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Values, prices, and wage–profit curves in the US economy

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1. Introduction

This paper presents actual labour values and production prices inherent in the structure of the US economy during the years 1947, 1958, 1961, 1963, 1967–70, and 1972, using a 71-industry fixed-capital model of the economy. This will be done in order to measure the deviations of production prices from labour values and actual market prices, as well as their relationship over time. In addition, US wage–profit curves for five of these years will be presented.

The above project acquires its significance when viewed in the context of the emergence of neoRicardian economic theory. Sraffa's analysis of the relationship between prices and distribution was aimed at a critique of the reigning marginalist orthodoxy (Sraffa, 1960). In this purpose, it was eminently successful. In a well-known debate carried out in a series of articles originating primarily from both Cambridges, the aggregate neoclassical theory of production and distribution[1] was dealt a severe—some would say fatal—blow.

This was due to two results of the Sraffian model. First, it showed that the neoclassical concept of capital as a 'real' factor of production whose quantity is measurable prior to exchange and distribution, was unsustainable, since the prices used to measure the value of capital goods depend on distribution. Second, by raising the possibility of reswitching of techniques as the rate of profit (and distribution) varies, it showed that the results of marginal productivity theory—inverse relation between capital intensity and the rate of profit; the rate of profit as the scarcity price or efficient allocator of capital—were theoretically untenable.

The same analysis, however, contained an implicit criticism of the Marxian labour theory of value. The criticism soon turned explicit (Steedman, 1977). The Sraffa system was said to make Marx's value analysis (and the subsequent transformation to prices of production) redundant. Moreover, the economic structure which emerged from a labour-value analysis, it was argued, did not necessarily match the observed structure in terms of

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[1] See Ferguson (1969) for a comprehensive statement of the neoclassical aggregate production and distribution theory. As was argued by Samuelson and others during the debate between the two Cambridges, reswitching did not affect the internal consistency of neoclassical theory in its full generality, in the Walrasian manner developed by Arrow and Hahn (1971), among others. But for a 'classical' critique of even this general form, see Dumenil and Levy (1985).

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prices. Specifically, the rate of profit in value terms did not equal the rate of profit in terms of production prices, the sum of prices did not equal the sum of values, and the sum of profits did not equal the sum of surplus value. Moreover, their deviations were not systematic, but depended on the vagaries of distribution. In fact, the adoption of new techniques did not follow a clear pattern in terms of value quantities (such as a rising organic composition of capital) but again depended on distribution and the ensuing price changes, since the decisions of individual capitalists are based on the observable prices alone.

The possibility of reswitching was once again invoked as an illustration of adjustments in the system in response to price changes which are independent of any changes in value magnitudes. This argument relies on the logical or algebraic possibility of magnitudes of the technical coefficients such that price-value deviations will be very large, and wage–profit rate curves will have the shapes required for reswitching of techniques, or distributional effects so large that value and price analyses will lead to different conclusions.

There is no question that these are mathematical possibilities of the model. Whether they are real possibilities of any actual economy, however, is another, more questionable matter. This paper will attempt to shed some light on this last question.

2. The Sraffian critique of Marxian value theory

The reswitching phenomenon in a Sraffian system can be briefly characterised as follows. Given a capitalist economy which produces n commodities using n single-product processes, the production-determined equilibrium prices which are formed depend on the distribution of the net product between wages and profits. Moreover, it is possible to construct two viable matrices of technical coefficients differing only in one column (set of sectoral inputs) such that one of the matrices (technologies) is more profitable than the other at high average rates of profit, less profitable at intermediate rates of profit, and once again more profitable at very low rates of profit. This is described as a switch out of, and a re-switch back to, the original technology as π assumes its range of values.

The criticism of Marxian value theory implicit here is that, since changes in distribution can reverse the ranking of techniques in price terms, their ranking according to which produces the given use-value at the lowest value (i.e., embodied labour time) is irrelevant to economic decision-making by individual capitals, and hence to the aggregate of those decisions. It follows that aggregate value analysis ('capital in general') cannot predict any dynamic tendencies operating at the price level (i.e., the laws of motion of capitalist production as a whole).

If the examples constructed to illustrate reswitching are representative of the techniques of production present in real economies, then this criticism has powerful practical force. However, if a careful study of all the available data on actual technology matrices which have existed over time yields no such cases, then the likelihood increases that reswitching is a mathematical curiosum of no real significance, rather like negative prices. By examining the technologies of the US economy for the period for which data are available (1947–1972), and the associated production prices and wage–profit rate curves, this paper seeks to make a contribution to this necessary empirical determination of the character of the reswitching phenomenon.

On a more general level, the neoRicardian criticism of Marxian value analysis rests on an emphasis on the quantitative divergence of prices from values. In this criticism, prices of production are identified with market prices; in other words, the economy is assumed to be in equilibrium. No real economy, however, is ever in equilibrium. The distinction between actual prices (market prices) and equilibrium prices (prices of production for all but the marginalists) is therefore crucial. The argument usually made against value analysis is that, since individual capitalists base their decisions on observed prices, value quantities which differ markedly from prices yield incorrect predictions of capitalists' behaviour.

A number of methodological objections which are independent of the magnitude of price-value deviations have elsewhere been made to this argument (Steedman, 1981). In this paper, however, we shall focus on two narrower points on which empirical evidence can be brought to bear. First, the divergence of theoretical predictions and actual behaviour of capitalists is equally likely to occur when comparing prices of production and market prices. It would seem in fact quite likely that values and prices of production are quantitatively closer to each other than either is to market prices, which would imply that value analysis is as good a practical tool of analysis of real economies as equilibrium-price analysis. Empirical confirmation of this hypothesis would be highly significant.

Second, one of the purposes of the value category in Marx's analysis is to show that the amount of socially necessary labour time required to produce commodities is the essential reality behind the formation of prices, and that therefore changes in the former will be the main long-run determinants of changes in the latter. This is likewise an empirically testable hypothesis.

