\sum_{t=1}^{T} \hat{\lambda}_t} = e^* E\left(\frac{\hat{\delta}}{\hat{\lambda}}\right), \tag{32}$$

where $e^* = \frac{\sum_{t=1}^{T} \hat{\theta}_t}{\sum_{t=1}^{T} \hat{\delta}_t}$. Hence, the expected value $E\left(\Omega\right)$ is given by

$$E\left(\Omega\right) = (1 + e^*)E\left(w\right). \tag{33}$$

Since the average hourly wage $E(w)$ is used as unit of account, that is $E(w) = 1$, expression (33) reduces to

$$E\left(\Omega\right) = 1 + e^*. \tag{34}$$

For that reason $E(\Omega)$ depends on the "general" profit-wage rate e^*. But this means that (34) is the stochastically counterpart to (9) and (20). To state more precisely: If we displace the assumption of uniform profit rates by considering the PDF instead, the theory of value holds as a *statistical law*, even in a state of equilibrium.

To put the discussion in concrete terms Farjoun and Machover assumed Ω to be described by the following normal distribution:

$$\mathcal{N}\left(1 + e^*; \sigma\right) = \mathcal{N}\left(2; \tfrac{1}{3}\right). \tag{35}$$

The authors make this suggestion because they observed some evidence that in developed capitalist countries all value added is split "fifty-fifty" between profits and wages, at least approximately. Furthermore, in their view the standard deviation σ should be $\frac{1}{3}$ because in this case the probability of $\Omega_t < 1$ would be less than $\frac{1}{1000}$ which they suppose to be quite realistic (Farjoun and Machover, 1983, pp. 123–124).

3 Empirical framework

3.1 Data

Data is taken from the German Federal Bureau of Statistics which offers IO tables including information on 71 sectors. Because statistics on German capital stocks only contain 57 sectors, the relevant columns and rows of IO tables have to be merged in such a way that every sector meets a figure from capital stocks. The data refers to the years 2000 and 2004.

Since the labour theory of value implies the distinction of productive and unproductive labour, the following rows are treated as being surplus value: Finance, assurance, real estate, educational and social services including all other kind of public or non-commercial services.[4] Moreover, taxes are taken as being profits and sectoral outputs are evaluated at producer prices to avoid confusion caused by trade margins (Shaikh and Tonak, 1994, pp. 78–81). Some sectors were removed from the analysis because they are outliers. This procedure is harmless since all of these sectors are either highly state-regulated (coal, water supply), rent-biased (oil) or offer non-market goods. Finally, there remain 38 sectors.

Now monetary labour values can be obtained by applying (2) and (5) to an appropriate IO table. Because the German National Accounts provide no information on direct labour inputs but on wages it is necessary to use a common dummy wage rate $w := 1$. This procedure implies that all empirical inter-sectoral wage differentials are considered to be caused solely by different skill levels. Or, rephrased, skilled labour is expressed in units of simple labour (Cockshott and Cottrell, 1997, p. 546). Of course, the empirical monetary labour values refer to aggregates, that is $\hat{\boldsymbol{\lambda}}_m := w\hat{\boldsymbol{\lambda}}$. Market prices or observed prices $\hat{\boldsymbol{p}}_o$ are obtained by taking the money value of sectoral net output.