3. Price systems

We now present labour values and three different types of price systems for the US economy for the years 1947, 1958, 1961, 1963, 1967, 1968, 1969, 1970 and 1972.

3.1. Labour values

Labour values are here assumed to be identical to total (direct and indirect) labour requirements for unit of output. The approach followed in dealing with the problem of heterogeneous labour is discussed in the appendix. In addition, we are ignoring dynamic effects such as rapidly changing technology and changing patterns of demand. The former effect would make the socially-necessary labour time—as determined in the sphere of production by the state-of-the-art technology—different from the (average) total labour requirements for the industry as a whole. The latter effect would imply a disjunction between the two senses of socially-necessary labour time (the second sense being the amount of social labour time which 'society' is willing to allocate to the production of a given good, as evidenced by the level of demand). Assuming these effects away is an abstraction comparable to assuming that the rate of profit is uniform across industries. Moreover, it should be possible to infer the real importance of these effects from the empirical results.

The system of 71 linear equations which define labour values is:

$$\mathbf{v}=\mathbf{a}_0+\mathbf{v}(A+D) \tag{1}$$

where $\mathbf{a}_0$ is the homogeneous-labour-coefficients row vector, A is the matrix of input coefficients whose elements a_{ij} represent the amount of good i used by industry j, and D is the physical capital depreciation matrix (see the appendix for a discussion of data sources and methods). The solution to (1) is given by

$$\mathbf{v} = \mathbf{a}_0(I - A - D)^{-1} \tag{2}$$

Equation (2) defines the row vector $\mathbf{v}$ as the amount of labour required directly and indirectly to produce one unit of sectoral output. That physical unit is a 'market-dollar's worth', given the way $\mathbf{a}_0$, A and D are defined. When we use their deflated versions, the unit is a constant (1972) market-dollar's worth, which is then a constant measure of physical output. The vector $\mathbf{v}$ thus has the units of labour + time per physical unit of output.

3.2. *Direct prices*

We define the row vector of direct prices $\mathbf{d}$, following Shaikh (1977), as the set of prices directly proportional to labour values, where the constant of proportionality relates the money unit to a unit of labour time (i.e. worker-hours/dollar). We define the money unit by requiring that the sum of sectoral outputs at direct prices equal the sum of sectoral output at market prices. In vector notation,

$$\mathbf{dq} = \mathbf{mq} \tag{3}$$

where the vector of market prices $\mathbf{m}$ is the unit vector, since $\mathbf{q}$ is measured in market prices. The proportionality constant μ (the value of money) is therefore given by:

$$\mathbf{d} = (1/\mu)\mathbf{v} = \frac{\mathbf{mq}}{\mathbf{vq}}\mathbf{v} \tag{4}$$

3.3. *Marxian prices of production*

Marx defines prices of production as the sum of costs plus an intersectorally uniform rate of profit on capital advanced. These magnitudes are seen to be the centres of gravity of the continually-fluctuating market prices. By capital advanced, we mean the capital invested in plant and equipment (fixed capital), plus the accumulated investment in inventories of materials, plus the stock of money necessary to pay out wages. The level of material inventories and wages fund is related to their flows by the turnover time of circulating capital t_j in the jth industry where $j = 1, \ldots 71$. If we have the flows per year, and the turnover time in (fractions of) years, then the necessary stocks are simply (annual flows × turnover time). This would give us the stock levels necessary to produce one year's output. Dividing through by the output level, we obtain the stocks required per unit of output.

In order to specify the wage, we must include the real counterpart of the value of labour power. This is given by the column vector $\mathbf{b}'$, which is the real wage basket per unit of homogeneous labour time. The expression for $\mathbf{p}$, the Marxian prices of production vector, is then as given below:

$$\mathbf{p} = \mathbf{p}(\mathbf{b}'\mathbf{a}_0 + A + D + \langle g \rangle) + \pi\mathbf{p}[K + (A + \mathbf{b}'\mathbf{a}_0)\langle \mathbf{t} \rangle] \tag{5}$$

where $\mathbf{b}'\mathbf{a}_0$ is the matrix of wage-good inputs, $\langle g \rangle$ is the diagonal matrix of indirect tax coefficients (see the appendix), and π is the uniform rate of profit. Let $A^+ = (\mathbf{b}'\mathbf{a}_0 + A + D + \langle g \rangle)$ (i.e. total costs) and $\mathbb{K} + [K + (A + \mathbf{b}'\mathbf{a}_0)\langle t \rangle]$ (total capital advanced). Then (5) reads

$$\mathbf{p} = \mathbf{p}A^+ + \pi\mathbf{p}K^+$$

or

$$(1/\pi)\mathbf{p} = \mathbf{p}K^{+}(I-A^{+})^{-1} \tag{6}$$

Equation (6) is the eigenvalue problem for the matrix $K^{+}(\mathrm{I}-A^{+})^{-1}$).

The economically meaningful solution requires $\mathbf{p}$ to be a strictly positive, real vector. If we assume that $K^{+}(\mathrm{I}-A^{+})^{-1}$ is an indecomposable matrix—and we know it is non-negative—then the Perron-Frobenius theorem ensures that the only such eigenvector is the one associated with the largest eigenvalue $(1/\pi)_{max}$ (to which corresponds the lowest π).

Since $\mathbf{p}$ is an eigenvector, it is defined up to a constant. In other words, (6) only defines a set of relative prices. To set the price level, we need a normalisation condition similar to (3):

$$\mathbf{pq} = \mathbf{mq} \tag{7}$$

Let $\mathbf{p}^{\star}$ and $\mathbf{p}$ be the unnormalised and normalised eigenvectors, respectively. Then we define the normalisation constant β such that $\mathbf{p} = \beta\mathbf{p}^{\star}$. We can then rewrite (7) as

$$\beta\mathbf{p}^{\star}\mathbf{q} = \mathbf{mq} \tag{8}$$

which implies

$$\beta = \frac{\mathbf{mq}}{\mathbf{p}^{\star}\mathbf{q}} \tag{9}$$

Therefore,

$$\mathbf{p} = \frac{\mathbf{mq}}{\mathbf{p}^{\star}\mathbf{q}}\mathbf{p}^{\star} \tag{10}$$

Summing up, Marxian prices of production are given by solving for the eigenvector associated with the maximum eigenvalue of the system of equations (6), and then normalising this eigenvector according to (10).

3.4. *Sraffian prices of production*

The generalised price system developed by Sraffa in Part II of his book (Sraffa, 1960) treats fixed capital as a joint product. Consider the following price system with joint products:

$$(1+\pi)\mathbf{s}K + \mathbf{s}A + \mathbf{a}_0\omega = \mathbf{s}B \tag{11}$$

K is the capital stock (which becomes a flow in a joint-product model) and A is the flow of circulating capital (materials), B is the matrix of joint products, ω is the scalar wage, and $\mathbf{s}$ is the price vector.

We shall present the conditions under which the Sraffian joint-product model of fixed capital reduces to a standard depreciation model. Define $\hat{B} = B - I$. Then $B = \hat{B} + I$, so that the joint-product matrix B is the sum of the identity matrix (i.e. the unit output of each industry) and the matrix $\hat{B}$, now interpreted to be the used-machinery coefficients (i.e. capital as a joint product). Then the above equation can be rewritten as follows:

$$\mathbf{s}(K-\hat{B}) + \pi\mathbf{s}K + \mathbf{s}A + \mathbf{a}_0\omega = \mathbf{s} \tag{12}$$

The matrix $(K-\hat{B})$ represents the difference between the stock of capital going into the production process and the stock of capital which emerges out of it; in other words, the scrappage matrix. This is precisely the series which we use to construct the matrix D in our price systems, so the above equation is none other than

$$\mathbf{s}=\mathbf{s}(A+D)+\pi\mathbf{s}K+\mathbf{a}_0\omega \tag{13}$$

This approach to fixed-capital models is discussed and criticised by Varri (1980) because the scrappage matrix $(K-B)$ cannot be defined independently of prices. Nevertheless, in any discussion of *actual* economies and calculations using empirically-obtained coefficients, this is all that is available. What the above discussion shows is that a joint-production treatment of fixed capital is equivalent to the standard treatment provided we assume that the price of old machines (and hence the decision to scrap) is not sensitive to changes in the distribution of income.

Even if this is the case, however, Sraffa's model is still different from Marx's, since he computes the profit rate on fixed capital advanced only. He also does not have a concept of the value of labour power, so the distribution of the surplus product between wages and profits is left open as a degree of freedom of the system. We can solve for the resultant system as follows:

$$\mathbf{s}=\omega\mathbf{a}_0(I-A-D-\pi K)^{-1} \tag{14}$$

This is a system of n linear equations and $n+2$ unknowns: $\mathbf{s}$, ω, and π. We also have our usual normalisation condition

$$\mathbf{sq}=\mathbf{mq} \tag{15}$$

Combining (14) and (15) and specifying the level of π, we get:

$$\omega\mathbf{a}_0(I-A-D-\pi K)^{-1}\mathbf{q}=\mathbf{mq}$$

or

$$\omega=\frac{\mathbf{mq}}{\mathbf{a}_0(I-A-D-\pi K)^{-1}\mathbf{q}} \tag{16}$$

Solving (16), we obtain the money wage, which we can use in (14) to obtain the price vector $\mathbf{s}$. This can be done for a number of values of π in the range (φ,R), where R is the inverse of the maximal eigenvalue of the system given below, which is (13) with $\omega=0$:

$$\mathbf{s}_R=\mathbf{s}_R(A-D)+R\mathbf{s}_RK \tag{17}$$

4. Sectoral price-value deviations

Following the definitions presented above, labour values $\mathbf{v}$, direct prices $\mathbf{d}$, and Marxian production prices $\mathbf{p}$ for the nine years studied were calculated. This section of the paper will present results based on those basic computations.

In order to measure the extent of the pair-wise cross-sectional deviation between direct prices, Marxian prices of production and market prices in a real economy, we developed the following statistics (here illustrated for the production price-direct price case of Table 1):

Mean Absolute Deviation (%): (18)

$$\mathrm{MAD}(\mathbf{p},\mathbf{d}) = (1/n)\sum_i \frac{|\mathbf{p}_i - \mathbf{d}_i|}{\mathbf{d}_i}(100)$$

Mean Absolute Weighted Deviation (%): (19)

$$\mathrm{MAWD}(\mathbf{p},\mathbf{d}) = \sum_i \frac{|\mathbf{p}_i - \mathbf{d}_i|}{\mathbf{d}_i}\cdot\frac{\mathbf{q}_i}{\sum_j \mathbf{q}_j}(100)$$

Normalised Vector Distance (%): (20)

$$\mathrm{NVD}(\mathbf{p},\mathbf{d}) = \frac{[\sum_i(\mathbf{p}_i\mathbf{q}_i - \mathbf{d}_i\mathbf{q}_i)^2]^{1/2}}{[\sum_i(\mathbf{d}_i\mathbf{q}_i)^2]^{1/2}}(100)$$

Both MAD and MAWD are the mean of the absolute value of the fractional deviations of one set of prices from another. The NVD measures the vector distance between the priced output vectors using two sets of prices as a fraction of the vector length of one of the former.

We also computed the cross-sectional correlation coefficient, which we report squared as R^2. In order to minimise spurious correlation—since prices and values must be correlated as $\mathbf{p}_i\mathbf{q}_i$ and $\mathbf{v}_i\mathbf{q}_i$ to have any variation in the market-price data—we computed R^2 on the logged data points. Nevertheless, as has been shown elsewhere (Ochoa, 1984; Petrovic, 1987), measures of covariance such as R^2 are not the proper statistics to assess the cross-sectional relation between alternative price systems. Rather, measures of deviation such as those presented above should be used.