Calculating prices of production $\hat{\boldsymbol{p}}_n$ needs some comment as well. In (14), $\boldsymbol{A}$ is based on flow terms. But in reality, obviously, there are stocks, too. Actually, this would require to add a matrix of capital coefficients and to calculate r with respect to these information. Or, rephrased, the "true" prices of production are not observable on the sole basis of IO tables. Unfortunately, in case of German data a capital matrix is not available. There is only knowledge about the money value of sectoral capital stocks. Therefore, the following procedure is made: neoricardian prices are computed despite of lacking capital coefficient but by using the money value of sectoral capital stocks for calculating r. Then the arithmetic mean of all 38 sectoral profit rates is used to get an estimate of r because this single numerical value should be the crucial empirical signal which would drive profit rates towards equalization. In doing so, another crucial point occurs: Applying stocks depends on defining turnover times. Because sectors in IO tables include a broad mix of different production periods, this is a serious problem which is hardly to handle in a satisfying way (Tsoulfidis and Maniatis, 2002, pp. 368–369). Thus, two polar assumptions can be introduced to make the empirical analysis possible: Firstly, individual time differences effectively cancel out. Profit rates then should only base on capital stocks. Secondly, the production period takes one year as it is implemented in national accounts. In this case, r should be better estimated with respect to capital stocks and sectoral inputs known from $\boldsymbol{A}$, too. Probably, the truth lies somewhere in the middle. Both possibilities, however, lead to almost the same empirical results, so there is no need

[4] This terminology is rather misleading. It would be more precise to speak of surplus-creating labour and surplus-consuming labour instead. See Shaikh and Tonak (1994, pp. 20–32, 74) and Mohun (2003) for further explanations.

to be concerned about these things too much. Instead, it is appropriate to choose this method whose fit is (marginally) better.[5] Therefore the uniform "production period" – which is in fact an accounting period – is defined to be one year.

3.2 Measuring deviations from prices to values

There are several ways to analyse the empirical relationship between labour values, prices of production and market prices. Common measures widely used in the literature are correlation coefficients and linear regression estimations.[6] But some authors deny the general possibility of estimating the empirical price-value deviations because in their view the results would be always affected by the physical units and the specific numériare behind the aggregated data from IO tables (Díaz and Osuna, 2005–06, 2007, 2009).[7] But the following steps will show that this is not true. To clarify that issue it will be useful to come back to a measurement procedure supposed by Steedman and Tomkins (1998, pp. 381–382). The starting point is the price system already known from equation (15). Rearranging this formula yields the neoricardian interpretation of the general price system given by (18):

$$\boldsymbol{p}_n = w\boldsymbol{\lambda} + r\,\boldsymbol{p}_n\boldsymbol{H}, \tag{36}$$

with $\boldsymbol{H} := \boldsymbol{A}\,(\boldsymbol{I} - \boldsymbol{A})^{-1}$. Further manipulation gives

$$w^{-1}\boldsymbol{p}_n\boldsymbol{\Lambda}^{-1} = \boldsymbol{i} + r\left(w^{-1}\boldsymbol{p}_n\boldsymbol{\Lambda}^{-1}\right)\left(\boldsymbol{\Lambda}\boldsymbol{H}\boldsymbol{\Lambda}^{-1}\right), \tag{37}$$

where $\boldsymbol{i}$ is the $(1 \times n)$-vector of dimensionless unit elements. The expression $\left(w^{-1}\boldsymbol{p}_n\boldsymbol{\Lambda}^{-1}\right)$ is another vector of pure numbers and $\left(\boldsymbol{\Lambda}\boldsymbol{H}\boldsymbol{\Lambda}^{-1}\right)$ is a matrix of pure numbers, too. For notational convenience additionally define $\bar{\boldsymbol{p}} := w^{-1}\boldsymbol{p}_n\boldsymbol{\Lambda}^{-1}$. Now the angle α between $\bar{\boldsymbol{p}}$ and $\boldsymbol{i}$ is an appropriate way to measure the deviations from prices to values because it is only based on *dimensionless numbers* and not the length but the *direction* of the relevant vectors, that is, it is based on relative prices. Hence, α must be independent of any physical unit and it is also not affected by the arbitrary choice of numériare.[8] Applying the inner product immediately gives

$$\sqrt{n(\bar{\boldsymbol{p}}\bar{\boldsymbol{p}}^T)}\cos\alpha = \bar{\boldsymbol{p}}\boldsymbol{i}^T. \tag{38}$$

As before, n indicates the number of sectors and commodities. Now it is possible to use the mean $\mu_{\bar{\boldsymbol{p}}}$ and the standard deviation (SD) $\sigma_{\bar{\boldsymbol{p}}}$ to rewrite (38) as

$$\sqrt{\left(\mu_{\bar{\boldsymbol{p}}}^2 + \sigma_{\bar{\boldsymbol{p}}}^2\right)}\cos\alpha = \mu_{\bar{\boldsymbol{p}}}. \tag{39}$$

[5] See Fröhlich (2009, pp. 276–278) for details.