The choice of numeraire should be the money unit, since that is how the exchange value of goods is actually measured. But the money unit's exchange value is in effect given by the equation of purchase price of total output to the quantity of total output. In effect, the unit of value is represented by a quantity vector with the proportions of the gross output vector, whose price is one dollar:

$$\mathbf{q} = \mathbf{Q}/\mathbf{mQ} = \mathbf{Q}/T \tag{21}$$

where $\mathbf{Q}$ = gross output vector; $\mathbf{p}$ = computed-price vector; T = scalar market price of output vector = $\mathbf{mQ}$; $\mathbf{q}$ = composite numeraire (vector); $\mathbf{m}$ = market-price vector.

Then the normalisation condition is

$$\mathbf{pq} = 1 \text{ or } \mathbf{pQ}/\mathbf{mQ} = 1 \text{ or } \mathbf{pQ} = \mathbf{mQ}$$

which is in fact the condition adopted in this study.

To measure the extent to which individual values determine the behaviour of production prices over time, we also performed 71 time-series linear regressions of values on Marxian production prices, using constant-dollar market prices as a pseudo-quantity measure. The associated correlation coefficients were averaged and squared to obtain a general measure of explained variation which is dimensionally comparable to the cross-sectional R^2. (The reason we squared after averaging was that we did not want to take credit for negative correlations over time. Using this procedure, the latter actually reduce the magnitude of the R^2-like time-series statistic.) Unlike in the case of cross-sectional deviations, here it is legitimate to use R^2 as a measure of correlation, because all three sets

Table 1. *Marxian production price—direct price relations*

Year	1947	1958	1961	1963	1967	1968	1969	1970	1972	AVE
MAD(**p**,**d**) (%)	18·2	13·8	14·9	16·3	17·7	18·1	17·8	16·9	17·9	16·9
MAWD(**p**,**d**) (%)	14·5	15·1	16·4	17·5	18·7	18·6	18·6	17·8	19·8	17·4
NVD(**p**,**d**) (%)	13·4	14·3	15·4	16·5	18·3	18·3	18·6	18·3	18·4	16·8
X-sect. R^2	0·972	0·979	0·976	0·972	0·967	0·965	0·966	0·971	0·970	0·971
T-sers. R^2	—	—	—	—	—	—	—	—	—	0·926

of prices (market, direct, and production prices) are varying over time independently of output levels.[1]

Calculated values for all these measures are presented in Table 1.[2] These results show clearly that labour values have a very high degree of cross-sectional correlation with prices of production. This admittedly inconclusive result is similar to results reported elsewhere (Wolff, 1979; Petrovic, 1987). More significantly, the average correlation over time is also quite high: approximately 93% of the variation in individual prices of production over time is due to changes in the underlying labour values, suggesting of all things a Ricardian 93% labour theory of value. In addition, average price-value deviations—whether weighted or unweighted—are quite small: around 17%. The 'transformation problem', therefore, appears to be of limited empirical significance.

[1] The following set of relations were estimated:

$$\mathbf{d}_i(t) = a_i + \beta_i \mathbf{m}_i(t) + \mathbf{u}_i(t);\ i = 1, \dots, 71;\ E(\mathbf{u}_i) = 0. \tag{i}$$

The relation above refers to unit prices, where the physical unit remains constant throughout the time-series. Our actual results use a current market dollar's worth as the physical unit; we needed to convert it to a constant (1972) market dollar's worth to obtain prices for an unchanging physical unit.

Our actual results were

$$D_i(t) = \mathbf{d}_i(t)\mathbf{q}_i(t);\ M_i(t) = \mathbf{m}_i(t)\mathbf{q}_i(t) \tag{ii}$$

We could not regress $D_i(t)$ on $M_i(t)$ without spurious correlation, due to the appearance of $\mathbf{q}_i(t)$ on both sides. So we can divide through by $M_i(t)$:

$$\frac{D_i(t)}{M_i(t)} = \frac{\mathbf{d}_i(t)\mathbf{q}_i(t)}{\mathbf{m}_i(t)\mathbf{q}_i(t)} = \frac{\mathbf{d}_i(t)}{\mathbf{m}_i(t)};\ \frac{M_i(t)}{M_i(t)} = 1 \tag{iii}$$

This is the form in which we have presented direct prices: $\mathbf{d}_i/\mathbf{m}_i$. While this eliminates $\mathbf{q}_i(t)$ from both expressions, it turns market prices into a constant (one), which means there is no variation left to correlate. Equation (i) is equally valid when divided through by a constant ($\mathbf{m}_i(1972)$), with suitable redefinition of a_i and $\mathbf{u}_i$

$$\frac{\mathbf{d}_i(t)}{\mathbf{m}_i(1972)} = a_i + \beta_i \frac{\mathbf{m}_i(t)}{\mathbf{m}_i(1972)} + \mathbf{u}_i(t) \tag{iv}$$

which gives prices per 1972 market dollar's worth of sectoral output. But

$$\frac{\mathbf{d}_i(t)}{\mathbf{m}_i(1972)} = \frac{\mathbf{d}_i(t)}{\mathbf{m}_i(t)} \cdot \frac{\mathbf{m}_i(t)}{\mathbf{m}_i(1972)}$$

Since $\mathbf{m}_i(t)/\mathbf{m}_i(1972)$ is nothing but the price-index $\mathbf{e}_i(t)$, we may write (iv) as

$$\frac{\mathbf{d}_i(t)}{\mathbf{m}_i(t)} \mathbf{e}_i(t) = a_i + \beta_i \mathbf{e}_i(t) + \mathbf{u}_i(t) \tag{v}$$

Equation (v) is equivalent to (i). The price-index $\mathbf{e}_i(t)$ is clearly a time-series of market prices; but the left-hand side appears to contain the same variable. A glance at (iv), however, quickly dispels that impression: the $\mathbf{e}_i(t)$ is there precisely to eliminate the influence of $\mathbf{m}_i(t)$ in the denominator of the left-hand side of (v).