[6] See especially Shaikh (1984), Cockshott and Cottrell (1997, 2003) and Tsoulfidis and Maniatis (2002) for details.

[7] A related topic is discussed by Cockshott and Cottrell (2005), Cockshott et al. (2009, pp. 201–202) and Kliman (2002, 2005).

[8] Both the independence from physical units and the independence from the choice of numériare are important. Although Díaz and Osuna (2009, p. 435) mention the second point, their critique do not cover the first one. Instead, they present an example in which the physical units are defined in such a way that $w\boldsymbol{\lambda} = \boldsymbol{i}$ (Díaz and Osuna, 2009, pp. 437–438). As a consequence, α depends on this arbitrary method of measurement. But this procedure is not in line with the approach described above: In equation (37) all physical units cancel out. Therefore, it is impossible that physical dimensions have any influence on α whenever (37) is applied.

Since, by elementary trigonometry, $\tan\alpha = \left(\sqrt{1-\cos^2\alpha}\right)(\cos\alpha)^{-1}$, it follows that

$$\tan\alpha = \frac{\sigma_{\bar{\boldsymbol{p}}}}{\mu_{\bar{\boldsymbol{p}}}}. \tag{40}$$

From equation (40) it becomes obvious that $\tan\alpha$ equals the coefficient of variation (CV) of $\bar{\boldsymbol{p}}$.[9] As mentioned above, neither α nor CV depends on the choice of numéraire or physical units. In the next step we are using the above expression to define the "distance measure" d (Steedman and Tomkins, 1998, p. 382):

$$d := 2\sin\left(\frac{\alpha}{2}\right) = \sqrt{2(1-\cos\alpha)}. \tag{41}$$

Due to its adequate properties and its utilisation in recent studies (Tsoulfidis and Maniatis, 2002; Tsoulfidis and Mariolis, 2007; Tsoulfidis, 2008) α and d will be employed on the German data later on. Of course this procedure is not restricted to the price system defined in (36) but can also be applied to other economic interpretations of equation (18).

3.3 Probability density functions

Because Farjoun's and Machover's idea of gamma distributed profit rates and their statements (34) and (35) offer theoretical reasons for a strong relationship between prices and values their main arguments should be empirically checked as well. To define appropriate empirical distributions for Ω and r the necessary parameters can be computed by some suitable Maximum-Likelihood estimations. Afterwards it is possible to test whether the estimated density functions fit the empirical data by using a one-sample Kolmogorov-Smirnov (KS) test. Because the ordinary KS test requires a fully specified null distribution, location, scale, and shape parameters cannot be estimated from sample data. For that reason a Monte-Carlo based KS test is used instead.[10]

4 Results

4.1 Price-value deviations

Table 1 shows the deviations from monetary labour values and neoricardian prices to market prices based on the German IO tables in 2000 and 2004. It turns out that values and prices are very close to each other. Moreover, labour theory of value and neoricardian theory show nearly identical fits in explaining market prices. This is consistent with previous empirical studies.[11]

[9]Note that formula (39) and (40) are based on the population variance $\sigma^2_{\bar{\boldsymbol{p}}} = \frac{1}{n}\left(\bar{\boldsymbol{p}}\bar{\boldsymbol{p}}^T - \frac{1}{n}\left(\bar{\boldsymbol{p}}\boldsymbol{i}^T\right)^2\right)$. Because variables generated from empirical IO tables can be considered as a kind of sample data, it would be more appropriate to apply the unbiased sample variance $s^2_{\bar{\boldsymbol{p}}} = \frac{n}{n-1}\sigma^2_{\bar{\boldsymbol{p}}}$. The information given in Table 1 is computed in this way. However, since $n = 38$, the differences caused by using $s^2_{\bar{\boldsymbol{p}}}$ instead of $\sigma^2_{\bar{\boldsymbol{p}}}$ are pretty small.

[10]All estimations are performed by using the statistical computer language R which can handle Monte-Carlo based distribution tests. See Wolter (2009, pp. 11–14) for details.

[11]See Cockshott and Cottrell (1997, p. 547), Tsoulfidis and Maniatis (2002, p. 366), Zachariah (2006, p. 9), Tsoulfidis and Mariolis (2007, p. 428), Tsoulfidis (2008, p. 715).

Table 1: Deviation statistics, Germany 2000 and 2004.

	2000		2004	
	α°	d	α°	d
$\hat{\boldsymbol{p}}_o, \hat{\boldsymbol{\lambda}}_m$	11.134	0.194	12.592	0.219
$\hat{\boldsymbol{p}}_o, \hat{\boldsymbol{p}}_n$	10.907	0.190	12.471	0.217
$\hat{\boldsymbol{p}}_n, \hat{\boldsymbol{\lambda}}_m$	3.159	0.055	3.480	0.061

Table 2: Summary statistics, Germany 2000 and 2004.

Ω	2000	2004
Mean	1.835	1.988
Weighted mean	1.696	1.804
SD	0.361	0.444
CV	0.197	0.223

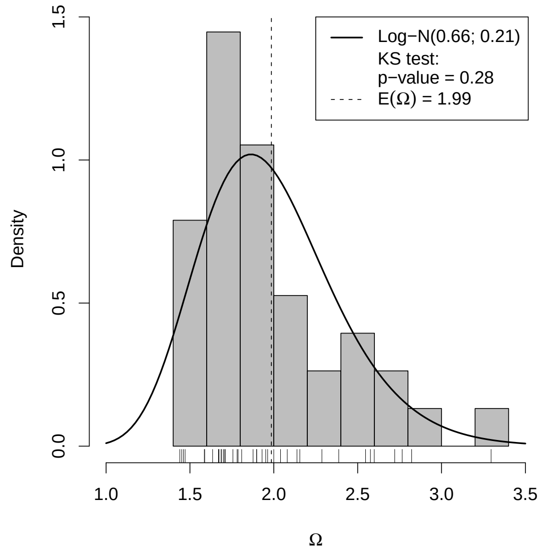
Notes: The mean values given in the second row are weighted by monetary labour values.

The summary statics given in Table 2 provides the other side of the coin because the mean and the standard deviation presented here constitutes the coefficient of variation which is just another way to express the small deviation measures. Obviously, the dispersion of the distribution of Ω is quite narrow. Moreover, as Farjoun and Machover have put it, the mean of Ω is close to 2 in 2000 as well as in 2004. But, more important, Ω is rather log-normal distributed than normal distributed. The reason becomes clear by remembering equation (24), that is influences on prices are multiplicative, not additive. We can see this in Figure 1 showing both the relevant histogram and the corresponding PDF estimation. Running a Jarque-Bera test shows that these values are likely to be normally distributed. Afterwards, a one-sample KS test was used to check if the resulting log-normal distribution meets the data. From there it is known that Ω could be described by the log-normal distribution $\Omega \sim \text{Log-}\mathcal{N}(0.66; 0.21)$ which contradicts equation (35). We can use the estimated PDF to calculate the expected value of deviation. This yields $E(\Omega) = 1.986$. Presenting similar estimations for the year 2000 would be redundant because the results do not differ significantly from the information given in Figure 1.

It is quite remarkable how well the labour theory of value is in line with the empirical knowledge. Though the same holds for neoricardian theory, this is not very surprising since nearly the whole body of both marxian and neoricardian literature supposed it to be state of the art. But hardly anybody of the theorists expects *both* theories to fit the data. However, on the basis of Section 2.3 and 2.4 there are good reasons to be not far too surprised.