[2] Empirical results not presented in this paper are available to readers from the author upon request.

Table 2. *Marxian production price—market price relations*

Year	1947	1958	1961	1963	1967	1968	1969	1970	1972	AVE
MAD(p,m) (%)	18·5	13·1	12·7	12·6	13·7	13·2	12·8	12·5	13·0	13·6
MAWD(p,m) (%)	16·8	13·4	14·1	14·3	15·0	14·5	14·1	13·1	14·5	14·6
NVD(p,m) (%)	19·6	15·5	16·4	16·7	17·4	16·8	16·1	15·3	17·6	16·8
X-sect. R^2	0·963	0·987	0·986	0·987	0·983	0·983	0·984	0·986	0·982	0·982
T-sers. R^2	—	—	—	—	—	—	—	—	—	0·760

Table 3. *Direct price—market price relations*

Year	1947	1958	1961	1963	1967	1968	1969	1970	1972	AVE
MAD(d,m) (%)	19·9	11·8	12·1	11·8	10·8	10·7	10·2	10·3	12·0	12·2
MAWD(d,m) (%)	16·0	11·8	12·7	12·5	11·8	11·1	11·5	11·1	13·8	12·5
NVD(d,m) (%)	17·3	12·0	13·6	13·4	13·2	12·2	13·2	12·7	15·8	13·7
X-sect. R^2	0·957	0·978	0·975	0·974	0·975	0·974	0·977	0·978	0·974	0·974
T-sers. R^2	—	—	—	—	—	—	—	—	—	0·754

It might be objected that our deviation measures are 'small' because we are using market prices (one dollar's worth) as the quantity unit of each sector, so that all computed prices cluster around unity. However, just as the mean value of a distribution is irrelevant when we measure its coefficient of variation (standard deviation/mean), so are the absolute values of the percentage deviation of one set of prices from another (see equations (18) and (19) above) independent of the scaling factor used. Whether we have actual $\mathbf{p}_i$ or $\mathbf{p}_i/\mathbf{m}_i$ as we do, the terms being summed in equations (18) and (19) would be the same:

$$\frac{|\mathbf{d}_i/\mathbf{m}_i - \mathbf{p}_i/\mathbf{m}_i|}{\mathbf{p}_i/\mathbf{m}_i} = \frac{|\mathbf{d}_i - \mathbf{p}_i|}{\mathbf{p}_i}$$

Moreover, in the case of our NVD measure, market prices are eliminated from the expression of computed prices because we use the value of the full sectoral output: $(\mathbf{p}_j/\mathbf{m}_j)(\mathbf{m}_j\mathbf{q}_j) = \mathbf{p}_j\mathbf{q}_j$.

The results presented above become even stronger when we consider the actual extent of the deviation between production prices and market prices, shown in Table 2.

A comparison of the results of Tables 1 and 2 shows that prices of production are nearly as far away from market prices as they are from direct prices. Since there is no reason to expect that these deviations are correlated, and from theoretical considerations, we would expect that the deviation between direct and market prices would be more than that of production prices from market prices, but substantially less than the sum of the two deviations. It was surprising to find that, in fact, the observed direct price-market price deviations are smaller than the production price-market price deviations (with the exception of the MAD in one year) as shown in Table 3.

While both MAD and MAWD measures are smaller between direct and market prices than between production and market prices, their R^2s are smaller as well. This suggests the possibility of systematic bias on the computation of production prices, leaving them farther away from market prices, even though more covariant with the latter than direct prices (we see here the usefulness of calculating R^2s as a means of estimating relative degrees of correlation, even though the absolute values are biased upward by the spurious correlation of output levels). The likely source of bias in the production-price computation is the capital stock series. The capital stock series issued by the Bureau of Industrial Economics of the US Department of Commerce (1983) is based on the 'perpetual inventory' method. The latter is essentially an integration of past investment flows coupled with a probabilistic scrappage function. This function uses a distribution centred around estimated asset lives. The results of the integration are very sensitive to the asset lives used, and the values used are known to be unreliable estimates.

Over time, prices of production account for somewhat more of the variation in market prices than do direct prices (R^2s equal 0·760 and 0·754, respectively), but the improvement is quite small compared to the amount of variation left unaccounted for. This suggests that labour values are the dominant deterministic influence over market prices, with the distribution effects of production prices playing a far less important role. Moreover, the distribution effects—the improvement obtained by the 'transformation' from values to prices of production—are themselves a fraction of the 'noise' component of market prices (the stochastic disequilibrium effects).

These results suggest that the labour theory of value is not only a powerful methodology for a critical understanding of the social relations of production in capitalism. It also appears that labour values are quantitatively dominant influences in the formation of market prices. The conceptual equalisation of the rate of profit which defines prices of production as the centre of gravity of market prices thus provides only a marginally better approximation to the latter than labour values themselves.

5. Price-value deviations and wage shares

The results presented above relative to price-value deviations are so startling that there has been and will be a tendency to minimise their importance by attributing them to the high wage shares in the US economy. In the limit, when profits go to zero and wages take all the net product, prices of production would become identical to values. Hence, if the distribution of the net product between profits and wages were close to this limit, prices of production would be close to values regardless of the manner in which the structure of the economy would transform values.

In order to investigate this possibility, we should vary the distribution of the net product between wages and profits, derive the corresponding prices of production, and observe the price-value deviations which ensue. To do this, we begin by replacing the real wage vector b′ with a variable scalar wage ω. From (5) we have:

$$\mathbf{p}=\mathbf{p}\mathbf{b}'\mathbf{a}_0+\mathbf{p}(A+D+<g>)+\pi\mathbf{p}\mathbf{b}'\mathbf{a}_0<t>+\pi\mathbf{p}(K+A<t>)$$

But $\mathbf{pb}'$ is a scalar (the production-price of the real wage vector); so we designate it ω. Solving the above equation for $\mathbf{p}$ we thus obtain:

$$\mathbf{p}=\omega\mathbf{a}_0(I+\pi<t>)[I-A-D-<g>-\pi(K+A<t>)]^{-1}$$

This is no longer an eigenvalue problem, but a linear system with two degrees of freedom. Combining the above expression with our usual normalisation condition leaves one degree of freedom. By setting π equal to values from 0 to R, we can obtain the

Table 4. *US price-value deviations as a function of wage share: 1967*

Wage share	π/R	MAD (%)	MAWD (%)	R^2
1·000	0·000	4·3	4·7	0·997
0·858	0·100	7·9	8·4	0·990
0·729	0·200	11·6	12·3	0·988
0·610	0·300	15·0	15·9	0·985
0·501	0·400	18·3	19·3	0·981
0·401	0·500	21·3	22·4	0·975
0·308	0·600	24·2	25·8	0·969
0·222	0·700	27·0	28·9	0·961
0·142	0·800	29·6	31·9	0·953
0·069	0·900	32·0	34·7	0·945
0·000	1·000	34·4	37·4	0·935
		Actual wage–profit point		
0·522	0·382	17·7	18·7	0·982

desired prices of production. The resultant price-value deviations for 1967 (a typical year) are shown in Table 4. It should be noted that it is the presence of indirect taxes as a component of costs in the price equation (5) that prevents the measures of deviation between production prices and direct prices from vanishing when $\pi = 0$.

These results show several things. First, the wage share of income in the US was by no means as high as is usually implied: the year 1967 shows an average wage share of income (after depreciation and indirect business taxes) of 52·2%. Second, the measures of deviation predictably become larger as the wage share is reduced, but they increase only moderately: even when the wage share drops to zero, the MAD is only 34·4%. The cross-sectional R^2s average 0·939 even for this extreme case. Similar results were obtained for the remaining eight years in this study. We conclude that the remarkably close correspondence between direct prices and prices of production is a feature of the actual US economy which is not sensitive to the level of the wage share.

6. Empirical wage–profit curves

Implicit in Marxian theory is the belief that an analysis of technical change carried out in value terms is not qualitatively modified in actual capitalist economies owing to the deviations of prices from values. It is the essence of the reswitching argument, however, that such 'distributional' sources of variation can generate phenomena all of their own, and thus not only destroy the neoclassical parable, but also make any analysis of choice of technique in terms of labour values inconsistent with the results in terms of prices (of production).[1]

There is no question but that it is possible to construct numerical examples of 2- or n-sector economies which will exhibit reswitching properties. The question emerges, however, as to whether such cases are merely logical curiosities or real possibilities for actual economic systems.

Unfortunately, we do not possess the information about all available techniques in each sector of production which would be necessary to calculate wage–profit frontiers. Using

[1] In this regard, see Parys (1982).

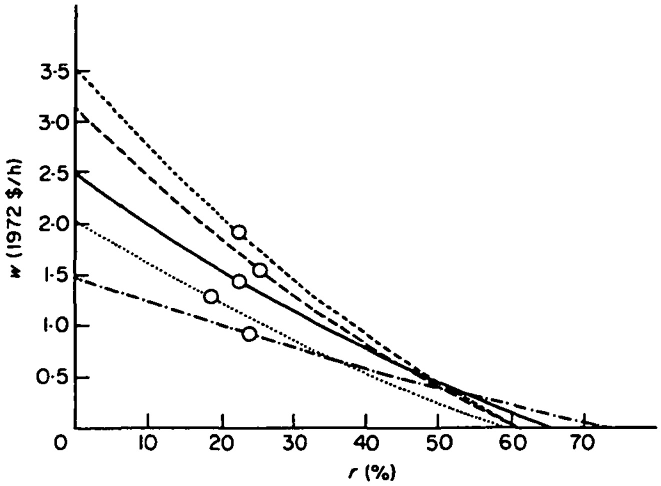

Fig. 1. *US wage–profit frontiers.* —·—·—· *1947;* ······· *1958;* ——— *1963;* — — — *1967;* - - - - *1972;* O *actual* (w,r).

our 'Sraffian' prices of production,[1] we can generate wage–profit *curves* for each year's economy. While these are not the wage–profit frontiers which are invoked in the reswitching argument, they are the only computable wage curves, and provide suggestive evidence on the empirical relevance of reswitching. To obtain them, we simply solve equations (14) and (16) above for values of π from φ to R. By using coefficients and output levels measured in constant dollars, we obtain values of the money wage which are directly comparable from year to year. The wages are, in effect, deflated.[2]

The results of this procedure for the years 1947, 1958, 1963, 1967, 1972 are plotted in Fig. 1. This shows the wage-profit curves of the US economy for the five years in question, spanning a period of 25 years. The most striking feature of all of them is how nearly linear they are: they exhibit very slight convexity, with extremely small 'wiggles' close to the π-intercept.[3] This is due to the remarkable closeness of labour values to prices of production for the US economy.

[1] We chose to use what we call Sraffian prices because these most closely approximate the terms of the reswitching debate. Marxian prices of production, however, yielded nearly identical curves.

[2] They are not, however, deflated by the equivalent of the GNP deflator. Rather, by what we might call 'gross output deflator', in the input–output sense of the term.

[3] Our description of the wage–profit curves as nearly linear can be justified by a simple quantitative measure: a stepwise linear regression of w on powers of π using our calculated points. For 1967—a typical year—we observed levels of explained sum of squares of deviations shown below (sum of squares explained by each variable when entered in the order given):

Due to	Sum of squares
π	9·682445
π^2	0·108906
π^3	0·000975
π^4	0·000009
Residual	0·000001
Total	9·792335

Alternatively, when we regress w on π alone (i.e., approximate the wage–profit curve with a straight line) we obtain for 1967 an R^2 of 0·989. Similar results hold for the other years.