It might be argued that there are indeed differences, however small, so neoricardian theory should be preferred. But this is not true. Because differences are small, we should rather take Occam's razor and favour that theory which is less complex. For two reasons, this is the labour theory of value: First of all, labour values can be computed without any need for data on capital stocks or capital coefficients. Secondly, we do not have to struggle with production periods

Figure 1: Histogram and PDF of Ω, Germany 2004.



because labour values do not depend on profit rates. Thus, using neoricardian prices for empirical research is more error-prone than applying labour values.

4.2 The economics of profit rates and surplus rates

We continue the empirical study by analyzing profit rates and surplus rates. Capital intensity q may also be of interest, that is capital advanced divided by paid wages. Table 3 gives a summary statistic. Obviously, none of these variables have narrow distributions. In case of capital intensity this is not amazing. But profit rates should cluster narrowly around the average because from a classical point of view there must be uniform profit rate. A similar argument applies to surplus rates. For instance, Farjoun and Machover (1983, pp. 32, 70) and Cockshott and Cottrell (1998, p. 77) argue that e does not vary strongly by sectors. In fact, the corresponding coefficient of variation is greater than the profit rate's is. Now recall Farjoun's and Machover's statements (34) and (35). They maintain that

$$E(\Omega) = 1 + e^* = 2. \tag{42}$$

For the German economy there is a weighted mean surplus rate $e^* = 0.691$ and $e^* = 0.788$ in 2000 and 2004, respectively (see Table 3). We therefore get an acceptable approximation of the expected value $E(\Omega) = 1.99$ presented in Figure 1. But note that these numerical values are based on the distinction between productive and unproductive labour, contrary to the considerations given by Farjoun and Machover. Of course, without this procedure the mean of e would be clearly smaller. However, from the perspective of the aforementioned probabilistic approach, this is just a methodological detail which do not alter the overall argument.

Farjoun's & Machover's hypothesis of gamma distributed profit rates is well founded, too. They originally proposed that profit rates in the British non-oil manufacturing industries in 1972

Table 3: Summary statistics of r, q and e, Germany 2000 and 2004.

	2000			2004		
	r	q	e	r	q	e
Mean	0.130	11.177	0.969	0.137	12.177	1.187
Weighted mean	0.103	6.699	0.691	0.115	6.879	0.788
SD	0.102	12.412	0.785	0.090	14.526	1.024
CV	0.787	1.111	0.809	0.659	1.193	0.862

Notes: The mean values given in the second row are weighted by denominator in each case.

Table 4: Correlation coefficients (weighted by capital stock), different countries.

Country	Year	N	cor(r, q)	cor(e, q)
USA	1987	47	−0.454	—
Sweden	1995	48	−0.478	0.538
Sweden	2000	48	−0.520	0.438
Germany	2000	38	−0.476	0.744
Germany	2004	38	−0.445	0.807

Notes: All correlation coefficients are significant at the 1% level. USA 1987 is taken from Cockshott and Cottrell (2003, p. 752). Sweden 1995 and 2000 are taken from Zachariah (2006, p. 15).

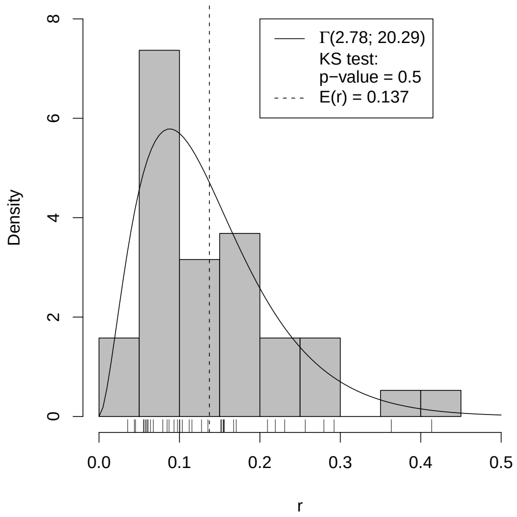
could be well described by the gamma distribution $r \sim \Gamma(a; b)$ with shape $a = 4.72$ and rate $b = 32.00$ (Farjoun and Machover, 1983, p. 173). As Figure 2a demonstrates, in case of Germany we have $r \sim \Gamma(2.78; 20.29)$ in 2004 with pretty goodness of fit. Calculating $E(r)$ with respect to the estimated PDF yields

$$E(r) = \frac{a}{b} = 0.137, \tag{43}$$

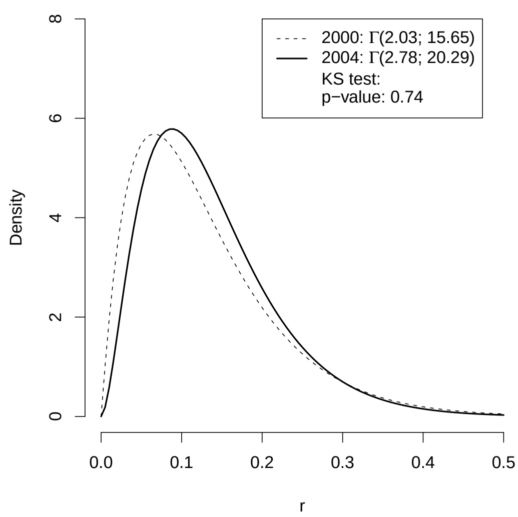
which means that the mean profit rate which was used for calculating neoricardian prices equals the expected value taken from (43), at least rounded to three digits. Moreover, the equilibrium assumption seems to be true as well since applying the data for 2000 yields $r \sim \Gamma(2.03; 15.65)$. Using a two-sample KS test shows that there is no significant difference to 2004 (see Figure 2b) so there is no need to present the estimated PDF for 2000 on its own. What we may conclude from here is that there is neither evidence for equalising profit rates nor do profit rates cluster nearly around the mean as it is typically expected in classical economics.

Now let us take functional relationship into account. Table 4 provides correlation coefficients for the relevant variables. In addition to the German data former estimations for the USA and Sweden were taken from the literature to give a complete picture. All coefficients are weighted by capital stock. In the case of USA this procedure refers to fixed capital plus one month's circulating capital (Cockshott and Cottrell, 2003, p. 752). The Swedish data is based on fixed capital solely (Zachariah, 2006, p. 14). As explained above, the German capital stock is defined as fixed capital plus one year's circulating capital. Note that the number of observations N differs

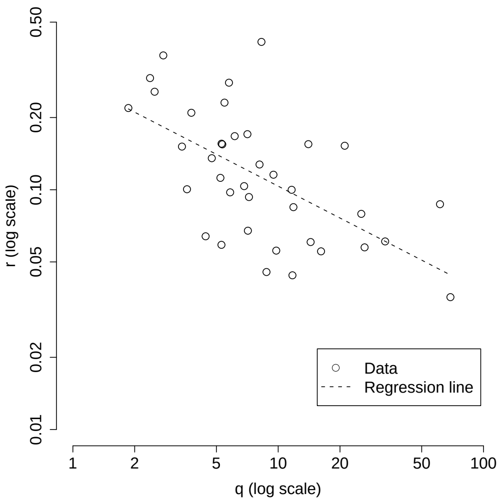
Figure 2: Evidence for a negligible transformation problem.



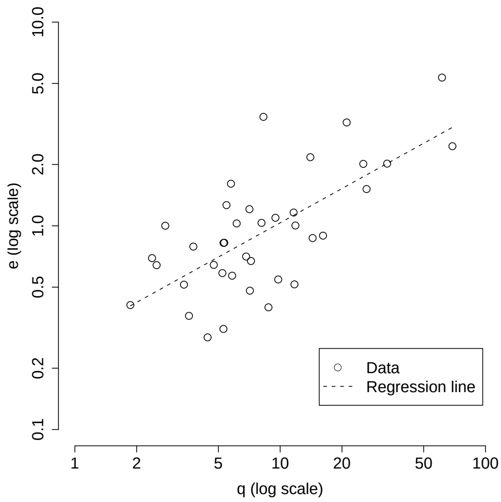
(a) Histogram and PDF of profit rate, 2004.