Table 5. *US rate of surplus value*

Year	1947	1958	1963	1967	1972
s/v (%)	95·1	86·9	101·4	116·7	111·7

(Standard deviation = 12·1; mean = 102·4; standard deviation/mean = 0·12).

There is also a strong trend toward higher net product per worker, as evidenced by the rising ω-intercept. The output-capital ratio (or maximum rate of profit R) fell dramatically from 1947 to 1958; after which it rose steadily until 1967. Between then and 1972 it fell once again.[1] In other words, the character of technical change from 1947 to 1958 and from 1967 to 1972 follows Marx's characterisation (falling unit values and falling R); from 1958 to 1967, however, the wage-profit curve moved strictly outward, so that the new techniques were unambiguously more profitable regardless of the wage rate. This implies that the new techniques were both capital- and labour-saving in this period.

Another noteworthy feature is the actual position of the economy during these years, as shown by the small circles on the curves. Notice that in every single instance, the new curve would have yielded a higher rate of profit if the wage rate had remained unchanged from one time period to the next. The higher wage rates alone were responsible for the fall in the profit rate in the two periods when this happened; in the other two periods, the rise in the wage rate was more than compensated for by the outward expansion of the wage–profit curve (specifically, by the rise in R).

In fact, the only periods where the actual π fell are precisely those for which the associated R fell; also, the rise in the wage rate closely mirrors the rise in output per worker. Both of these characteristics are consequences of the fact that the rate of surplus value remained relatively steady throughout this entire period, as shown in Table 5.

7. Conclusion

This paper has shown that labour-values and prices of production for the US economy in the post World War II period were remarkably close to each other as well as to market prices. The scale of the errors to be expected in the available data suggests that little if any accuracy is to be gained by calculating prices of production, so that either value or market-price series should be adequate in studying the behaviour of the economy in the aggregate and over time. This is a startling empirical postcript to the long-standing debate on the 'transformation problem' which now appears to involve relatively insignificant magnitudes in real economies.

Equally remarkably, the wage–profit curves implicit in the input-output coefficients for the period all were very nearly linear. Moreover, this latter result does not depend on the composition of output or the weighting scheme used to homogenise labour inputs. As is well known, linear wage—profit curves are obtained when Sraffa's standard product is used as the numeraire, so if the US economy was producing in near-standard proportions, this could explain our results. However, when we calculated the standard product proportions and compared them to the actual ones, we found them drastically different: for 1967—a typical year—the MAD between standard and actual products was 386%, and the

[1] Correcting for capacity utilisation does not alter these results (Ochoa, 1984).

R^2 0·037. The weighting scheme used for labour inputs was uncorrelated with capital intensity; moreover, the results with unweighted labour inputs are not significantly different (Ochoa, 1984).

While the wage–profit curves calculated cannot be brought directly to bear on the theoretical reswitching debate, the fact remains that over a period of 25 years the economy has exhibited wage—profit curves (i.e. techniques) which are a far cry from the apparently unlikely shapes required for reswitching and capital reversing to occur. While the presence of heterogeneous capital goods and fixed proportions dealt fatal blows to the neoclassical concept of aggregate physical capital, the near-linearity of actual wage–profit curves appears to support the labour theory of value as a powerful practical tool to analyse and understand the global character of production and growth in capitalist economies.

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Appendix: data sources

This is an account of the sources and methods used to perform the calculations reported in this paper. For further details, see Ochoa (1984). We have, from the US Department of Commerce, A, the matrix of input–output coefficients for the US economy for the years mentioned above, at various levels of disaggregation. We worked at the 82-industry level, aggregated to 71 industries, in order to match the capital-stock data available. Unlike the technical coefficients matrix used in Sraffian models, these matrices show the input requirements in dollars per dollar of output (the Sraffian technical-coefficients matrix shows the interindustry requirements in real terms). From the same source we have **q**, the dollar value of output-by-industry vector, at 82-order.

We have, from the US Department of Labor, the direct-labour requirements, in worker hours, per dollar of output, at 82-order, for the years listed above. Since these labour times are heterogeneous (different skills and intensities), we used the relative wage structure in these industries, obtained from the same source, to reduce skilled or more intense labour to unskilled labour of minimum intensity, as exemplified by the lowest-wage sector. We thereby assume that the labour markets have no significant barriers to entry, so that the relative wage structure is a good measure of the relative values of heterogeneous labour powers and the rate of surplus value is uniform across industries. We call the resultant reduced-labour requirements vector, $\mathbf{a}_0$.

From the US Department of Commerce (1983), we have the vector $\mathbf{k}$, the dollar value of gross capital stock in current dollars for each industry. Since this capital stock is heterogeneous, we used asset weights w to disaggregate $\mathbf{k}$ in G, the gross capital-stock matrix, whose elements G_{ij} show the dollar value of the stock of the ith capital good in the jth industry. Dividing each column j of G_{ij} by q_j, we obtain the matrix of capital-stock coefficients K_{ij}. The set of asset weights were derived by assuming that the stock of every capital good has a uniform age distribution, so that the fraction scrapped each year is the inverse of the asset lifetime (given in US Dept. of Commerce, 1979). We then assumed further that the composition of the capital stock for each industry, in terms of the 71-commodity structure we are using, changes slowly over time. It follows that there is a straightforward relation between the composition of gross investment—which is given for 1963, 1967, and 1972—and the composition of the capital stocks. The composition of net investment will be the same as that of replacement investment, and clearly the same as their sum (gross investment). By assumption, the following relation holds between replacement investment (depreciation) and stock of the ith good in the jth industry:

$$D_{ij}=\frac{K_{ij}}{l_i} \text{ or } K_{ij}=l_iD_{ij} \tag{A1}$$

where l_i is the lifetime of the ith asset. The relative composition of the capital stock w_{ij} is therefore given by

$$w_{ij}=\frac{K_{ij}}{\sum_i K_{ij}}=\frac{l_iD_{ij}}{\sum_i l_iD_{ij}} \tag{A2}$$

and we obtain K from $\mathbf{k}$ using the expression below:

$$K_{ij}=w_{ij}k_j \tag{A3}$$

We also have 71-order depreciation vectors $\mathbf{d}_j$ (gross discards) of capital stock for the relevant years (US Dept. of Labor, 1980). These depreciation levels are straightforwardly disaggregated—given our assumption of constant composition—as follows:

$$D_{ij} = \frac{H_{ij}}{\sum_i H_{ij}} \mathbf{d}_j \tag{A4}$$

where H_{ij} is the known gross investment matrix. This also allows us to compute w_{ij} as outlined above.

We proceed to discuss the conditions under which it is appropriate to use gross discards as a measure of the physical wear-and-tear of capital goods. Assume a uniform distribution of ages for each kind of asset held by each industry. If this assumption is reasonable, then each year's gross discards of equipment by each industry will equal total physical wear-and-tear for all assets held by that industry. The latter quantity (which is defined prior to distribution) need not be obtained by assuming linear depreciation: in fact, in the statistics which we used, it is not. This proposition can be shown to be true as follows. Let $\mathbf{d}_i$, $i = 1, \ldots, n$ be the fractional physical wear-and-tear which occurs for a given kind of asset in a given kind of industry in year i of this asset's life (which is n years). The $\mathbf{d}_i$ may all be different, but we require that $\sum_i \mathbf{d}_i = 1$ (i.e. the machine transfers its value to the product exactly over its life). Given our assumption that we have a uniform distribution of ages, the fraction of the total assets of this kind in this industry of age i years is $f_i = 1/n$, for all i. Now we show that the physical amount of gross discards equals the total amount of physical wear-and-tear on all the assets (of this kind in this industry).

1. Every year, $1/n$ of all the assets are discarded (as given by our gross-discards series).
2. Also every year, the total amount of physical wear-and-tear is given by

$$\sum_i f_i \mathbf{d}_i = \sum_i (1/n)\mathbf{d}_i = (1/n)\sum_i \mathbf{d}_i = (1/n)$$

Hence, the physical amount of gross discards equals the physical amount of wear-and-tear, assuming a uniform distribution of vintages for this kind of asset, but without having to assume linear wear-and-tear.

It might be questioned how we could define wear-and-tear in purely physical terms, or rather gross discards. This was done the same way we measured all other physical quantities in this project: by making one current-market-dollar's-worth the physical unit of measurement. Hence the gross discards were measured at replacement cost in current dollars. Then depreciation is given as $\mathbf{s}D$ (i.e. not before distribution).

To obtain the real unit wage vector $\mathbf{b}$, we use the sectoral proportions in the Personal Consumption Expenditures component of final demand, which is part of the input–output data developed by the Department of Commerce. By multiplying it by the wage rate in current dollars of the lowest-wage sector, we obtain the dollar amount of each consumer good required per unit of reduced labour.

The treatment of indirect business taxes in this study assumes that they represent a cost like any other in the formation of a uniform rate of profit. We therefore compiled a vector of indirect-tax coefficients, $\mathbf{g}$, from US Department of Commerce (1981).

The turnover-time vector $\mathbf{t}$ was estimated by using the inventory-output ratios for input-output industries for 1963 in US Department of Commerce (1973).

We required a 71-order price-index vector, $\mathbf{e}$, to factor out changes in input–output coefficients which are due to market price changes. This is necessary if we are to compare technologies over time, since we are using a 'dollar's worth' as a measure of physical quantity of a product (see Carter, 1970, p. 21). The US Department of Labor (1979) provides price indices for most input–output industries. Except for 1947, all the required indices are included in the data tape of Research Data Associates (1982). We used the price index vector implicit in US Department of Commerce (1970)—which gives 1947 input–output data in 1947 and 1958 dollars—to obtain the 1947 values.

Summarising, we have

A = Input–output coefficients matrix;
$\mathbf{k}$ = Fixed-capital coefficients row vector;
$\mathbf{a}_0$ = Direct reduced-labour coefficients row vector;
w = Capital-assets weights matrix;
G = Gross-capital-stocks matrix;
$\mathbf{b}'$ = Real-wage column vector per unit of reduced labour;
$\mathbf{l}$ = Asset-lifetime column vector (per asset type);
$\mathbf{e}$ = Price-index row vector;
$\mathbf{q}$ = Dollar value of output-by-industry row vector;
K = Capital-stocks coefficients matrix;

D = Depreciation coefficients matrix;
H = Gross-investment coefficients matrix;
$\mathbf{g}$ = Indirect-tax coefficients row vector;
$\mathbf{t}$ = Turnover-time row vector.

Given this data base, we could perform all the computations outlined in the paper. In all cases, we can compute prices and values using the data in current dollars or in constant dollars. The former is more accurate when investigating relations in a single year; the latter is necessary for intertemporal comparisons.

To deflate the A matrix we computed:

$$A^* = \langle e^{-1} \rangle A \langle e \rangle \tag{A5}$$

where $\langle e \rangle$ is the diagonal matrix obtained from the vector $\mathbf{e}$.

To deflate the output row vector $\mathbf{q}$:

$$\mathbf{q}^* = \mathbf{q} \langle e^{-1} \rangle \tag{A6}$$

To handle the capital valuation problem, we took the capital coefficients matrix K at current dollars and deflated it like A. This is valid since current dollar gross-stocks measure current replacement value of assets still in place.

The real unit-wage vector $\mathbf{b}$, also measured in 'dollar's worth', was deflated as $\mathbf{q}$ above.