(b) Profit rate equilibrium, 2000 and 2004.



(c) Relation between capital intensity and profit rates, 2004.



(d) Relation between capital intensity and surplus rates, 2004.

across the aforementioned countries due to different numbers of sectors in national capital stock statistics. But all estimations are based on a similar procedure to distinguish productive labour from unproductive labour as it is assumed throughout this paper. Therefore, the different numerical values in Table 4 are comparable to each other.

Regarding Table 4, there are several points are of interest. First of all, and most important, we can recognize a negative correlation between profit rate and capital intensity for all countries.[12] Figure 2c offers a graphical impression of this issue, showing a negatively sloped regression line. This is a very remarkable result challenging the whole body of literature on the transformation problem because it is always taken for granted that profit rates must be independent of capital intensity. Only the labour theory of value predicts this incidence and it is precisely due to this fact that it is usually thought to be fundamentally flawed.

It should be emphasised that the key point in Table 4 is rather the missing statistically independence between r and q, not the statement that there is a quit strong negative correlation coefficient. Podkaminer (2005) argues that this kind of empirical results are not surprising because if profits, capital advanced and wage costs are pairwise independent random variables the correlation between r and q would be expected to be negative and relatively large. But this is a misunderstanding: Nearly the whole body if literature expects these variables *not* to be independent from each other. Instead, they are supposed to be functional related in such a way that in a state of equilibrium there is a uniform profit rate across the whole economy. For example, equation (13) is based on that assumption.

At least, there is another unexpected effect, namely the positive correlation between surplus rate and capital intensity. This seems to be the counterpart to the negative relationship between r and r (see Figure 2d). Those Sectors producing with a relatively high capital equipment per working hour partially compensate the comparatively lower profitability of capital stock by arranging an appropriate surplus rate. This observation seems to indicate a specific kind of transformation process.[13] On the basis of Section 2.1, this is a serious problem because in traditional marxian economics e should be uniform having no systemic influence on its own. In this respect, the deterministic or "mechanical" version of the labour theory is effectively wrong. But reformulated as a kind of Probabilistic Political Economy, it is not affected. More important, though a certain transformation tendency appears, this phenomena is not strong enough to fully compensate the effects of different capital intensities. Obviously, this is a strong provocation for all theories based on the assumption of non-dependency between profit rate and capital intensity.

5 Conclusion

This paper gives similar results to those of the previous studies concerning labour values and market prices. It is argued that both the labour theory of value and the neoricardian theory yield very good results in explaining data. Differences in estimated outcomes are mainly negligible. Therefore, noticing the approaches developed by Shaikh (1984) and Farjoun and Machover (1983)

[12] As mentioned before, the results do not depend on the assumption of a yearly production period, that is the definition of capital stocks. Quite the contrary, in the case of German data calculating r and q with respect only to fixed capital yields $\text{cor}(r, q) = -0.536$ in 2000 and $\text{cor}(r, q) = -0.556$ in 2004.

[13] Cockshott and Cottrell (1998, p. 82) also recognise this outcome but their results are only based on flow terms.

and having Occam's razor in mind, we should prefer the labour theory of value for analyzing real world phenomena. In addition, there is one critical point for neoricardian theory: The basis of the transformation debate seems to be wrong because profit rates and capital intensity are negatively correlated. Moreover, there are hints on gamma distributed profit rates which are therefore not uniform. The corresponding density functions do not change significantly during 2000 and 2004. Hence, in terms of Probabilistic Political Economy, it seems that the German economy was in a state of statistical equilibrium.

After all, although most of the classical authors deny the relevance of labour values for explaining prices, there are good reasons to argue that the law of value is correct in a stochastic sense. Without going into detail, this implies that the famous marxian invariant postulates are justifiable in a similar way. Profit, therefore, is not based on the marginal product of capital but on exploited labour. But with due respect to the reader's resources, questions like this may be discussed some other time.

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